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CONTROLLING AND SYNCHRONIZATION IN CHAOTIC SYSTEMS USING PARAMETER IDENTIFICATION METHOD

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Abstract. In this manuscript, we investigate projective synchronization (PS) between identical chaotic new Hamiltonian systems based on Hénon-Heiles Model using parameter identification method (PIM). Initially, Lyapunov's theory of stability is used to design the proper adaptive controllers in view of master-slave configuration to achieve the global asymptotic stability. Also, the proposed technique establishes identification of parameter simultaneously via PS scheme. Additionally, numerical simulations are performed using MATLAB software for visualizing the efficiency and feasibility of the proposed methodology. Furthermore, the discussed approach has numerous significant applications in encryption and secure communication.

Keywords: Chaotic system; Hamiltonian system; Lyapunov stability theory; parameter identification method; projective synchronization; MATLAB

2010 AMS Subject Classification: 34K23, 34K35, 37B25, 37N35.

1. Introduction

Chaos theory is undoubtedly an important and intriguing field of applied mathematics that deals with highly complex nonlinear dynamical systems. This theory has played a vital role in several disciplines, including physics [1], biomedical engineering [2], chemistry [3], ecology

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[4], economics [5], image encryption [6], secure communication [7] and so on. As a result, chaos synchronization and chaos control have now become the most promising fields for researchers and scientists.

Chaos phenomenon has been a fundamental topic of applied mathematics ever since its introduction in 1963 due to the pioneering work of E.N. Lorenz [8]. He observed the chaotic system for the first time while examining weather prediction model and also proposed the term 'butterfly effect' which is the sensitive dependency to the initial conditions. Afterwards, Pecora and carroll [9] firstly in 1990 pronounced the chaos synchronization among chaotic systems using master-slave configuration. Furthermore, Ott et al. [10] in 1990 introduced a technique known as OGY method to control chaotic systems.

Synchronization is a procedure to adjust two or more (identical or non-identical) chaotic systems in a manner that both (all) exhibit the similar behavior owing to pairing to gain stability. Till now, several synchronization schemes for chaotic systems are introduced such as complete synchronization[11], hybrid synchronization [12], anti-synchronization [13, 14], function projective synchronization [15], hybrid projective synchronization [16], phase synchronization [17], projective synchronization [18], modified projective synchronization [19], combination synchronization [20], compound synchronization [21] etc. Up to now, numerous techniques like active control [22, 23, 24], adaptive control [16, 25, 20], backstepping design [26], feedback control [27], sliding mode control [28, 21], impulsive control [29] etc have been initiated to achieve chaos control in chaotic systems.

Synchronization among chaotic systems using parameter identification method (PIM) or adaptive control method was introduced in 1989 by Hubler [30]. In [16, 25, 31, 32, 33, 34, 35, 36, 37, 38], many control techniques are studied for controlling and synchronizing the chaotic/ hyperchaotic systems. Vaidyanathan et al. [39] defined and studied a new chaotic Hamiltonian system based on Hénon-Heiles system, describing the nonlinear complex motion of a star around a galactic centre with the motion restricted to a plane, which was modeled by Hénon and Heiles [40] in 1964.

Considering the above literature review and discussion, the main objective of this manuscript is to carry out an investigation of projective synchronization (PS) scheme in a new Hamiltonian chaotic systems using PIM. The underlying idea here is to use Lyapunov's theory of stability to design proper adaptive controllers to achieve global asymptotic stability.

The remainder of this manuscript is organized as: Sect. 2 comprising of few mathematical preliminaries having essential notations and terminology to be utilised within the manuscript. In Sect. 3, basic structured properties of Hamiltonian system to be synchronized, are mentioned. Sect. 4 investigates the active control method to stabilize the given chaotic system asymptotically. Sect. 5 deals with the numerical simulations illustrating the effectiveness and feasibility of the discussed complete synchronization approach using active control method. Finally, Sect. 6 concludes the paper.

2. Mathematical Preliminaries

Considering the master system and corresponding slave system as:

$$\dot{v}_m = \phi_1(v_m), \tag{1}$$

$$\dot{v}_s = \phi_2(v_s) + \eta, \tag{2}$$

where $v_m = (v_{m1}, v_{m2}, \ldots, v_{mn})^T$, $v_s = (v_{s1}, v_{s2}, \ldots, v_{sn})^T$ are state variables of (1) and (2) respectively, $\phi_1, \phi_2 : R^n \to R^n$ are nonlinear continuous vector functions and $\eta = (\eta_1, \eta_2, \ldots\ldots, \eta_n) \in R^n$ is the properly constructed controller.

We define the projective synchronization (PS) error as:

$$\lim_{t\to\infty} \|E(t)\| = \lim_{t\to\infty} \|v_s(t) - Av_m(t)\| = 0, \tag{3}$$

for some matrix $A = \mathrm{diag}(\xi, \xi, \xi, \ldots, \xi)$ and $\|\cdot\|$ represents vector norm.

Remark 1. *For $\xi = 1$, complete synchronization is achieved.*

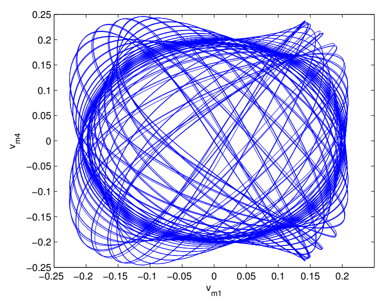
Remark 2. *For $\xi = -1$, anti-synchronization is attained.*

3. MODEL ANALYSIS

Reported by Sundarapandian Vaidyanathan et al. [39], the discussed chaotic system can be written as:

$$
\begin{cases}
\dot{v}_{m1} = v_{m2} \\
\dot{v}_{m2} = -v_{m1} - 2v_{m1}v_{m3} + pv_{m1}^2 \\
\dot{v}_{m3} = v_{m4} \\
\dot{v}_{m4} = -v_{m3} - v_{m1}^2 + v_{m3}^2 + qv_{m3}^4,
\end{cases} \tag{4}
$$

where $(v_{m1}, v_{m2}, v_{m3}, v_{m4})^T \in R^4$ is the state vector and p and q are parameters. When $p = -1.95$ and $q = 1.48$, the system (4) displays chaos. Also, the Lyapunov exponents of system (4) are $LE_1 = 0.0015$, $LE_2 = 0$, $LE_3 = 0$, $LE_1 = -0.0015$. Further, Fig. 1(a-d) display the phase graphs of (4). Moreover, the detailed analytic study and simulation results for system (4) may be found in [39].



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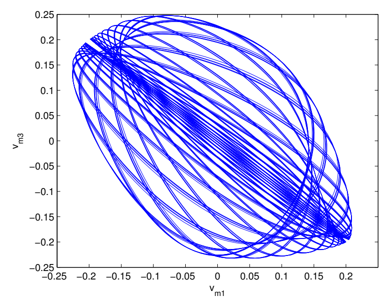

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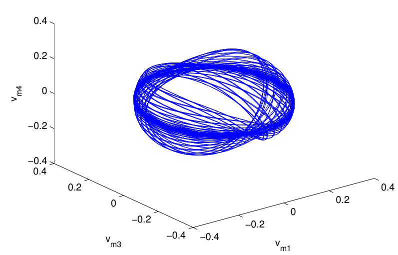

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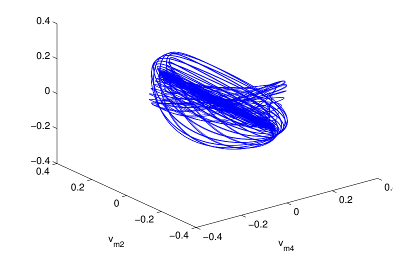


(D)

FIGURE 1. Phase graphs of Hamiltonian chaotic system in (a) $v_{m1} - v_{m4}$ plane, (b) $v_{m1} - v_{m3}$ plane, (c) $v_{m1} - v_{m3} - v_{m4}$ space, (d) $v_{m4} - v_{m2} - v_{m1}$ space

4. Stability Analysis

In this section, we study PS scheme for the new chaotic Hamiltonian system to design proper adaptive controllers using PIM in such a way that each state variable v_{m1}, v_{m2}, v_{m3} and y_{m4} approaching to equilibrium points as t tending to infinity.

The system (4) is chosen as master system and corresponding slave system is defined as:

$$\begin{cases} \dot{v}_{s1} = v_{s2} + \eta_1 \\ \dot{v}_{s2} = -v_{s1} - 2v_{s1}v_{s3} + pv_{s1}^2 + \eta_2 \\ \dot{v}_{s3} = v_{s4} + \eta_3 \\ \dot{v}_{s4} = -v_{s3} - v_{s1}^2 + v_{s3}^2 + qv_{s3}^4 + \eta_4, \end{cases} \tag{5}$$

where η_1, η_2, η_3 and η_4 are adaptive nonlinear controllers to be constructed so that PS scheme between two identical Hamiltonian chaotic systems will be attained.

State errors are defined as

$$\begin{cases} E_1 = v_{s1} - \xi v_{m1} \\ E_2 = v_{s2} - \xi v_{m2} \\ E_3 = v_{s3} - \xi v_{m3} \\ E_4 = v_{s4} - \xi v_{m4} \end{cases} \tag{6}$$

The main goal in this manuscript is to design controllers v_i, $(i = 1,2,3,4)$ so that state errors mentioned in eq. (6) satisfy

$$\lim_{t\to\infty} E_i(t) = 0, \quad \text{for } (i = 1,2,3,4).$$

The resulting error dynamics would be written as:

$$\begin{cases} \dot{E}_1 = E_2 + \eta_1 \\ \dot{E}_2 = -E_1 - 2(v_{s1}v_{s3} - \xi v_{m1}v_{m3}) + p(v_{s1}^2 - \xi v_{m1}^2) + \eta_2 \\ \dot{E}_3 = E_4 + \eta_3 \\ \dot{E}_4 = -E_3 - (v_{s1}^2 - \xi v_{m1}^2) + (v_{s3}^2 - \xi v_{m3}^2) + q(v_{s3}^4 - \xi v_{m3}^4) + \eta_4. \end{cases} \tag{7}$$

Next, we describe the adaptive controllers by

$$\begin{cases} \eta_1 = -E_2 - M_1 E_1 \\ \eta_2 = E_1 + 2(v_{s1}v_{s3} - \xi v_{m1}v_{m3}) - \hat{p}(v_{s1}^2 - \xi v_{m1}^2) - M_2 E_2 \\ \eta_3 = -E_4 - M_3 E_3 \\ \eta_4 = E_3 + (v_{s1}^2 - \xi v_{m1}^2) - (v_{s3}^2 - \xi v_{m3}^2) - \hat{q}(v_{s3}^4 - \xi v_{m3}^4) - M_4 E_4, \end{cases} \tag{8}$$

where $M_1 > 0$, $M_2 > 0$, $M_3 > 0$, $M_4 > 0$ are gain constants.

By substituting the controllers as defined in eq. (8) in error dynamics eq. (7), one finds that

$$\begin{cases} \dot{E}_1 = -M_1 E_1 \\ \dot{E}_2 = (p - \hat{p})(v_{s1}^2 - \xi v_{m1}^2) - M_2 E_2 \\ \dot{E}_3 = -M_3 E_3 \\ \dot{E}_4 = (q - \hat{q})(v_{s3}^4 - \xi v_{m3}^4) - M_4 E_4 \end{cases} \tag{9}$$

where $\hat{p}$, $\hat{q}$ are estimated quantities of unknown parameter p, q respectively.

Defining the parameter estimation error as:

$$\tilde{p} = p - \hat{p}, \;\; \tilde{q} = q - \hat{q}. \tag{10}$$

Using eq. (10), the error dynamics eq. (9) is written as:

$$\begin{cases} \dot{E}_1 = -M_1 E_1 \\ \dot{E}_2 = \tilde{p}(v_{s1}^2 - \xi v_{m1}^2) - M_2 E_2 \\ \dot{E}_3 = -M_3 E_3 \\ \dot{E}_4 = \tilde{q}(v_{s3}^4 - \xi v_{m3}^4) - M_4 E_4. \end{cases} \tag{11}$$

On differentiating eq. (12), one finds that

$$\dot{\tilde{p}} = -\dot{\hat{p}}, \;\; \dot{\tilde{q}} = -\dot{\hat{q}}. \tag{12}$$

Lyapunov function is defined as:

$$V = \frac{1}{2}[E_1^2 + E_2^2 + E_3^2 + E_4^2 + \tilde{p}^2 + \tilde{q}^2], \tag{13}$$

which implying that V is positive definite.

Derivative of V may be written as:

$$\dot{V} = E_1\dot{E}_1 + E_2\dot{E}_2 + E_3\dot{E}_3 + E_4\dot{E}_4 - \tilde{p}\dot{\hat{p}} - \tilde{q}\dot{\hat{q}}. \tag{14}$$

Keeping eq. (14) in view, we formulate the parameter estimates laws as:

$$\begin{cases} \dot{\hat{p}} = (v_{s1}^2 - \xi v_{m1}^2)E_2 + M_5\tilde{p} \\ \dot{\hat{q}} = (v_{s3}^4 - \xi v_{m3}^4)E_4 + M_6\tilde{q}, \end{cases} \tag{15}$$

where $M_5 > 0$ and $M_6 > 0$ are gain constants.

Theorem 1. *The chaotic systems eqs.* (4)-(5) *are asymptotically projective synchronized for all initial states* $(v_{m1}(0), v_{m2}(0), v_{m3}(0), v_{m4}(0)) \in R^4$ *by the designed adaptive controller eq.* (8) *and the parameter update law eq.* (15).

Proof. The Lyapunov functional V as defined in eq. (13) is a positive definite function. By solving eq. (11), eq. (14) and eq. (15), we have

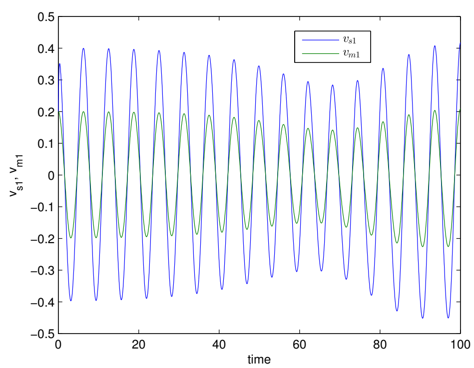
$$\begin{aligned} \dot{V} &= -M_1E_1^2 - M_2E_2^2 - M_3E_3^2 - M_4E_4^2 - M_5\tilde{p}^2 - M_6\tilde{q}^2 \\ &< 0, \end{aligned}$$

confirming that $\dot{V}$ is negative definite.

Thus, by Lyapunov's theory of stability [41], we conclude that projective synchronized error $e(t) \to 0$ exponentially as $t \to \infty$ for each initial conditions $e(0) \in R^4$. □

5. Numerical Simulation and Discussion

This section presents some simulation experiments to illustrate the efficiency of the proposed PS scheme via PIM. Here, we use the fourth order Runge-Kutta methodology to solve system of differential equations. The initial states of master(4) and slave systems (5) are $(0.2, 0, -0.2, 0)$ and $(0.2, 0.2, 0.2, 0)$ respectively. For scaling matrix A is selected as $\xi = 2$, we have achieved complete PS in master(4) and slave (5) systems. The control gains are selected as $K_i = 6$ for $i = 1, 2, 3, 4$. Simulation results are displayed in Fig. 2(a-d) which depict state trajectories of master(4) and slave systems(5) and Fig. 3(a) displays that synchronization error $(E_1, E_2, E_3, E_4) = (-0.2, 0.2, 0.6, 0)$ tends to zero as t tends to infinity. Further, Fig. 3(b) displays the estimated values $(\hat{p}, \hat{q})$ asymptotically with time. Therefore, the investigated PS scheme among master and slave systems has been attained computationally. In addition, if scaling matrix A is selected as $\xi = -5$, we have achieved anti-PS scheme in master(4) and slave(5) systems.The Fig. 4(a-e) shows that anti-PS scheme in systems(4) and (5) is achieved numerically. Also, Fig. 5(a) shows that synchronization error $(E_1, E_2, E_3, E_4) = (1.2, 0.2, -0.8, 0)$ tends to zero as t tends to infinity. Moreover, Fig. 5(b) shows the estimated values $(\hat{p}, \hat{q})$ asymptotically with time. Thus, the investigated PS technique in the sense of both complete and anti between chaotic master and slave systems has been obtained computationally.



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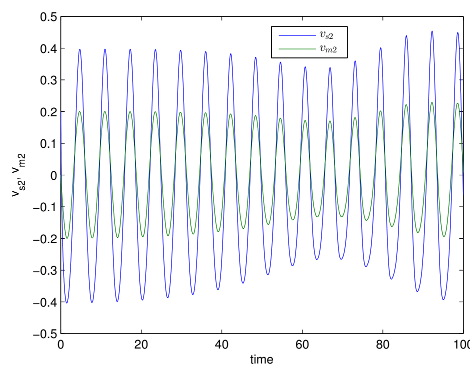

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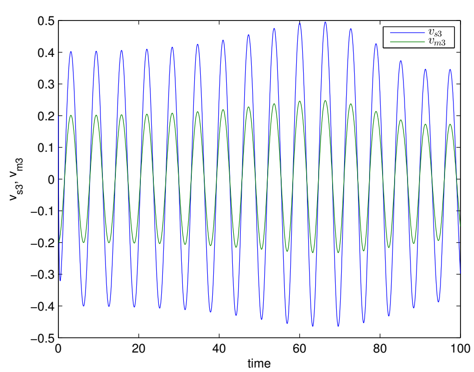

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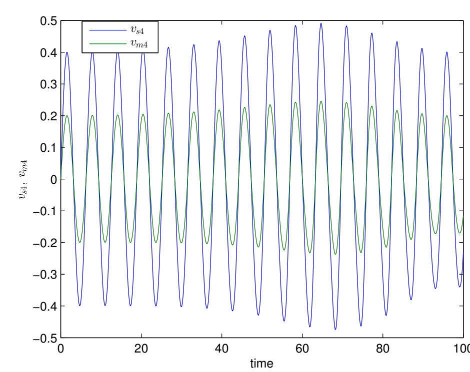
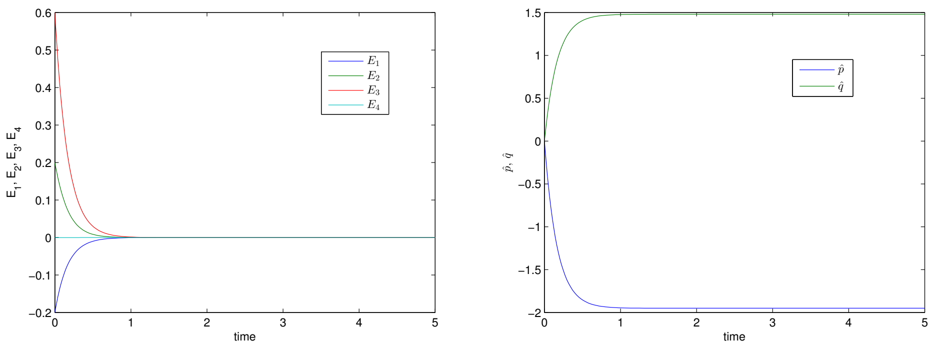


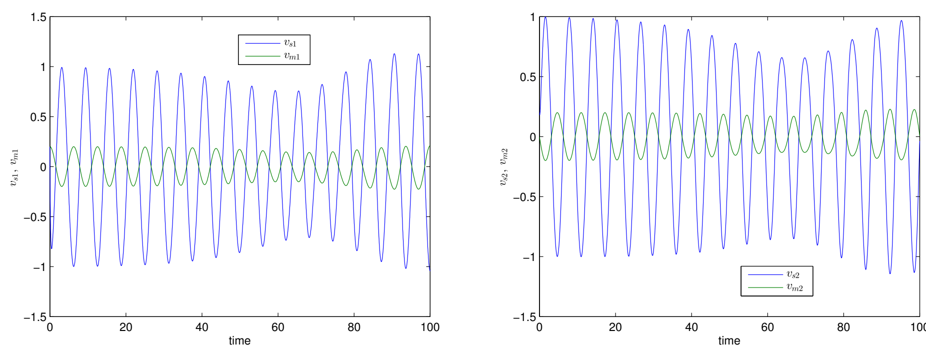
(D)

FIGURE 2. Projective complete synchronization between Hamiltonian chaotic systems (a) between $v_{m1}(t) - v_{s1}(t)$, (b) between $v_{m2}(t) - v_{s2}(t)$, (c) between $v_{m3}(t) - v_{s3}(t)$, (d) between $v_{m4}(t) - v_{s4}(t)$

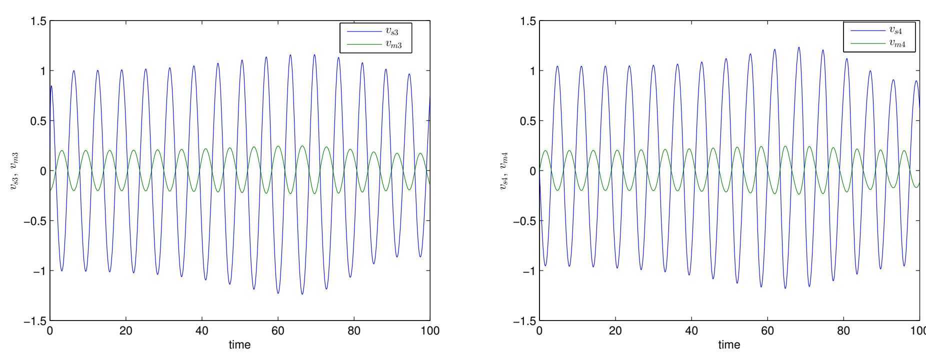


(A) (B)

FIGURE 3. Dynamics in (a) synchronization error states $(t, E_1(t), E_2(t), E_3(t), E_4(t))$, (b) Parameter identification

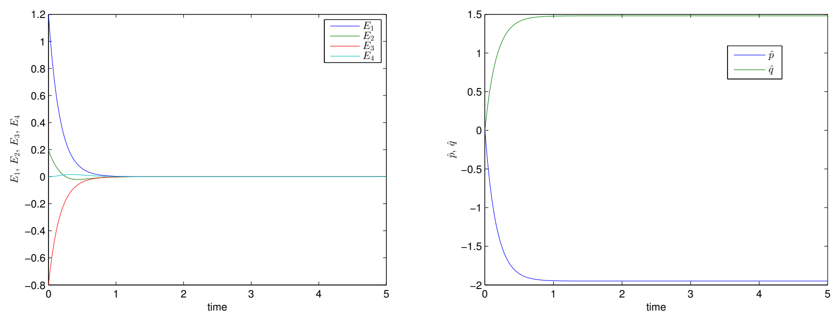


(A) (B)



(C) (D)

FIGURE 4. Projective anti-synchronization of Hamiltonian chaotic systems (a) between $v_{m1}(t) - v_{s1}(t)$, (b) between $v_{m2}(t) - v_{s2}(t)$, (c) between $v_{m3}(t) - v_{s3}(t)$, (d) between $v_{m4}(t) - v_{s4}(t)$



(A) (B)

FIGURE 5. Dynamics in (a) synchronization error states $(t, E_1(t), E_2(t), E_3(t), E_4(t))$, (b) Parameter identification

6. CONCLUSION

In this manuscript, the proposed projective synchronization scheme in identical chaotic new Hamiltonian systems via parameter identification method has been investigated. By designing proper controllers based on master-slave configuration and Lyapunov's theory of stability, the considered projective synchronization technique is achieved. The efficacy and superiority of theoretic results are justified in numerical simulations by using MATLAB. Such techniques would be utilised to control the nonlinear motion of a star around a galactic centre with motion restricted to a plane. Moreover, the considered scheme is very effective as it has numerous applications in encryption and secure communication. Furthermore, we understand that our proposed projective synchronization scheme in identical chaotic new Hamiltonian systems may be generalized by utilising other control and synchronization techniques.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

REFERENCES

[1] D. Ghosh, A. Mukherjee, N. Das, B. Biswas, Generation & control of chaos in a single loop optoelectronic oscillator, Optik 165 (2018) 275–287.

[2] A. Provata, P. Katsaloulis, D. Verganelakis, Dynamics of chaotic maps for modelling the multifractal spectrum of human brain diffusion tensor images, Chaos Solitons Fractals 45 (2) (2012) 174–180.

[3] S. Han, C. Kurrer, Y. Kuramoto, Dephasing and bursting in coupled neural oscillators, Phys. Rev. Lett. 75 (17) (1995) 3190.

[4] B. Sahoo, S. Poria, The chaos and control of a food chain model supplying additional food to top-predator, Chaos Solitons Fractals 58 (2014) 52–64.

[5] X.-J. Tong, M. Zhang, Z. Wang, Y. Liu, J. Ma, An image encryption scheme based on a new hyperchaotic finance system, Optik 126 (20) (2015) 2445–2452.

[6] G. Wu, D. Baleanu, Z.-X. Lin, Image encryption technique based on fractional chaotic time series, J. Vibrat. Control 22 (8) (2016) 2092–2099.

[7] B. Naderi, H. Kheiri, Exponential synchronization of chaotic system and application in secure communication, Optik 127 (5) (2016) 2407–2412.

[8] E. Lorenz, Deterministic nonperiodic flow, J. Atmosph. Sci. 20 (2) (1963), 130–141.

[9] L. Pecora, T. Carroll, Synchronization in chaotic systems, Phys. Rev. Lett. 64 (8) (1990) 821.

[10] T. Shinbrot, E. Ott, C. Grebogi, J. Yorke, Using chaos to direct trajectories to targets, Phys. Rev. Lett. 65 (26) (1990) 3215.

[11] A. Singh, V. Yadav, S. Das, Synchronization between fractional order complex chaotic systems, Int. J. Dyn. Control 5 (3) (2017) 756–770.

[12] K. Sudheer, M. Sabir, Hybrid synchronization of hyperchaotic lu system, Pramana 73 (4) (2009) 781.

[13] G.-H. Li, S.-P. Zhou, Anti-synchronization in different chaotic systems, Chaos Solitons Fractals 32 (2) (2007) 516–520.

[14] T. Khan, H. Chaudhary, Estimation and identifiability of parameters for generalized lotka-volterra biological systems using adaptive controlled combination difference anti-synchronization, Differ. Equ. Dyn. Syst. 28 (2020) 515–526.

[15] P. Zhou, W. Zhu, Function projective synchronization for fractional-order chaotic systems, Nonlinear Anal.: Real World Appl. 12 (2) (2011) 811–816.

[16] A. Khan, H. Chaudhary, Adaptive control and hybrid projective combination synchronization of chaos generated by generalized lotka-volterra biological systems, Bloomsbury India (2019) 174.

[17] J. Ma, L. Mi, P. Zhou, Y. Xu, T. Hayat, Phase synchronization between two neurons induced by coupling of electromagnetic field, Appl. Math. Comput. 307 (2017) 321–328.

[18] Z. Ding, Y. Shen, Projective synchronization of nonidentical fractional-order neural networks based on sliding mode controller, Neural Netw. 76 (2016) 97–105.

[19] G.-H. Li, Modified projective synchronization of chaotic system, Chaos Solitons Fractals 32 (5) (2007) 1786–1790.

[20] T. Khan, H. Chaudhary, Controlling chaos generated in predator-prey interactions using adaptive hybrid combination synchronization, in: Proceedings of 3rd International Conference on Computing Informatics and Networks: ICCIN 2020, Springer Singapore, 2021, pp. 449–459.

[21] L. Jahanzaib, P. Trikha, H. Chaudhary, S. H. SM, et al., Compound synchronization using disturbance observer based adaptive sliding mode control technique, J. Math. Comput. Sci. 10 (5) (2020) 1463–1480.

[22] H. Hadi, M. Mohadeszadeh, Hybrid complex projective synchronization of complex chaotic systems using active control technique with nonlinearity in the control input, J. Control Eng. Appl. Inform. 20 (1) (2018) 67–74.

[23] T. Khan, H. Chaudhary, An investigation on hybrid projective combination difference synchronization scheme between chaotic prey-predator systems via active control method, Poincare J. Anal. Appl. 7 (2) (2020) 211–225.

[24] T. Khan, H. Chaudhary, Controlling and synchronizing combined effect of chaos generated in generalized lotka-volterra three species biological model using active control design, Appl. Appl. Math. 15 (2) (2020), 1135-1148.

[25] A. Khan, H. Chaudhary, Hybrid projective combination–combination synchronization in non-identical hyperchaotic systems using adaptive control, Arab. J. Math. 9 (3) (2020), 597-611.

[26] S. Rasappan, S. Vaidyanathan, Synchronization of hyperchaotic liu system via backstepping control with recursive feedback, in: International Conference on Eco-friendly Computing and Communication Systems, Springer, 2012, pp. 212–221.

[27] M. Chen, Z. Z. Han, Controlling and synchronizing chaotic genesio system via nonlinear feedback control, Chaos Solitons Fractals 17 (4) (2003) 709–716.

[28] S. Vaidyanathan, S. Sampath, Anti-synchronization of four-wing chaotic systems via sliding mode control, Int. J. Autom. Comput. 9 (3) (2012) 274–279.

[29] D. Li, X. Zhang, Impulsive synchronization of fractional order chaotic systems with time-delay, Neurocomputing 216 (2016) 39–44.

[30] A. Hubler, Adaptive control of chaotic system, Helv. Phys. Acta 62 (1989) 343–346.

[31] T.-L. Liao, S.-H. Tsai, Adaptive synchronization of chaotic systems and its application to secure communications, Chaos Solitons Fractals 11 (9) (2000) 1387–1396.

[32] M. Yassen, Adaptive control and synchronization of a modified chua's circuit system, Appl. Math. Comput. 135 (1) (2003) 113–128.

[33] Z. Li, D. Xu, A secure communication scheme using projective chaos synchronization, Chaos Solitons Fractals 22 (2) (2004) 477–481.

[34] S.-Y. Li, C.-H. Yang, C.-T. Lin, L.-W. Ko, T.-T. Chiu, Adaptive synchronization of chaotic systems with unknown parameters via new backstepping strategy, Nonlinear Dyn. 70 (3) (2012) 2129–2143.

[35] Z. Wu, J. Duan, X. Fu, Complex projective synchronization in coupled chaotic complex dynamical systems, Nonlinear Dyn. 69 (3) (2012) 771–779.

[36] S. Vaidyanathan, Adaptive biological control of generalized lotka-volterra three-species biological system, Int. J. PharmTech Res. 8 (4) (2015) 622–631.

[37] A. Khan, A. Tyagi, Hybrid projective synchronization between two identical new 4-d hyper-chaotic systems via active control method, Int. J. Nonlinear. Sci. 23 (3) (2017) 142–150.

[38] S. Kumar, A. Matouk, H. Chaudhary, S. Kant, Control and synchronization of fractional-order chaotic satellite systems using feedback and adaptive control techniques, Int. J. Adapt. Control Signal Proc. 2020 (2020), https://doi.org/10.1002/acs.3207.

[39] S. Vaidyanathan, A. Sambas, S. Zhang, M. Mohamed, M. Mamat, A new hamiltonian chaotic system with coexisting chaotic orbits and its dynamical analysis, International J. Eng. Technol. 7 (4) (2018) 2430–2436.

[40] M. Hénon, C. C. Heiles, The applicability of the third integral of motion: some numerical experiments, Astronomic. J. 69 (1964) 73.

[41] L. Perko, Differential equations and dynamical systems, Vol. 7, Springer Science & Business Media, 2013.