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ON CO-PRIME ORDER GRAPHS OF FINITE ABELIAN p-GROUPS

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Abstract. For a finite group G, the co-prime order graph $\Theta(G)$ of G is defined as the graph with vertex set G, the group itself, and two distinct vertices u, v in $\Theta(G)$ are adjacent if and only if $gcd(o(u), o(v)) = 1$ or a prime number. In this paper, some properties and some topological indices such as Wiener, Hyper-Wiener, first and second Zagreb, Schultz, Gutman and eccentric connectivity indices of the co-prime order graph of finite abelian p-group are studied. We also figure out the metric dimension and resolving polynomial of the co-prime order graph of finite abelian p-group.

Keywords: resolving polynomial of a graph; co-prime order graph; finite abelian p-group; Wiener index; Zagreb indices; Schultz index.

2010 AMS Subject Classification: 05C25, 05C50.

1. Introduction

An obvious phenomenon is to generate graphs from groups. The notion of ”co-prime order graph of a finite group” has been introduced by Subarsha Banerjee in 2019 [8]. They defined it as a simple graph with vertex set as the elements of a group, and there is adjacency between two

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vertices u and v if and only if $gcd(o(u),o(v)) = 1$ or a prime number. For more studies about co-prime order graphs, we refer the reader to see [1, 10].

Suppose that Γ is a simple graph,which is undirected and contains no multiple edges or loops. Here, the set of vertices of Γ is denoted by $V(\Gamma)$ and the corresponding set of edges is denoted by $E(\Gamma)$. We write $uv \in E(\Gamma)$ if u and v form an edge in Γ. The *size* of the vertex-set is denoted by $|V(\Gamma)|$ for the set Γ and the number of its edges is denoted by $|E(\Gamma)|$. The *degree* of a vertex is defined as number of vertices adjacent to u and is represented as $deg(u)$, The *distance* between any pair of vertices u and v denoted by $d(u,v)$, is the shortest $u-v$ path in graph Γ and the *eccentricity* of any vertex u is given as $ecc(u)$ and it is the largest distance between u and any other vertex in Γ. The diameter of the graph Γ, denoted by $diam(\Gamma)$, is given by $diam(\Gamma) = max\{ecc(u) : u \in V(\Gamma)\}$. A graph Γ is called *complete* if every pair of vertices of Γ are adjacent. If $D \subseteq V(\Gamma)$ and no vertices of D are adjacent, then D is called an *independent set*. The cardinality of the largest independent set is called an *independent number* of the graph Γ. A graph Γ is called *bipartite* one if $V(\Gamma)$ can be partitioned in such a way into two disjoint independent sets that each edge in Γ has its ends in different independent sets. A graph Γ is called *split* if its vertex set can be splitted up into two different sets U and K such that U is an independent set and the induced subgraph by K is a complete graph.

Let $W = \{v_1, v_2, v_3, ..., v_k\} \subseteq V(\Gamma)$ and let v be any vertex of Γ. The representation of v with respect to W is the k-vector $r(v|W) = (d(v,v_1), d(v,v_2), ..., d(v,v_k))$. If different vertices have different representations with respect to W, then W is called a *resolving set* of Γ. A *basis* of Γ is a minimum resolving set for Γ and the cardinality of a basis of Γ is called *metric dimension* of Γ and it is denoted by $\beta(\Gamma)$ [3]. Suppose r_i is the number of resolving sets for $\Gamma$of cardinality i. Then the *resolving polynomial*of a graph Γ of order n, denoted by $\beta(\Gamma,x)$, is defined as $\beta(\Gamma,x) = \sum_{i=\beta(\Gamma)}^{n} r_i x^i$. The sequence $(r_{\beta(\Gamma)}, r_{\beta(\Gamma)+1}, ..., r_n)$ formed from the coefficients of $\beta(\Gamma,x)$ is called the *resolving sequence*.

For a graph Γ, the *Wiener index* and $Hyper-Wiener$ *index* are defined by $W(\Gamma) = \sum_{\{u,v\}\subset V(\Gamma)} d(u,v)$ [5] and $WW(\Gamma) = \frac{1}{2}W(\Gamma) + \frac{1}{2}\sum_{\{u,v\}\subset V(\Gamma)} d(u,v)^2$ [2]. The *Zagreb indices* mainly first and second are defined by $M_1(\Gamma) = \sum_{v\in V(\Gamma)}(deg(v))^2$ and $M_2(\Gamma) = \sum_{uv\in E(\Gamma)}[deg(u) \times deg(v)]$ [6]. The *Schultz index* of Γ is defined by $MTI(\Gamma) =$

$\sum_{\{u,v\}\subset V(\Gamma)} d(u,v)[deg(u)+deg(v)]$ [4]. The *Gutman index* of Γ is defined by $Gut(\Gamma) = \sum_{\{u,v\}\subset V(\Gamma)} d(u,v)[deg(u)\times deg(v)]$ [7]. The *eccentric connectivity index* of Γ is defined by $\xi^c(\Gamma) = \sum_{v\in V(\Gamma)} deg(v)ecc(v)$ [9].

In [10], Xuanlong Ma and Zhonghua Wang studied $\Theta(G)$ of all finite groups which are complete and classify all finite groups which are planar. In this paper, the focus will be on the co-prime order graph of a finite abelian p-group which is defined as $G_p = \{\prod_{i=1}^{i=r} x_i | o(x_i) = p^{\alpha_i}, x_i x_j = x_j x_i \text{ where } i,j = 1,2,\ldots,r\} \cong Z_{p^{\alpha_1}} \times Z_{p^{\alpha_2}} \times \ldots \times Z_{p^{\alpha_r}}$ where p is a prime number. When $\alpha_i = 0$ for all i, then co-prime order graph is null graph, which is not of our interest.

Throughout sections 2,3 and 4, we use notations and assumptions given below

p is a prime number, $r \geq 1$, $\alpha_i \geq 1$ for all $i = 1,2,\ldots,r$, $G_p = Z_{p^{\alpha_1}} \times Z_{p^{\alpha_2}} \times \ldots \times Z_{p^{\alpha_r}}$, $\Omega_1 = \{x \in G_p | o(x) = 1 \text{ or } p\}$ and $\Omega_2 = G - \Omega_1$.

This paper is organized as follows. In Section 2, some basic properties of the graph $\Theta(G_p)$ are investigated. We see that the graph $\Theta(G_p)$ is split. In Section 3, we find some topological indices of the graph $\Theta(G_p)$ such as the Wiener, Hyper-Wiener and Zagreb indices. In Section 4, we find the metric dimension and the resolving polynomial of the graph $\Theta(G_p)$.

2. Some Properties of the Graph $\Theta(G_p)$

Lemma 1. *Let G_p be a finite abelian p-group. Then $|\Omega_1| = p^r$ and $|\Omega_2| = p^{\sum_{i=1}^{i=r}\alpha_i} - p^r$.*

Proof. Firstly, we count elements of order 1 and p in the group G_p.
Let x be arbitrary element of order 1 or p. Then $x = x_1 x_2 \cdots x_r$ such that $x_i \in Z_{p^{\alpha_i}}$ and $o(x_i) = 1$ or p where $i = 1,2,\ldots,r$.
Possibilities for each x_i are p, hence possibilities for x are p^r. Remaining $p^{\sum_{i=1}^{i=r}\alpha_i} - p^r$ has neither order 1 nor p. Hence, we get desired result. □

Lemma 2. *A sub-graph induced by Ω_1 from the graph $\Theta(G_p)$ is K_{p^r} and a sub-graph induced by Ω_2 from the graph $\Theta(G_p)$ has no edges. Furthermore, each vertex of the set Ω_1 must be adjacent with every vertex in Ω_2.*

Proof. Using the definition of co-prime order graph, every pair of vertices in Ω_1 must be adjacent because of the order of every vertex is either 1 or p. Also, each vertex of the set Ω_1 must be adjacent with every vertex in Ω_2. If $\Omega_2 \neq \phi$, then no pair of vertices in Ω_2 are adjacent because of the order of each vertex is a power of p other than 1 or p. □

Using above lemmas, we can state the following results

Theorem 1. *Let $\Theta(G_p)$ be the co-prime order graph on G. Then $\Theta(G_p) = K_{p^r} \vee (p^{\sum_{i=1}^{i=r} \alpha_i} - p^r)K_1$.*

Theorem 2. *In the graph $\Theta(G_p)$, $deg(u) = \begin{cases} |\Omega_1|+|\Omega_2|-1, \ if \ u \in \Omega_1 \\ |\Omega_1|, \ if \ u \in \Omega_2 \end{cases}$*

Corollary 1. *In the graph $\Gamma = \Theta(G_p)$, $|E(\Gamma)|= \binom{|\Omega_1|}{2} + |\Omega_1||\Omega_2|$*

Proof. Every pair of vertices $u, v \in \Omega_1$ are adjacent, and the total number of edges in Ω_1 is $\binom{|\Omega_1|}{2}$. Also, there is an edge between any vertex $u \in \Omega_1$ and every vertex $v \in \Omega_2$, and the total number of such edges is $|\Omega_1||\Omega_2|$. No pairs of vertices $u, v \in \Omega_2$ are adjacent. In total, we get $|E(\Gamma)|= \binom{|\Omega_1|}{2} + |\Omega_1||\Omega_2|$. □

Theorem 3. *In the graph $\Theta(G_p)$, $ecc(u) = \begin{cases} 1, \ if \ u \in \Omega_1, \\ 2, \ if \ u \in \Omega_2 \end{cases}$*

Proof. If $u \in \Omega_1$, then u is directly connected with every vertex of the graph because of the order of u is either 1 or p. So, the maximum distance between u and any vertex of the graph is 1. If $u \in \Omega_2$, then the order of u is p^k, where $k \geq 2$. So, there exists at least $\phi(p^k)$ vertices of order the same as u. By the use of the concept $k \geq 2$, we know that $\phi(p^k) \geq 2$. Take v be another element different from u whose order is neither 1 nor p, then the maximum distance between u and v is 2. □

Corollary 2. *The diameter of the graph $\Gamma = \Theta(G_p)$ is 2, that is $diam(\Gamma) = 2$.*

Corollary 3. *In the graph $\Gamma = \Theta(G_p)$, $d(u,v) = \begin{cases} 2, \ if \ u,v \in \Omega_2 \\ 1, \ otherwise \end{cases}$*

Corollary 4. *In the graph $\Gamma = \Theta(G_p)$, the independent set S of Γ is either a singleton set or a set with the property that no vertex has order 1 or p.*

Proof. If S is a singleton set, then it is an independent set.

If S contains more than one element with at least one element of order 1 or p, say x, then using the definition of the co-prime order graph, x must be adjacent with every vertex of S except x. So, S is not an independent set.

We conclude that if S is an independent set with more than one element, then S does not contains any element of order 1 or p. □

Corollary 5. *In the graph $\Gamma = \Theta(G_p)$, the largest independent set is Ω_2 or $\{e\}$ according as group G_p has an element of order p^2 or not, respectively.*

Proof. If every element of the group G_p has order 1 or p, then by the above corollary, the set $\{e\}$ is largest independent set.

If the group G_p has at least one element of order p^2, then by the above corollary, the set $S = \{x \in G_p | o(x) \neq 1, o(x) \neq p\} = \Omega_2$ is an independent set. The set S is the largest independent set because the remaining elements are either of order 1 or p, which cannot belong to the independent set. □

Corollary 6. *If $H = \{x \in G_p | o(x) = 1 \text{ or } p\} \cup L$, where $L = \phi$ or $L = \{y\}$ such that $o(y) \neq 1$ or p then $\Theta(H)$ is the maximal clique of the graph $\Theta(G_p)$.*

Proof. By Corollary 2, $d(u,v) = 1$ if one of u or v has order 1 or p, then $\Theta(H)$ is a clique. Now we show that $\Theta(H)$ is a maximal clique.

Using the definition of the co-prime order graph, two vertices with orders neither 1 nor p can be adjacent. Hence, they cannot a part of a clique. So, clique can have at most one vertex whose order is neither 1 nor p.

Take L contains singleton element if at-least one of $\alpha_i \geq 2$ otherwise ϕ and set $\{x \in G_p | o(x) = 1 \text{ or } p\}$ contains all elements of order 1 or p. Therefore, H is maximal clique. □

Corollary 7. *The graph $\Gamma = \Theta(G_p)$ is a complete split graph .*

Proof. There are two cases to consider.

Case 1:- If $\alpha_i \geq 2$ for some i.

The vertex set of the graph Γ can be partitioned as $K = \{x \in G_p | o(x) = 1 \text{ or } p\}$ and $L =$

$G_p - K$. Here K is non-empty because K must contains the identity element of G_p and the largest independent set which is determined in Corollary 4. Hence K must be an independent set.

It is given that at least one of the $\alpha_i \geq 2$, so the group G_p contains at least two elements of order not equal to 1 or p. So L is non-empty. Also, L contains a maximal clique of the co-prime order graph of the group G_p which is determined in Corollary 6, hence the subgraph induced by L must be a clique. Also, each vertex of K must be adjacent to every vertex of L, so the graph Γ is a complete split graph.

Case 2:- If $\alpha_i = 1$ for all i.

In this case the group G_p is a complete graph with more than one vertex. Take $K = \{e\}$ and $L = G_p - K$. Then K is independent because K is a singleton set which is not adjacent with any vertex of K, and L is a clique because it induces a subgraph of a complete graph. □

3. Some Topological Indices of the Graph $\Theta(G_p)$

Theorem 4. *Let $\Gamma = \Theta(G_p)$ be a co-prime order graph of the group G_p. Then* $W(\Gamma) = \binom{|\Omega_1|}{2} + |\Omega_1||\Omega_2| + 2\binom{|\Omega_2|}{2}$

Proof. Let $u, v \in \Gamma$. It follows from the Corollary 3 that the number of possibilities of $d(u,v) = 1$ is $\binom{|\Omega_1|}{2} + |\Omega_1||\Omega_2|$ and number of possibilities of $d(u,v) = 2$ is $\binom{|\Omega_2|}{2}$. Thus, $W(\Gamma) = \binom{|\Omega_1|}{2} + |\Omega_1||\Omega_2| + 2\binom{|\Omega_2|}{2}$ □

Theorem 5. *Let $\Gamma = \Theta(G_p)$ be a co-prime order graph of the group G_p. Then* $WW(\Gamma) = \binom{|\Omega_1|}{2} + |\Omega_1||\Omega_2| + 3\binom{|\Omega_2|}{2}$

Proof. From Theorem 4 and Corollary 3, we can see that $WW(\Gamma) = \frac{1}{2}(\binom{|\Omega_1|}{2} + |\Omega_1||\Omega_2| + 2\binom{|\Omega_2|}{2}) + \frac{1}{2}(\binom{|\Omega_1|}{2} + |\Omega_1||\Omega_2| + 4\binom{|\Omega_2|}{2}) = \binom{|\Omega_1|}{2} + |\Omega_1||\Omega_2| + 3\binom{|\Omega_2|}{2}$ □

In the next two theorems, the first and second Zagreb indices for the co-prime order graph of the group G_p are presented.

Theorem 6. *Let $\Theta(G_p)$ be a co-prime order graph of the group G_p. Then* $M_1(\Theta(G_p)) = |\Omega_1|(|\Omega_1| + |\Omega_2| - 1)^2 + (|\Omega_1|)^2|\Omega_2|$

Proof. It follows from Theorem 2 that $M_1(\Theta(G_p)) = |\Omega_1|(|\Omega_1|+|\Omega_2|-1)^2 + (|\Omega_1|)^2|\Omega_2|$ □

Theorem 7. *Let $\Theta(G_p)$ be a co-prime order graph of the group G_p. Then $M_2(\Theta(G)) = \binom{|\Omega_1|}{2}(|\Omega_1|+|\Omega_2|-1)^2 + (|\Omega_1|)^2|\Omega_2|(|\Omega_1|+|\Omega_2|-1)$*

Proof. From Theorem 2 and Corollary 1, we have $\binom{|\Omega_1|}{2}$ edges with end vertices of degree $|\Omega_1|+|\Omega_2|-1$, and $|\Omega_1||\Omega_2|$ edges with one end vertex of degree $|\Omega_1$ and the other end vertex of degree $|\Omega_1|+|\Omega_2|-1$. Hence we get desired result. □

Theorem 8. *Let $\Theta(G_p)$ be a co-prime order graph of the group G_p. Then $MTI(\Theta(G_p)) = 2\binom{|\Omega_1|}{2}(|\Omega_1|+|\Omega_2|-1) + |\Omega_1||\Omega_2|(2|\Omega_1|+|\Omega_2|-1) + 4|\Omega_1|\binom{|\Omega_2|}{2}$.*

Proof. There are three possibilities for $u, v \in G_p$.

Case 1:- $u, v \in \Omega_1$.

It follows from Theorem 2 and Corollary 1 that $d(u,v) = 1$ and $deg(u) + deg(v) = 2(|\Omega_1|+|\Omega_2|-1$. There are $\binom{|\Omega_1|}{2}$ possibilities for this case.

Case 2:-$u \in \Omega_1$ and $v \in \Omega_2$.

It follows from Theorem 2 and Corollary 1 that $d(u,v) = 1$ and $deg(u) + deg(v) = (2|\Omega_1|+|\Omega_2|-1$. There are $|\Omega_1||\Omega_2|$ possibilities for this case.

Case 3:- $u, v \in \Omega_2$.

It follows from Theorem 2 and Corollary 1 that $d(u,v) = 2$ and $deg(u)+deg(v) = 2|\Omega_1|$. There are $\binom{|\Omega_2|}{2}$ possibilities for this case.

Hence, we get $MTI(\Theta(G_p)) = 2\binom{|\Omega_1|}{2}(|\Omega_1|+|\Omega_2|-1) + |\Omega_1||\Omega_2|(2|\Omega_1|+|\Omega_2|-1) + 4|\Omega_1|\binom{|\Omega_2|}{2}$. □

Theorem 9. *Let $\Theta(G_p)$ be a co-prime order graph of the group G_p. Then $Gut(\Theta(G_p)) = \binom{|\Omega_1|}{2}(|\Omega_1|+|\Omega_2|-1)^2 + |\Omega_1|^2|\Omega_2|(|\Omega_1|+|\Omega_2|-1) + 2|\Omega_1|^2\binom{|\Omega_2|}{2}$.*

Proof. There are three possibilities for $u, v \in G_p$.

Case 1:- $u, v \in \Omega_1$.

It follows from Theorem 2 and Corollary 1 that $d(u,v) = 1$ and $deg(u)deg(v) = (|\Omega_1|+|\Omega_2|-1)^2$. There are $\binom{|\Omega_1|}{2}$ possibilities for this case.

Case 2:-$u \in \Omega_1$ and $v \in \Omega_2$.

It follows from Theorem 2 and Corollary 1 that $d(u,v) = 1$ and $deg(u)deg(v) = |\Omega_1|(|\Omega_1|+|\Omega_2|-1)$. There are $|\Omega_1||\Omega_2|$ possibilities for this case.

Case 3:- $u,v \in \Omega_2$.

It follows from Theorem 2 and Corollary 1 that $d(u,v) = 2$ and $deg(u)deg(v) = |\Omega_1|^2$. There are $\binom{|\Omega_2|}{2}$ possibilities for this case.

Hence, we get $Gut(\Theta(G_p)) = \binom{|\Omega_1|}{2}(|\Omega_1|+|\Omega_2|-1)^2 + |\Omega_1|^2|\Omega_2|(|\Omega_1|+|\Omega_2|-1) + 2|\Omega_1|^2\binom{|\Omega_2|}{2}$. □

Theorem 10. *Let $\Theta(G_p)$ be a co-prime order graph of the group G_p. Then $\xi^c(\Theta(G_p)) = (|\Omega_1|^2 + 3|\Omega_1||\Omega_2| - |\Omega_1|)$.*

Proof. There are two possibilities for $u \in G_p$.

Case 1:- $u \in \Omega_1$.

It follows from Theorem 2 and Theorem 3 that $deg(u) = (|\Omega_1|+|\Omega_2|-1)$ and $ecc(u) = 1$. There are $|\Omega_1|$ possibilities for this case.

Case 2:-$u \in \Omega_2$.

It follows from Theorem 2 and Theorem 3 that $deg(u) = |\Omega_1|$ and $ecc(u) = 2$. There are $|\Omega_2|$ possibilities for this case.

Hence, we get $\xi^c(\Theta(G_p)) = (|\Omega_1|^2 + 3|\Omega_1||\Omega_2| - |\Omega_1|)$. □

4. Metric Dimension and Resolving Polynomial of the Graph $\Theta(G_p)$

For a graph Γ, we define the open neighborhood of a vertex $u \in \Gamma$, $N(u)$, by $N(u) = \{v \in V(\Gamma) : uv \in E(\Gamma)\}$ and the closed neighborhood of u, $N[u]$, by $N[u] = N(u) \cup \{u\}$. Two distinct vertices u and v of Γ are said to be twins if $N(u) = N(v)$ or $N[u] = N[v]$. A subset S of $V(\Gamma)$ is said to be a twin set in Γ if every pair of vertices in S are twins.

Lemma 3. *If Γ is a connected graph of order n and $S \subseteq V(\Gamma)$ is a twin in Γ set of size $l \geq 2$, then every resolving set of Γ contains at least $l-1$ vertices of S.*

Corollary 8. *If S is a resolving set for a connected graph Γ and u and v are twin vertices in Γ, then $u \in S$ or $v \in S$. Furthermore, if $u \in S$ and $v \notin S$, then $(S \setminus \{u\}) \cup \{v\}$ is also a resolving set for Γ.*

Theorem 11. *Let $\Gamma = \Theta(G_p)$ be a co-prime order graph of group G_p. Then $\beta(\Gamma) = p^{\sum_{i=1}^{r} \alpha_i} - 2$.*

Proof. We see that the sets Ω_1 and Ω_2 are twin sets in Γ of cardinality p^r and $p^{\sum_{i=1}^{r} \alpha_i}$, respectively. From Lemma 3, we have $\beta(\Gamma) \geq p^{\sum_{i=1}^{r} \alpha_i} - 2$. In addition, we see that the set $W := \Omega_1 \cup \Omega_2 - \{e, x\}$, where $x \in G_p$ with $o(x)$ is neither 1 nor p, is a resolving set for Γ of cardinality $p^{\sum_{i=1}^{r} \alpha_i} - 2$, this implies that $\beta(\Gamma) \leq p^{\sum_{i=1}^{r} \alpha_i} - 2$. □

Lemma 4. *For a connected graph Γ of order n, $r_n = 1$ and $r_{n-1} = n$.*

Theorem 12. *Let $\Gamma = \Theta(G_p)$ be a co-prime order graph of group G_p. Then $\beta(\Gamma, x) = x^{p^{\sum_{i=1}^{r} \alpha_i} - 2}(|\Omega_1||\Omega_2| + (|\Omega_1| + |\Omega_2|)x + x^2)$.*

Proof. From Theorem 11, we have that $\beta(\Gamma) = |\Omega_1| + |\Omega_2| - 2$. We need to find the resolving sequence $(r_{|\Omega_1|+|\Omega_2|-2}, r_{|\Omega_1|+|\Omega_2|-1}, r_{|\Omega_1|+|\Omega_2|})$ of length three.
For $r_{|\Omega_1|+|\Omega_2|-2}$: By the principal of multiplication and Corollary 8, we see that

$$r_{|\Omega_1|+|\Omega_2|-2} = \binom{|\Omega_1|}{|\Omega_1|-1}\binom{|\Omega_2|}{|\Omega_2|-1} = |\Omega_1||\Omega_2|.$$

By Lemma 4, we have $r_{|\Omega_1|+|\Omega_2|-1} = |\Omega_1| + |\Omega_2|$ and $r_{|\Omega_1|+|\Omega_2|} = 1$. □

Conflict of Interests

The author(s) declare that there is no conflict of interests.

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