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SOME GENERALIZATION OF (1,2)S_P- LOCALLY CLOSED SETS IN BITOPOLOGICAL SPACES

S. DHANALAKSHMI*, N. DURGA DEVI

Department of Mathematics, Sri Parasakthi College for Women, Courtallam, India

(Affiliated to Manonmaniam Sundaranar University Tirunelveli, Tamilnadu, India)

Abstract: In this paper, the notion of (1,2)S_P-locally-closed sets are introduced and discuss some of their properties and also the generalization of the sets is studied.

Keywords: (1,2)S_P-open sets; (1,2)S_P-closed sets; (1,2)S_Pg-open sets; (1,2)S_Pg-closed sets; (1,2)S_P-locally closed sets; (1,2)S_P-generalized-locally closed sets.

AMS Subject Classification: 54A05.

1. INTRODUCTION

The study of locally closed sets was introduced by Bourbaki [1] in 1966 then the authors Ganster and Reilly [3] have studied it extensively. A subset A of a topological space X is called locally closed if A = U ∩ F, where U is open and F is closed. It is interesting that a locally closed set is a generalization of both open sets and closed sets. In 1963 Kelly [5] define a bitopological spaces (X, τ_1, τ_2) with two topologies τ_1 and τ_2 on X. Raja Rajeswari [8] defined and studied the concepts of ultra-locally closed sets in bitopological spaces. In this paper, a new notion of locally

*Corresponding author
E-mail address: prithira456@gmail.com
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closed as (1,2)S_P-Locally closed sets in bitopological spaces are defined and study some of their properties.

2. PRELIMINARIES

Throughout this paper by X, we mean bitopological space (X, τ_1, τ_2). For a subset A of a bitopological space (X, τ_1, τ_2), then cl(A), int(A) and A^c denote the closure of A, the interior of A and the complement of A in X respectively.

Definition 2.1. [6] A subset A of a bitopological space X is called a

(i) (1,2)semi-open if A ⊆ $\tau_1\tau_2$Cl($\tau_1 Int(A)$).

(ii) (1,2)pre-open if A ⊆ τ_1Int($\tau_1\tau_2 Cl(A)$).

(iii) (1,2)regular-open if A = τ_1Int($\tau_1\tau_2 Cl(A)$).

The collection of all (1,2)semi-open, (1,2)pre-open and (1,2)regular-open sets are denoted by (1,2)SO(X), (1,2)PO(X) and (1,2)RO(X) respectively.

Definition 2.2. [6] A subset A of a bitopological space X is called a

(i) (1,2)α-closed if τ_1Cl($\tau_1\tau_2$Int(τ_1Cl(A))) ⊆ A.

(ii) (1,2)semi-closed if $\tau_1\tau_2$Int(τ_1Cl(A)) ⊆ A.

(iii) (1,2)pre-closed if τ_1Cl($\tau_1\tau_2$Int(A)) ⊆ A.

(iv) (1,2)regular-closed if A = τ_1Cl($\tau_1\tau_2$Int(A)).

The set of all (1,2)α-closed, (1,2)semi-closed, (1,2)pre-closed and (1,2)regular-closed sets are defined in the usual sense and denoted as (1,2)αCL(X), (1,2)SCL(X), (1,2)PCL(X) and (1,2)RCL(X) respectively.

Also, for any subset A of X, the (1,2)α-closure, (1,2)semi-closure, (1,2)pre-closure and (1,2)regular-closure of A is denoted as (1,2)αCl(A), (1,2)SCl(A), (1,2)PCl(A) and (1,2)RCl(A) respectively.

Definition 2.3. [4] A (1,2) semi-open set A of a bitopological space X is called (1,2)S_P-open set if for each x ∈ A, there exists a (1,2) pre-closed set F such that x ∈ F ⊆ A.

Definition 2.4. [2] A subset A of X is called a (1,2)S_P-generalized-closed (briefly (1,2)S_P-closed) set if (1,2)S_PCl(A) ⊆ U whenever A ⊆ U and U ∈ (1,2)S_PO(X). The family of all (1,2)S_Pg-closed sets is denoted by (1,2)S_PGCL(X).

Definition 2.5. [2] A subset A of a bitopological space X is called a (1,2)S_P-generalized-open (briefly (1,2)S_Pg-open) set if A^c is (1,2)S_Pg-closed. The set of all (1,2)S_Pg-open set is denoted by (1,2)S_PGO(X).

Remark 2.6. [4] Any intersection of (1,2)S_P-closed sets of a bitopological space X is (1,2)S_P-closed.

Theorem 2.7. [2] Every (1,2)S_P-closed set is a (1,2)S_Pg-closed.

Definition 2.8. [7] A subset A of a space (X, τ) is called generalized-closed (briefly g-closed), if cl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ) and the complement of a g-closed set is called g-open.

3. (1,2)S_p-LOCALLY CLOSED SETS

Definition 3.1. A subset A of a bitopological space X is said to be (1,2)S_P-locally closed (briefly (1,2)S_P-LC) if A = C ∩ D, where C is a (1,2)S_P-open set and D is a (1,2)S_P-closed set in X. The family of (1,2)S_P-locally closed sets is denoted by (1,2)S_P-LC(X).

Proposition 3.2. For a bitopological space X,

(i) a subset A of X is (1,2)S_P-locally closed if and only if its complement X −A is the union of a (1,2)S_P-closed set and a (1,2)S_P-open set in X.

(ii) every (1,2)S_P-open (resp. (1,2)S_P-closed) subset of X is (1,2)S_P-locally closed.

(iii) the complement of (1,2)S_P-LC set need not be an (1,2)S_P-LC set.

Proof. **(i)** Let A be a subset of a bitopological space X and let A be (1,2)S_P-locally closed. Then A = C ∩ D where C is (1,2)S_P-open set and D is (1,2)S_P-closed set in X which implies Cl(A) = Cl(C ∩ D) ⊆ Cl(D) = D that implies A ⊆ C ∩ Cl(A) ⊆ C ∩ D = A. Thus A = C ∩ Cl(A). That is, A = C ∩ D. Hence X − A = $C^c \cup D^c$. Hence, X − A is the union of a (1,2)S_P-closed set and a (1,2)S_P-open set in X.

(ii) Let A be a (1,2)S_P-open subset of X. Then A = A ∩ CI(A) ⊆ Int[A ∪ (X – CI(A))] ∩ CI(A) = A ∩ CI(A) = A. Therefore A = Int[A ∪ (X – CI(A))] ∩ CI(A). Hence A is (1,2)S_P-locally closed.

(iii) Let A be (1,2)S_P-locally closed set. Then A = C ∩ D where C is a (1,2)S_P-open set and D is a (1,2)S_P-closed set in X which implies X – A = $(C \cap D)^c$. That is X – A = (X – D) ∪ (X – C), where (X – D) is (1,2)S_P-open set and (X – C) is (1,2)S_P-closed set in X. Thus (X – A) is not (1,2)S_P-locally closed. Hence, the complement of an (1,2)S_P-LC set need not be an (1,2)S_P-LC set.

Example 3.3. Let X = {a, b, c, d} with two topologies τ_1 = {ϕ, X, {a, c}, {a, c, d}}, τ_2 = {ϕ, X, {b}, {b, c}}. Then (1,2)S_PO(X) = {ϕ, X, {a, c}, {a, c, d}}. (1,2)S_PCL(X) = {X, ϕ, {b, d}, {b}}. (1,2)S_p-LC(X) = (1,2)S_PO(X) ∩ (1,2)S_PCL(X) = {X, ϕ, {a, c}, {a, c, d}, {d}, {b, d}, {b}}. Here {d} ∈ (1,2)S_P-LC(X), but its complement {a, b, c} does not belongs to (1,2)S_P-LC(X). Hence the complement of an (1,2)S_P-LC set need not be an (1,2)S_P-LC set.

Proposition3.4. A subset A of a bitopological space X is (1,2)S_P-closed if and only if it is both (1,2)$S_P g$-closed and (1,2)S_P-locally closed.

Proof. Let A be a subset of a bitopological space X which is both (1,2)$S_P g$-closed and (1,2)S_P-locally closed. Then A = C ∩ D where C is (1,2)S_P-open set and D is (1,2)S_P-closed set. Hence A ⊆ C and A ⊆ D. As A is (1,2)$S_P g$-closed implies (1,2)S_PCl(A) ⊆ C and as D is (1,2)$S_P g$-closed implies (1,2)S_PCl(A) ⊆ D. Consequently (1,2)S_PCl(A) ⊆ A. Hence A is (1,2)S_P-closed.

Also, let A be (1,2)S_P-closed set. They by Theorem 2.7, A is (1,2)$S_P g$-closed set. Therefore (1,2)S_PCI(A) ⊆ U, whenever A ⊆ U and U is (1,2)S_PO(X). Now A ⊆ C implies (1,2)S_PCl(A) ⊆ C and A ⊆ D implies (1,2)S_PCl(A) ⊆ D that implies (1,2)S_PCl(A) ⊆ A. Therefore, A = C ∩ D where C is (1,2)S_P-open and D is (1,2)S_P-closed. Hence A is (1,2)$S_P g$-closed and (1,2)S_P-locally closed.

Theorem 3.5. If X is a bitopological space, then the following are equivalent

(i) A is (1,2)S_P-locally closed.

(ii) A = C ∩ (1,2)S_PCI(A) for some (1,2)S_P-open set C.

(iii) [(1,2)S_PCl(A) − A] is (1,2)S_P-closed.

(iv) A ∪ [X − (1,2)S_PCl(A)] is (1,2)S_P-open.

(v) A ⊆ (1,2)S_PInt(A ∪ [X −(1,2)S_PCl(A)]).

Proof. (i) => (ii): Assume that A is (1,2)S_P-locally closed. Therefore A = C ∩ D, where C is (1,2)S_P-open and D is (1,2)S_P-closed. If A ⊆ D, then (1,2)S_P Cl(A) ⊆ D, that is A ⊆ C ∩ (1,2)S_PCl(A) ⊆ C ∩ D = A. Hence A = C ∩ (1,2)S_PCl(A) for some (1,2)S_P-open set C.

(ii) => (iii): Assume A = C ∩ (1,2)S_P Cl(A) for some (1,2)S_P-open set C. Then [(1,2)S_PCl(A) – A] = C^c ∩ (1,2)S_PCl(A) = (X – C) ∩ (1,2)S_PCl(A), which is (1,2)S_P-closed [by Remark 2.6].

(iii) => (iv): $[(1,2)S_P\text{Cl(A)} - A]^c = C \cup ((1,2)S_P\text{Cl}(A))^c$ is $(1,2)S_P$-open. If we take $F = [(1,2)S_P\text{Cl(A)} - A]$, then $X - F = [(1,2)S_P\text{Cl(A)} - A]^c = C \cup [(1,2)S_P\text{Cl(A)}]^c$ is $(1,2)S_P$-open which implies $X - [(1,2)S_P\text{Cl(A)} - A] = A \cup [X - (1,2)S_P\text{Cl(A)}]$ is $(1,2)S_P$-open.

(iv) => (v): By assumption, $A \cup [X - (1,2)S_P\text{Cl(A)}]$ is $(1,2)S_P$-open. Again $A \subseteq A \cup [X - (1,2)S_P\text{Cl(A)}] = (1,2)S_P\text{Int}(A \cup [X - (1,2)S_P\text{Cl(A)}])$. Hence $A \subseteq (1,2)S_P\text{Int}(A \cup [X - (1,2)S_P\text{Cl(A)}])$.

(v) => (i): Let $A \subseteq (1,2)S_P\text{Int}(A \cup [X - (1,2)S_P\text{Cl(A)}])$, which implies $A = (1,2)S_P\text{Int}(A \cup [X - (1,2)S_P\text{Cl(A)}]) \cap (1,2)S_P\text{Cl(A)}$. Hence A is $(1,2)S_P$-locally closed.

Definition 3.6. A subset A of a bitopological space X is said to be an $(1,2)S_P$-Q-set if $(1,2)S_P\text{Int}[(1,2)S_P\text{Cl(A)}] = (1,2)S_P Cl[(1,2)S_P Int(s)]$.

Example 3.7. Follow the Example 3.3, $(1,2)S_P\text{O(X)} = \{\phi, X, \{a, c\}, \{a, c, d\}$, $(1,2)S_P Cl(X) = \{X, \phi, \{b, d\}, \{b\}\}$. Here A = {b}, {d}, {a, c}, {b, d}, {a, b, c}, {a, c, d} are an $(1,2)S_P$-Q-set.

Theorem3.8. Let X be an $(1,2)S_P$-topology and A be an $(1,2)S_P$-Q-set. If A is an $(1,2)S_P$-locally closed set, then

(i) $(1,2)S_P Int(A)$ is $(1,2)S_P$-closed.

(ii) $(1,2)S_P Cl(A)$is contained in a $(1,2)S_P$-closed set.

Proof. (i) Let X be an $(1,2)S_P$-topology and A be an $(1,2)S_P$-Q-set. Assume that A is an $(1,2)S_P$-locally closed. Then $A = B \cap (1,2)S_P Cl(\text{A})$, where B is $(1,2)S_P$-open. $(1,2)S_P\text{Int(A)} = (1,2)S_P\text{Int(B)} \cap (1,2)S_P\text{Int}[(1,2)S_P Cl(\text{A})] = (1,2)S_P\text{Int(B)} \cap (1,2)S_P\text{Cl}[(1,2)S_P\text{Int(A)}]$. Thus $(1,2)S_P\text{Int(A)}$ is $(1,2)S_P$-locally closed. Hence $(1,2)S_P\text{Int(A)}$ is $(1,2)S_P$-closed.

(ii) Let $A = B \cap (1,2)S_P\text{Cl(A)}$. Then $(1,2)S_P\text{Cl(A)} = (1,2)S_P\text{Cl}[B \cap (1,2)S_P\text{Cl(A)}] \subseteq (1,2)S_P\text{Cl(B)} \cap (1,2)S_P\text{Cl(A)}$. By Remark 2.6, $(1,2)S_P\text{Cl(A)}$ is contained in a $(1,2)S_P$-closed set.

Proposition 3.9. If $A \subset B \subset X$ and B is $(1,2)S_P$-locally closed, then there exists an $(1,2)S_P$-locally closed set C such that $A \subset C \subset B$.

Proof. Let $A \subset B \subset X$ and B be an $(1,2)S_P$-locally closed. Take $B = S \cap (1,2)S_P\text{Cl(B)}$, where S is $(1,2)S_P$-open. Now $A \subset B = S \cap (1,2)S_P\text{Cl(B)}$ which implies $A \subset S$ and $A \subset (1,2)S_P\text{Cl(B)}$. Hence $A \subseteq S \cap (1,2)S_P\text{Cl(A)} = C$, where C is an $(1,2)S_P$-locally closed set such that $A \subset C \subset B$.

Definition 3.10. A subset A of X is called $(1,2)S_P$-generelized locally closed (briefly $(1,2)S_P - glc$) if $A = U \cap F$, where U is $(1,2)S_P g$-open and F is a $(1,2)S_P g$-closed set of X. The family of all $(1,2)S_P$-glc sets is denoted as $(1,2)S_P$-GLC(X).

Remark 3.11. Every (1,2)S_P-open, (1,2)S_P-closed and (1,2)S_P-LC sets are (1,2)S_Pg-LC. But the converse is not true and is justified in the following example.

Example 3.12. In Example 3.3, (1,2)S_p-GC(X) = {X, ϕ, {b}, {a, b}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {b, c, d}} and (1,2)S_PGO(X) = {ϕ, X, {a, c, d}, {c, d}, {a, d}, {a, c}, {d}, {c}, {a}}. (1,2)S_P-GLC(X) = (1,2)S_P-GO(X) ∩ (1,2)S_P-GC(X) = {X, ϕ, {b}, {a, b}, {a}, {b, c}, {c}, {b, d}, {d}, {a, b, c}, {a, c}, {a, b, d}, {a, d}, {b, c, d}, {c, d}, {a, c, d}}. Here, {a, b, c} ∈ (1,2)S_P-GLC(X), but {a, b, c} ∉ (1,2)S_PLC(X) and also {a, b, c} is neither (1,2)S_PO(X) nor (1,2)S_PCL(X).

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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