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A NOTE ON $(\in, \in \vee q)$-FUZZY PRIME IDEALS OF NEAR-RINGS

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Abstract. Using the idea of quasi-coincidence of a fuzzy point with a fuzzy set, we discuss a new kind of $(\in, \in \vee q)$-fuzzy prime (semiprime) ideals of near-rings. The concept of $(\in, \in \vee q_0^\delta)$-fuzzy prime (semiprime) ideals of near-rings are introduced and some of its related properties are investigated.

Keywords: near-rings; subnear-rings (ideals); $(\in, \in \vee q_0^\delta)$-fuzzy ideals; $(\in, \in \vee q_0^\delta)$-fuzzy prime (semiprime) ideals.

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1. Introduction

In 1965, Zadeh [16] introduced the concept of a fuzzy set. Using this concept, Rosenfeld [15] defined fuzzy subgroups in 1971. Since then, the study of fuzzy algebraic structure has been pursued in many directions such as groups, rings, near-rings, modules and so on. Abou-zaid [1] introduced the concept of fuzzy subnear-rings (ideals) and defined fuzzy prime ideals of near-rings. The concept of quasi-coincidence of a fuzzy point with a fuzzy set was introduced by Ming and Ming [14]. Using the idea of quasi-coincidence of a fuzzy point with a fuzzy set, Bhakat and Das [2] defined different types of fuzzy subgroups called (α, β)-fuzzy subgroups. In particular, they introduced $(\in, \in \vee q)$-fuzzy subgroup as the most viable generalization of

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a fuzzy subgroup defined by Rosenfeld. Narayanan and Manikantan [13] defined $(\in, \in \vee q)$-fuzzy subnear-rings (ideals) of near-rings. The notions of $(\in, \in \vee q_0^\delta)$-fuzzy subgroups, q_0^δ-level sets and $(\in \vee q_0^\delta)$-level sets were introduced by Jun *et al.* [9]. The notion of $(\in, \in \vee q_0^\delta)$-fuzzy subnear-rings (ideals) of near-rings was introduced by Gangmei and Devi [8]. Using the idea of quasi-coincidence of a fuzzy point with a fuzzy set, Bhakat and Das [4] defined $(\in, \in \vee q)$-fuzzy semiprime (prime) of rings. Jian-ming and Davvaz [10] defined $(\in, \in \vee q)$-fuzzy prime ideals of near-rings in the context of a fuzzy point with a fuzzy set. In this paper, $(\in, \in \vee q_0^\delta)$-fuzzy semiprime (prime) ideals of near-rings in context of a fuzzy point with a fuzzy set are introduced and some of its related properties are investigated.

2. Preliminaries

We first recall the definition of a near-ring. A non-empty subset N with two binary operations "+"(addition) and "$\cdot$" (multiplication) is called a near-ring if it satisfies the following axioms:

i) $(N, +)$ is a group,

ii) $(N, \cdot)$ is a semigroup,

iii) $(x+y)\cdot z = x\cdot z + y\cdot z \ \forall \ x, y, z \in N$. It is a right near-ring because it satisfies the right distributive law. If it satisfies left distributive law it is called left near-ring.

Unless otherwise stated, we shall consider only right near-rings throughout this paper.

Definition 2.1. [1] Let N be a near-ring. A normal subgroup I of $(N, +)$ is called

i) a right ideal if $IN \subseteq I$,

ii) a left ideal if $n(m+i) - nm \in I \ \forall \ n, m \in N$ and $i \in I$,

iii) an ideal if it is both right and left ideal.

An ideal I of a near-ring N is called prime if $xy \in I \Rightarrow x \in I$ or $y \in I \ \forall \ x, y \in N$.

Definition 2.2. [16] A fuzzy set in a set X is a function $\mu : G \to [0,1]$.

Definition 2.3. [14] A fuzzy set μ in a set X of the form

$$\mu(y) = \begin{cases} t \in (0,1] & \text{if } y = x, \\ 0 & \text{if } y \neq x, \end{cases}$$

is said to be a fuzzy point with support x and value t and is denoted by x_t.

Definition 2.4. [14] For a fuzzy point x_t and a fuzzy set μ in a set X, we say that

$i)$ $x_t \in \mu$ (resp. $x_t q\mu$) if $\mu(x) \geq t$ (resp. $\mu(x)+t > 1$),

$ii)$ $x_t \in \vee q\mu$ if $x_t \in \mu$ or $x_t q\mu$.

Definition 2.5. [2],[3] A fuzzy set μ of a group G is said to be an $(\in, \in \vee q)$-fuzzy subgroup of G if $\forall\, x,y \in G$ and $t,r \in (0,1]$,

$i)$ $x_t, y_r \in \mu \Rightarrow (xy)_{min\{t,r\}} \in \vee q\mu$ and

$ii)$ $x_t \in \mu \Rightarrow (-x)_t \in \vee q\mu$.

Definition 2.6. [13] A fuzzy set μ of a near-ring N is said to be an $(\in, \in \vee q)$-fuzzy subnear-ring of N if $\forall\, x,y \in N$ and $t,r \in (0,1]$

$i)$ $x_t, y_r \in \mu \Rightarrow (x+y)_{min\{t,r\}} \in \vee q\mu$.

$ii)$ $x_t \in \mu \Rightarrow (-x)_t \in \vee q\mu$.

$iii)$ $x_t, y_r \in \mu \Rightarrow (xy)_{min\{t,r\}} \in \vee q\mu$.

Definition 2.7. [13] A fuzzy set μ of a near-ring N is said to be an $(\in, \in \vee q)$-fuzzy ideal of N if

$i)$ μ is an $(\in, \in \vee q)$-fuzzy subnear-ring of N,

$ii)$ $x_t \in \mu$ and $y \in N \Rightarrow (y+x-y)_t \in \vee q\mu$,

$iii)$ $x_t \in \mu$ and $y \in N \Rightarrow (xy)_t \in \vee q\mu$,

$iv)$ $a_t \in \mu$ and $x,y \in N \Rightarrow (y(x+a)-yx)_t \in \vee q\mu\ \forall\, x,y,a \in N$.

Definition 2.8. [10] An $(\in, \in \vee q)$-fuzzy ideal μ of a near-ring N is prime if $\forall\, x,y \in N$ and $t \in (0,1]$, we have $(xy)_t \in \mu \Rightarrow x_t \in \vee q\mu$ or $y_t \in \vee q\mu$.

An $(\in, \in \vee q)$-fuzzy ideal μ of a near-ring N is semiprime if $\forall\, x,y \in N$ and $t \in (0,1]$, we have $(x^2)_t \in \mu \Rightarrow x_t \in \vee q\mu$.

Jun *et al.* [9] introduced the concept of δ-quasi-coincidence of a fuzzy point with a fuzzy set as a generalization of quasi-coincidence of a fuzzy point with a fuzzy set in view of Bhakat and Das [2]. Let $\delta \in (0,1]$. For a fuzzy point x_t and a fuzzy set μ in a set X, we say that

• x_t is a δ-quasi-coincident with μ, written $x_t q_0^{\delta}\mu$, if $\mu(x)+t > \delta$.

• $x_t \in \vee q_0^{\delta}\mu$, if $x_t \in \mu$ or $x_t q_0^{\delta}\mu$.

If $\delta = 1$, then the δ-quasi-coincident with μ is the quasi-coincident with μ, i,e $x_t q_0^1\mu = x_t q\mu$.

Definition 2.9. [8] A fuzzy set μ of a near-ring N is called an $(\in, \in \vee q_0^\delta)$-fuzzy subnear-ring of N if $\forall\, x, y \in N$ and $t, r \in (0, \delta]$,

$i)$ $x_t \in \mu, y_r \in \mu \Rightarrow (x-y)_{min\{t,r\}} \in \vee q_0^\delta \mu$ and

$ii)$ $x_t \in \mu, y_r \in \mu \Rightarrow (xy)_{min\{t,r\}} \in \vee q_0^\delta \mu$.

Definition 2.10. [8] A fuzzy set μ of a near-ring N is called an $(\in, \in \vee q_0^\delta)$-fuzzy ideal in N if $\forall\, x, a \in N$ and $t \in (0, \delta]$,

$i)$ it is an $(\in, \in \vee q_0^\delta)$-fuzzy subnear-ring of N,

$ii)$ $x_t \in \mu, y \in N \Rightarrow (y+x-y)_t \in \vee q_0^\delta \mu$,

$iii)$ $x_t \in \mu, y \in N \Rightarrow (xy)_t \in \vee q_0^\delta \mu$ and

$iv)$ $a_t \in \mu, x, y \in \mu \Rightarrow (y(x+a)-yx)_t \in \vee q_0^\delta \mu$.

A fuzzy set with condition $i), ii), iii)$ is called an $(\in, \in \vee q_0^\delta)$-fuzzy right ideal of N and if it satisfies $i), ii), iv)$, then it is called an $(\in, \in \vee q_0^\delta)$-fuzzy left ideal of N.

3. Main Results

Throughout this section, $\delta \in (0, 1]$.

Definition 3.1. An $(\in, \in \vee q_0^\delta)$-fuzzy ideal μ of a near-ring N is said to be

$i)$ an $(\in, \in \vee q_0^\delta)$-fuzzy prime, if $\forall\, x, y \in N$ and $t \in (0, \delta]$,

$(xy)_t \in \mu \Rightarrow x_t \in \vee q_0^\delta \mu$ or $y_t \in \vee q_0^\delta \mu$.

$ii)$ an $(\in, \in \vee q_0^\delta)$-fuzzy semiprime, if $\forall\, x \in N$ and $t \in (0, \delta]$,

$(x^2)_t \in \mu \Rightarrow x_t \in \vee q_0^\delta \mu$.

Example 3.2. Let $N = \{0, a, b, c\}$ with $(N, +)$ as the Klein 4-group and $(N, .)$ as given in table below,

·	0	a	b	c
0	0	0	0	0
a	0	a	a	a
b	0	b	b	b
c	0	c	c	c

Then $(N,+,\cdot)$ is a right near-ring.

Define a fuzzy set μ in N as $\mu(0)=0.7, \mu(a)=0.8, \mu(b)=0.6, \mu(c)=0.5$. Then μ is an $(\in, \in \vee q_0^{\delta})$-fuzzy prime of N with $\delta \in (0,1]$.

Theorem 3.3. An $(\in, \in \vee q_0^{\delta})$-fuzzy ideal μ of a near-ring N is an $(\in, \in \vee q_0^{\delta})$-fuzzy prime if and only if $max\{\mu(x), \mu(y)\} \geq min\{\mu(xy), \frac{\delta}{2}\} \; \forall \, x, y \in N$.

Proof: Let μ be an $(\in, \in \vee q_0^{\delta})$-fuzzy prime.

If possible, let there exist $x, y \in N$ such that $max\{\mu(x), \mu(y)\} < min\{\mu(xy), \frac{\delta}{2}\}$.

Choose t such that $max\{\mu(x), \mu(y)\} < t < min\{\mu(xy), \frac{\delta}{2}\}$. Then

$(xy)_t \in \mu$ but $x_t \overline{\in \vee q \mu}$ and $y_t \overline{\in \vee q \mu}$ which is a contradiction.

Therefore, $max\{\mu(x), \mu(y)\} \geq min\{\mu(xy), \frac{\delta}{2}\}$.

Conversely, suppose $max\{\mu(x), \mu(y)\} \geq min\{\mu(xy), \frac{\delta}{2}\}$.

Let $(xy)_t \in \mu$, now $max\{\mu(x), \mu(y)\} \geq min\{\mu(xy), \frac{\delta}{2}\} \geq min\{t, \frac{\delta}{2}\}$.

$\Rightarrow max\{\mu(x), \mu(y)\} \geq t$ if $t \leq \frac{\delta}{2}$ or $max\{\mu(x), \mu(y)\} + t > \delta$ if $t > \frac{\delta}{2}$.

So, either $x_t \in \vee q_0^{\delta} \mu$ or $y_t \in \vee q_0^{\delta} \mu$. Hence, μ is an $(\in, \in \vee q_0^{\delta})$-fuzzy prime.

Proposition 3.4. Let μ be an $(\in, \in \vee q_0^{\delta})$-fuzzy prime of a near-ring N, then

$i)$ $\mu(x) \geq min\{\mu(x^n), \frac{\delta}{2}\}$ and

$ii)$ if $\mu(x) < \frac{\delta}{2}$, then $\mu(x) \geq \mu(x^n) \; \forall \, n$ belong to natural number.

Proof: i) Since μ is an $(\in, \in \vee q_0^{\delta})$-fuzzy prime ideal of N,

$max\{\mu(x), \mu(x)\} \geq min\{\mu(xx), \frac{\delta}{2}\}$. This implies that $\mu(x) \geq min\{\mu(x^2), \frac{\delta}{2}\}$.

Thus, for $n=2$ is true. Suppose for $n=k$ is true, that is, $\mu(x) \geq min\{\mu(x^k), \frac{\delta}{2}\}$.

Now, $max\{\mu(x), \mu(x^k)\} \geq min\{\mu(x^{k+1}), \frac{\delta}{2}\}$. If $min\{\mu(x^k), \frac{\delta}{2}\} = \mu(x^k)$, then, $\mu(x) \geq \mu(x^k)$.

$\Rightarrow max\{\mu(x), \mu(x^k)\} = \mu(x)$. This gives $\mu(x) \geq min\{\mu(x^{k+1}), \frac{\delta}{2}\}$.

If $min\{\mu(x^k), \frac{\delta}{2}\} = \frac{\delta}{2}$, then $\mu(x) \geq \frac{\delta}{2} \geq min\{\mu(x^{k+1}, \frac{\delta}{2}\}$.

$\Rightarrow \mu(x) \geq min\{\mu(x^{k+1}), \frac{\delta}{2}\}$. Hence, by principle of mathematical induction,

$\mu(x) \geq min\{\mu(x^n), \frac{\delta}{2}\} \; \forall \, n$ belong to natural number.

ii) If $\mu(x) < \frac{\delta}{2}$, then $\mu(x) \geq min\{\mu(x^n), \frac{\delta}{2}\}$ [by (i)].

$\Rightarrow \mu(x) \geq \mu(x^n) \; \forall \, n$ belong to natural number.

Theorem 3.5. An $(\in, \in \vee q_0^{\delta})$-fuzzy ideal μ of a near-ring N is $(\in, \in \vee q_0^{\delta})$-fuzzy semiprime if and only if $\mu(x) \geq min\{\mu(x^2), \frac{\delta}{2}\} \; \forall \, x \in N$.

Proof: Let μ be an $(\in,\in\vee q_0^{\delta})$-fuzzy semiprime. Let $x\in N$ such that $\mu(x)<t<min\{\mu(x^2),\frac{\delta}{2}\}$.

Then $\mu(x^2)\geq t$. This implies that $(x^2)_t\in\mu$. That is, $x_t\in\vee q_0^{\delta}\mu$ which is a contradiction.

Therefore, $\mu(x)\geq min\{\mu(x^2),\frac{\delta}{2}\}$.

Conversely, we assume that $\mu(x)\geq min\{\mu(x^2),\frac{\delta}{2}\}$. Let $x\in N$ and $(x^2)_t\in\mu$.

Then $\mu(x^2)\geq t$. Now, $\mu(x)\geq min\{\mu(x^2),\frac{\delta}{2}\}\geq min\{t,\frac{\delta}{2}\}$.

$\Rightarrow\mu(x)\geq t$ if $t\leq\frac{\delta}{2}$ or $\mu(x)\geq\frac{\delta}{2}$ if $t>\frac{\delta}{2}$.

$\Rightarrow\mu(x)\geq t$ if $t\leq\frac{\delta}{2}$ or $\mu(x)+t>\delta$ if $t>\frac{\delta}{2}$.

Therefore, $(x)_t\in\vee q_0^{\delta}\mu$. Hence, μ is an $(\in,\in\vee q_0^{\delta})$-fuzzy semiprime.

Definition 3.6. [6] Let μ be a fuzzy set of a group G. Then $\forall\ t\in(0,1]$, the set $\mu_t=\{x\in G\,|\,\mu(x)\geq t\}$ is called level set of μ.

Definition 3.7. [5] The subset $\bar{\mu}_t=\{x\in X\,|\,\mu(x)\geq t$ or $\mu(x)+t>1\}$ is called $(\in\vee q)$-level set of X determined by μ and t.

Theorem 3.8. [8] A fuzzy set μ of a near-ring N is an $(\in,\in\vee q_0^{\delta})$-fuzzy subnear-ring (ideal) of N if and only if the levet set μ_t is a subnear-ring (ideal) of $N\ \forall\ t\in(0,\frac{\delta}{2}]$ and $\delta\in(0,1]$.

Collorary 3.9. [13] A fuzzy set μ of a near-ring N is an $(\in,\in\vee q)$-fuzzy subnear-ring(ideal) of N if and only if the levet set μ_t is a subnear-ring(ideal) of $N\ \forall\ t\in(0,0.5]$.

Theorem 3.10. An $(\in,\in\vee q_0^{\delta})$-fuzzy ideal μ of a near-ring N is $(\in,\in\vee q_0^{\delta})$-fuzzy prime if and only if μ_t is a prime ideal $\forall\ t\in(0,\frac{\delta}{2}]$ and $\delta\in(0,1]$.

Proof: Let μ be an $(\in,\in\vee q_0^{\delta})$-fuzzy prime.

Then by Theorem 3.8., μ_t is an ideal of $N\ \forall\ t\leq\frac{\delta}{2}$.

Let $xy\in\mu_t$. Since μ is an $(\in,\in\vee q_0^{\delta})$-fuzzy prime, by Theorem 3.3.,

$max\{\mu(x),\mu(y)\}\geq min\{\mu(xy),\frac{\delta}{2}\}\geq min\{t,\frac{\delta}{2}\}=t$.

Then $\mu(x)\geq t$ or $\mu(y)\geq t$. This implies that $x\in\mu_t$ or $y\in\mu_t$.

Hence, μ_t is a prime ideal.

Conversely, let μ_t be a prime ideal. Then by Theorem 3.8.,

μ is an $(\in,\in\vee q_0^{\delta})$-fuzzy ideal of N. Let $(xy)_t\in\mu$, then $xy\in\mu_t$.

$\Rightarrow x\in\mu_t$ or $y\in\mu_t$.[since μ_t is a prime].

$\Rightarrow\mu(x)\geq t$ or $\mu(y)\geq t$.

$\Rightarrow x_t\in\mu$ or $y_t\in\mu$.

$\Rightarrow x_t \in \vee q_0^\delta \mu$ or $y_t \in \vee q_0^\delta \mu$.

Therefore, μ is an $(\in, \in \vee q_0^\delta)$-fuzzy prime.

If $t > \frac{\delta}{2}$, then μ_t may not be a prime ideal.

Example 3.11. Let $N = \{0, a, b, c\}$ be a near-ring with $(N, +)$ as the Klein 4-group and $(N, .)$ as given in table below,

·	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	0	0	0	0
c	a	a	a	a

Define a fuzzy set μ in N as $\mu(0) = 0.7, \mu(a) = 0.8, \mu(b) = 0.6, \mu(c) = 0.5$.

Then μ is an $(\in, \in \vee q_0^\delta)$-fuzzy prime ideal of N with $\delta \in (0, 1]$.

When $t = 0.65 > \frac{\delta}{2}, \mu_t = \{0, a\}$. Then μ_t is an ideal of N,

but μ_t is not a prime ideal since $c \cdot b = a \in \mu_t$ but $c \notin \mu_t$ and $b \notin \mu_t$.

Collorary 3.12. [13] An $(\in, \in \vee q)$-fuzzy ideal μ of a near-ring N is prime if and only if the level set μ_t is a prime ideal of $N \; \forall \, t \in (0, 0.5]$.

Theorem 3.13. An $(\in, \in \vee q_0^\delta)$-fuzzy ideal μ of a near-ring N is an $(\in, \in \vee q_0^\delta)$-fuzzy semiprime if and only if μ_t is a semiprime ideal $\forall \, t \in (0, \frac{\delta}{2}]$.

Proof is similar to the proof of Theorem 3.10.

Definition 3.14. [9] Let μ be a fuzzy set of a group G. Then the subset $\bar{\mu}_t^\delta = \{x \in G; \mu(x) \geq t$ or $\mu(x) + t > \delta\}$ is called $(\in \vee q_0^\delta)$-level set of G.

Theorem 3.15. [8] A fuzzy set μ of a near-ring N is an $(\in, \in \vee q_0^\delta)$-fuzzy ideal of N if and only if $\bar{\mu}_t^\delta (\neq \phi)$ is an ideal of $N \; \forall \, t \in (0, \delta]$ and $\delta \in (0, 1]$.

Theorem 3.16. [10] A fuzzy set μ of a near-ring N is an $(\in, \in \vee q)$-fuzzy ideal of N if and only if $\bar{\mu}_t (\neq \phi)$ is an ideal of $N \; \forall \, t \in (0, 1]$.

Theorem 3.17. A fuzzy set μ of a near-ring N is an $(\in, \in \vee q_0^\delta)$-fuzzy prime of N if and only if $\bar{\mu}_t^\delta (\neq \phi)$ is a prime ideal of $N \; \forall \, t \in (0, \delta]$.

Proof: Let μ be an $(\in, \in \vee q_0^\delta)$-fuzzy prime of N.

Then by Theorem 3.15., $\bar{\mu}_t^{\delta}$ is an ideal of N $\forall\, t \in (0,\delta]$.

Let $xy \in \bar{\mu}_t^{\delta}$. Then $\mu(xy) \geq t$ or $\mu(xy)+t > \delta$. This implies that $(xy)_t \in \mu$ or $(xy)_t q_0^{\delta}\mu$.

Case 1. Let $(xy)_t \in \mu$.

a) if $t > \frac{\delta}{2}$, then $max\{\mu(x),\mu(y)\} \geq min\{\mu(xy),\frac{\delta}{2}\} \geq min\{t,\frac{\delta}{2}\} = \frac{\delta}{2}$

$\Rightarrow max\{\mu(x),\mu(y)\}+t > \delta$.

$\Rightarrow \mu(x)+t > \delta$ or $\mu(y)+t > \delta$.

$\Rightarrow x \in \bar{\mu}_t^{\delta}$ or $y \in \bar{\mu}_t^{\delta}$.

b) if $t \leq \frac{\delta}{2}$, then $max\{\mu(x),\mu(y)\} \geq min\{\mu(xy),\frac{\delta}{2}\} \geq min\{t,\frac{\delta}{2}\} = t$

$\Rightarrow \mu(x) \geq t$ or $\mu(y) \geq t$

$\Rightarrow x \in \bar{\mu}_t^{\delta}$ or $y \in \bar{\mu}_t^{\delta}$.

Case 2. Let $\mu(xy) < t$ and $\mu(xy)+t > \delta$.

a) if $\mu(xy) \leq \frac{\delta}{2}$, then $max\{\mu(x),\mu(y)\}+t \geq min\{\mu(xy),\frac{\delta}{2}\}+t = \mu(xy)+t > \delta$, implies that $\mu(x)+t > \delta$ or $\mu(y)+t > \delta$. That is, $x \in \bar{\mu}_t^{\delta}$ or $y \in \bar{\mu}_t^{\delta}$

b) if $\mu(xy) > \frac{\delta}{2}$, then $max\{\mu(x),\mu(y)\} \geq min\{\mu(xy),\frac{\delta}{2}\} = \frac{\delta}{2}$.

$\Rightarrow max\{\mu(x),\mu(y)\}+t > \delta$ [since $t > \mu(xy) > \frac{\delta}{2}$]

$\Rightarrow \mu(x)+t > \delta$ or $\mu(y)+t > \delta$. That is, $x \in \bar{\mu}_t^{\delta}$ or $y \in \bar{\mu}_t^{\delta}$.

Thus, in both cases, $x \in \bar{\mu}_t^{\delta}$ or $y \in \bar{\mu}_t^{\delta}$. Hence, $\bar{\mu}_t^{\delta}$ is a prime ideal of N.

Conversely, suppose $\bar{\mu}_t^{\delta}$ is a prime ideal of N.

Then by Theorem 3.15., μ is an $(\in,\in\vee q_0^{\delta})$-fuzzy ideal of N.

Let $(xy)_t \in \mu$. Then $\mu(xy) \geq t$, this implies that $xy \in \bar{\mu}_t^{\delta}$.

$\Rightarrow x \in \bar{\mu}_t^{\delta}$ or $y \in \bar{\mu}_t^{\delta}$ [since $\bar{\mu}_t^{\delta}$ is a prime ideal].

$\Rightarrow x_t \in \vee q_0^{\delta}\mu$ or $y_t \in \vee q_0^{\delta}\mu$. Thus, μ is an $(\in,\in\vee q_0^{\delta})$-fuzzy prime ideal of N.

Collorary 3.18. [10] A fuzzy set μ of a group G is an $(\in,\in\vee q)$-fuzzy prime of G if and only if $\bar{\mu}_t(\neq \phi)$ is a prime ideal of N $\forall\, t \in (0,1]$.

Definition 3.19. [8] For a subset A of a near-ring N, a fuzzy set χ_A^{δ} in N defined by $\chi_A^{\delta} : N \rightarrow [0,\delta]$ as

$$\chi_A^{\delta}(x) = \begin{cases} \delta & \text{if } x \in A, \\ 0 & \text{otherwise,} \end{cases}$$

is called a δ-characteristic fuzzy set of A in N.

Theorem 3.20. [8] A non-empty subset A of a near-ring N is a subnear-ring(ideal) of N if and only if χ_A^δ is an $(\in, \in \vee q_0^\delta)$-fuzzy subnear-ring(ideal) of N.

Theorem 3.21. A non-empty subset A of a near-ring N is a prime (semiprime) ideal if and only if χ_A^δ is an $(\in, \in \vee q_0^\delta)$-fuzzy prime (semiprime) ideal of N.
The proof is straightforward

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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