Available online at http://scik.org
J. Math. Comput. Sci. 11 (2021), No. 6, 7040-7051
https://doi.org/10.28919/jmcs/5980
ISSN: 1927-5307

CONTINUOUS MONOTONIC DECOMPOSITION OF JUMP GRAPH OF PATHS AND COMPLETE GRAPHS

M. JENISHA[1,†,*], P. CHITHRA DEVI[2]

[1]Department of Mathematics, Sri Parasakthi College for Women, Courtallam, Manonmaniam Sundaranar University, Tirunelveli-627 012, Tamil Nadu, India

[2]Department of Mathematics, Sri Parasakthi College for Women, Courtallam, India

Abstract. The Jump graph $J(G)$ of a graph G is the graph whose vertices are edges of G and two vertices of $J(G)$ are adjacent if and only if they are not adjacent in G. In this article, we have given characterization for the Jump graph of paths into Continuous monotonic star decomposition. Also we have given characterization for the Jump graph of complete graphs into Continuous monotonic tree decomposition.

Keywords: decomposition; jump graph; path; complete graph.

2010 AMS Subject Classification: 05C30, 05C76.

1. Introduction

Let $G = (V,E)$ be a simple undirected graph without loops or multiple edges. A *path* on n vertices is denoted by P_n, *cycle* on n vertices is denoted by C_n and *complete graph* on n vertices is denoted by K_n. The *neighbourhood* of a vertex v in G is the set $N(v)$ consisting of all vertices that are adjacent to v. $|N(v)|$ is called the degree of v and is denoted by $d(v)$. A *complete bipartite graph* with partite sets V_1 and V_2, where $|V_1| = r$ and $|V_2| = s$, is denoted by $K_{r,s}$. The graph

*Corresponding author
E-mail address: jenisha.m92@gmail.com
†Research Scholar, Reg. No: 18111202092001
Received May 04, 2021

$K_{1,r}$ is called a *star* and is denoted by S_r. *Claw* is a star with three edges. For any set S of points of G, *induced subgraph* $< S >$ is the maximal subgraph of G with point set S. An edge induced subgraph $< E' >$ of G is the subgraph of G whose vertex set is the set of ends of edges in E' and whose edge set is E'. The terms not defined here are used in the sense of [3].

A *decomposition* of a graph G is a family of edge-disjoint subgraphs $\{G_1, G_2, \ldots, G_k\}$ such that $E(G) = E(G_1) \cup E(G_2) \cup \ldots \cup E(G_k)$. If each G_i is isomorphic to H for some subgraph H of G, then the decomposition is called a H-decomposition of G. A decomposition, $\{G_1, G_2, \ldots, G_k\}$ for all $k \in N$ is said to be a Continuous Monotonic Decomposition (CMD) if each G_i is connected and $|E(G_i)| = i$ for all $i \in N$. The concept of CMD was introduced by Paulraj Joseph and Gnanadhas [4].

The *Jump graph* $J(G)$ of a graph G is the graph whose vertices are edges of G and two vertices of $J(G)$ are adjacent if and only if they are not adjacent in G. Equivalently complement of line graph $L(G)$ is the Jump graph $J(G)$ of G. This concept was introduced by Chartrand in [1]. Coconut tree $CT(m,n)$ is a graph obtained from the path P_n by appending m new pendant edges at an end vertex of P_n. Double coconut tree $D(n,r,m)$ is a graph obtained by attaching $n > 1$ pendant vertices to one end of the path P_r and $m > 1$ pendant vertices to the other end of the path P_r.

2. CONTINUOUS MONOTONIC STAR DECOMPOSITION OF JUMP GRAPH OF PATHS

Let $J(P_n)$ denote the Jump graph of paths. Then $J(P_n)$ is a connected graph if and only if $n \geq 5$. Let us consider the connected jump graph of paths. Let the edges of path P_n be labelled as $x_1, x_2, \ldots, x_{n-1}$. Since the number of edges of path P_n is $(n-1)$, the number of vertices of $J(P_n)$ is $(n-1)$. The number of edges of Jump graph of paths $J(P_n)$ is $\binom{n-2}{2}$.

Definition 2.1. [4] If G admits a CMD $\{G_1, G_2, \ldots, G_k\}$ for all $k \in N$, where each G_i is a star, then we say that G admits a Continuous Monotonic Star Decomposition (CMSD).

Theorem 2.2. [4] *Let G be a connected simple graph of order p and size q. Then G admits a CMD $\{H_1, H_2, \ldots, H_n\}$ if and only if $q = \binom{n+1}{2}$.*

Lemma 2.3. *Let $m \geq 2$. The set $\{1, 2, \ldots, m\}$ can be partitioned into two sets B_1 and B_2 such that $\sum_{x \in B_1} x = \frac{(n-3)(n-4)}{2}$ and $\sum_{y \in B_2} y = n-3$ where $\frac{m(m+1)}{2} = \binom{n-2}{2}$.*

Proof. Let $m \geq 2$. and $n = m+3$. Let us prove this lemma by induction on m. When m = 2, n = 5. If $B_1 = 1$ and $B_2 = 2$ then $\sum_{x \in B_1} x = 1$ and $\sum_{y \in B_2} y = 2$. Hence the result is true for $m = 2$.

Assume that the result is true for $m-1$. Hence the set $\{1,2,\ldots,m-1\}$ can be partitioned into two sets B_1 and B_2 such that $\sum_{x \in B_1} x = \frac{(m-1)(m-2)}{2}$ and $\sum_{y \in B_2} y = m-1$. Then the set $\{1,2,\ldots,m\}$ can be partitioned into two sets B_1' and B_2' where $B_1' = B_1 \cup \{m-1\}$ and $B_2' = m$.

Clearly $\sum_{x \in B_1'} x = \sum_{x \in B_1} x + \{m-1\} = \frac{(m-1)(m)}{2} = \frac{(n-3)(n-4)}{2}$ and $\sum_{y \in B_2} y = m = n-3$. Hence the induction and lemma holds. □

Theorem 2.4. *Jump graph of path $J(P_n)$ admits Continuous monotonic star decomposition $\{S_1, S_2, \ldots, S_m\}$ if and only if there exists an integer m such that (i) $m = n-3$ and (ii) $\frac{m(m+1)}{2} = \binom{n-2}{2}$.*

Proof. We have $|E[J(P_n)]| = \binom{n-2}{2}$.

Assume that $J(P_n)$ admits Continuous monotonic star decomposition $\{S_1, S_2, \ldots, S_m\}$. By theorem 2.2, $|E[J(P_n)]| = \frac{m(m+1)}{2}$. Hence, $\frac{m(m+1)}{2} = \binom{n-2}{2}$. Hence (ii).

Since $J(P_n)$ admits Continuous monotonic star decomposition $\{S_1, S_2, \ldots, S_m\}$, $\binom{n-2}{2} = 1+2+\ldots+m = \frac{m(m+1)}{2}$. This implies $\frac{m(m+1)}{2} = \frac{(n-2)(n-3)}{2}$. Thus $m = n-3$. Hence (i).

Conversely, assume that $m = n-3$ and $\frac{m(m+1)}{2} = \binom{n-2}{2}$.

Define $T_1 = \left\{x_i x_j / 3 \leq i \leq n-2; i > j; 1 \leq j \leq n-3; j \neq i-1\right\}$ and

$T_2 = \left\{x_{n-1} x_j / 1 \leq j \leq n-3\right\}$.

Now, $|T_1| = \frac{(n-3)(n-4)}{2}$ and $|T_2| = n-3$. Thus $|T_1| + |T_2| = \binom{n-2}{2} = \frac{m(m+1)}{2} = 1+2+\ldots+m$.

By lemma 2.3, $\{1,2,\ldots,m\} = B_1 \cup B_2$ where $\sum_{x \in B_1} x = \frac{(n-3)(n-4)}{2}$ and $\sum_{y \in B_2} y = n-3$.

Decompose T_1 and T_2 into stars S_i as follows:

$T_1 = \bigcup S_i$ where $i \in B_1$ and $S_i = \{x_{i+2}; x_1, x_2, \ldots, x_i\}$. Here x_{i+2} forms center of the star S_i. $T_2 = S_m$ where $m \in B_2$ and $S_m = \{x_{n-1}; x_1, x_2, \ldots, x_{n-3}\}$. Here x_{n-1} forms center of the star S_m. Also$|E(S_i)| = i$; $1 \leq i \leq m$. Thus $J(P_n)$ admits Continuous monotonic star decomposition $\{S_1, S_2, \ldots, S_m\}$. □

Illustration 2.5. As an illustration let us decompose $J(P_{12})$.

Let $E(P_{12}) = \{1,2,\ldots,11\}$. Therefore $V[J(P_{12})] = \{1,2,\ldots,11\}$.

P_{12} and $J(P_{12})$ are given in Figure 2.1 and Figure 2.2 respectively.

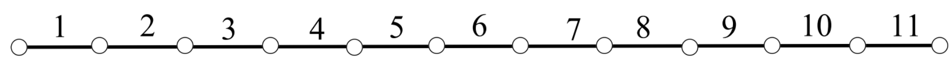

FIGURE 2.1. P_{12}

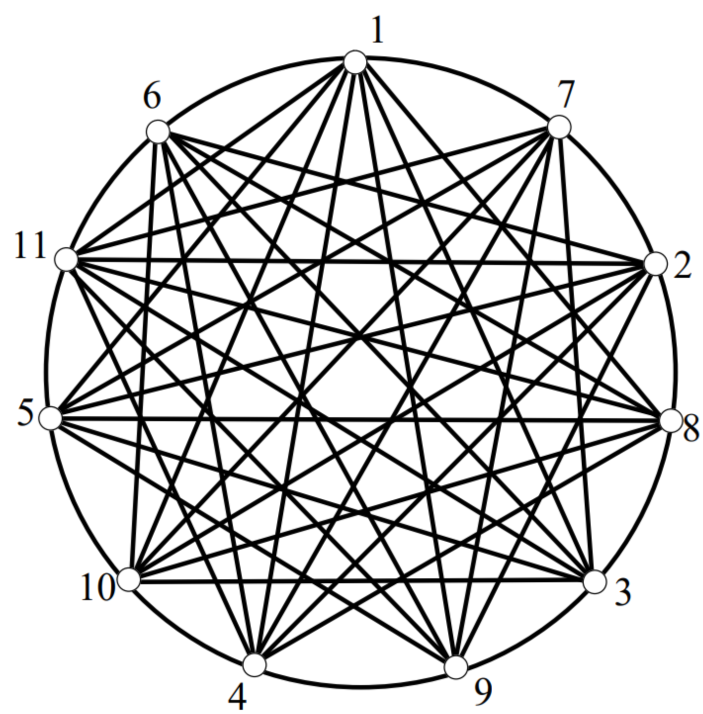

FIGURE 2.2. $J(P_{12})$

Here $|E[J(P_{12})]| = 45$.

Define $T_1 = \{31,41,42,51,52,53,61,62,63,64,71,72,73,74,75,81,82,83,84,85,86,91,92,93, 94,95,96,97,(10)1,(10)2,(10)3,(10)4,(10)5,(10)6,(10)7,\ (10)8\}$ and

$T_2 = \{(11)1,(11)2,(11)3,(11)4,(11)5,(11)6,(11)7,(11)8,(11)9\}$.

$|T_1| + |T_2| = 45 = 1+2+3+4+5+6+7+8+9 = \binom{10}{2}$.

Here $B_1 = \{1,2,3,4,5,6,7,8\}$ and $B_2 = \{9\}$.

T_1 is decomposed as $S_1 \cup S_2 \cup \ldots \cup S_8$ where $S_1 = \langle\{31\}\rangle$, $S_2 = \langle\{41,42\}\rangle$,
$S_3 = \langle\{51,52,53\}\rangle$, $S_4 = \langle\{61,62,63,64\}\rangle$, $S_5 = \langle\{71,72,73,74,75\}\rangle$,
$S_6 = \langle\{81,82,83,84,85,86\}\rangle$, $S_7 = \langle\{91,92,93,94,95,96,97\}\rangle$,
$S_8 = \langle\{(10)1,(10)2,(10)3,(10)4,(10)5,(10)6,(10)7,(10)8\}\rangle$.
T_2 is decomposed as S_9 where $S_9 = \langle\{(11)1,(11)2,(11)3,(11)4,(11)5,(11)6,(11)7,(11)8,(11)9\}\rangle$.
Clearly $\{S_1, S_2, \ldots, S_9\}$ forms a CMSD of $J(P_{12})$.

3. Continuous Monotonic Tree Decomposition of Jump Graph of Complete Graphs

Let $J(K_n)$ denote the Jump graph of complete graphs. Then $J(K_n)$ is a connected graph if and only if $n \geq 5$. Let us consider the connected jump graph of complete graphs. Let $V(K_n) = \{1,2,\ldots,n\}$ and $E(K_n) = \{12,13,\ldots,1n,23,24,\ldots,2n,\ldots,(n-2)(n-1),(n-2)n,(n-1)n\}$. Since the number of edges of complete graphs K_n is $\frac{n(n-1)}{2}$, the number of vertices of $J(K_n)$ is $\frac{n(n-1)}{2}$. The number of edges of Jump graph of complete graphs $J(K_n)$ is $\frac{n(n-1)(n-2)(n-3)}{8}$.

Theorem 3.1. *Let $n \geq 5$. Then $J(K_n)$ is decomposed into $\{T_2S_1, T_3S_2, \ldots, T_{n-2}S_{n-3}\}$ where $T_l = \frac{l(l+1)}{2}$; $2 \leq l \leq n-2$.*

Proof. Let $n \geq 5$. Let $V(K_n) = \{1,2,\ldots,n\}$ and
$E(K_n) = \{12,13,\ldots,1n,23,24,\ldots,2n,\ldots,(n-2)(n-1),(n-2)n,(n-1)n\}$.
Since edges of K_n are taken as vertices of $J(K_n)$ we have, $V[J(K_n)] = \{12,13,\ldots,1n,23,\ldots,2n,\ldots,(n-2)(n-1),(n-2)n,(n-1)n\}$. Two vertices uv, xy in $J(K_n)$ are adjacent if u, v, x, y all are distinct elements.
Define $U_1 = \{(n-3)(n-2),(n-3)(n-1),(n-3)n\}$ and
$V_1 = \{(n-2)(n-1),(n-2)n,(n-1)n\}$. Thus $|U_1| = 3$ and $|V_1| = 3$.
Now $\langle\{(n-3)(n-2),(n-3)(n-1),(n-3)n,(n-2)(n-1),(n-2)n,(n-1)n\}\rangle \cong 3(S_1)$.
Thus we get T_2 copies of S_1.
Define $U_2 = \{(n-4)(n-3),(n-4)(n-2),(n-4)(n-1),(n-4)n\}$ and $V_2 = \{U_1, V_1\}$. Thus $|U_2| = 4$ and $|V_2| = 6$. Now $\langle\{U_1,U_2\}\rangle \cong 3(S_2)$. Also, $\langle\{V_1,U_2\}\rangle \cong 3(S_2)$. Therefore we get T_3 copies of S_2.
Define $U_3 = \{(n-5)(n-4),(n-5)(n-3),(n-5)(n-2),(n-5)(n-1),(n-5)n\}$ and

$V_3 = \{U_2, V_2\}$. Thus $|U_3| = 5$ and $|V_3| = 10$.

Now, $\langle\{U_2, U_3\}\rangle \cong |U_2|(S_3)$. Also, $\langle\{U_1, U_3\}\rangle \cong |U_1|(S_3)$ and $\langle\{V_1, U_3\}\rangle \cong |V_1|(S_3)$. $|U_1| + |U_2| + |V_1| = 3 + 3 + 4 = \frac{(4)(5)}{2} = T_4$. Thus we get T_4 copies of S_3. Proceed like this. Finally we define $U_{(n-3)} = \{12, 13, \ldots 1n\}$ and $V_{(n-3)} = \left\{U_{(n-4)}, V_{(n-4)}\right\}$.

Thus $|U_{(n-3)}| = n-1$ and $|V_{(n-3)}| = |U_{(n-4)}| + |V_{(n-4)}|$.

Now, $\left\langle\left\{U_{(n-4)}, U_{(n-3)}\right\}\right\rangle \cong |U_{(n-4)}|(S_{n-3})$

Also, $\left\langle\left\{U_{(n-5)}, U_{(n-3)}\right\}\right\rangle \cong |U_{(n-5)}|(S_{n-3})$,

$\vdots$

$\left\langle\left\{U_1, U_{(n-3)}\right\}\right\rangle \cong |U_1|(S_{n-3})$,

$\left\langle\left\{V_1, U_{(n-3)}\right\}\right\rangle \cong |V_1|(S_{n-3})$.

Now $|V_1| + |U_1| + |U_2| + \ldots |U_{(n-5)}| + |U_{(n-4)}| = 3 + 3 + 4 + 5 + \ldots + (n-3) + (n-2) = \frac{(n-2)(n-1)}{2} = T_{n-2}$. Thus we get $T_{(n-2)}$ copies of $S_{(n-3)}$. Hence we have $E[J(K_n)] = \underbrace{S_1 \cup S_1 \cup S_1}_{T_2\ times} \cup \underbrace{S_2 \cup \ldots \cup S_2}_{T_3\ times} \cup \underbrace{S_3 \cup \ldots \cup S_3}_{T_4\ times} \ldots \cup \underbrace{S_{(n-3)} \cup \ldots \cup S_{(n-3)}}_{T_{n-2}\ times}$. Thus $J(K_n)$ is decomposed into $\{T_2S_1, T_3S_2, \ldots, T_{n-2}S_{n-3}\}$ where $T_l = \frac{l(l+1)}{2}; 2 \leq l \leq n-2$. $\square$

Lemma 3.2. *Let $m = \frac{n(n-3)}{2}$ where $n \geq 5$. The set $\{1, 2, \ldots, m\}$ can be partitioned into two sets B_1 and B_2 such that $\sum_{x \in B_1} x = \frac{n(n-3)(n^2-3n-2)}{8}$ and $\sum_{y \in B_2} y = \frac{n(n-3)}{2}$ where $\frac{m(m+1)}{2} = \frac{n(n-1)(n-2)(n-3)}{8}$.*

Proof. Let $n \geq 5$. Let us prove this lemma by induction on n. When n = 5, m = 5. If $B_1 = \{1, 2, 3, 4\}$ and $B_2 = 5$ then $\sum_{x \in B_1} x = 10$ and $\sum_{y \in B_2} y = 5$. Hence the result is true for $n = 5$.

Assume that the result is true for $n-1$. Hence the set $\left\{1, 2, \ldots, \frac{(n-1)(n-4)}{2}\right\}$ can be partitioned into two sets B_1 and B_2 such that $\sum_{x \in B_1} x = \frac{(n-1)(n-4)[(n-1)^2-3(n-1)-2]}{8}$ and $\sum_{y \in B_2} y = \frac{(n-1)(n-4)}{2}$.

Then the set $\{1, 2, \ldots, m\}$ can be partitioned into two sets B_1' and B_2' where $B_1' = B_1 \cup B_2 \cup \left\{\frac{(n-1)(n-4)}{2} + 1, \frac{(n-1)(n-4)}{2} + 2, \ldots, \frac{(n-1)(n-4)}{2} + (n-3)\right\}$ and $B_2' = \frac{(n-1)(n-4)}{2} + (n-2)$. Clearly $\sum_{x \in B_1'} x = \sum_{x \in B_1} x + \sum_{y \in B_2} y + \frac{(n-1)(n-4)}{2} + 1 + \frac{(n-1)(n-4)}{2} + 2 + \ldots + \frac{(n-1)(n-4)}{2} + (n-3) = \frac{n(n^3-6n^2+7n+6)}{8} = \frac{n(n-3)(n^2-3n-2)}{8}$. Also $\sum_{y \in B_2'} y = \frac{(n-1)(n-4)}{2} + (n-2) = \frac{n(n-3)}{2}$.

Hence the induction and lemma holds. $\square$

Theorem 3.3. *Let $n > 5$. Then the jump graph of complete graph $J(K_n)$ is $\{3DCT(2,2,2), S_{T_l}, CT(T_l - 1,t), DCT(T_l - 1,r,s)\}$ decomposable where $T_l = \frac{l(l+1)}{2}; l = 3,4,\ldots,n-3$.*

Proof. Let $V(K_n) = \{1,2,\ldots,n\}$ and $E(K_n) = \{12,13,\ldots,1n,23,24,\ldots,2n,\ldots,(n-2)(n-1),(n-2)n,(n-1)n\}$. Then $V[J(k_n)] = \{12,\ldots,1n,23,\ldots,2n,\ldots,(n-2)(n-1),(n-2)n,(n-1)n\}$.

Take $D_1 = \{(n-4)(n-3),(n-4)(n-2),(n-4)(n-1),(n-4)n\}$,

$D_2 = \{(n-3)(n-2),(n-1)n\}$,

$D_3 = \{(n-3)(n-1),(n-2)n\}$ and

$D_4 = \{(n-3)n,(n-2)(n-1)\}$.

Now, $\langle\{D_1,D_2\}\rangle \cong DCT(2,2,2)$,

$\langle\{D_1,D_3\}\rangle \cong DCT(2,2,2)$,

$\langle\{D_1,D_4\}\rangle \cong DCT(2,2,2)$.

Now $((n-5)(n-4);(n-3)(n-2),(n-3)(n-1),(n-3)n,(n-2)(n-1),(n-2)n,(n-1)n) \cong S_6$, $((n-5)(n-3);(n-4)(n-2),(n-4)(n-1),(n-2)(n-1),(n-2)n,(n-1)n) \cong S_5$ and $\langle\{(n-5)(n-3),(n-4)n,(n-5)(n-2)\}\rangle \cong P_3$. Thus $\langle E(S_5) \cup E(P_3)\rangle \cong CT(5,3)$.

Now $((n-5)(n-1);(n-4)(n-2),(n-3)(n-2),(n-3)n,(n-4)n,(n-2)n) \cong S_5$ and $\langle\{(n-5)(n-1),(n-4)(n-3),(n-5)(n-2),(n-4)(n-1)\}\rangle \cong P_4$.

Thus $\langle E(S_5) \cup E(P_4)\rangle \cong CT(5,4)$.

Now $((n-5)n;(n-4)(n-3),(n-4)(n-2),(n-4)(n-1),(n-3)(n-2),(n-2)(n-1)) \cong S_5$, $\langle\{(n-5)n,(n-3)(n-1),(n-5)(n-2)\}\rangle \cong P_3$ and $((n-5)(n-2);(n-3)n,(n-1)n) \cong S_2$.

Thus $\langle E(S_5) \cup E(P_3) \cup E(S_2)\rangle \cong DCT(5,3,2)$.

Now $((n-6)(n-5);((n-4)(n-3),(n-4)(n-2),(n-4)(n-1),(n-4)n,(n-3)(n-2),(n-3)(n-1),(n-3)n,(n-2)(n-1),(n-2)n,(n-1)n) \cong S_{10}$,

$((n-6)(n-4);(n-5)(n-3),(n-5)(n-2),(n-5)n,(n-3)(n-2),(n-3)(n-1),(n-3)n,(n-2)(n-1),(n-2)n,(n-1)n) \cong S_9$ and

$\langle\{(n-6)(n-4),(n-5)(n-1),(n-6)(n-3)\}\rangle \cong P_3$.

Thus $\langle E(S_9) \cup E(P_3)\rangle \cong CT(9,3)$.

Now $((n-6)(n-2);(n-5)(n-3),(n-5)(n-1),(n-5)n,(n-4)(n-3),(n-4)(n-1),(n-4)n,(n-2)(n-1),(n-3)n,(n-1)n) \cong S_9$ and

$\langle\{(n-6)(n-2),(n-5)(n-4),(n-6)(n-3),(n-5)(n-2)\}\rangle \cong P_4$.

Thus $\langle E(S_9)\cup E(P_4)\rangle \cong CT(9,4)$.

Now $((n-6)(n-1);(n-5)(n-4),(n-5)(n-3),(n-5)(n-2),$ $(n-4)(n-3),(n-4)(n-2),(n-4)n,(n-3)(n-2),(n-3)n,(n-2)n) \cong S_9$,

$\langle\{(n-6)(n-1),(n-5)n,(n-6)(n-3)\}\rangle \cong P_3$ and

$((n-6)(n-3);(n-2)(n-1),(n-4)(n-1)) \cong S_2$.

Thus $\langle E(S_9)\cup E(P_3)\cup E(S_2)\rangle \cong DCT(9,3,2)$.

Now $((n-6)n;(n-5)(n-4),(n-5)(n-3),(n-5)(n-2),(n-5)(n-1),$ $(n-4)(n-3),(n-4)(n-1),(n-3)(n-2),(n-3)(n-1),(n-2)(n-1)) \cong S_9$,

$\langle\{(n-6)n,(n-4)(n-2),(n-6)(n-3)\rangle \cong P_3$ and

$((n-6)(n-3);(n-4)n,(n-2)n,(n-1)n) \cong S_2$. Thus $\langle E(S_9)\cup E(P_3)\cup E(S_2)\rangle \cong DCT(9,3,3)$.

Proceed like this,

Now $(12;34,35,\ldots,3n,\ldots,(n-2)(n-1),(n-2)n,(n-1)n) \cong S_{\frac{(n-3)(n-2)}{2}}$.

Also, $(13;24,25,\ldots,2n,\ldots,(n-2)(n-1),(n-2)n,(n-1)n) \cong S_{\frac{(n-3)(n-2)}{2}-1}$ and

$\langle\{(13),(26),(14)\}\rangle \cong P_3$.

Thus $\left\langle E(P_3)\cup E\left(S_{\frac{(n-3)(n-2)}{2}-1}\right)\right\rangle \cong CT(\frac{(n-3)(n-2)}{2}-1,3)$.

$\vdots$

$(1n;23,24,\ldots 2(n-1),\ldots,(n-2)(n-1)) \cong S_{\frac{(n-3)(n-2)}{2}-1}$,

$\langle\{(1n),(35),(14)\}\rangle \cong P_3$ and $(14;3n,5n,\ldots,(n-1)n) \cong S_{n-4}$.

Thus we get

$\left\langle E(S_{\frac{(n-3)(n-2)}{2}-1})\cup E(P_3)\cup E(S_{(n-4)})\right\rangle \cong DCT\left(\frac{(n-3)(n-2)}{2}-1,3,n-4\right)$.

Thus $E[J(K_n)] = E[3DCT(2,2,2)] \cup E[S_{T_l}] \cup E[CT(T_l-1,3)] \cup E[CT(T_l-1,4)] \cup$ $E[DCT(T_l-1,3,2)]\cup E[DCT(T_l-1,3,3)]\cup\ldots\cup E[DCT(T_l-1,3,n-4)]$ where $T_l=\frac{l(l+1)}{2}$; $l=3,4,\ldots,n-3$. □

Theorem 3.4. *Jump graph of complete graph $J(K_n)$ admits Continuous Monotonic tree decomposition $\{H_1,H_2,\ldots,H_m\}$ if and only if there exists an integer m such that (i) $m=\frac{n(n-3)}{2}$ and (ii) $\frac{m(m+1)}{2}=\frac{n(n-1)(n-2)(n-3)}{8}$.*

Proof. Let $|E[J(K_n)]| = \frac{n(n-1)(n-2)(n-3)}{8}$.

Assume $J(K_n)$ admits Continuous Monotonic decomposition $\{H_1, H_2, \ldots, H_m\}$. By theorem 2.2, $|E[J(K_n)]| = \frac{m(m+1)}{2}$. Hence, $\frac{m(m+1)}{2} = \frac{n(n-1)(n-2)(n-3)}{8}$.

Since $J(K_n)$ admits Continuous Monotonic decomposition $\{H_1, H_2, \ldots, H_m\}$, $\frac{n(n-1)(n-2)(n-3)}{8} = 1+2+\ldots+m = \frac{m(m+1)}{2}$. This implies $\frac{m(m+1)}{2} = \frac{n(n-1)(n-2)(n-3)}{8}$. Hence (ii).

(ii) implies $m(m+1) = \frac{n(n-3)}{2}[\frac{n(n-3)}{2}+1]$. Thus $m = \frac{n(n-3)}{2}$. Hence (i).

Conversely, assume that $m = \frac{n(n-3)}{2}$ and $\frac{m(m+1)}{2} = \frac{n(n-1)(n-2)(n-3)}{8}$.

By theorem 3.3, $J(K_n)$ is decomposed into $\{3DCT(2,2,2), S_{T_l}, CT(T_l-1,t), DCT(T_l-1,r,s)\}$ where $T_l = \frac{l(l+1)}{2}; l = 3,4,\ldots,n-3$.

Define $T_1 = \{E[3DCT(2,2,2)] \cup E[S_{T_l}] \cup E[CT(T_l-1,3)] \cup E[CT(T_l-1,4)] \cup E[DCT(T_l-1,3,2)] \cup E[DCT(T_l-1,3,3)] \cup \ldots \cup E[DCT(T_l-1,3,n-5)]\}$ where $l = 3,4,\ldots,n-3$ and $T_2 = E[DCT(T_l-1,3,n-4)]$ where $l = n-3$.

Here $|T_1| = \frac{n(n-3)(n^2-3n-2)}{8}$ and $|T_2| = \frac{n(n-3)}{2}$. Also $|T_1|+|T_2| = \frac{n(n-3)(n^2-3n-2)}{8} + \frac{n(n-3)}{2} = \frac{m(m+1)}{2} = 1+2+\ldots+m$.

By lemma 3.2, $\{1,2,\ldots,m\} = B_1 \cup B_2$ where $\sum_{x\in B_1} x = \frac{n(n-3)(n^2-3n-2)}{8}$ and $\sum_{y\in B_2} y = \frac{n(n-3)}{2}$.

Decompose T_1 and T_2 into trees H_i. For $i \in B_1$ and B_2, we choose H_i in such a way that $|E(H_i)| = i$.

$T_1 = \bigcup H_i; i \in B_1$ where $H_i \in \{3DCT(2,2,2), S_{T_l}, CT(T_l-1,3), CT(T_l-1,4), DCT(T_l-1,3,2), DCT(T_l-1,3,3), \ldots, DCT(T_l-1,3,n-5)\}$.

$T_2 = \bigcup H_i; i \in B_2$ where $H_i = \{DCT(T_l-1,3,n-4)\}$ and $|E(H_i)| = i$ for all $1 \le i \le m$. Clearly $J(K_n)$ admits Continuous Monotonic tree decomposition $\{H_1, H_2, \ldots, H_m\}$. Hence the theorem. $\square$

Illustration 3.5. As an illustration let us decompose $J(K_6)$.

Let $E(K_6) = \{12,13,14,15,16,23,24,25,26,34,35,36,45,46,56\}$.

Thus $V[J(K_6)] = \{12,13,14,15,16,23,24,25,26,34,35,36,45,46,56\}$.

K_6 and $J(K_6)$ are given in Figure 3.1 and Figure 3.2 respectively.

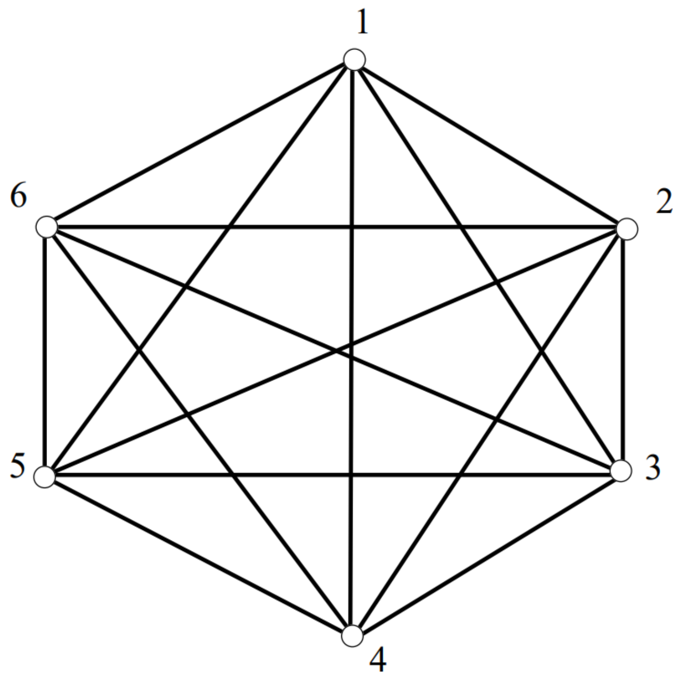

FIGURE 3.1. K_6

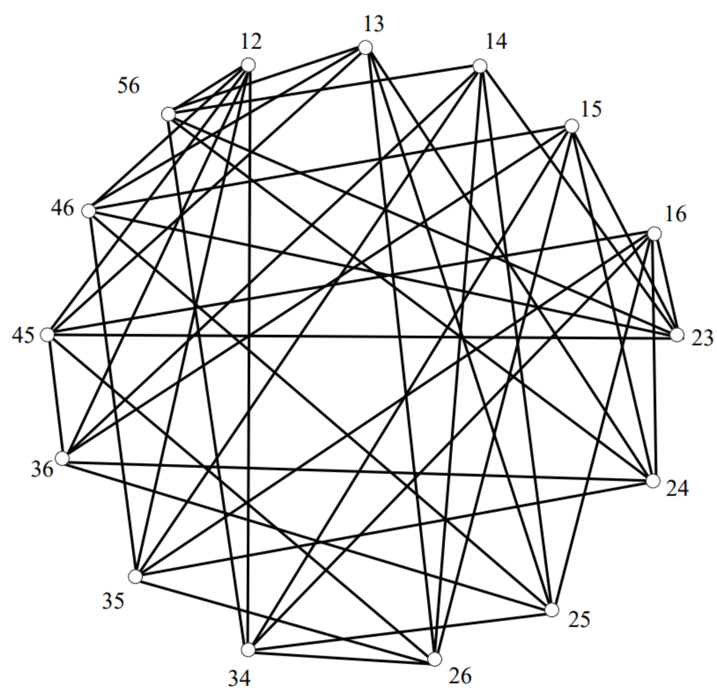

FIGURE 3.2. $J(K_6)$

Here $|E[J(K_6)]| = 45$.

Define $T_1 = \{(34)(25),(34)(26),(34)(56),(56)(23),(56)(24),(35)(24),(35)(26),(35)(46),(46)(23),(46)(25),(36)(24),(36)(25),(36)(45),(45)(23),(45)(26),(12)(34),(12)(35),(12)(36),(12)(45),(12)(46),(12)(56),(13)(24),(13)(25),(13)(45),(13)(46),(13)(56),(13)(26),(14)(26),(15)(24),(15)(34),(15)(36),(15)(26),(15)(46),(15)(23),(14)(23),(14)(25)\}$ and

$T_2 = \{(16)(23),(16)(24),(16)(25),(16)(34),(16)(45),(16)(35),(14)(36),(14)(56),(14)(35)\}$.

Now $|T_1|+|T_2| = 45 = 1+2+3+4+5+6+7+8+9 = \frac{n(n-1)(n-2)(n-3)}{8}$.

Here $B_1 = \{1,2,3,4,5,6,7,8\}$ and $B_2 = \{9\}$.

T_1 is decomposed as $H_1 \cup H_2 \cup \ldots \cup H_8$ where $H_1 = \langle\{(34)(25)\}\rangle$,

$H_2 = \langle\{(35)(24),(35)(26)\}\rangle$, $H_3 = \langle\{(35)(46),(46)(23),(46)(25)\}\rangle$,

$H_4 = \langle\{(34)(26),(34)(56),(56)(23),(56)(24)\}\rangle$,

$H_5 = \langle\{(36)(24),(36)(25),(36)(45),(45)(23),(45)(26)\}\rangle$,

$H_6 = \langle\{(12)(34),(12)(35),(12)(36),(12)(45),(12)(46),(12)(56)\}\rangle$,

$H_7 = \langle\{(13)(24),(13)(25),(13)(45),(13)(46),(13)(56),(13)(26),(14)(26)\}\rangle$,

$H_8 = \langle\{(15)(23),(15)(34),(15)(36),(15)(26),(15)(46),(15)(24),(14)(23),(14)(25)\}\rangle$.

Also T_2 is decomposed as

$H_9 = \langle\{(16)(23),(16)(24),(16)(25),(16)(34),(16)(45),(16)(35),(14)(36),(14)(56),(14)(35)\}\rangle$.

Clearly $\{H_1,H_2,\ldots,H_9\}$ forms a CMTD of $J(K_6)$.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

REFERENCES

[1] G.T. Chartrand, H. Hevia, E.B. Jarette, and M. Schulty, Subgraph distance in graphs defined by edge transfer, Discrete Math. 170 (1997), 63-79.

[2] E.E.R. Merly and N. Gnanadhas, Arithmetic ODD decomposition of extended Lobester, Glob. J. Math. Sci. 4(1) (2012), 35-42.

[3] F. Harary, Graph Theory, Addison-Wesley, Reading, (1969).

[4] J.P. Joseph, N. Gnanadhas, Continuous monotonic decomposition of graphs, Int. J. Manage. Syst. 16 (2000), 333-334.

[5] V. Brishni, V. Maheswari and A. Nagarajan, Even decomposition of graphs, J. Emerging Technol. Innov. Res. 6 (2019), 107-115.

[6] S. Asha and R. Kala, Continuous monotonic decomposition of some special class of graph, Int. J. Math. Anal. 4(51) (2010), 2535-2546.