Available online at http://scik.org
J. Math. Comput. Sci. 11 (2021), No. 5, 5670-5681
https://doi.org/10.28919/jmcs/6089
ISSN: 1927-5307

ON HINGE DOMINATION IN GRAPHS

ANJANEYULU MEKALA[1], U. VIJAYA CHANDRA KUMAR[2,*], R. MURALI[3]

[1]Department of Mathematics, Guru Nanak Institutions Technical Campus (Autonomous), Hyderabad, Telangana, India

[2]School of Applied Sciences (Mathematics), REVA University, Bengaluru, Karnataka, India

[3]Department of Mathematics, Dr. Ambedkar Institute of Technology, Bengaluru, Karnataka, India

Abstract. A set D_h of vertices in a graph $G = (V,E)$ is a hinge dset if every vertex u in $V - D_h$ is adjacent to some vertex v in D_h and a vertex w in $V - D_h$ such that (v,w) is not an edge in E. The hinge domination number $\gamma_h(G)$ is the minimum size of a hinged dset. In this paper we determine hinge domination number $\gamma_h(G)$ for standard graphs and some shadow distance graphs.

Keywords: dominating set; hinge domination number; minimal dominating set.

2010 AMS Subject Classification: 05C69.

1. Introduction

A graph $G = (V,E)$, we mean a finite, nontrivial and undirected graph without loops and multiple edges. The concept of a dset is well known in graph theoretic literature and various domination parameters have been studied. A set D_h of vertices in G is called a hinge dominating set [1] if every $u \in V - D_h$ is adjacent to some vertex $v \in D_h$ and a vertex w in $V - D_h$ such that

*Corresponding author

E-mail address: upparivijay@gmail.com

Received May 22, 2021

(v,w) is not an edge in E. The hinge domination number $\gamma_h(G)$ [1] is the minimum size of a hinge dominating set. Throughout this paper we will denote *dominating set* by *dset*.

Let D be the set of all possible distances in $G = (V,E)$ and let $D_s \subset D$. The distance graph associated with G denoted by $D(G,D_s)$ [7] is the graph with vertex set V and two vertices u and v are adjacent in it if $d(u,v) \in D_s$. The shadow distance graph of G, denoted by $D_{sd}(G,D_s)$ is obtained from G by considering two copies of G namely G itself and $G^{'}$ such that if $u \in V(G)$ then the corresponding vertex $u^{'}$ is in $V(G^{'})$ and $E(D_{sd}(G,D_s)) = E(G) \cup E(G^{'}) \cup E_{DS}$ where E_{DS} consists of the set of all edges of the form $e = (u,v^{'})$ with the condition $d(u,v) \in D_s$ in G.

In this paper we determine the hinge domination number for some standard graphs and shadow distance graphs. We also show that the hinge domination number of the cycle graph provided in [1] is incorrect and provide the exact value.

2. Main Results

We begin this section with the following result which gives the condition for a minimal hinge dset.

Theorem 2.1. *A hinge dset D_h is minimal if and only if for every $v \in D_h$, one of the following condition holds:*

(i) deg(v) = 0 in D_h

(ii) $\exists$ a vertex u in $V - D_h$ such that $N(u) \cap D_h = \{v\}$.

(iii) $< (V - D_h) \cup \{v\} >$ is connected

Proof. For every $u \in D_h$, if $D_h - \{u\}$ is not a hinge dset in G, it follows that either u is an isolated vertex of D_h or there exists a vertex $v \in V - D_h$ such that $N(v) \cap D_h = \{u\}$. Further, for $v \in D_h$, it is clear that the induced graph of $[(V - D_h) \cup \{v\}]$ is connected.

Conversely, if D_h is not minimal, there exist $u \in D_h$ such that $D_h - \{u\}$ is also a hinge dset. Thus, for at least one $v \in D_h - \{u\}$ there is a path between u and v in G. This contradicts condition (i). Also, If $D_h - \{v\}$ is a hinge dset, then every $u \in V - D_h$ is adjacent to at least one vertex in $D_h - \{v\}$, so that condition (ii) also fails. Now, let us consider $v \in D_h$ such that v does not satisfy conditions (i) and (ii). Then from conditions (i) and (ii), $D_{h_1} = D_h - v$ is hinge dset. Also by condition (iii), $< V - D_h >$ is disconnected, so that D_{h_1} is a hinge dset of G. This contradicts condition (iii). Hence the proof. □

Theorem 2.2. *For any graph G, $\gamma_h(G) \geq \frac{n+1}{\Delta(G)+1}$.*

Proof. Let D_h be a minimum hinge dset of G and the number of edges t in G having one $v \in D_h$ and the other in $V - D_h$. Since $\Delta(G) \geq degv \ \forall \ v \in D_h$. For every $v \in D_h$ has at least one unique neighbor in D_h, $t \leq \gamma_h(G).\Delta(G) - 1$. Also $t \geq |V - D_h| = n - \gamma_h(G)$. Hence $n - \gamma_h(G) \leq \Delta(G)\gamma_h(G) - 1$. This gives $\gamma_h(G) \geq \frac{n+1}{\Delta(G)+1}$. □

Theorem 2.3. *For any graph $G = (V,E)$ such that $|V| = p$ and $|E| = q$, $p - q \leq \gamma(G) \leq \gamma_h(G)$.*

Proof. Suppose $q \geq p - 1$, then $1 \geq p - q$ since $\gamma_h(G) \geq 1$, $\gamma_h(G) \geq p - q$. So assume $q \leq p$ then G has atleast $p - q$ components. At least one vertex per component is requried in any hinge dset. Therefore $p - q \leq \gamma(G) \leq \gamma_h(G)$. □

Theorem 2.4. *For any graph G, $\lceil \frac{p}{1 + \triangle(G)} \rceil \leq \gamma(G) \leq \gamma_h(G)$.*

Proof. Let D_h be a hinge dset of G. Each vertex dominates atmost itself and $\triangle(G)$ other vertices. From the proof of the theorem, it follows that $\gamma(G) = \frac{p}{1 + \triangle(G)} = \gamma_h(G)$ if and only if γ_h set D_h such that $N[u] \cap N[v] = \phi$ for all $u, v \in D_h$ and $|N(v)| = \triangle(G)$ for all $v \in D_h$. For example, the cycle C_6 has $\gamma(G) = 2 = \gamma_h(G)$ and $\frac{p}{1 + \triangle(G)} = 2$. □

Theorem 2.5. *Let D_h be a hinge dset of G such that $|D_h| = \gamma_h(G)$. Then $|V(G) - D| \leq deg(v)$.*

Proof. Let D_h be a hinge dset of G, then $|degv - degu| \leq 1 \ \forall \ v \in D_h, u \in V - D_h$ and every vertex $v \in V - D_h$ is adjacent to one vertex in D_h. Hence each vertex in $V - D_h$ contributes at least one to the sum of degrees of the vertex of D_h. Hence $|V(G) - D| \leq deg(v)$ □

The following result is from [1] related to the cycle graph C_n.

Proposition 2.2 For $n \geq 3$, $\gamma_h(C_n)$= $\begin{cases} k & if \quad n = 3k \\ k+1 & if \quad n = 3k+1 \\ k+2 & if \quad n = 3k+2 \end{cases}$.

From this result, it is clear that $\gamma(C_3) = 1$. As a counter example we observe that the graph C_3 illustrated in figure 1 has hinge domination number 3.

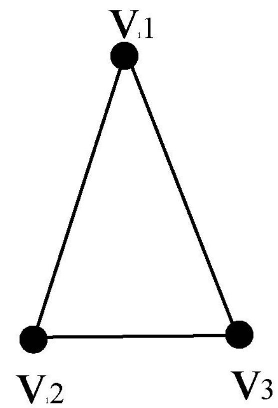

FIGURE 1. $\gamma_h(C_3) = 3$, $D_h = \{v_1, v_2, v_3\}$

We now provide the correct value of $\gamma_h(C_n)$ in our next result.

Theorem 2.6. *If $n \geq 3$, then* $\gamma_h(C_n) = \begin{cases} 2 & n = 4, 6 \\ 3 & n = 3, 5 \\ \frac{n}{3}, & n \equiv 0 (mod 3) \\ \lceil \frac{n}{3} \rceil, & n \equiv 1 (mod 3) \\ \lceil \frac{n}{3} \rceil + 1, & n \equiv 2 (mod 3) \end{cases}$

Proof. Let $V(C_n) = \{v_i | 1 \leq i \leq n\}$ and $E(C_n) = \{e_i | 1 \leq i \leq n\}$ where $e_i = (v_i, v_{i+1})$, $i = 1, 2, ...n$, where computation is under modulo n.

If $n = 4$ and 6, the sets $D_h = \{v_1, v_2\}$ and $D_h = \{v_2, v_5\}$ are minimal so that $\gamma_h(G) = 2$. Also, for $n = 3$ and 5, the set $D_h = \{v_1, v_2, v_3\}$ is minimal so that $\gamma_h(G) = 3$. Let $n \geqslant 7$. Then, for

case(i): $n = 3i + 4$, $i = 1, 2, 3 \ldots$, we consider the set $D_h = \{v_{3s-2}\}$, $1 \leq s \leq \lceil \frac{n}{3} \rceil$.

case(ii): $n = 3j + 5$, $j = 1, 2, 3 \ldots$, we consider the set $D_h = \{v_{3t-2}\} \cup \{v_{n-1}\} \cup \{v_n\}$, $1 \leq t \leq \lceil \frac{n}{3} \rceil - 1$.

and for

case(iii): $n = 3k + 6$, $k = 1, 2, 3 \ldots$, we consider the set $D_h = \{v_{3r-2}\}$, $1 \leq r \leq \frac{n}{3}$.

It is clear that the sets D_h in cases $(i), (ii)$ and (iii) are minimal hinge dsets. Thus, some vertex $v \in D_h$ is adjacent to only one vertex $u \in V - D_h$ and not to any other vertex.

Therefore, since $|D_h|$ = $\begin{cases} 2 & n=4,6 \\ 3 & n=5 \\ \frac{n}{3}, & n\equiv 0(mod3) \\ \lceil\frac{n}{3}\rceil, & n\equiv 1(mod3) \\ \lceil\frac{n}{3}\rceil+1, & n\equiv 2(mod3) \end{cases}$,

we immediately obtain $\gamma_h(C_n)$ = $\begin{cases} 2 & n=4,6 \\ 3 & n=5 \\ \frac{n}{3}, & n\equiv 0(mod3) \\ \lceil\frac{n}{3}\rceil, & n\equiv 1(mod3) \\ \lceil\frac{n}{3}\rceil+1, & n\equiv 2(mod3) \end{cases}$.

Hence the proof.

□

For the path graph P_n, the following result can be found in [1].

Proposition 2.2 $\gamma_h(P_n)$ = $\begin{cases} 2, & n=2 \\ k+2, & n=3k \\ \lceil\frac{n-1}{3}\rceil+1, & n\neq 3k \end{cases}$

In the next theorem, a modified version of this result is provided.

Theorem 2.7. *If $n\geq 3$, then* $\gamma_h(P_n)$ = $\begin{cases} \frac{n}{3}+2, & n\equiv 0(mod3) \\ \lceil\frac{n}{3}\rceil, & n\equiv 1(mod3) \\ \lceil\frac{n}{3}\rceil+1, & n\equiv 2(mod3) \end{cases}$

Proof. Let $V(P_n)$ = $\{v_i/1\leq i\leq n\}$ and $E(P_n)$= $\{e_i/1\leq i\leq n-1\}$ where e_i = (v_i,v_{i+1}), i = $1,2,.....n-1$, where computation is under modulo n.

If $n=3$ and 4, the sets D_h = $\{v_1,v_2,v_3\}$ and D_h = $\{v_1,v_4\}$ are minimal so that $\gamma_h(P_3)$ = 3 and $\gamma_h(P_4)$ = 2 respectively. Let $n\geqslant 5$. Then, for

case(i): n = $3i+2$, i = $1,2,3\ldots$, we consider the set D_h = $\{v_{3s-2}\}\cup\{v_n\}$, $1\leq s\leq\lceil\frac{n}{3}\rceil$.

case(ii): $n = 3j+3$, $j = 1,2,3\ldots$, we consider the set $D_h = \{v_{3t-2}\} \cup \{v_{n-1}\} \cup \{v_n\}$, $1 \leq t \leq \frac{n}{3}$.

and for

case(iii): $n = 3k+4$, $k = 1,2,3\ldots$, we consider the set $D_h = \{v_{3r-2}\}$, $1 \leq r \leq \lceil \frac{n}{3} \rceil$.

It is clear that the sets D_h in cases $(i),(ii)$ and (iii) are minimal hinge dsets. Thus, some vertex $v \in D_h$ is adjacent to only one vertex $u \in V - D_h$ and not to any other vertex.

Therefore, since $|D_h| = \begin{cases} \frac{n}{3}+2, & n \equiv 0(mod3) \\ \lceil \frac{n}{3} \rceil, & n \equiv 1(mod3) \\ \lceil \frac{n}{3} \rceil + 1, & n \equiv 2(mod3) \end{cases}$,

we immediately obtain $\gamma_h(P_n) = \begin{cases} \frac{n}{3}+2, & n \equiv 0(mod3) \\ \lceil \frac{n}{3} \rceil, & n \equiv 1(mod3) \\ \lceil \frac{n}{3} \rceil + 1, & n \equiv 2(mod3) \end{cases}$

Hence the proof. □

We now determine the hinge domination number for some shadow distance graphs.

Theorem 2.8. *If* $n \geq 2$, *then* $\gamma_h(D_2\{P_n\}) = \begin{cases} 2 & n = 2 \\ n-1 & n \geq 3 \end{cases}$

Proof. Let $V(P_n)= \{v_i/1 \leq i \leq n\}$ and $V(P_n^{'}) = \{v_i^{'}/\{v_i/1 \leq i \leq n\}$. Let $E(P_n) = \{e_i/1 \leq i \leq n-1\}$ and $E(P_n^{'})= \{e_i^{'}/1 \leq i \leq n-1\}$, where $e_i = (v_i, v_{i+1})$, $e_i^{'} = (v_i^{'}, v_{i+1}^{'})$ for $i = 1,2,.....n-1$.

Let $G = (D_2\{P_n,\})$.

If $n = 2$, $D_h = \{v_1, v_2^{'}\}$ is minimal so that $\gamma_h(G) = 2$.

Let $n \geq 3$

Consider $D_h = \{v_{2j-1}\} \cup \{v_{2k}^{'}\}$, where $1 \leq j \leq \lfloor \frac{n}{2} \rfloor$, $1 \leq k \leq \frac{n}{2} - 1$, when n is even and $1 \leq k \leq \lfloor \frac{n}{2} \rfloor$ when n is odd

If D_h is not a hinge dset of G, there exists a vertex $v \in D_h$ such that $D_{h_1} = D_h - \{v\}$ is a hinge dset of G and also, $< V - D_h >$ is disconnected. This implies that D_{h_1} is a hinge dset of G, which contradicts condition (iii). Therefore, D_h is minimal and since

$|D_h| = \begin{cases} 2, & n = 2 \\ n-1, & n \geq 3 \end{cases}$, so that $\gamma_h(D_2\{P_n\}) = \begin{cases} 2, & n = 2 \\ n-1, & n \geq 3 \end{cases}$

Hence the proof.

□

Theorem 2.9. *If* $n \geq 3$*, then* $\gamma_h(D_2\{C_n\}) = \begin{cases} \frac{2n}{3}, & n \equiv 0(mod3) \\ \frac{2(n-1)}{3}+2, & n \equiv 1(mod3) \\ \frac{2(n-2)}{3}+4, & n \equiv 2(mod3) \end{cases}$

Proof. Let $V(C_n)$= $\{v_i/1 \leq i \leq n\}$ and $V(C_n^{'}) = \{v_i^{'}/\{v_i/1 \leq i \leq n\}$. Let $E(C_n) = \{e_i/1 \leq i \leq n\}$ and $E(C_n^{'})$= $\{e_i^{'}/1 \leq i \leq n\}$, where $e_i = (v_i, v_{i+1})$ and $e_i^{'} = (v_i^{'}, v_{i+1}^{'})$ for $i = 1, 2, ...n$, where computation is under modulo n.

Let $G = (D_2\{C_n\})$.

Let $n \geq 3$. Then, for

case(i): $n = 3a$, $a = 1, 2, 3\ldots$, we consider the set $D_h = \{v_{3j-2}\} \cup \{v_{3k-2}^{'}\}$, $1 \leq j, k \leq \frac{n}{3}$.

case(ii): $n = 3b+1$, $b = 1, 2, 3\ldots$, we consider the set $D_h = \{v_{3j-2}\} \cup \{v_{3k-2}^{'}\}$, $1 \leq j, k \leq \lceil \frac{n}{3} \rceil$

and for

case(iii): $n = 3c+2$, $c = 1, 2, 3\ldots$, we consider the set $D_h = \{v_{3j-2}\} \cup \{v_{3k-2}^{'}\} \cup \{v_n\} \cup \{v_n^{'}\}$, $1 \leq j, k \leq \lceil \frac{n}{3} \rceil$.

If D_h is not a hinge dset of G, there exists a vertex $v \in D_h$ such that $D_{h_1} = D_h - \{v\}$ is a hinge dset of G and also, $< V - D_h >$ is disconnected. This implies that D_{h_1} is a hinge dset of G, which contradicts condition (iii). Therefore, D_h is minimal and since

$$|D_h| = \begin{cases} \frac{2n}{3}, & n \equiv 0(mod3) \\ \frac{2(n-1)}{3}+2, & n \equiv 1(mod3) \\ \frac{2(n-2)}{3}+4, & n \equiv 2(mod3) \end{cases},$$

so that $\gamma_h(D_2\{C_n\}) = \begin{cases} \frac{2n}{3}, & n \equiv 0(mod3) \\ \frac{2(n-1)}{3}+2, & n \equiv 1(mod3) \\ \frac{2(n-2)}{3}+4, & n \equiv 2(mod3) \end{cases}$

Hence the proof.

□

Theorem 2.10. *If $n \geq 3$, then* $\gamma_h(D_h\{P_n,\{2\}\}) = \begin{cases} 2 & n=3 \\ 3 & n=4 \\ n-1 & n \geq 5 \end{cases}$

Proof. Let $V(P_n)= \{v_i/1 \leq i \leq n\}$ and $V(P_n^{'}) = \{v_i^{'}/\{v_i/1 \leq i \leq n\}$. Let $E(P_n) = \{e_i/1 \leq i \leq n-1\}$ and $E(P_n^{'})= \{e_i^{'}/1 \leq i \leq n-1\}$, where $e_i = (v_i, v_{i+1})$, $e_i^{'} = (v_i^{'}, v_{i+1}^{'})$ for $i = 1,2,.....n-1$.

Let $G = (D_{sd}\{P_n,\{2\}\})$.

If $n = 3,4$, the sets $D_h = \{v_1, v_1^{'}\}$ and $D_h = \{v_1, v_4, v_2^{'}\}$ are minimal so that $\gamma_h(G) = 2$ and 3 respectively

Let $n \geq 5$

Consider $D_h = \{v_{2j-1}\} \cup \{v_{2k+1}^{'}\}$, where $1 \leq j \leq \lfloor \frac{n}{2} \rfloor$, $1 \leq k \leq \frac{n}{2} - 1$ where n is even , $1 \leq k \leq \lfloor \frac{n}{2} \rfloor$ where n is odd.

Let D_h is not hinge dset of G, there exists a vertex $v \in D_h$, then $D_{h_1} = D_h - v$ is dset of G, also $< V - D_h >$ is disconnected. This implies that D_{h_1} is a hinge dset of G, This contradicts condition (iii).

Therefore, D_h is minimum and

$|D_h| = \begin{cases} 2 & n=3 \\ 3 & n=4 \\ n-1 & n \geq 5 \end{cases}$, so that $\gamma_h(D_h\{P_n,\{2\}\}) = \begin{cases} 2 & n=3 \\ 3 & n=4 \\ n-1 & n \geq 5 \end{cases}$

Hence the proof. □

Theorem 2.11. *if $n \geq 4$, then* $\gamma_h(D_h\{C_n,\{2\}\}) = \begin{cases} \frac{2n}{3}, & n \equiv 0(mod3) \\ \frac{2(n-1)}{3} + 2, & n \equiv 1(mod3) \\ \frac{2(n-2)}{3} + 2, & n \equiv 2(mod3) \end{cases}$

Proof. Let $V(C_n)= \{v_i/1 \leq i \leq n\}$ and $V(C_n^{'}) = \{v_i^{'}/\{v_i/1 \leq i \leq n\}$. Let $E(C_n) = \{e_i/1 \leq i \leq n\}$ and $E(C_n^{'})= \{e_i^{'}/1 \leq i \leq n\}$, where $e_i = (v_i, v_{i+1})$ and $e_i^{'} = (v_i^{'}, v_{i+1}^{'})$ for $i = 1,2,.....n$, where computation is under modulo n.

Let $G = (D_{sd}\{C_n,\{2\}\})$.

Let $n \geq 4$. Then for

case(i): $n = 3a+1$, $a = 1,2,3\ldots$, we consider the set $D_h = \{v_{3j-2}\} \cup \{v'_{3k}\} \cup \{v'_n\}$, $1 \le j \le \lceil \frac{n}{3} \rceil$, $1 \le k \le \lfloor \frac{n}{3} \rfloor$

case(ii): $n = 3b+2$, $b = 1,2,3\ldots$, we consider the set $D_h = \{v_{3j-2}\} \cup \{v'_{3k-2}\}$, $1 \le j,k \le \lceil \frac{n}{3} \rceil$ and for

case(iii): $n = 3c+3$, $c = 1,2,3\ldots$, we consider the set $D_h = \{v_{3j-2}\} \cup \{v'_{3k-2}\}$, $1 \le j,k \le \frac{n}{3}$

Let D_h is not hinge dset of G, there exists a vertex $v \in D_h$, then $D_{h_1} = D_h - v$ is dset of G, also $< V - D_h >$ is disconnected. This implies that D_{h_1} is a hinge dset of G, This contradicts condition (iii).

Therefore, D_h is minimal and

$$|D_h| = \begin{cases} \frac{2n}{3}, & n \equiv 0(mod3) \\ \frac{2(n-1)}{3}+2, & n \equiv 1(mod3) \\ \frac{2(n-2)}{3}+2, & n \equiv 2(mod3) \end{cases},$$

so that $$\gamma_h(D_h\{C_n,\{2\}\}) = \begin{cases} \frac{2n}{3}, & n \equiv 0(mod3) \\ \frac{2(n-1)}{3}+2, & n \equiv 1(mod3) \\ \frac{2(n-2)}{3}+2, & n \equiv 2(mod3) \end{cases}$$ Hence the proof. □

Theorem 2.12. *If $n \ge 4$, then* $$\gamma_h(D_h\{P_n,\{3\}\}) = \begin{cases} 4, & n = 4,5 \\ \frac{2n}{3}, & n \equiv 0(mod3) \\ \frac{2(n-1)}{3}+1, & n \equiv 1(mod3) \\ \frac{2(n-2)}{3}+2, & n \equiv 2(mod3) \end{cases}$$

Proof. Let $G = (D_{sd}\{P_n,\{3\}\})$.

If $n = 4,5$, the set $D_h = \{v_1, v_4, v'_1, v'_4\}$ is minimal so that $\gamma_h(G) = 4$.

Let $n \ge 6$. Then for

case(i): $n = 3a+1$, $a = 1,2,3\ldots$, we consider $D_h = \{v_{3j-2}\} \cup \{v'_{3k}\} \cup \{v'_n\}$, $1 \le j \le \lceil \frac{n}{3} \rceil$, $1 \le k \le \lfloor \frac{n}{3} \rfloor$

case(ii): $n = 3b+2$, $b = 1,2,3\ldots$, we consider $D_h = \{v_{3j}\} \cup \{v'_{3k-2}\}$, $1 \le j \le \lfloor \frac{n}{3} \rfloor$, $1 \le k \le \lceil \frac{n}{3} \rceil$

and for

case(iii): $n = 3c+3$, $c = 1,2,3\ldots$, we consider $D_h = \{v_{3j}\} \cup \{v'_{3k-2}\}$, $1 \le j,k \le \frac{n}{3}$

Let D_h is not hinge dset of G, there exists a vertex $v \in D_h$, then $D_{h_1} = D_h - v$ is dset of G, also $< V - D_h >$ is disconnected. This implies that D_{h_1} is a hinge dset of G, This contradicts condition (iii).

Therefore, D_h is minimal and

$$|D_h| = \begin{cases} 4, & n = 4,5 \\ \frac{2n}{3}, & n \equiv 0(mod3) \\ \frac{2(n-1)}{3} + 1, & n \equiv 1(mod3) \\ \frac{2(n-2)}{3} + 2, & n \equiv 2(mod3) \end{cases},$$

so that $\gamma_h(D_h\{P_n, \{3\}\}) = \begin{cases} 4, & n = 4,5 \\ \frac{2n}{3}, & n \equiv 0(mod3) \\ \frac{2(n-1)}{3} + 1, & n \equiv 1(mod3) \\ \frac{2(n-2)}{3} + 2, & n \equiv 2(mod3) \end{cases}$ Hence the proof □

Theorem 2.13. *If $n \geq 6$, then* $\gamma_h(D_h\{C_n, \{3\}\}) = \begin{cases} 4, & n = 6 \\ \frac{2n}{3}, & n \equiv 0(mod3) \\ \frac{2(n-1)}{3} + 2, & n \equiv 1(mod3) \\ \frac{2(n-2)}{3} + 2, & n \equiv 2(mod3) \end{cases}$

Proof. Let $G = (D_{sd}\{C_n, \{3\}\})$.

If $n = 6$, $D_h = \{v_1, v_4, v_1', v_4'\}$ is minimal so that $\gamma_h(G) = 4$.

Let $n \geq 7$. Then for

case(i): $n = 3a + 4$, $a = 1,2,3\ldots$, we consider $D_h = \{v_{3j-2}\} \cup \{v_{3k}'\} \cup \{v_n'\}$, $1 \leq j \leq \lceil \frac{n}{3} \rceil$, $1 \leq k \leq \lfloor \frac{n}{3} \rfloor$

case(ii): $n = 3b + 5$, $b = 1,2,3\ldots$, we consider $D_h = \{v_{3j-2}\} \cup \{v_{3k-2}'\}$, $1 \leq j,k \leq \lceil \frac{n}{3} \rceil$

and for

case(iii): Let $n = 3c + 6$, $c = 1,2,3\ldots$, we consider $D_h = \{v_{3j-2}\} \cup \{v_{3k-2}'\}$, $1 \leq j,k \leq \frac{n}{3}$

Let D_h is not hinge dset of G, there exists a vertex $v \in D_h$, then $D_{h_1} = D_h - v$ is dset of G, also $< V - D_h >$ is disconnected. This implies that D_{h_1} is a hinge dset of G, This contradicts condition (iii).

Therefore, D_h is minimal and

$$|D_h| = \begin{cases} 4, & n = 6 \\ \frac{2n}{3}, & n \equiv 0(mod3) \\ \frac{2(n-1)}{3} + 2, & n \equiv 1(mod3) \\ \frac{2(n-2)}{3} + 2, & n \equiv 2(mod3) \end{cases},$$

so that $$\gamma_h(D_h\{C_n, \{3\}\}) = \begin{cases} 4, & n = 6 \\ \frac{2n}{3}, & n \equiv 0(mod3) \\ \frac{2(n-1)}{3} + 2, & n \equiv 1(mod3) \\ \frac{2(n-2)}{3} + 2, & n \equiv 2(mod3) \end{cases}$$

□

3. CONCLUSION

In this paper, the hinge domination number of some standard graphs and shadow distance graphs related to the path and cycle graphs is determined. The hinge domination number related to the cycle C_n which was provided in [1] is corrected and, a more generalized result for the hinge domination number of the path P_n is provided.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

REFERENCES

[1] B. N. Kavitha, K. Indrani, Hinge domination number of a graph, J. Eng. Res. Appl. 8 (2018), 70-71.

[2] S. T. Hedetniemi, R. C. Laskar, Bibliography on domination in graphs and some basic definitions of domination parameters, Discr. Math. 86 (1990), 257–277.

[3] V. R. Kulli, Theory of domination in graphs, Vishwa International Publications, 2013.

[4] F. Harary, Graph Theory, Addison-Wesley Publications, 1969.

[5] U. Vijaya Chandra Kumar, R. Murali, Edge domination in shadow distance graphs, Int. J. Math. Appl. 4 (2016), 125-130.

[6] U. Vijaya Chandra Kumar, R. Murali, Edge domination in shadow distance graph of some star related graphs, Ann. Pure Appl. Math. Appl. 2 (2017), 33-40.

[7] B. Sooryanarayana, Certain Combinatorial connections between groups, graphs and surfaces, Ph.D. Thesis, (1998).