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SOME TYPES OF θ-FEEBLY CLOSED SETS IN TOPOLOGICAL SPACES

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Abstract: The aim of this paper is to introduce and study the classes of feebly θg-closed set, feebly θg^*-closed sets and feebly θg^*s-closed sets in topological spaces and analyze its basic properties.

Keywords: $f\theta g$-closed sets; $f\theta g^*$-closed sets; $f\theta g^*s$-closed sets.

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1. Introduction

In 1963, Levine [8] showed that since the semi-open set of a topological space need not be closed under finite intersection, then the collection of all semi-open sets need not be a topology on the base set. This result raised questions about the collection of feebly open sets for a given topological space, which led to the following discoveries. The concept of feebly open and feebly closed sets are introduced by Maheswari and Jain [9]. Ibraheem [3, 4] introduced feebly generalized closed (briefly fg-closed) sets, generalized feebly closed (briefly gf-closed) sets.

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Dhana Balan and Bhuvaneswari [5] introduced feebly regular closed sets in 2015. Sathishmohan [14] et al introduced and investigates the properties of $f\theta$-closed sets. Many authors used the concepts of feebly open and feebly closed sets to study many concepts in topological spaces and gave several interesting results about that. Besides we initiate a new class of sets called $f\theta g$-closed sets, $f\theta g^*$-closed sets and $f\theta g^*s$-closed sets in topological spaces and analyze some of their properties in this paper.

2. Preliminaries

Definition 2.1. *A subset A of space (X,τ) is called*

(1) semi-closed set [2]*, if $int(cl(A)) \subseteq A$*

(2) α-closed set [10]*, if $cl(int(cl(A)) \subseteq A$*

(3) regular closed set [15]*, if $A = cl(int(A))$*

The complements of the above mentioned closed sets are called their respective open sets.

Definition 2.2. [17] *A point x of a space (X,τ) is called θ-adherent point of a subset A of X if $cl(U) \cap A \neq \phi$, for every open set U containing x. The set of all θ-adherents points of A is called the θ-closure of A and is denoted by $cl_\theta(A)$. A subset A of a space X is called θ-closed if and only if $A = cl_\theta(A)$. The complement of a θ-closed set is called θ-open.*

Definition 2.3. [6] *A point x of a space (X,τ) is called semi θ-cluster point of A if $A \cap scl(U) \neq \phi$, for every semi-open set U containing x. The set of all semi θ-cluster points of A is called semi θ-closure of A and is denoted by $scl_\theta(A)$. Hence, a subset A is called semi θ-closed if $scl_\theta(A) = A$. The complement of a semi θ-closed set is called semi θ-open set.*

Definition 2.4. *A subset A of a space (X,τ) is called*

(1) a regular generalized closed set (briefly. rg-closed) set [12]*, if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is r-open in (X,τ).*

(2) a α-generalized regular closed (briefly. αgr-closed) set [16]*, if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is r-open in (X,τ).*

(3) a generalized pre regualr closed (briefly. gpr-closed) set [7]*, if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is r-open in (X,τ).*

(4) *a regular ω-generalized closed (briefly. $r\omega g$-closed) set* [11] *if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is r-open in (X,τ).*

(5) *a θ-generalized star semi-closed (briefly θg^*s-closed) set* [13] *if $scl_\theta(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open.*

Definition 2.5. [9] *A subset A of a topological space X is said to be feebly open (resp. feebly closed) if $A \subset scl(int(A))$ (resp. $sint(cl(A)) \subset A$).*

Remark 2.6. [1] *Every open set (resp. closed set) is feebly open (resp. feebly closed set) .*

Definition 2.7. [14] *A subset A of X is said to be feebly θ-open if $A \subset s\theta cl(int(A))$ and it is denoted by $f\theta$-open set. The complement of $f\theta$-open sets is called $f\theta$-closed sets.*

3. Some θ-Feebly Closed Sets

In this section, we introduce and study the notion of $f\theta g$-closed sets, $f\theta g^*$-closed sets and $f\theta g^*s$-closed set in topological spaces.

Definition 3.1. *A subset A of a topological space (X,τ) is called feebly θ-generalized closed set (briefly $f\theta g$-closed set) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is feebly θ-open in X.*

Theorem 3.2. *Every closed set (resp. r-closed and θ-closed) is $f\theta g$-closed set but not conversely.*

Proof: Let A be any closed set (resp. r-closed and θ-closed) in X such that $A \subseteq U$, where U is $f\theta$-open. Since $cl(A) = A \subseteq U$, we have $cl(A) \subseteq U$ (resp. $cl(A) \subseteq rcl(A) \subseteq cl_\theta(A) \subseteq U$). Hence A is $f\theta g$-closed set in X.

Example 3.3. *Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{b\}, \{a, c\}, X\}$. Let $A = \{b, c\}$. Then A is $f\theta g$-closed but not closed, r-closed and θ-closed.*

Theorem 3.4. *Let (X,τ) be a topological space, then*

(1) *Every $f\theta g$-closed set is rg-closed.*

(2) *Every $f\theta g$-closed set is gpr-closed.*

(3) *Every $f\theta g$-closed set is αgr-closed.*

(4) Every $f\theta g$-closed set is rwg-closed.

Proof: (1) Let A be any $f\theta g$-closed set in X and U be any regular open set containing A. Since every r-open set is $f\theta$-open set we have $cl(A) \subseteq U$. Hence A is rg-closed.
The proof of (2) to (4) are obvious.
The converse of the above theorem need not be true in general as seen in the following example.

Example 3.5. *Let X = $\{a,\ b,\ c\}$ and τ = $\{\phi,\ \{b\},\ \{a,\ b\},\ \{b,\ c\},\ X\}$. Let A = $\{b\}$. Then A is rg-closed, αgr-closed, gpr-closed and rwg-closed but not $f\theta g$-closed.*

Theorem 3.6. *If A and B are $f\theta g$-closed sets in X then $A \cup B$ is $f\theta g$-closed in X.*
Proof: Let A and B are $f\theta g$-closed sets in X and U be any $f\theta$-open set such that $A \cup B \subseteq U$. Therefore $cl(A) \subseteq U$, $cl(B) \subseteq U$. Hence $cl(A \cup B) = cl(A) \cup cl(B) \subseteq U$. Therefore $A \cup B$ is $f\theta g$-closed in X.

Remark 3.7. *The intersection of any two subsets of $f\theta g$-closed sets in X is also $f\theta g$-closed set in X.*

Example 3.8. *Let X = $\{a,\ b,\ c\}$ and τ = $\{\phi,\ \{a\},\ \{a,\ b\},\ \{a,\ c\},\ X\}$. Let A = $\{a,\ b\}$ and B = $\{a,\ c\}$ are $f\theta g$-closed sets. Then $A \cap B$ is also $f\theta g$-closed.*

Theorem 3.9. *If a set A is $f\theta g$-closed set then $cl(A) - A$ contains no non empty $f\theta$-closed set.*
Proof: Let F be a $f\theta$-closed set in X such that $F \subseteq cl(A) - A$. Then $A \subseteq X - F$. Since A is $f\theta g$-closed and $X - F$ is $f\theta$-open then $cl(A) \subseteq X - F$. (i.e.) $F \subseteq X - cl(A)$. So $F \subseteq (X - cl(A)) \cap (cl(A) - A)$. Therefore $F = \phi$.

Definition 3.10. *A subset A of a topological space (X, τ) is called feebly θ-generalized star closed set (briefly $f\theta g^*$-closed set) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is feebly θg-open in X.*

Theorem 3.11. *Every closed set (resp. r-closed and θ-closed) is $f\theta g^*$-closed set but not conversely. Proof: Let A be any closed set (resp. r-closed and θ-closed) in X such that $A \subseteq U$, where U is $f\theta g$-open. Since $cl(A) \subseteq A$ (resp. $rcl(A) \subseteq cl_\theta(A) \subseteq U$). Hence A is $f\theta g^*$-closed set in X.*

Example 3.12. *Let* $X = \{a,\ b,\ c\}$ *and* $\tau = \{\phi,\ \{a\},\ \{a,\ b\},\ X\}$. *Let* $A = \{a,\ c\}$. *Then* A *is* $f\theta g^*$*-closed but not closed, r-closed and* θ*-closed.*

Theorem 3.13. *Let* (X,τ) *be a topological space, then*

(1) Every $f\theta g^*$*-closed set is rg-closed.*

(2) Every $f\theta g^*$*-closed set is gpr-closed.*

(3) Every $f\theta g^*$*-closed set is* α*gr-closed.*

(4) Every $f\theta g^*$*-closed set is rwg-closed.*

Proof: (1) Let A be any $f\theta g^*$*-closed set in* X *and* U *be any regular open set containing A. Since every r-open set is* $f\theta g$*-open set, we have* $cl(A) \subseteq U$*. Hence A is rg-closed.*

The proof of (2) to (4) are obvious.

The converse of the above theorem need not be true in general as seen in the following example.

Example 3.14. *In above example, Let* $A = \{a,\ b\}$*. Then A is rg-closed,* α*gr-closed, gpr-closed and rwg-closed but not* $f\theta g^*$*-closed.*

Theorem 3.15. *If A and B are* $f\theta g^*$*-closed sets in* X *then* $A\cup B$ *is* $f\theta g^*$*-closed in* X*.*

Proof: Let A and B are $f\theta g^*$*-closed sets in* X *and* U *be any* $f\theta g$*-open set such that* $A\cup B \subseteq U$*. Therefore* $cl(A) \subseteq U$*,* $cl(B) \subseteq U$*. Hence* $cl(A\cup B) = cl(A)\cup cl(B) \subseteq U$*. Therefore* $A\cup B$ *is* $f\theta g^*$*-closed set in* X*.*

Remark 3.16. *The intersection of any two subsets of* $f\theta g^*$*-closed sets in* X *is also a* $f\theta g^*$*-closed set in* X*.*

Example 3.17. *Let* $X = \{a,\ b,\ c\}$ *and* $\tau = \{\phi,\ \{a\},\ \{b\},\ \{a,\ b\},\ \{a,\ c\},\ X\}$*. Let* $A = \{a,\ c\}$ *and* $B = \{b,\ c\}$ *are* $f\theta g^*$*-closed sets. Then* $A\cap B$ *is* $f\theta g$*-closed.*

Theorem 3.18. *If a set A is* $f\theta g^*$*-closed set then* $cl(A) - A$ *contains no non empty* $f\theta g$*-closed set.*

Proof: Let F *be a* $f\theta g$*-closed set in* X *such that* $F \subseteq cl(A) - A$*. Then* $A \subseteq X - F$*. Since A is* $f\theta g^*$*-closed set and* $X - F$ *is* $f\theta g$*-open then* $cl(A) \subseteq X - F$*. (i.e.)* $F \subseteq X - cl(A)$*. So* $F \subseteq (X - cl(A)) \cap (cl(A) - A)$*. Therefore* $F = \phi$*.*

Definition 3.19. *A subset A of a topological space (X,τ) is called feebly θ-generalized star semi closed set (briefly $f\theta g^*s$-closed set) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is feebly θg-open in X.*

Theorem 3.20. *Every closed set (resp. r-closed and θ-closed) is $f\theta g^*s$-closed set but not conversely. Proof: Let A be any closed set (resp. r-closed and θ-closed) in X such that $A \subseteq U$, where U is $f\theta g$-open. Since $scl(A) \subseteq cl(A) \subseteq A$ (resp. $rcl(A) \subseteq cl_\theta(A) \subseteq U$). Hence A is $f\theta g^*s$-closed set in X.*

Example 3.21. *Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{b\}, \{b, c\}, X\}$. Let $A = \{a, b\}$. Then A is $f\theta g^*s$-closed but not closed, r-closed, g^*sr-closed and θ-closed.*

Theorem 3.22. *Let (X,τ) be a topological space, then*

(1) Every $f\theta g^*s$-closed set is gpr-closed.
(2) Every $f\theta g^*s$-closed set is αgr-closed.
(3) Every $f\theta g^*s$-closed set is rwg-closed.

Proof: (1) Let A be any $f\theta g^*s$-closed set in X and U be any regular open set containing A. Since every r-open set is $f\theta g$-open set, we have $pcl(A) \subseteq cl(A) \subseteq U$. Hence A is gpr-closed. The proof of (2) and (3) are obvious.
The converse of the above theorem need not be true in general as seen in the following example.

Example 3.23. *In the above example, Let $A = \{c\}$. Then A is gpr-closed, αgr-closed and rwg-closed but not $f\theta g^*s$-closed.*

Theorem 3.24. *If A and B are $f\theta g^*s$-closed sets in X then $A \cup B$ is $f\theta g^*s$-closed in X.*
Proof: Let A and B are $f\theta g^*s$-closed sets in X and U be any $f\theta g$-open set such that $A \cup B \subseteq U$. Therefore $scl(A) \subseteq U$, $scl(B) \subseteq U$. Hence $scl(A \cup B) = scl(A) \cup scl(B) \subseteq U$. Therefore $A \cup B$ is $f\theta g^*s$-closed set in X.

Remark 3.25. *The intersection of any two subsets of $f\theta g^*s$-closed sets in X is also a $f\theta g^*s$-closed set in X.*

Example 3.26. *Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{b\}, \{c\}, \{a, c\}, \{b, c\}, X\}$. Let $A = \{a, b\}$ and $B = \{a, c\}$ are $f\theta g^*s$-closed sets. Then $A \cap B$ is $f\theta g^*s$-closed.*

Theorem 3.27. *If a set A is $f\theta g^*s$-closed set then $scl(A)-A$ contains no non empty $f\theta g$-closed set.*

Proof: Let F be a $f\theta g$-closed set in X such that $F\subseteq scl(A)-A$. Then $A\subseteq X-F$. Since A is $f\theta g^*s$-closed set and $X-F$ is $f\theta g$-open then $scl(A)\subseteq X-F$. (i.e.) $F\subseteq X-scl(A)$. So $F\subseteq (X-scl(A))\cap(scl(A)-A)$. Therefore $F=\phi$.

Theorem 3.28. *If A is θg^*s-closed set, then it is $f\theta g$-closed set.*

Proof: Let A be θg^*s-closed set. Let $A\subseteq U$ and U be $f\theta$-open. Since every $f\theta$-open is g-open. Then $cl(A)\subseteq scl_\theta(A)\subseteq U$. Hence A is a $f\theta g$-closed set.

Theorem 3.29. *If A is θg^*s-closed set, then it is $f\theta g^*$-closed set and $f\theta g^*s$-closed set.*

Proof: Let A be θg^*s-closed set. Let $A\subseteq U$ and U be $f\theta g$-open. Since every $f\theta g$-open is g-open. Then $scl(A)\subseteq cl(A)\subseteq scl_\theta(A)\subseteq U$. Hence A is a $f\theta g^*$-closed set and $f\theta g^*s$-closed set.

Example 3.30. *Let $X=\{a,\ b,\ c\}$ and $\tau=\{\phi,\ \{a\},\ \{a,\ b\},\ \{a,\ c\},\ X\}$. Let $A=\{c\}$. Then A is $f\theta g$-closed, $f\theta g^*$-closed and $f\theta g^*s$-closed but not θg^*s-closed.*

Theorem 3.31. *If A is $f\theta$-closed set, then it is $f\theta g$-closed set.*

Proof: Obvious by the definition..

Theorem 3.32. *If A is $f\theta$-closed set, then it is $f\theta g^*$-closed set and $f\theta g^*s$-closed set. Proof: Let A be $f\theta$-closed set. Let $A\subseteq U$ and U be $f\theta g$-open. Then $scl(A)\subseteq cl(A)\subseteq s\theta cl(int(A))\subseteq U$. Hence A is a $f\theta g^*$-closed set and $f\theta g^*s$-closed set.*

Example 3.33. *Let $X=\{a,\ b,\ c\}$ and $\tau=\{\phi,\ \{a,\ c\},\ X\}$. Let $A=\{a,\ b\}$. Then A is $f\theta g$-closed, $f\theta g^*$-closed and $f\theta g^*s$-closed but not $f\theta$-closed.*

Theorem 3.34. *If A is $f\theta g$-closed set, then it is $f\theta g^*$-closed set and $f\theta g^*s$-closed set.*

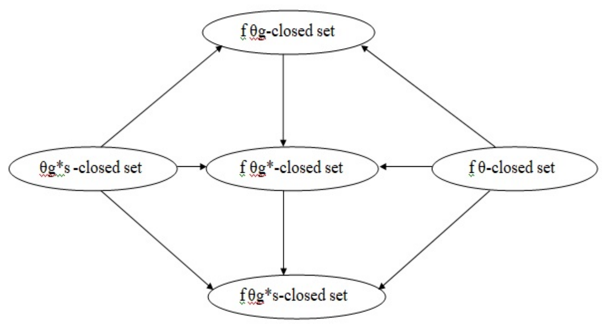
Proof: Let A be $f\theta g$-closed set. Let $A\subseteq U$ and U be $f\theta g$-open. Then $scl(A)\subseteq cl(A)\subseteq U$. Hence A is $f\theta g^*$-closed set and $f\theta g^*s$-closed set.

Example 3.35. *Let $X=\{a,\ b,\ c\}$ and $\tau=\{\phi,\ \{c\},\ \{a,\ c\},\ X\}$. Let $A=\{b,\ c\}$. Then A is $f\theta g^*$-closed and $f\theta g^*s$-closed but not $f\theta g$-closed.*

Theorem 3.36. *If A is $f\theta g^*$-closed set, then it is $f\theta g^*s$-closed set. Proof: Let A be $f\theta g^*$-closed set. Let $A \subseteq U$ and U be $f\theta g$-open. Then $scl(A) \subseteq cl(A) \subseteq U$. Hence A is $f\theta g^*s$-closed set.*

Example 3.37. *Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, \{a, c\}, X\}$. Let $A = \{c\}$. Then A is $f\theta g^*s$-closed but not $f\theta g^*$-closed.*

Remark 3.38. *From the above discussions, we have the following implications.*



None of the implications are reversible.

Conflict of Interests

The author(s) declare that there is no conflict of interests.

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