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DETECTING A RANDOMLY MOVING COVID-19 USING WEIGHT TCHEBYCHEFF OPTIMIZATION TECHNIQUE

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Abstract: This paper completely characterizes the search for randomly moving COVID-19 among a finite set of different states (cells). A search effort at each fixed number of time intervals is a random variable with a given distribution. Both of probability of detecting the COVID-19 in a certain state j at a certain time i and the detection functions are supposed to be known to the searcher or robot. We seek for the optimal distribution of the effort to deduce an explicit formula for the optimality distribution of the random variable effort using weighted Tchebycheff optimization technique. We find the necessary conditions for the optimal search technique. An algorithm is constructed for generated weighted Tchebycheff optimization technique. The effectiveness of these techniques is illustrated by numerical results which indicate that the proposed techniques are promising.

Keywords: COVID-19; search theory; probability of undetected; moving target; a random variable effort; lost target; cells; microorganisms; optimality; Tchebycheff technique**.**

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1. INTRODUCTION

The Searching Process for lost COVID-19 that moves randomly becomes very interesting contemporary problem. Before starting the search for the missing goal, the search team should put some questions under considerations to facilitate detecting the COVID-19 correctly and quickly. Is this target valuable or not? Is this target static or moving? What is the nature of the location where the target is expected to be lost? What is the way to minimize the cost of searching for the target to be detected quickly? Can the missing target be detected quickly by using only one or more researchers? Using more than one researcher will affect your search cost or not? Most of previous questions have already been answered by presenting different models that have been studied previously using different strategies and are also interesting at the same time. To mention some important models, better to start with a linear search strategy, which has many life and mission applications, for example searching for a damaged unit in a large linear organ (electrical power lines, telephone lines and petrol or gas support lines), whether this linear organ is independent or intersecting has been studied by El-Rayes, Mohamed, and Fergani [2], El-Rayes, Mohamed [1], Mohamed [3] and Mohamed, and Abou Gabal [4] on the other side Balkhi [20] obtained the generalized optimal search paths for continuous univariate Random variable Also, the coordinated search technique has been studied in case of linear search has been analyzed by Mohamed and Afifi [5], and with different models by Mohamed, Abou Gabal, and Afifi [6] and [7], the authors discussed the coordinated search technique for a located target on two intersected lines, when the located target has symmetric and unsymmetrical distribution Mohamed, Abou Gabal, and Afifi [8], presented more advanced work when they applied the coordinated search technique for a moving target on one of two independent lines. The coordinated search technique in the plane has been discussed by Afifi, El-Bagoury and AL-Aziz [19]. Recently, El-Hadidy, and Fakharany [9], Afifi and EL-Bagoury et al. [10] proposed and studied a modern search model in the three-dimensional space to find a 3-D randomly located target by one searcher, two searchers and four searchers. Also, recently Afifi, El-Bagoury and AL-Aziz [17] illustrated a random walker target on one of n disjoint lines. Also, In the discrete search problems W. Afifi et al. [18]. obtained an optimal discrete search for a randomly moving COVID-19 between several cells in the part in human nasopharynx using monitoring system. More recently Afifi, El-Bagoury calculated the optimal search plan of a discrete Randomly Located Target [16].

In this article, the authors mainly discuss the following answers using weighted Chebycheff optimization technique, the location of the COVID-19 which assumed to be in consists of several cells in additions the searcher must distributes his effort (for time, energy,..., etc.) among the cells, the probability of being a COVID-19 in a certain cell at a certain time, and the detection function are supposed to be known to the searcher, the optimal distribution of the effort over the set of possible cells such that the overall probability of detecting the target is maximal.

The case of the target moving between two states according to a discrete parameter Markov chain has been studied by Kress,Kyle and Szechhtman [11]. Also, Stone [13] and BROWN [14], the authors discussed the case of the target moving among a finite number of states according to a Markov chain with a discrete parameter. El-Rayes and Mohamed [1], calculated the optimal distribution of the effort in problems, where the target moves randomly from state to state following any process, and the effort has a fixed. The distribution of effort which made the probability of undetected for unrestricted effort when the states are not identical and the cost of finding the target are minimum has been studied by Bahgat [12]. Moshe Kress, Kyle and Roberto [15] and Afifi [18] studied problem of located and a randomly moving target among a finite set of different states and used a surveillance organ in this case to search the lost target which hidden in one of n cells in each fixed number of time intervals m. the target located in a certain state j at a certain time i. and move to a new state in each time interval. They introduced the expected time of the search in each time interval. Also, they found the optimal search plan which minimizes the expected time of the search in each time interval for more studies.

2. PRELIMINARIES

The COVID-19 moves randomly in the respiratory system which divided into organs (states), we will suppose every organ represents a searching state. Here, we aim to find the optimal distribution of the random variable effort using weighted Tchebycheff optimization technique, which provides a clear interpretation of minimizes the largest difference between the probability of undetected the lost COVID-19 and its utopia point.

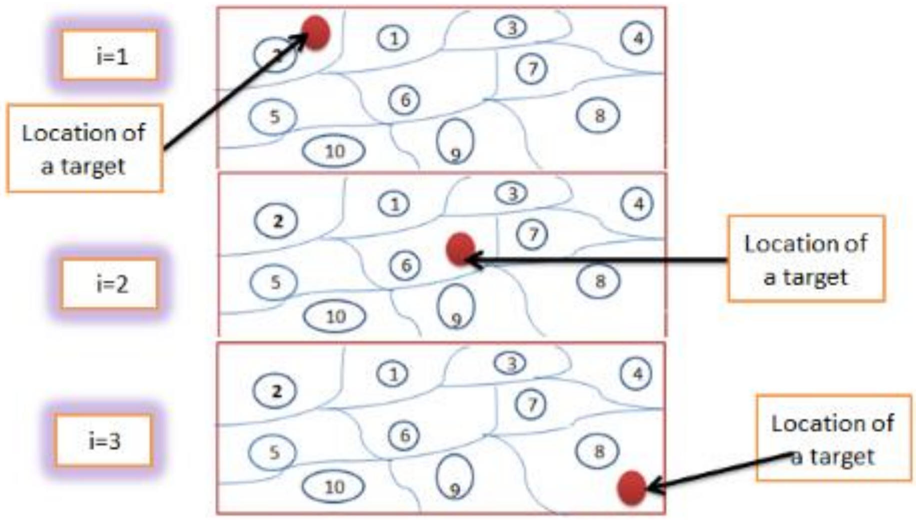

Figure (1): The location of the lost COVID-19 (target) in each time interval i.

Figure 1 shows the respiratory system which divided into organs (states), the lost COVID-19 moves from one state to another in each new time interval i=1, 2, 3. Here, each state indicates the expected organ of the lost COVID-19.

The search space in our model is a several states not necessarily identical region. Let the number of states be *m*. The COVID-19 is a randomly moving and occupies one state during each of *n* time intervals.

The searcher must distribute his effort among the states, the probability that the COVID-19 is in state *j*, $j = 1, 2, ..., m$ at time interval i, $i = 1, 2, ..., n$ is denoted by P_{ij}, during the i^{th} time interval, the effort is given by $0 \le L_i(Z) \le v_i$ which will be divided among the state.

In this paper, we will assume that the lost COVID-19 is the target and the states are the cells which in organs of the human body. The COVID-19 moves from one cell to another every time interval, and due to the importance of our target the value of effort is bounded by a random variable.

3. THE SEARCHING TECHNIQUE AND MAIN RESULTS

In this technique, we have only one searcher (robot), the main task is to distribute his effort in an ideal way to discover the lost goal as soon as possible with skill and speed.

The allocation of search effort is $Z_{i\,j}$ where $i=1, 2, ..., n$ and $j=1, 2, ..., m$, which gives the effort to be put into state *j* at time *i*, we call $Z_{i\,j}=Z$ a search plan, the conditional probability of finding the target at time given that it is located in state *j*, is given by the detection function $[1-b(i,\ j,\ Z_{i\,j})]$. We assume that the searches at distinct time intervals are independent of searcher's action.

The most importance factor in discovering the COVID-19 in the cells of the human body is calculation the following weight Tchebycheff optimization technique

$$\left.\begin{aligned} &\min_{Z\in R^{n+m}} U(Z)=\max_j\left\{p_{ij}\left[H(Z)-H^0(Z)\right]\right\},\\ &where\\ &\min_{Z\in R^{n+m}} :\lambda, \qquad 0\le\lambda\le 1\\ &S.t: p_{ij}\left[H(Z)-H^0(Z)\right]-\lambda\le 0, \forall j=1,2,\dots,m \end{aligned}\right\} \qquad (1)$$

Where, $H^0(Z)$ is an utopia point, and we can defined it as the following:

<u>Definition</u> 1:

A point $H_i^0(Z)$ in criterion space is called utopia point if $H_i^0(Z)=\min\left\{H_i(Z)/Z\in R^{n+m}\right\}$, i=1, 2…, n, and n is the number of the time intervals.

The probabilities of undetection of the target over the whole time (see El-Rayes, Mohamed (1986)).

$$H(Z)=\prod_{i=1}^{n}\sum_{j=1}^{m}P_{i\,j}\,b(i,\ j,\ Z^0{}_{i\,j}), \qquad (2)$$

And the effort is

$$L^0(Z)=\sum_{i=1}^{n}\sum_{j=1}^{m} Z^0{}_{ij} \le \sum_{i=1}^{n} v_i = v, \qquad L_i(Z) \le v_i, \qquad (3)$$

Where v_i is a random variable whose probability density function $f(x)$ and distribution function is $F(x)$, $i=1, 2, ..., n$ and $j=1, 2, ..., m$ and we can suppose $Z^0{}_{ij}$ is the initial distribution of the effort to be put into state *j* at time i before optimize it to calculate $H(Z)$.

Our aim is to minimize the value of U(Z), so we need to find the optimal value of $Z=(Z_{ij})$ which minimize $H_i(Z)$ subject to constraints

$$L_i(Z) \le v_i, \quad \sum_{i=1}^{n} L_i(Z) \le v, \quad z_{ij} \ge 0 \text{ and} \sum_{j=1}^{m} P_{ij} = 1. \qquad (4)$$

If the detection function is exponential function that is $1-b(i, j, Z_{ij}) = 1-e^{-\left(\frac{Z_{ij}}{T_j}\right)}$ and the target is located in one of several states no transitions we will consider the following stochastic nonlinear programming problem:

$$\left.\begin{array}{l} \min H(Z)=\sum_{j=1}^{m} P_j e^{-\left(\frac{Z_j}{T_j}\right)}, \\ \text{where} \\ Z(v)=\left\{ Z\in R^m \;/\; L(Z)=\sum_{j=1}^{m} Z_j \le v \right\}, \\ Z_j \ge 0 \text{ and } \sum_{j=1}^{m} P_j = 1, \text{ where } j=1, 2, ..., m, \end{array}\right\} (5)$$

Where $H(Z)$ and $Z(v)$ are real valued convex functions and T_j is a factor due to the search in cell *j* and the dimensions of it. Let $E(v)$ and $\text{var}(v)$ denote the mean and variance of the normally distributed random variable *v*. when the target moves among *m* states, where the detection function is exponential the problem becomes

$$\left.\begin{array}{l} H^0(Z) = \min\ H(Z) = \prod_{i=1}^{n} \sum_{j=1}^{m} P_{ij}\, e^{-\left(\frac{Z_{ij}}{T_j}\right)}, \quad \text{where} \\ Z(v_i) = \left\{ \begin{array}{l} Z \in R^{n+m} \ / \ L_i(Z) \le v, \\ L(Z) - \sum_{i=1}^{n} \sum_{j=1}^{m} Z_{ij} \le v = \sum_{i=1}^{n} v_i \end{array} \right\}, \\ Z_j \ge 0 \ \text{ and } \sum_{j=1}^{m} P_{ij} = 1, \text{ where } v = \sum_{i=1}^{n} v_i,\ i = 1, 2, ..., n, \text{ and } j = 1, 2, ..., m. \end{array}\right\} \quad (6)$$

<u>Definition</u> 2:

$\tilde{Z} \in Z(v_i)$ is said to be an optimal solution for problem (6) if there does not exist $Z \in Z(v_i)$ such that $H(Z) \le H(\tilde{Z})$ with strict inequality hold with $P(L_i(\tilde{Z}) \le v_i) \le \alpha$, $\alpha \in [0, 1]$.

The corresponding chance-constraints problem is:

$$\left.\begin{array}{l} H^0(Z) = \min\ H(Z) = \prod_{i=1}^{n} \sum_{j=1}^{m} P_{ij}\, e^{-\left(\frac{Z_{ij}}{T_j}\right)}, \quad \text{where} \\ P(L_i(Z) \le v_i) \le \alpha, \qquad \alpha \in [0, 1], \\ Z_{ij} \ge 0 \ \text{ and } \sum_{i=1}^{n} \sum_{j=1}^{m} P_{ij} = 1 \\ \text{where } i = 1, 2, ..., n, \text{ and } j = 1, 2, ..., m. \end{array}\right\} \quad (7)$$

The constrain $\tilde{P}(L_i(Z) \le v_i) \ge 1 - \alpha$ has to be satisfied with an un probability of at least $(1-\alpha)$ and can be restated as

$$\tilde{P}\left(\frac{L_i(Z) - E(v_i)}{\sqrt{\operatorname{var}(v_i)}} \le \frac{v_i - E(v_i)}{\sqrt{\operatorname{var}(v_i)}} \right) \ge 1 - \alpha,$$

and for complement probability we have

$$P\left(\frac{L_i(Z)-E(v_i)}{\sqrt{\operatorname{var}(v_i)}} \geq \frac{v_i-E(v_i)}{\sqrt{\operatorname{var}(v_i)}}\right) \leq \alpha,$$

Where,

$$\frac{v_i-E(v_i)}{\sqrt{\operatorname{var}(v_i)}} = k_p \tag{8}$$

is a standard normal random variable.

If K_p represent the value of the standard normal random variable at $\varphi(K_p)=\alpha$, then this constraint can be expressed as

$$\varphi = \left(\frac{L_i(Z)-E(v_i)}{\sqrt{\operatorname{var}(v_i)}}\right) \leq \varphi(K_p),$$

this inequality will be satisfied only if

$$\frac{L_i(Z)-E(v_i)}{\sqrt{\operatorname{var}(v_i)}} \leq K_p,$$

i.e.

$$L_i(Z)-E(v_i) \leq K_p \sqrt{\operatorname{var}(v_i)},$$

Thus, the probabilistic programming problem stated (7) is equivalent to the following deterministic programming problem

$$\left.\begin{array}{l} H^0(Z) = \min\ H_i(Z) = \prod_{i=1}^{n} \sum_{j=1}^{m} P_{ij}\, e^{-\left(\frac{Z_{ij}}{T_j}\right)}, \text{ where} \\ \\ L_i(Z)-E(v_i) \leq K_p \sqrt{\operatorname{var}(v_i)}, \qquad \sum_{j=1}^{m} P_{ij} = 1 \\ \\ \text{where } i = 1, 2, \ldots, n, \text{ and } j = 1, 2, \ldots, m. \end{array}\right\} \tag{9}$$

Which written in the form

$$\left.\begin{aligned} & H^0(Z) = \min\ H(Z) = \prod_{i=1}^{n} \sum_{j=1}^{m} P_{ij}\, e^{-\left(\frac{Z_{ij}}{T_j}\right)}, \quad \text{where} \\ & Z(v_i) = \left\{ \begin{aligned} & Z \in R^{n+m}\ /\ g(Z) = \\ & \qquad L_i(Z) - E(v_i) - K_p \sqrt{\operatorname{var}(v_i)} \le 0 \end{aligned} \right\} \end{aligned}\right\} \quad (10)$$

The optimal solution of problem (4) can be characterized in terms of optimal solution of the problem (10). $H(Z)$ is a convex function and the using Kuhn-Tucker theorem, we obtain the following results,

let

$$C = \sum_{j=1}^{m} T_j ,$$

and

$$r_i = E(v_i) + K_p \sqrt{\operatorname{var}(v_i)} , \qquad (11)$$

we get

$$Z_{ij} = \ln \left[\frac{P_{ij}}{\left[\left(\prod_{j=1}^{m} (P_{ij})^{T_j} \right) \left(\prod_{j=1}^{m} (T_j)^{T_j} \right)^{-1} e^{-r_i} \right]^{\frac{1}{c}} T_j} \right]^{T_j} \qquad (12)$$

Which takes the form

$$Z_{ij} = T_j \left\{ \ln \frac{P_{ij}}{T_j} - \frac{1}{C} \left(\sum_{j=1}^{m} T_j \ln \frac{P_{ij}}{T_j} \right) + \frac{r_i}{C} \right\}, \qquad (13)$$

if $T_j = 1$ for $j = 1, 2, ..., m$; $L_i(Z) \le r_i$, then

$$Z_{ij} = \ln P_{ij} + \frac{r_i}{m} - \frac{1}{m} \sum_{j=1}^{m} \ln P_{ij}$$

$$= \ln P_{i\,j} + \frac{r_i}{m} - \ln\left(\prod_{j=1}^{m} P_{i\,j}\right)^{\frac{1}{m}}$$

$$= \ln P_{i\,j} + \frac{r_i}{m} - \ln G_i\,,$$

hence

$$Z_{i\,j} = \ln\frac{P_{i\,j}}{G_i} + \frac{r_i}{m}\,, \qquad (14)$$

where G_i is the geometric mean, and

$$G_i = \sqrt[m]{P_{i\,1}\,P_{i\,2}\,\ldots\,P_{i\,m}} = (P_{i\,1}\,P_{i\,2}\,\ldots\,P_{i\,m})^{\frac{1}{m}}\,, \qquad (15)$$

by substituting from equation (12) in $H(Z)$ we get

$$H^{0}(Z) = \prod_{i=1}^{n}\sum_{j=1}^{m} P_{i\,j}\left[e^{(1/T_j)\left[\ln\left[\frac{P_{i\,j}}{\left[\left(\prod_{k=1}^{m}(P_{ik})^{T_k}\right)\left(\prod_{k=1}^{m}(T_k)^{T_k}\right)^{-1} e^{-r_i}\right]^{\frac{1}{c}} T_j}\right]^{-1}\right]}\right]$$

$$= \prod_{i=1}^{n}\sum_{j=1}^{m} P_{i\,j}\left[e^{\ln\left[\frac{P_{i\,j}}{\left[\left(\prod_{k=1}^{m}(P_{i\,k})^{T_k}\right)\left(\prod_{k=1}^{m}(T_k)^{T_k}\right)^{-1} e^{-r_i}\right]^{\frac{1}{c}} T_j}\right]^{-1}}\right]$$

$$=\prod_{i=1}^{n}\sum_{j=1}^{m}P_{ij}\left[\frac{\left[\left(\prod_{k=1}^{m}(P_{ik})^{T_k}\right)\left(\prod_{k=1}^{m}(T_k)^{T_k}\right)^{-1}e^{-r_i}\right]^{\frac{1}{c}}T_j}{P_{ij}}\right]$$

$$=\prod_{i=1}^{n}\sum_{j=1}^{m}\left[\left[\left(\prod_{k=1}^{m}(P_{ik})^{T_k}\right)\left(\prod_{k=1}^{m}(T_k)^{T_k}\right)^{-1}e^{-r_i}\right]^{\frac{1}{c}}T_j\right]$$

$$=\prod_{i=1}^{n}\left\{\left(\sum_{j=1}^{m}T_j\right)\left[\left(\prod_{k=1}^{m}(P_{ik})^{T_k}\right)\left(\prod_{k=1}^{m}(T_k)^{T_k}\right)^{-1}e^{-r_i}\right]^{\frac{1}{c}}\right\} \quad (16)$$

Since $Z_{ij} > 0$ and $0 \le H_i(Z) \le 1$ we can obtain the following conditions

$$Z_{ij} = T_j\left\{\ln P_{ij} - \frac{1}{c}\left\{\ln\left(\prod_{j=1}^{m}P_{ij}\right)^{T_j} - \ln\prod_{j=1}^{m}(T_j)^{T_j} - r_i\right\} - \ln T_i\right\}$$

$$= T_j\left\{\ln P_{ij} - \frac{1}{c}\left\{\sum_{j=1}^{m}(T_j \ln P_{ij}) - \sum_{j=1}^{m}T_j \ln T_j\right\} + \frac{r_i}{C} - \ln T_i\right\}$$

$$= T_j\left\{\ln P_{ij} - \frac{1}{c}\sum_{j=1}^{m}T_j \ln\left(\frac{P_{ij}}{T_j}\right) + \frac{r_i}{C} - \ln T_i\right\}, \quad (17)$$

since $Z_{ij} \ge 0$, we get

$$r_i \ge c\ln\frac{T_j}{P_{ij}} + \sum_{j=1}^{m}T_j \ln\left(\frac{P_{ij}}{T_j}\right), \quad \text{for all} \quad i = 1, 2, ..., n, \quad (18)$$

if $T_j = 1$ for all $j = 1, 2, ..., m$ we get

$$r_i \ge -m\ln P_{ij} + \ln\prod_{j=1}^{m}P_{ij}, \quad \text{for all} \quad i = 1, 2, ..., n,$$

$$r_i = -m\ln P_{ij} + m\left(\frac{1}{m}\ln\prod_{j=1}^{m}P_{ij}\right)$$

$$= -m \ln P_{ij} + m \ln \left(\prod_{j=1}^{m} P_{ij} \right)^{\frac{1}{m}}$$

$$= -m \ln P_{ij} + m \ln G_i = m \left(\ln \frac{G_i}{P_{ij}} \right),$$

then

$$r_i \geq m \ \max_j \left(\ln \frac{G_i}{P_{ij}} \right), \qquad (19)$$

from $H^0(Z) \leq 1$, we get $\ln H^0(Z) \leq \ln 1$, hence $\ln H^0(Z) \leq 0$

there for equation (16) becomes

$$\ln \left(\prod_{i=1}^{n} \left(\sum_{j=1}^{m} T_j \right) \left[\left(\prod_{j=1}^{m} (P_{ij})^{T_j} \right) \left(\prod_{j=1}^{m} (T_j)^{T_j} \right)^{-1} e^{-r_i} \right]^{\frac{1}{c}} \right)$$

$$= \sum_{i=1}^{n} Ln\, c + \ln \prod_{i=1}^{n} \left(\sum_{j=1}^{m} T_j \right) \left(\prod_{j=1}^{m} (P_{ij})^{T_j} \right)^{\frac{1}{c}} + \sum_{i=1}^{n} \ln \left(\sum_{j=1}^{m} T_j \right) \left(\prod_{j=1}^{m} (T_j)^{T_j} \right)^{-\frac{1}{c}} - \ln \prod_{i=1}^{n} e^{\frac{r_i}{c}}$$

$$= n \ln c - \frac{n}{c} \sum_{j=1}^{m} T_j \ln T_j + \ln \prod_{i=1}^{n} \left(\sum_{j=1}^{m} T_j \right) \left(\prod_{j=1}^{m} (P_{ij})^{T_j} \right)^{\frac{1}{c}} - \frac{\sum_{i=1}^{n} r_i}{c} \leq 0,$$

hence

$$\frac{\sum_{i=1}^{n} r_i}{c} \geq n \ln c - \frac{n}{c} \sum_{j=1}^{m} T_j \ln T_j + \ln \prod_{i=1}^{n} \left(\sum_{j=1}^{m} T_j \right) \left(\prod_{j=1}^{m} (P_{ij})^{T_j} \right)^{\frac{1}{c}}, \qquad (20)$$

and

$$\ln \prod_{i=1}^{n} \left(\sum_{j=1}^{m} T_j \right) \left(\prod_{j=1}^{m} (P_{ij})^{T_j} \right)^{\frac{1}{c}} = \ln \prod_{i=1}^{n} \left(\sum_{j=1}^{m} T_j \right) [(P_{i1})^{T_1} (P_{i2})^{T_2} \ldots (P_{im})^{T_m}]^{\frac{1}{c}}$$

$$= \ln \prod_{i=1}^{n} \left(\sum_{j=1}^{m} T_j \right) [(P_{i1})^{\frac{T_1}{c}} (P_{i2})^{\frac{T_2}{c}} \ldots (P_{im})^{\frac{T_m}{c}}]$$

$$= \ln \left(\sum_{j=1}^{m} T_j \right) [(p_{11}\ p_{21} \ldots p_{n1})^{\frac{T_1}{c}} (p_{12}\ p_{22} \ldots p_{n2})^{\frac{T_2}{c}} \ldots$$

$$(p_{1m}\ p_{2m} \ldots p_{nm})^{\frac{T_m}{c}}$$

$$= \frac{ncT_1}{c} \sum_{i=1}^{n} \ln p_{i1} + \frac{T_2}{c} \sum_{i=1}^{n} \ln p_{i2} + \ldots + \frac{T_m}{c} \sum_{i=1}^{n} \ln p_{im}$$

$$= n \sum_{i=1}^{n} \frac{T_1}{c} \ln p_{i1} + \sum_{i=1}^{n} \frac{T_2}{c} \ln p_{i2} + \ldots + \sum_{i=1}^{n} \frac{T_m}{c} \ln p_{im} ,$$

hence

$$\ln \prod_{i=1}^{n} \left(\sum_{j=1}^{m} T_j \right) \left(\prod_{j=1}^{m} (P_{ij})^{T_j} \right)^{\frac{1}{c}} = n \sum_{i=1}^{n} \sum_{j=1}^{m} T_j \ln p_{ij} , \qquad (21)$$

substitute from equation (21) in (20), we get

$$\sum_{i=1}^{n} r_i \geq n c (\ln c) - n \sum_{j=1}^{m} T_j \ln T_j + n \left[\sum_{i=1}^{n} \sum_{j=1}^{m} \frac{T_j}{c} \ln p_{ij} \right]. \qquad (22)$$

At $T_j = 1$ for all $j = 1, 2, \ldots, m$, we get

$$\sum_{i=1}^{n} r_i \geq n m (\ln m) + m \left[\sum_{i=1}^{n} \sum_{j=1}^{m} \ln p_{ij} \right]$$

$$= n m (\ln m) + m \left[\sum_{i=1}^{n} \sum_{j=1}^{m} (\ln p_{ij}) \right] = n m (\ln m) + m \sum_{i=1}^{n} \ln \left(\prod_{j=1}^{m} \ln p_{ij} \right)$$

$$= n m (\ln m) + m \sum_{i=1}^{n} \ln G_i = m \left(n \ln m + \sum_{i=1}^{n} \ln G_i \right), \qquad (23)$$

hence

$$\sum_{i=1}^{n} r_i = m \left(\sum_{i=1}^{n} \ln m G_i \right). \qquad (24)$$

In case of located target, equation (18) becomes

$$r_0 \geq c \ \max_j \ \ln \frac{T_j}{P_{0j}} + \sum_{j=1}^{m} T_j \ln\left(\frac{P_{0j}}{T_j} \right). \qquad (25)$$

ALGORITHM

We construct an algorithm to minimize the largest difference between the probability of undetected the lost COVID-19 and its utopia point after choosing the optimal values of Z_{ij}, where i = 1,2, 3, ….,n and j = 1,2, 3, ….,m to calculate $H^0(Z)$.Also, we can suppose the initial distribution of Z^0_{ij} before optimize it to calculate $H(Z)$. The steps of the algorithm can be summarized as in the following:

Step 1. Input the values of the following:

M the number of states j.

N the number of time intervals i.

The j states Marcove chain transition matrix which the lost COVID-19 moves according to it.

p_{0j} the initial probabilities that the lost COVID-19 is in state j at time interval 0.

p_{ij} the probability that the lost COVID-19 is in state j at time interval i.

Z^0_{ij} the initial search effort, which gives the effort to be put into state j at time interval i before optimize it.

The value of λ, where $0 \leq \lambda \leq 1$.

$E(v_i)$ and $\mathrm{var}(v_i)$ the mean and variance of normally distributed random variable effort v_i .

Step 2. Compute K_p from (8)

Step 3. Compute r_i from (11), If it is satisfied conditions (18) and (22), then go to step 4. Elsewhere, stop the process and reenter other values of r_i that fulfills conditions (18) and (22).

Step 4. Generate the optimal effort Z_{ij} from (13), and repeat the above steps from 2 to 4 again in each time interval i, and then go to step 5.

Step 5. Calculate $H^0(Z)$ the utopia points of the probability of undetected the lost COVID-19 from (16).

Step 6. Calculate $H(Z)$ the probability of undetected the lost COVID-19 from (2).

Step 7. Compute min U from (1), and then go to step 8.

Step 8. End (stop).

APPLICATION

Suppose the COVID-19 moves according to a two states Markov chain with a transition matrix

$$A = \begin{bmatrix} 1-x & x \\ y & 1-y \end{bmatrix},$$

where $x = 0.4$ and $b = 0.3$, the initial probabilities are given by: $P_{01} = 0.2$ and $P_{02} = 0.8$, $T_j = 1$ for the cells j, $j = 1, 2$ and $i = 1, 2, 3, 4$. We obtain the following probabilities $P_{11} = 0.36$, $P_{12} = 0.64$, $P_{21} = 0.408$, $P_{22} = 0.592$, $P_{31} = 0.4224$, $P_{32} = 0.5776$, $P_{41} = 0.42672$, $P_{42} = 0.57328$ .The random variable v_i follows the normal distribution with mean $E(v_i)$ and variance $\text{var}(v_i)$, where $r_i = E(v_i) + k_p \sqrt{\text{var}(v_i)}$, The vales of Z_{i1} and Z_{i2}, which give the optimal solution of $H(Z)$ and $H^0(Z)$ are obtained from relation (13) and (16) . Also, from (11) we obtain a different value of r_i, which must satisfy conditions (18) and (22), in this example we use weighted Tchebycheff optimization technique from relation (1) and satisfy the optimal condition when $\lambda = 0.1$, which minimizes the largest difference between the probability of undetected the lost COVID-19 and its utopia point to see (Table 1).

Time interval I	p_{i1}	p_{i2}	$E(v_i)$	$\mathrm{var}(v_i)$	G_i	k_p	r_i	z_{i1}	z_{i2}	$H(Z)$	$H_0(Z)$	Min U
1	0.36	0.64	0.35	0.01	0.48	6.5	1	0.21231	0.78768	30.7E-3	21.6E-3	5.3721E-3
2	0.408	0.592	0.64	0.16	0.491	3.4	2	0.814008	1.18624			
3	0.4224	0.5776	0.47	0.25	0.493	5.06	3	1.343619	1.65654			
4	0.42672	0.57328	0.24	0.09	0.494	4.2	1.5	0.602378	0.89762			

(Table 1): The value of minimizing the largest difference between the probability of undetected the lost COVID-19 and its utopia point.

SPECIAL CASE

The optimal solution of problem (6) when the lost COVID-19 is located in one of *m* cell and the effort a random variable given by:

$$Z_j = \ln\left(\frac{P_{0j}}{G_0}\right) + \frac{k}{m},$$

where $G_0 = \sqrt[m]{P_{01}\ P_{02}\ \dots\ P_{0m}} = \left(\prod_{j=1}^{m} (P_{0j})\right)^{\frac{1}{m}}$, $j = 1, 2, \dots, m$, also

$$H^0(Z) = \left\{ \left(\sum_{j=1}^{m} T_j\right) \left[\left(\prod_{k=1}^{m} (P_{0k})^{T_k}\right) \left(\prod_{k=1}^{m} (T_k)^{T_k}\right)^{-1} e^{-r} \right]^{\frac{1}{c}} \right\}, \text{ and}$$

$$H(Z) = \sum_{j=1}^{m} P_{0j}\, b(i,\ j,\ Z^0{}_{0j}), \text{ hence}$$

$$\min_{Z \in R^{n+m}} U(Z) = \max_j \left\{ p_{0j} \left[H(Z) - H^0(Z) \right] \right\}.$$

CONCLUSION

In this paper, we investigated the search model for a randomly moving the lost COVID-19 among a finite set of different states, a search effort is available at each fixed number of time intervals and the bounded of it is a random variable with a given distribution. The probability of finding the lost COVID-19 in a certain state j at a certain time i and the detection functions are supposed to be known to the searcher. We got the optimal distribution of the effort, which minimizes the probability of undeception the lost COVID-19 and searching effort, and therefore we minimized the largest difference between the probability of undetected the lost COVID-19 and its utopia point using weighted Tchebycheff optimization technique.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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