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EFFECT OF THERMAL RADIATION ON MHD THREE DIMENSIONAL NATURAL CONVECTIVE COUETTE FLOW IN PRESENCE OF THERMO-DIFFUSION AND CHEMICAL REACTION

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Abstract: This paper analyzes the effect of thermal radiation on MHD three dimensional natural convective Couette flow of an incompressible, viscous and electrically conducting fluid with uniform magnetic field. The plates are taken vertically upward. The magnetic field is applied normal to the plates. The governing equations are solved using simple perturbation technique. The effect of various parameters like *M*, *Pr*, *R*, *Re*, *Gr*, *Gm* etc. are studied graphically on dimensionless velocity, temperature and concentration. Also the variations in shear stress (τ), Nusselt number (*Nu*) and Sherwood number (*Sh*) for different physical parameters are presented in tables.

Keywords: MHD; thermal radiation; chemical reaction; thermo-diffusion.

2010 AMS Subject Classification: 76W05.

1. INTRODUCTION

The fundamental concept behind MHD is that magnetic fields can induce currents in a moving conductive fluid, which in turn polarizes the fluid and reciprocally changes the magnetic field itself. There are huge applications of magnetohydrodynamics such as in astrophysics,

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geophysics, sensors etc. It is also useful in power generation and earthquake assumptions. Many investigators have made model studies on the effect of magnetic field in different branches of science. Some of them are Raju and Varma [1], Acharya et al. [2], Jha and Prasad [3], Magyari et al. [4], Sahoo et al. [5], Ravikumar et al. [6] and many others.

Alam et al. [7] have numerically studied the effects of thermal diffusion and heat generation effects on steady combined free-forced convection and mass transfer flow past a semi-infinite vertical porous plate embedded in a porous medium. Ahmed and Sengupta [8] also have investigated thermo-diffusion and diffusion-thermo effects on three dimensional MHD convective flow.

Thermal radiation is electromagnetic radiation generated by the thermal motion of particles in matter. All matter with a temperature greater than absolute zero emits thermal radiation. Some examples are like heating of the Earth by the Sun, the heating of a room by an open-hearth fire place etc.. Mahmoud [9] have considered thermal radiation effect on unsteady MHD free convection flow past vertical plate with temperature dependent viscocity. All other authors like Raju et al. [10] Takhar et al. [11], Muthucumarswamy and Kumar [12], Ahmed and Sarmah [13] etc. have researched on various effects of thermal radiations in fluid motion.

The effect of heat source or heat sink on heat transfer plays a crucial role in controlling the heat transfer and in cooling processes. In thermodynamics, a heat sink is a heat reservoir that can absorb an arbitrary amount of heat without significantly changing temperature. Practical heat sinks for electronic devices must have a temperature higher than the surroundings to transfer heat by convection, radiation and conduction. Sahoo et al. [14] have studied on unsteady hydromagnetic free convective flow past an infinite vertical porous plate in presence of constant suction and heat absorbing sinks. Several other authors like Chamaka [15], Mythreye et al. [16], Johari et al. [17] etc. have studied on different effects of heat sink on unsteady MHD convective flows.

The analysis of chemical reaction gives a mathematical model for the system to predict the reactor performance. In fact the study of heat and mass transfer with chemical reaction is of considerable importance in chemical and hydrometallurgical industries. Effect of chemical reaction is determined whether the reaction is homogeneous or heterogeneous. A large amount of work has been reported in this field. Ibrahim et al. [18] have studied the effect of the chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate with heat source and suction. The effect of chemical reaction on a moving isothermal vertical surface with suction has been investigated by Muthucumarswamy [19]. Chamaka et al. [20], Seddeek [21] Hossain et al. [22], Sarada and Shankar [23] etc. have worked extensively to study the effect of chemical reaction on various convective flows under different conditions.

2. MATHEMATICAL FORMULATION

We have considered a coordinate system with the plates lying vertically along $\bar{x}$ - $\bar{z}$ plane. The $\bar{x}$ - axis is oriented along the length of the plates in the direction of the buoyancy force and the $\bar{y}$ – axis is taken perpendicular to the plane of the plates as shown in Figure 1. In this paper, a 3 dimensional free convection Couette flow of a viscous incompressible and electrically conducting fluid with heat sink is considered. A uniform magnetic field is applied normal to the plane of the plates.

The following assumptions are made for the analysis of the problem:

(i) All the fluid properties are assumed to be independent of x except for the pressure.

(ii) Applied electric field and induced magnetic field are neglected.

(iii) A transverse sinusoidal injection velocity distribution is applied to the plate at rest which is of the form $\bar{v}\left(\bar{z}\right)=V\left(1+\varepsilon\cos\frac{\pi\bar{z}}{d}\right)$.

(iv) The moving plate has a uniform motion U and is subjected to a constant suction V under first order slip conditions.

(v) Joules dissipation and Viscous dissipation are neglected.

(vi) Without any loss of generality, it is assumed that the distance between the plates is equal to the wave length d of the injection velocity.

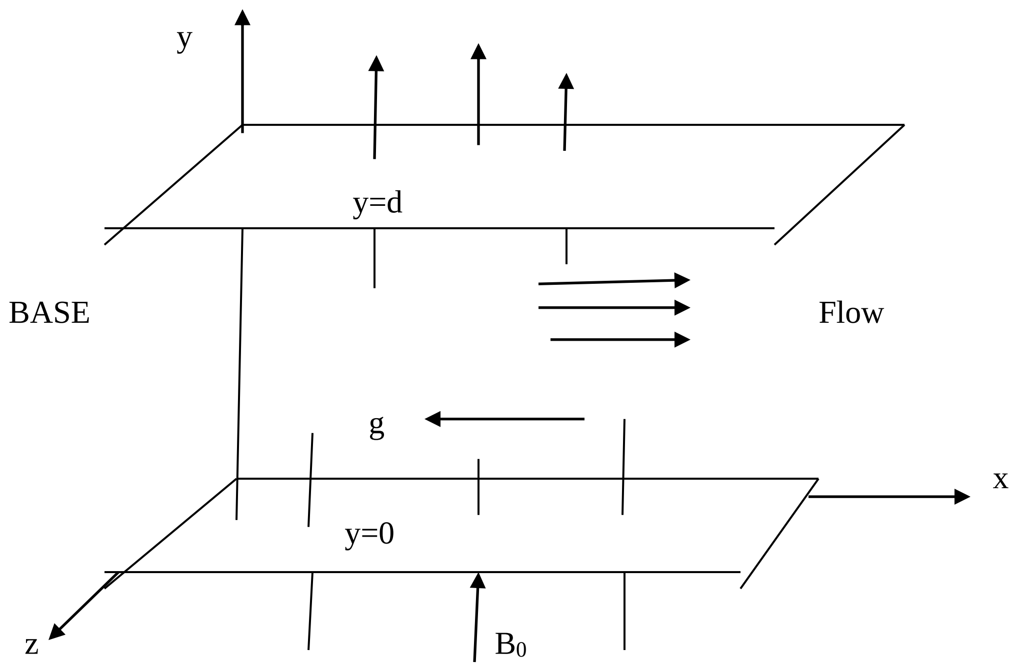


Figure 1: Flow configuration

The basic equations governing the flow are

Continuity equation:

$$\frac{\partial \bar{v}}{\partial \bar{y}} + \frac{\partial \bar{w}}{\partial \bar{z}} = 0\,; \tag{1}$$

Momentum equation:

$$\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} + \bar{w}\frac{\partial \bar{u}}{\partial \bar{z}} = g\beta\left(\bar{T} - \bar{T}_e\right) + g\bar{\beta}\left(\bar{C} - \bar{C}_e\right) + \upsilon\left(\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{z}^2}\right) - \frac{\sigma B_0^2 \bar{u}}{\rho}\,, \tag{2}$$

$$\bar{v}\frac{\partial \bar{v}}{\partial \bar{y}} + \bar{w}\frac{\partial \bar{v}}{\partial \bar{z}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{y}} + \upsilon\left(\frac{\partial^2 \bar{v}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{z}^2}\right)\,, \tag{3}$$

$$\bar{v}\frac{\partial \bar{w}}{\partial \bar{y}} + \bar{w}\frac{\partial \bar{w}}{\partial \bar{z}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{z}} + \upsilon\left(\frac{\partial^2 \bar{w}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{w}}{\partial \bar{z}^2}\right) - \frac{\sigma B_0^2 \bar{w}}{\rho}\,; \tag{4}$$

Energy equation:

$$\bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} + \bar{w}\frac{\partial \bar{T}}{\partial \bar{z}} = \frac{\kappa}{\rho C_p}\left(\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{z}^2}\right) - \frac{Q}{\rho C_p}\left(\bar{T}_e - \bar{T}\right) - \frac{1}{\rho C_p}\vec{\nabla}.\vec{q}_r\,; \tag{5}$$

Mass transfer equation:

$$\bar{v}\frac{\partial \bar{C}}{\partial \bar{y}}+\bar{w}\frac{\partial \bar{C}}{\partial \bar{z}}=D\left(\frac{\partial^2 \bar{C}}{\partial \bar{y}^2}+\frac{\partial^2 \bar{C}}{\partial \bar{z}^2}\right)+D_1\left(\frac{\partial^2 \bar{T}}{\partial \bar{y}^2}+\frac{\partial^2 \bar{T}}{\partial \bar{z}^2}\right)+D_2\left(\bar{C}-\bar{C_e}\right) . \tag{6}$$

We have $\vec{q}_r = q_r\hat{j}+q_r\hat{k}$.

Rooseland approximation on radiation is given by

$$q_r=-\frac{4\sigma^*}{3k^*}\left(\frac{\partial \bar{T}^4}{\partial \bar{y}}+\frac{\partial \bar{T}^4}{\partial \bar{z}}\right)$$

Equation (5) $\Rightarrow$ $\bar{v}\frac{\partial \bar{T}}{\partial \bar{y}}+\bar{w}\frac{\partial \bar{T}}{\partial \bar{z}}=\frac{\kappa}{\rho C_p}\left(\frac{\partial^2 \bar{T}}{\partial \bar{y}^2}+\frac{\partial^2 \bar{T}}{\partial \bar{z}^2}\right)-\frac{Q}{\rho C_p}\left(\bar{T_e}-\bar{T}\right)-\frac{16}{3}\frac{\sigma^*\bar{T_e}^3}{\rho C_p k^*}\left(\frac{\partial^2 \bar{T}}{\partial \bar{y}^2}+\frac{\partial^2 \bar{T}}{\partial \bar{z}^2}\right)$ (5.1)

The corresponding boundary conditions are:

$$\left.\begin{array}{l}\bar{y}=0;\bar{u}=0;\bar{v}=V\left(1+\varepsilon\cos\frac{\pi\bar{z}}{d}\right);\bar{w}=0;\bar{T}=\bar{T_0};\bar{C}=\bar{C_0}\\ \bar{y}=d;\bar{u}=U+L_1\frac{\partial\bar{u}}{\partial\bar{y}};\bar{v}=V;\bar{w}=0;\bar{T}=\bar{T_1};\bar{C}=\bar{C_1}\end{array}\right\} \tag{7}$$

where $L_1=\left(\frac{2-m_1}{m_1}\right)L$.

To normalize the governing equations we introduce the non-dimensional quantities as:

$$\left.\begin{array}{l}y=\frac{\bar{y}}{d},z=\frac{\bar{z}}{d},u=\frac{\bar{u}}{U},v=\frac{\bar{v}}{V},p=\frac{\bar{p}}{\rho V^2},\mathrm{Re}=\frac{Vd}{\upsilon},Pr=\frac{\mu C_p}{k},w=\frac{\bar{w}}{V},\\ Sc=\frac{\upsilon}{D},Sr=\frac{D_1\left(\bar{T_0}-\bar{T_e}\right)}{\upsilon\left(\bar{C_0}-\bar{C_e}\right)},\theta=\frac{\left(\bar{T}-\bar{T_e}\right)}{\left(\bar{T_0}-\bar{T_e}\right)},\phi=\frac{\left(\bar{C}-\bar{C_e}\right)}{\left(\bar{C_0}-\bar{C_e}\right)},M=\frac{\sigma B_0^2 d}{\rho V},\\ h=\frac{L_1}{d},\upsilon=\frac{\mu}{\rho},Gr=\frac{gd\beta\left(\bar{T_0}-\bar{T_e}\right)}{UV},Gm=\frac{gd\bar{\beta}\left(\bar{C_0}-\bar{C_e}\right)}{UV},n=\frac{\left(\bar{T_1}-\bar{T_e}\right)}{\left(\bar{T_0}-\bar{T_e}\right)},\\ r=\frac{\left(\bar{C_1}-\bar{C_e}\right)}{\left(\bar{C_0}-\bar{C_e}\right)},A=\frac{Qdv}{Vk},Kr=\frac{d^2D_2}{\upsilon Re},R=\frac{KK^*}{4\sigma^*T_e^3}\end{array}\right\} \tag{8}$$

Using (8) in the equations (1)-(6), we get the following non-dimensional equations as:

$$\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{9}$$

$$v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\left(Gr\theta+Gm\phi\right)+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)-Mu, \tag{10}$$

$$v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{\partial p}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right), \tag{11}$$

$$v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial z}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)-Mw, \tag{12}$$

$$v\frac{\partial \theta}{\partial y}+w\frac{\partial \theta}{\partial z}=\frac{\lambda}{\mathrm{Re}\,\mathrm{Pr}}\left(\frac{\partial^2 \theta}{\partial y^2}+\frac{\partial^2 \theta}{\partial z^2}\right)-\frac{A\theta}{\mathrm{Pr}}, \tag{13}$$

$$v\frac{\partial \phi}{\partial y}+w\frac{\partial \phi}{\partial z}=\frac{1}{\mathrm{Re}}\left[\frac{1}{Sc}\left(\frac{\partial^2 \phi}{\partial y^2}+\frac{\partial^2 \phi}{\partial z^2}\right)+Sr\left(\frac{\partial^2 \theta}{\partial y^2}+\frac{\partial^2 \theta}{\partial z^2}\right)\right]+Kr\phi, \tag{14}$$

Using the dimensionless quantities (8) in (7), we get the relevant boundary conditions as:

$$\left.\begin{aligned}&y=0, u=0, v=1+\varepsilon\cos\pi z, w=0, \theta=1, \phi=1\\&y=1, u=1+h\frac{\partial u}{\partial y}, v=1, w=0, \theta=n, \phi=r\end{aligned}\right\} \tag{15}$$

3. METHOD OF SOLUTION

The solution to the flow problem, using perturbation technique is assumed to be of the following form where higher powers of ε are neglected

$$\left.\begin{aligned}&v(y,z)=v_0(y)+\varepsilon v_1(y,z)\\&u(y,z)=u_0(y)+\varepsilon u_1(y,z)\\&w(y,z)=w_0(y)+\varepsilon w_1(y,z)\\&\theta(y,z)=\theta_0(y)+\varepsilon\theta_1(y,z)\\&\phi(y,z)=\phi_0(y)+\varepsilon\phi_1(y,z)\\&p(y,z)=p_0(y)+\varepsilon p_1(y,z)\end{aligned}\right\} \tag{16}$$

The problem reduces to a two dimensional flow for $\varepsilon=0$ which is governed by the following equations obtained from (9)-(14). Taking $w_0=0, v_0=1, p_0=$ constant, we have

$$\frac{dv_0}{dy}=0, \tag{17}$$

$$\frac{d^2u_0}{dy^2}-\mathrm{Re}\frac{du_0}{dy}-\mathrm{Re}\,Mu_0=-\mathrm{Re}\left(Gr\theta_0+Gm\phi_0\right) \quad , \tag{18}$$

$$\frac{dp_0}{dy}=0 \quad , \tag{19}$$

$$\frac{d^2w_0}{dy^2}-\mathrm{Re}\frac{dw_0}{dy}-Mw_0\,\mathrm{Re}=0 \ , \tag{20}$$

$$\frac{d^2\theta_0}{dy^2}-\frac{\mathrm{Pr}\,\mathrm{Re}}{\lambda}\frac{d\theta_0}{dy}+\frac{A\theta_0\,\mathrm{Re}}{\lambda}=0 \ , \tag{21}$$

$$\frac{d^2\phi_0}{dy^2}-Sc\,\mathrm{Re}\frac{d\phi_0}{dy}+SrSc\frac{d^2\theta_0}{dy^2}+\mathrm{Re}\,ScKr\phi_0=0 \ . \tag{22}$$

Using (16) in (15), the corresponding conditions are:

$$\left.\begin{aligned} &y=0; u_0=0, v_0=1, w_0=0, \theta_0=1, \phi_0=1 \\ &y=1; u_0=1+h\frac{du_0}{dy}, v_0=1, w_0=0, \theta_0=n, \phi_0=r \end{aligned}\right\} \tag{23}$$

Solving equations (17)-(22) using boundary conditions (23), the following solutions are obtained:

$$u_0=c_7e^{t_5y}+c_8e^{t_6y}+c_9e^{t_1y}+c_{10}e^{t_2y}+c_{11}e^{t_3y}+c_{12}e^{t_4y} \quad , \tag{24}$$

$$\phi_0=c_3e^{t_3y}+c_4e^{t_4y}+c_5e^{t_1y}+c_6e^{t_2y} \quad , \tag{25}$$

$$\theta_0=c_1e^{t_1y}+c_2e^{t_2y} \quad . \tag{26}$$

when $\varepsilon\neq 0$, substituting (16) in equations (9)-(14) respectively and equating the coefficients of like powers of ε and avoiding higher powers of ε, we obtain the following equations:

$$\frac{\partial v_1}{\partial y}+\frac{\partial w_1}{\partial z}=0 \quad , \tag{27}$$

$$\frac{\partial u_1}{\partial y}+v_1\frac{\partial u_0}{\partial z}=\left(Gr\theta_1+Gm\phi_1\right)+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2u_1}{\partial y^2}+\frac{\partial^2u_1}{\partial z^2}\right)-Mu_1 \ , \tag{28}$$

$$\frac{\partial v_1}{\partial y}=-\frac{\partial p_1}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2v_1}{\partial y^2}+\frac{\partial^2v_1}{\partial z^2}\right) , \tag{29}$$

$$\frac{\partial w_1}{\partial y}=-\frac{\partial p_1}{\partial z}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2w}{\partial y^2}+\frac{\partial^2w}{\partial z^2}\right)-Mw_1 \ , \tag{30}$$

$$\frac{\partial\theta_1}{\partial y}+v_1\frac{\partial\theta_0}{\partial y}=\frac{\lambda}{\mathrm{Re}\,\mathrm{Pr}}\left(\frac{\partial^2\theta_1}{\partial y^2}+\frac{\partial^2\theta_1}{\partial z^2}\right)-\frac{A\theta_1}{\mathrm{Pr}}\ , \tag{31}$$

$$\frac{\partial\phi_1}{\partial y}+v_1\frac{\partial\phi_0}{\partial y}=\frac{1}{\mathrm{Re}}\left[\frac{1}{Sc}\left(\frac{\partial^2\phi_1}{\partial y^2}+\frac{\partial^2\phi_1}{\partial z^2}\right)+Sr\left(\frac{\partial^2\theta_1}{\partial y^2}+\frac{\partial^2\theta_1}{\partial z^2}\right)\right]+Kr\phi_1\ . \tag{32}$$

The corresponding boundary conditions:

$$\left.\begin{array}{l} y=0; u_1=0, v=\cos\pi z, w_1=0, \theta_1=1, \phi_1=0 \\ y=1; u_1=h\dfrac{\partial u_1}{\partial y}, v_1=0, w_1=0, \theta_1=0, \phi_1=0 \end{array}\right\} \tag{33}$$

Solutions of cross flow

Equations (27), (29) and (30) are the governing equations for the cross flow. The solutions for $v_1(y,z)$, $w_1(y,z)$ and $p_1(y,z)$ are assumed to be of following form:

$$\left.\begin{array}{l} v_1(y,z)=v_{11}(y)\cos\pi z \\ w_1(y,z)=-\dfrac{1}{\pi}v_{11}'(y)\sin\pi z \\ p_1(y,z)=p_{11}(y)\cos\pi z \end{array}\right\} \tag{34}$$

The boundary conditions obtained by substituting (34) in (33) are:

$$\left.\begin{array}{l} y=0; v_{11}=1, v_{11}'=0 \\ y=1; v_{11}=0, v_{11}'=0 \end{array}\right\} \tag{35}$$

Substituting (34) in equations (27), (29) and (30), and solving them subject to the boundary conditions (35), the following solutions are obtained as:

$$v_1=\left(c_{13}e^{t_7 y}+c_{14}e^{t_8 y}+c_{15}e^{t_9 y}+c_{16}e^{t_{10} y}\right)\cos\pi z \tag{36}$$

$$w_1=\left(c_{17}e^{t_7 y}+c_{18}e^{t_8 y}+c_{19}e^{t_9 y}+c_{20}e^{t_{10} y}\right)\sin\pi z \tag{37}$$

Main flow, temperature and species concentration solutions

The equations for the main flow, temperature and species concentration fields are given by (28), (31) and (32) respectively.

The following assumptions are made for the solutions of $u_1(y,z)$, $\theta_1(y,z)$ and $\phi_1(y,z)$:

$$\left.\begin{aligned} u_1(y,z) &= u_{11}(y)\cos\pi z \\ \theta_1(y,z) &= \theta_{11}(y)\cos\pi z \\ \phi_1(y,z) &= \phi_{11}(y)\cos\pi z \end{aligned}\right\} \quad (38)$$

The boundary conditions obtained by substituting (38) in (33) are:

$$\left.\begin{aligned} y=0; & u_{11}=0, \theta_{11}=0, \phi_{11}=0 \\ y=1; & u_{11}=hu_{11}^{/}, \theta_{11}=0, \phi_{11}=0 \end{aligned}\right\} \quad (39)$$

Using the above substitutions (38) for equations (28), (31) and (32) and then solving them subject to the boundary conditions (39), we obtain the solutions for u_1, θ_1 and ϕ_1 as follows:

$$\begin{aligned} u_1 = (&c_{51}e^{t_{11}y} + c_{52}e^{t_{12}y} + c_{53}e^{t_{13}y} + c_{54}e^{t_{14}y} + c_{55}e^{t_{15}y} + c_{56}e^{t_{16}y} + c_{57}e^{t_{17}y} + c_{58}e^{t_{18}y} + c_{59}e^{t_{19}y} + c_{60}e^{t_{20}y} \\ &+ c_{61}e^{t_{21}y} + c_{62}e^{t_{22}y} + c_{63}e^{t_{23}y} + c_{64}e^{t_{24}y} + c_{65}e^{t_{25}y} + c_{66}e^{t_{26}y} + c_{67}e^{t_{27}y} + c_{68}e^{t_{28}y} + c_{69}e^{t_{29}y} + c_{70}e^{t_{30}y} \\ &+ c_{71}e^{t_{31}y} + c_{72}e^{t_{32}y} + c_{73}e^{t_{33}y} + c_{74}e^{t_{34}y} + c_{75}e^{t_{35}y} + c_{76}e^{t_{36}y} + c_{77}e^{t_{37}y} + c_{78}e^{t_{38}y} + c_{79}e^{t_{39}y} + c_{80}e^{t_{40}y})\cos\pi z \end{aligned} \quad (40)$$

$$\theta_1 = (c_{21}e^{t_{21}y} + c_{22}e^{t_{22}y} + c_{23}e^{t_{23}y} + c_{24}e^{t_{24}y} + c_{25}e^{t_{25}y} + c_{26}e^{t_{26}y} + c_{27}e^{t_{27}y} + c_{28}e^{t_{28}y} + c_{29}e^{t_{29}y} + c_{30}e^{t_{30}y})\cos\pi z$$

(41)

$$\begin{aligned} \phi_1 = (&c_{31}e^{t_{11}y} + c_{32}e^{t_{12}y} + c_{33}e^{t_{13}y} + c_{34}e^{t_{14}y} + c_{35}e^{t_{15}y} + c_{36}e^{t_{16}y} + c_{37}e^{t_{17}y} + c_{38}e^{t_{18}y} + c_{39}e^{t_{19}y} + c_{40}e^{t_{20}y} \\ &+ c_{41}e^{t_{21}y} + c_{42}e^{t_{22}y} + c_{43}e^{t_{23}y} + c_{44}e^{t_{24}y} + c_{45}e^{t_{25}y} + c_{46}e^{t_{26}y} + c_{47}e^{t_{27}y} + c_{48}e^{t_{28}y} + c_{49}e^{t_{29}y} + c_{50}e^{t_{30}y})\cos\pi z \end{aligned} \quad (42)$$

Hence the solutions for velocity, temperature and concentration of the fluid are obtained by substituting (24)-(26) and (40)-(42) in (16).

Skin friction

The shear stress τ in the main flow direction at the plate y = 0 based on Newton's law of viscosity is given as

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = u_0^{/}(0) + \varepsilon u_{11}^{/}(0)\cos\pi z \quad (43)$$

Nusselt number

The rate of heat transfer in terms of Nusselt number *Nu* quantified by Fourier law of conduction is given by

$$Nu = \left(\frac{\partial \theta}{\partial y}\right)_{y=0} = \theta_0^{/}(0) + \varepsilon \theta_{11}^{/}(0)\cos \pi z \tag{44}$$

Sherwood number

The rate of transfer of concentration of species *Sh* at the plate *y* = 0 based on Fick's law of diffusion is obtained as

$$Sh = \left(\frac{\partial \phi}{\partial y}\right)_{y=0} = \phi_0^{/}(0) + \varepsilon \phi_{11}^{/}(0)\cos \pi z \tag{45}$$

4. RESULTS AND DISCUSSION

The paper aims to investigate the effects of Soret number (*Sr*), thermal radiation (*R*), Schimdt number (*Sc*), Reynolds number (*Re*), thermal Grashof number (*Gr*), solutal Grashof number (*Gm*), heat absorption parameter (*A*), Prandtl number (*Pr*), Hartmann number (*M*), slip parameter (*h*) on main flow velocity *u*, temperature θ and concentration φ which are demonstrated through graphs. Also the effect of these parameters are studied on skin friction (τ), Nusselt number (*Nu*) and Sherwood number (*Sh*) which are exhibited through tables. For the present investigation, the values of ε and *z* are taken to be 0.01 and 0.3 respectively.

The fluid velocity is generally higher near the moving surface, and decreases to zero value far away from the plate surface, thereby satisfying the far field boundary condition for the values of the parameter which is seen from Figure 2-8. Figure 2 illustrates that the effect of magnetic field on the fluid motion retards the fluid motion. This agrees with the fact that magnetic field creates a drag force known as the Lorentz force which opposes the fluid motion. In Figure 3, it is seen that the thermal Grashof number accelerates the fluid velocity due to the enhancement in buoyancy force. From Figure 4 it is observed that as thermal radiation *R* rises, the velocity of the flow increases. It is seen that velocity *u* increases with the increase in Soret number (*Sr*) which is depicted in Figure 5, whereas this trend gets reversed for different values of *Kr* and *h* as shown in the figures 6 and 7. The fluid motion increases for increased values of *Re*, in the region given approximately by $y \in (0, 0.84)$ which is exhibited in Figure 8. But this behavior reverses in the region very close to the moving plate. It is noticed from Figure 9 that the velocity is accelerated by solutal Grashof number *Gm*. This is true because as *Gm* increases viscosity of the fluid decreases or buoyancy force increases.

The effects of *Re*, *R*, *Pr* and *A* on the temperature field are shown in the figures 10, 11, 12 and 13. From these figures it is clear that there is a comprehensive fall in the temperature for increasing the values of *Re*, *Pr* and *A* but rises with higher values of *R*. This is due to the reason that as thermal radiation increases, energy is emitted from the heated surface which results in the rise of the temperature. Figures 14, 15, 16 and 17 depict the effect of *A*, *Kr*, *R* and *Pr* on the concentration field of the fluid. These figures simulate that the species concentration of the fluid decreases with higher values of *A* and *R* but increases with the increments in *Kr* and *Pr*.

The behavior of skin friction (τ), Nusselt number (*Nu*) and Sherwood number (*Sh*) under the influence of *M*, *Gr*, *R*, *Kr*, *Pr* and *Re* are interpreted through tables 1, 2 and 3. From Table 1, it can be concluded that the shear stress at the wall reduces for increased values of Hartmann number *M* and thermal radiation *R*, whereas it increases with *Gr* and *Re*. The rate of heat transfer declines due to the effects of *Re*, *Pr* and *R* which is reflected in Table 2. It is observed from Table 3 that Sherwood number *Sh* decreases against the increasing values of *Re*, *Kr* and *R*.

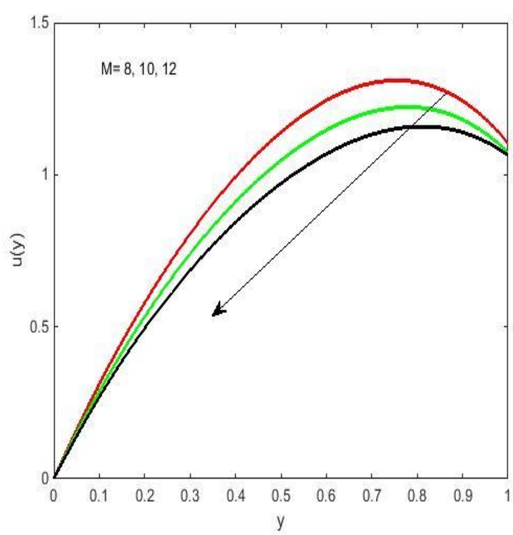


Figure 2: Velocity profile with variation in *M* for *Gr*=10,*Gm*=10,*Pr*=0.71,*A*=1.5,*h*=0.1, *r*=2,*n*=2,*R*=0.5,*Sr*=2,*Sc*=0.22,*Re*=0.5,*Kr*=5

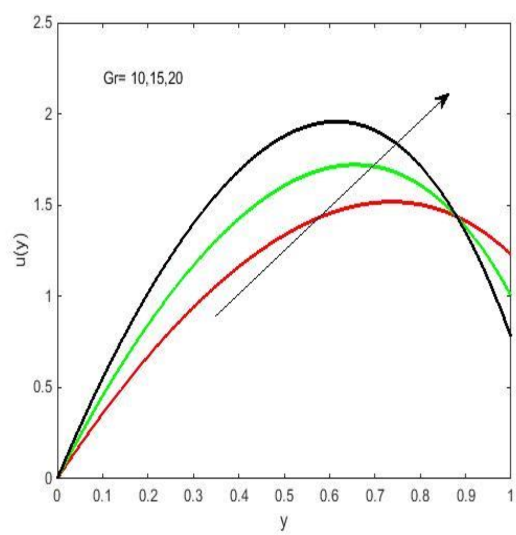


Figure 3: Velocity profile with variation in *Gr* for *M*=10,*Gm*=10,*Pr*=0.71,*A*=1.5,*h*=0.1 *r*=2,*n*=2,*R*=0.5,*Sr*=2,*Sc*=0.22,*Re*=0.5,*Kr*=5

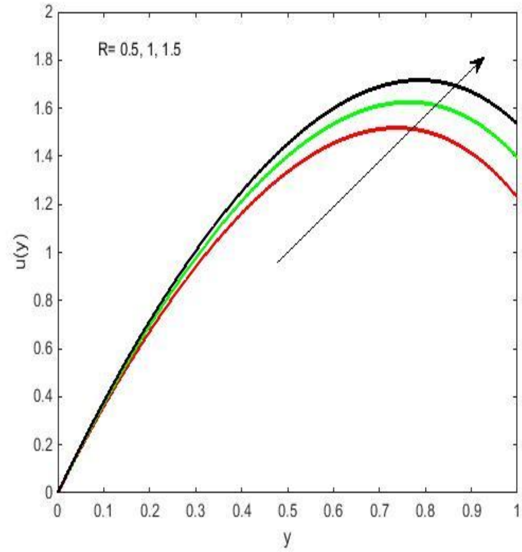


Figure 4: Velocity profile with variation in R for M=10,Gm=10,Pr=0.71,A=1.5,h=0.1, r=2,n=2,Gr=10,Sr=2,Sc=0.22,Re=0.5,Kr=5

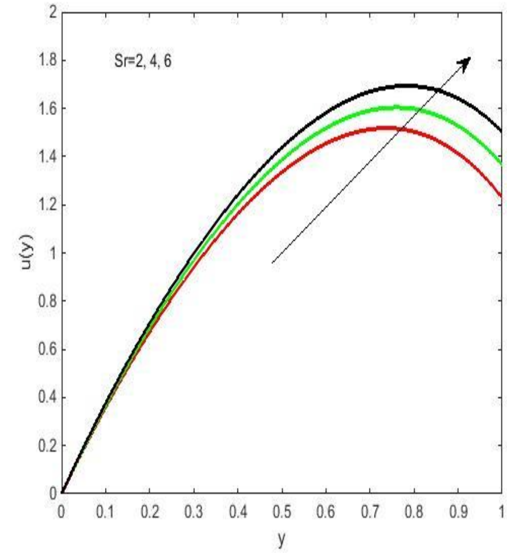


Figure 5: Velocity profile with variation in Sr for M=10,Gm=10,Pr=0.71,A=1.5,h=0.1, r=2,n=2,R=0.5,Gr=10,Sc=0.22,Re=0.5,Kr=5

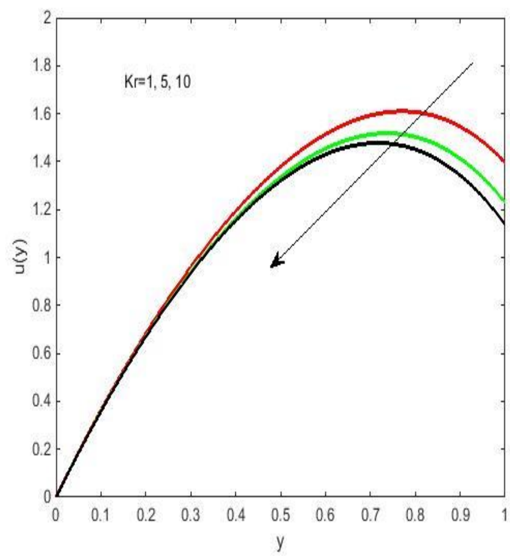


Figure 6: Velocity profile with variation in Kr for M=10,Gm=10,Pr=0.71,A=1.5,h=0.1, r=2,n=2,R=0.5,Gr=10,Sc=0.22,Re=0.5,Sr=2

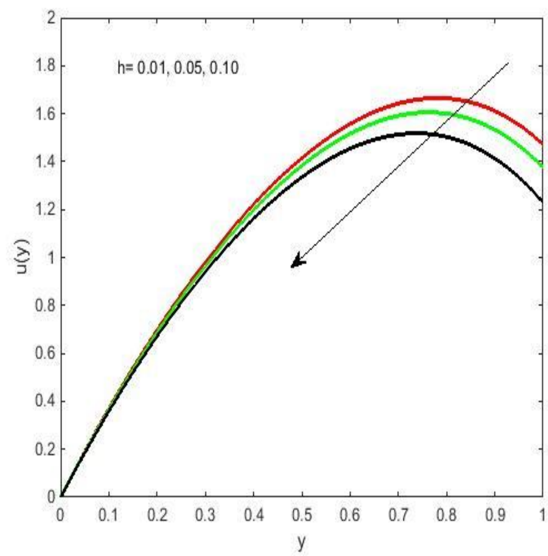


Figure 7: Velocity profile with variation in h for M=10,Gm=10,Pr=0.71,A=1.5,Sr=2, r=2,n=2,R=0.5,Gr=10,Sc=0.22,Re=0.5,Kr=5

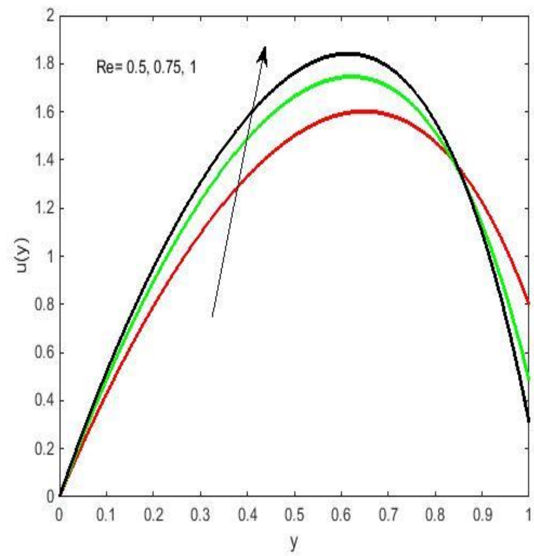


Figure 8: Velocity profile with variation in *Re* for M=10,Gm=10,Pr=0.71,A=1.5,h=0.1, r=2,n=2,R=0.5,Gr=10,Sc=0.22,Sr=2,Kr=5

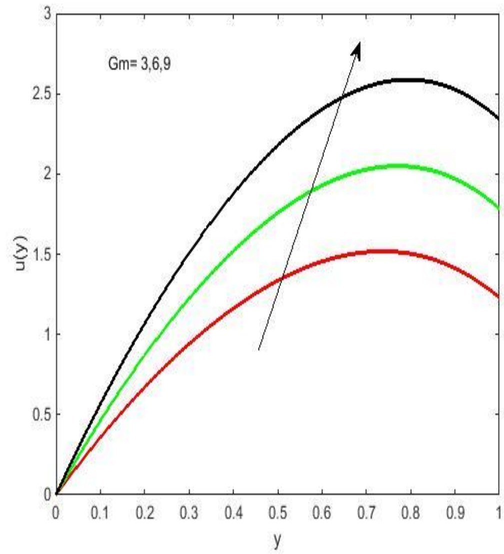


Figure 9: Velocity profile with variation in *Gm* for M=10,Gm=10,Pr=0.71,A=1.5,h=0.1, r=2,n=2,R=0.5,Gr=10,Sc=0.22,Re=0.5,Sr=2

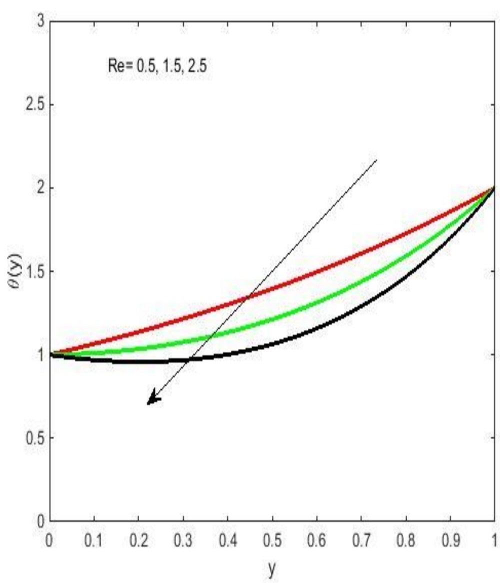


Figure 10: Temperature profile with variation in *Re* for R=0.5,Pr=0.71,A=1.5,n=2.

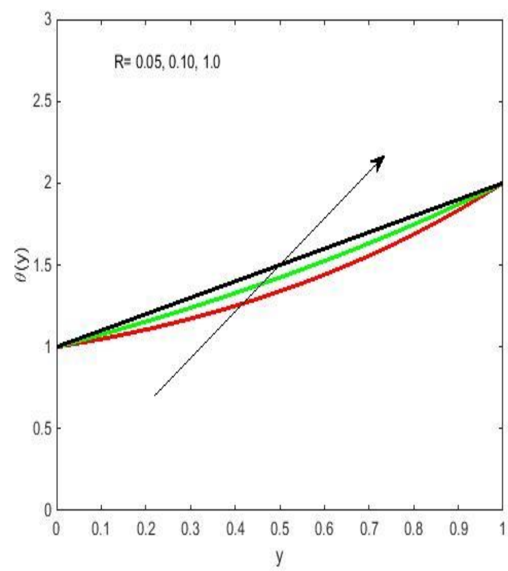


Figure 11:Temperature profile with variation in *R* for Re=0.5,Pr=0.71,A=1.5,n=2.

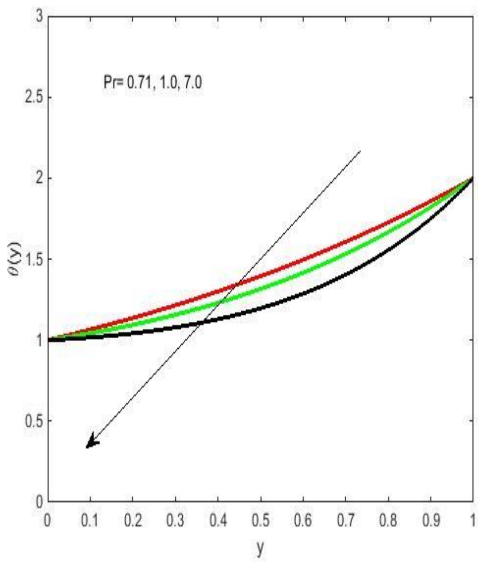


Figure 12:Temperature profile with variation in *Pr* for *R*=0.5,*Re*=0.5,*A*=1.5,*n*=2.

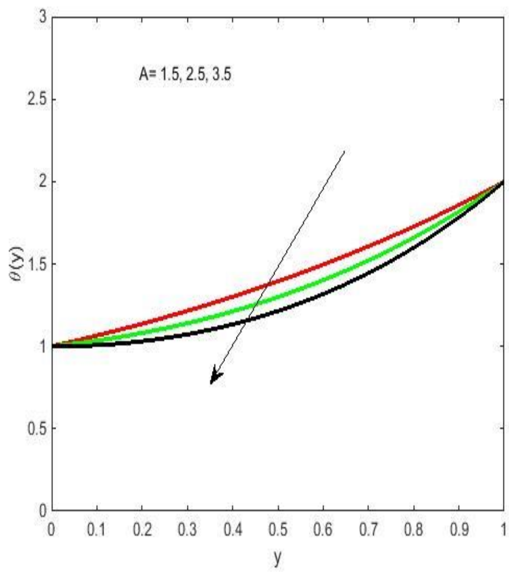


Figure 13:Temperature profile with variation in *A* for *R*=0.5,*Pr*=0.71,*Re*=0.5,*n*=2.

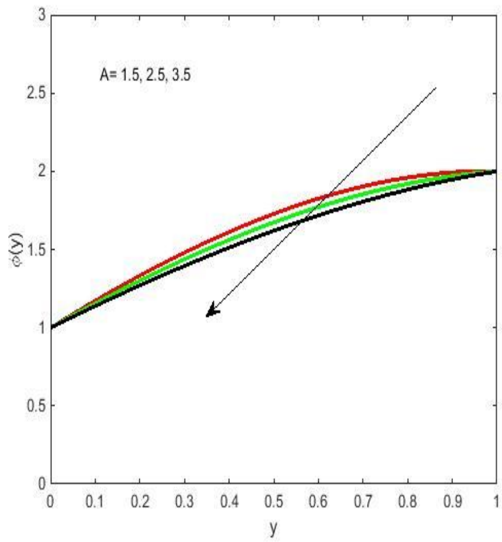


Figure 14:Concentration profile with variation in *A* for *R*=0.5,*Re*=0.5,*Pr*=0.71,*n*=2,*Kr*=5,*Sr*=2 *Sc*=0.22,*r*=2.

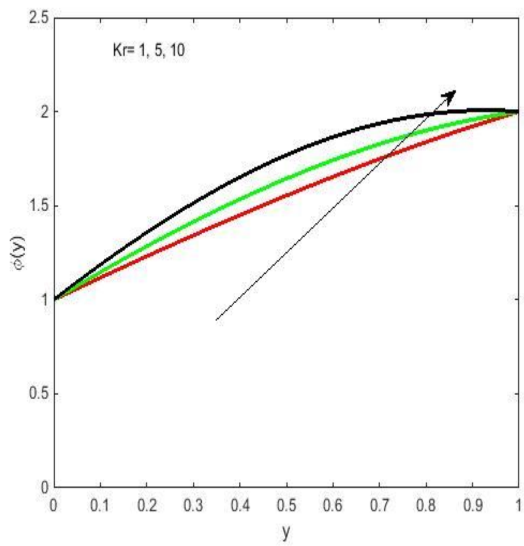
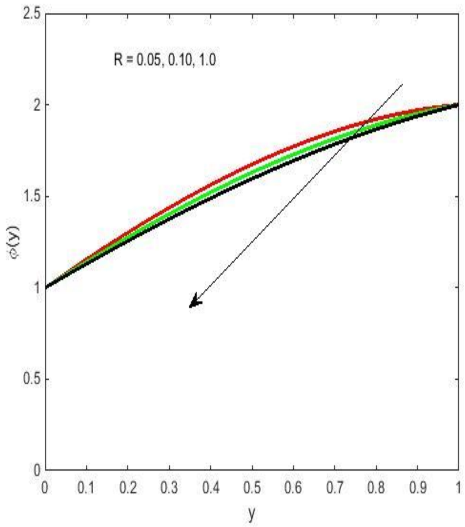


Figure 15:Concentration profile with variation in *Kr* for *R*=0.5,*Re*=0.5,*Pr*=0.71,*n*=2,*A*=1.5, *Sr*=2,*Sc*=0.22,*r*=2.



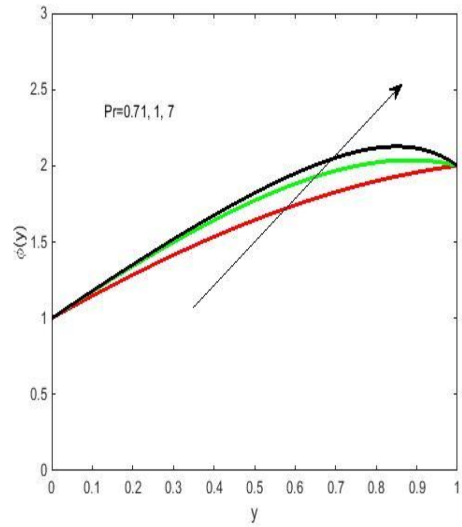


Figure 16:Concentration profile with variation in R for A=1.5,Re=0.5,Pr=0.71,n=2,Kr=5,Sr=2 Sc=0.22,r=2

Figure 17:Concentration profile with variation in Pr for A=1.5,Re=0.5,Pr=0.71,n=2,Kr=5,Sr=2 Sc=0.22,r=2

Table 1: Numerical values of Skin friction for different values M, Gr, R and Re

M	Gr	R	Re	τ
5	10	0.5	0.5	**4.0889**
10	10	0.5	0.5	**3.3436**
15	10	0.5	0.5	**2.8731**
5	**10**	0.5	0.5	**4.0889**
5	**15**	0.5	0.5	**5.1844**
5	**20**	0.5	0.5	**6.2799**
5	10	**0.5**	0.5	**4.0889**
5	10	**1.0**	0.5	**3.9256**
5	10	**1.5**	0.5	**3.8523**
5	10	0.5	**0.5**	**4.0889**
5	10	0.5	**1.0**	**5.2926**
5	10	0.5	**1.5**	**6.2382**

Table 2: Numerical values of Nusselt number for different values Re, Pr and R

Re	**Pr**	**R**	**Nu**
0.5	0.71	0.5	**0.8217**
1.0	0.71	0.5	**0.6558**
1.5	0.71	0.5	**0.5015**
0.5	**0.71**	0.5	**0.8217**
0.5	**1.0**	0.5	**0.8040**
0.5	**7**	0.5	**0.4930**
0.5	0.71	**0.5**	**0.8217**
0.5	0.71	**1.0**	**0.7255**
0.5	0.71	**1.5**	**0.6652**

Table3: Numerical values of Sherwood number for different values Re, Kr and R

Re	Kr	R	Sh
0.5	5	0.5	**1.4024**
1.0	5	0.5	**1.8265**
1.5	5	0.5	**2.2768**
0.5	**1**	0.5	**1.0957**
0.5	**5**	0.5	**1.4024**
0.5	**10**	0.5	**1.8251**
0.5	5	**0.5**	**1.4024**
0.5	5	**1.0**	**1.4461**
0.5	5	**1.5**	**1.4734**

5. CONCLUSIONS

(1) The velocity profiles decrease with increasing values of *M*, *Kr*, *h* but accelerated due to *R*, *Gm*, *Gr*, *Sr*.

(2) The temperature of the fluid rises substantially due to thermal radiation *R*.

(3) The species concentration at plate is decreased with higher values of *R* and *A*.

(4) An increase in *R* causes the skin friction and Sherwood number to increase whereas it shows an opposite behavior for Nusselt number.

NOMENCLATURE

$\bar{u}$ = component of velocity along $\bar{x}$.

$\bar{v}$ = component of velocity along $\bar{y}$.

$\bar{w}$ = component of velocity along $\bar{z}$.

g = acceleration due to gravity.

β = coefficient of volume expansion for heat transfer.

$\bar{\beta}$ = coefficient of volumetric expansion with species concentration.

$\bar{T}$ = fluid temperature.

$\bar{T}_e$ = equilibrium temperature of the fluid.

$\bar{C}$ = molar species concentration of the fluid.

$\bar{C}_e$ = equilibrium molar species concentration of the fluid.

C_p = specific heat at constant pressure.

$\bar{P}$ = fluid pressure.

υ = kinematic viscosity.

ρ = fluid density.

σ = electrical conductivity of the fluid.

B_0 = magnetic field component along y- axis.

κ = thermal conductivity.

Q = dimensionless heat absorption coefficient.

D = chemical molecular diffusivity.

D_1 = coefficient of thermal diffusivity.

D_2 = chemical reaction rate constant.

q_r = radiative heat flux.

$\bar{T}_0$ = temperature of the fluid at rest.

$\bar{T}_1$ = temperature of the fluid at the plate in motion.

$\bar{C}_0$ = molar species concentration of the fluid at the rest plate.

$\bar{C}_1$ = molar species concentration of the fluid at the plate in motion.

L = mean free path.

m_1 = Maxwell's reflection coefficient.

V = undisturbed part of the injection velocity.

d = wavelength of the periodic injection velocity.

ε = reference parameter.

p = dimensionless fluid pressure.

Re = Reynolds number.

Pr = Prandtl number.

Sc = Schmidt number.

M = Hartmann number.

A = heat absorption parameter.

Gr = thermal Grashof number.

Gm = solutal Grashof number.

Sr = Soret number.

Kr = chemical reaction parameter.

R = thermal radiation.

h = slip parameter.

A= heat absorption parameter.

n = wall temperature ratio.

r = wall concentration ratio.

θ = dimensionless temperature.

ϕ = dimensionless concentration.

ϕ_0 = dimensionless concentration at rest.

θ_0 = dimensionless temperature at rest.

u_0 = fluid velocity along x-axis at rest.

v_0 = fluid velocity along y-axis at rest.

w_0 = fluid velocity along z-axis at rest.

APPENDIX

$\lambda = 1 + \frac{4}{3R}$, $a_0 = \frac{\Pr \operatorname{Re}}{\lambda}$, $a_1 = a_0\left(\frac{A}{\Pr} - \frac{\pi^2}{a_0}\right)$, $b_0 = \frac{\operatorname{Re} A}{\lambda}$, $c_0 = Sc\operatorname{Re}$, $d_0 = Sc\operatorname{Re}Kr$, $e_0 = \operatorname{Re}$,

$f_0 = 4\operatorname{Re}M$, $g_0 = \sqrt{a_0^2 - 4\left(b_0^2 - \pi^2\right)}$, $h_0 = ScSr$, $A_1 = \sqrt{e_0^2 + f_0}$, $B = \frac{1}{2}\left(A_1 - e_0\right)$,

$C = \frac{1}{2}\left(A_1 + e_0\right)$, $F = \frac{e_0^2}{2} + \frac{f_0}{4} + 4\pi^2$, $G = \frac{1}{2}e_0 A_1$, $D = \sqrt{F - G}$, $E = \sqrt{F + G}$,

$t_1 = \frac{a_0 + \sqrt{a_0^2 - 4b_0}}{2}$, $t_2 = \frac{a_0 - \sqrt{a_0^2 - 4b_0}}{2}$, $t_3 = \frac{c_0 - \sqrt{c_0^2 - 4d_0}}{2}$, $t_4 = \frac{c_0 + \sqrt{c_0^2 - 4d_0}}{2}$,

$t_5 = \frac{e_0 + \sqrt{e_0^2 + f_0}}{2}$, $t_6 = \frac{e_0 - \sqrt{e_0^2 + f_0}}{2}$, $t_7 = \frac{1}{2}\left(D - B\right)$, $t_7 = \frac{1}{2}\left(D + B\right)$, $t_9 = \frac{1}{2}\left(E + C\right)$,

$t_{10} = \frac{1}{2}\left(E - C\right)$, $t_{11} = \frac{c_0 + \sqrt{c_0^2 - 4\left(d_0 - \pi^2\right)}}{2}$, $t_{12} = \frac{c_0 - \sqrt{c_0^2 - 4\left(d_0 - \pi^2\right)}}{2}$, $t_{13} = t_3 + t_7$,

$t_{14} = t_4 + t_7$, $t_{15} = t_3 + t_8$, $t_{16} = t_4 + t_8$, $t_{17} = t_3 + t_9$, $t_{18} = t_4 + t_9$, $t_{19} = t_3 + t_{10}$, $t_{20} = t_4 + t_{10}$,

$t_{21} = \frac{a_0 + \sqrt{a_0^2 - 4a_1}}{2}$, $t_{22} = \frac{a_0 - \sqrt{a_0^2 - 4a_1}}{2}$, $t_{23} = t_1 + t_7$, $t_{24} = t_2 + t_7$, $t_{25} = t_1 + t_8$, $t_{26} = t_2 + t_8$,

$t_{27} = t_1 + t_9$, $t_{28} = t_2 + t_9$, $t_{29} = t_1 + t_{10}$, $t_{30} = t_2 + t_{10}$, $t_{31} = t_5 + t_7$, $t_{32} = t_6 + t_7$, $t_{33} = t_5 + t_8$,

$t_{34} = t_6 + t_8$, $t_{35} = t_5 + t_9$, $t_{36} = t_6 + t_9$, $t_{37} = t_5 + t_{10}$, $t_{38} = t_6 + t_{10}$, $t_{39} = \frac{e_0 + \sqrt{e_0^2 + 4\pi^2 + f_0}}{2}$,

$t_{40} = \frac{e_0 - \sqrt{e_0^2 + 4\pi^2 + f_0}}{2}$, $c_1 = \frac{e^{t_2} - n}{e^{t_2} - e^{t_1}}$, $c_2 = \frac{n - e^{t_2}}{e^{t_2} - e^{t_1}}$, $c_5 = \frac{-ScSrc_1 t_1^2}{f_1\left(t_1\right)}$, $c_6 = \frac{-ScSrc_2 t_2^2}{f_1\left(t_2\right)}$,

$f_1\left(x\right) = x^2 - c_0 x + d_0$, $K_1 = 1 - c_5 - c_6$, $K_2 = r - c_5 e^{t_1} - c_6 e^{t_2}$, $c_3 = \frac{K_1 e^{t_4} - K_2}{e^{t_4} - e^{t_3}}$, $c_4 = \frac{K_2 - K_1 e^{t_3}}{e^{t_4} - e^{t_3}}$,

$c_9 = \frac{\operatorname{Re}Grc_1 + \operatorname{Re}Gmc_5}{f_2\left(t_1\right)}$, $c_{10} = \frac{\operatorname{Re}Grc_2 + \operatorname{Re}Gmc_6}{f_2\left(t_2\right)}$, $c_{11} = \frac{\operatorname{Re}Gmc_3}{f_2\left(t_3\right)}$, $c_{12} = \frac{\operatorname{Re}Gmc_4}{f_2\left(t_4\right)}$,

$f_2\left(x\right) = \operatorname{Re}M + \operatorname{Re}x - x^2$, $L_1 = c_9 + c_{10} + c_{11} + c_{12}$,

$L_2 = c_9 e^{t_1}\left(1 - ht_1\right) + c_{10}e^{t_2}\left(1 - ht_2\right) + c_{11}e^{t_3}\left(1 - ht_3\right) + c_{12}e^{t_4}\left(1 - ht_4\right) - 1$, $L_{21} = e^{t_5}\left(1 - ht_5\right)$,

$L_{22} = e^{t_6}\left(1 - ht_6\right)$, $c_7 = \frac{L_2 - L_1 L_{22}}{L_{22} - L_{21}}$, $c_8 = \frac{L_1 L_{21} - L_2}{L_{22} - L_{21}}$,

$$D_r=\left(e^{t_7}-e^{t_8}\right)\left(e^{t_{10}}-e^{t_9}\right)\left(t_7t_8+t_9t_{10}\right)+\left(e^{t_9}-e^{t_7}\right)\left(e^{t_{10}}-e^{t_8}\right)\left(t_7t_9+t_8t_{10}\right)$$
$$+\left(e^{t_8}-e^{t_9}\right)\left(e^{t_{10}}-e^{t_7}\right)\left(t_9t_8+t_7t_{10}\right),$$
$$N_1=e^{t_9+t_{10}}\left(t_8t_{10}-t_8t_9\right)+e^{t_8+t_{10}}\left(t_8t_9-t_{10}t_9\right)+e^{t_9+t_8}\left(t_9t_{10}-t_8t_{10}\right)$$
$$N_2=e^{t_9+t_{10}}\left(t_7t_9-t_7t_{10}\right)+e^{t_7+t_{10}}\left(t_{10}t_9-t_7t_9\right)+e^{t_9+t_7}\left(t_7t_{10}-t_9t_{10}\right),$$
$$N_3=e^{t_8+t_{10}}\left(t_7t_{10}-t_7t_8\right)+e^{t_7+t_{10}}\left(t_8t_7-t_{10}t_8\right)+e^{t_8+t_7}\left(t_8t_{10}-t_7t_{10}\right),$$
$$N_4=e^{t_8+t_9}\left(t_7t_8-t_7t_9\right)+e^{t_7+t_9}\left(t_8t_9-t_7t_8\right)+e^{t_8+t_7}\left(t_9t_7-t_8t_9\right),$$
$$c_{13}=\frac{N_1}{D_r},\quad c_{14}=\frac{N_2}{D_r},\quad c_{15}=\frac{N_3}{D_r},\quad c_{16}=\frac{N_4}{D_r},\quad a=a_0c_1c_{13}t_1,\quad b=a_0c_2c_{13}t_2,\quad c=a_0c_1c_{14}t_1,$$
$$d=a_0c_2c_{14}t_2,\ e=a_0c_1c_{15}t_1,\ f=a_0c_2c_{15}t_2,\ g=a_0c_1c_{16}t_1,\ h=a_0c_2c_{16}t_2,\ c_{17}=\frac{-t_7c_{13}}{\pi},\ c_{18}=\frac{-t_8c_{14}}{\pi},$$
$$c_{19}=\frac{-t_9c_{15}}{\pi},\ c_{20}=\frac{-t_{10}c_{16}}{\pi},\ f_3(x)=x^2-a_0x+a_1,\ c_{23}=\frac{a}{f_3(t_{23})},\ c_{24}=\frac{b}{f_3(t_{24})},\ c_{25}=\frac{c}{f_3(t_{25})},$$
$$c_{26}=\frac{d}{f_3(t_{26})},\quad c_{27}=\frac{e}{f_3(t_{27})},\quad c_{28}=\frac{f}{f_3(t_{28})},\quad c_{29}=\frac{g}{f_3(t_{29})},\quad c_{30}=\frac{h}{f_3(t_{30})},$$
$$M_1=c_{23}+c_{24}+c_{25}+c_{26}+c_{27}+c_{28}+c_{29}+c_{30},$$
$$M_2=c_{23}e^{t_{23}}+c_{24}e^{t_{24}}+c_{25}e^{t_{25}}+c_{26}e^{t_{26}}+c_{27}e^{t_{27}}+c_{28}e^{t_{28}}+c_{29}e^{t_{29}}+c_{30}e^{t_{30}},\quad c_{21}=\frac{M_2-M_1e^{t_{22}}}{e^{t_{22}}-e^{t_{21}}},$$
$$c_{22}=\frac{M_1e^{t_{21}}-M_2}{e^{t_{22}}-e^{t_{21}}},\quad f_4(x)=x^2-c_0x+\left(d_0-\pi^2\right),\quad c_{33}=\frac{Sc\operatorname{Re}c_3c_{13}t_3}{f_4(t_{13})},\quad c_{34}=\frac{Sc\operatorname{Re}c_4c_{13}t_4}{f_4(t_{14})},$$
$$c_{35}=\frac{Sc\operatorname{Re}c_3c_{14}t_4}{f_4(t_{15})},\quad c_{36}=\frac{Sc\operatorname{Re}c_4c_{14}t_4}{f_4(t_{16})},\quad c_{37}=\frac{Sc\operatorname{Re}c_3c_{15}t_4}{f_4(t_{17})},\quad c_{38}=\frac{Sc\operatorname{Re}c_4c_{15}t_4}{f_4(t_{18})},$$
$$c_{39}=\frac{Sc\operatorname{Re}c_3c_{16}t_3}{f_4(t_{19})},\ c_{40}=\frac{Sc\operatorname{Re}c_4c_{16}t_4}{f_4(t_{20})},\ c_{41}=\frac{-ScSrc_{21}\left(t_{21}^2-\pi^2\right)}{f_4(t_{21})},\ c_{42}=\frac{-ScSrc_{22}\left(t_{22}^2-\pi^2\right)}{f_4(t_{22})},$$
$$c_{43}=\frac{Sc\operatorname{Re}c_5c_{13}t_1-ScSrc_{23}\left(t_{23}^2-\pi^2\right)}{f_4(t_{23})},\quad c_{44}=\frac{Sc\operatorname{Re}c_6c_{13}t_2-ScSrc_{24}\left(t_{24}^2-\pi^2\right)}{f_4(t_{24})},$$
$$c_{45}=\frac{Sc\operatorname{Re}c_5c_{14}t_1-ScSrc_{25}\left(t_{25}^2-\pi^2\right)}{f_4(t_{25})},\quad c_{46}=\frac{Sc\operatorname{Re}c_6c_{14}t_2-ScSrc_{26}\left(t_{26}^2-\pi^2\right)}{f_4(t_{26})},$$
$$c_{47}=\frac{Sc\operatorname{Re}c_5c_{15}t_1-ScSrc_{27}\left(t_{27}^2-\pi^2\right)}{f_4(t_{27})},\quad c_{48}=\frac{Sc\operatorname{Re}c_6c_{15}t_2-ScSrc_{28}\left(t_{28}^2-\pi^2\right)}{f_4(t_{28})},$$
$$c_{49}=\frac{Sc\operatorname{Re}c_5c_{16}t_1-ScSrc_{29}\left(t_{29}^2-\pi^2\right)}{f_4(t_{29})},\quad c_{50}=\frac{Sc\operatorname{Re}c_6c_{16}t_2-ScSrc_{30}\left(t_{30}^2-\pi^2\right)}{f_4(t_{30})},$$
$$n_1=c_{33}+c_{34}+c_{35}+c_{36}+c_{37}+c_{38}+c_{39}+c_{40}+c_{41}+c_{42}+c_{43}+c_{44}+c_{45}+c_{46}+c_{47}+c_{48}+c_{49}+c_{50},$$
$$n_2=c_{33}e^{t_{13}}+c_{34}e^{t_{14}}+c_{35}e^{t_{15}}+c_{36}e^{t_{16}}+c_{37}e^{t_{17}}+c_{38}e^{t_{18}}+c_{39}e^{t_{19}}+c_{40}e^{t_{20}}+c_{41}e^{t_{21}}+c_{42}e^{t_{22}}+c_{43}e^{t_{23}}$$
$$+c_{44}e^{t_{24}}+c_{45}e^{t_{25}}+c_{46}e^{t_{26}}+c_{47}e^{t_{27}}+c_{48}e^{t_{28}}+c_{49}e^{t_{29}}+c_{50}e^{t_{30}},$$

$$c_{31}=\frac{n_1e^{t_{12}}-n_2}{e^{t_{11}}-e^{t_{12}}}\ ,\quad c_{32}=\frac{n_2-n_1e^{t_{11}}}{e^{t_{11}}-e^{t_{12}}}\ ,\qquad f_5\left(x\right)=x^2-\operatorname{Re}x-\left(\pi^2+\operatorname{Re}M\right)\ ,\quad c_{51}=\frac{-Gm\operatorname{Re}c_{31}}{f_5\left(t_{11}\right)}\ ,$$

$$c_{52}=\frac{-Gm\operatorname{Re}c_{32}}{f_5\left(t_{12}\right)}\ ,\quad c_{53}=\frac{\operatorname{Re}c_{13}c_{11}t_3-Gm\operatorname{Re}c_{33}}{f_5\left(t_{13}\right)}\ ,\quad c_{54}=\frac{\operatorname{Re}c_{13}c_{12}t_3-Gm\operatorname{Re}c_{34}}{f_5\left(t_{14}\right)}\ ,$$

$$c_{55}=\frac{\operatorname{Re}c_{14}c_{11}t_3-Gm\operatorname{Re}c_{35}}{f_5\left(t_{15}\right)}\ ,\quad c_{56}=\frac{\operatorname{Re}c_{14}c_{12}t_4-Gm\operatorname{Re}c_{36}}{f_5\left(t_{16}\right)}\ ,\quad c_{57}=\frac{\operatorname{Re}c_{15}c_{11}t_3-Gm\operatorname{Re}c_{37}}{f_5\left(t_{17}\right)}\ ,$$

$$c_{58}=\frac{\operatorname{Re}c_{15}c_{12}t_4-Gm\operatorname{Re}c_{38}}{f_5\left(t_{18}\right)}\ ,\quad c_{59}=\frac{\operatorname{Re}c_{16}c_{11}t_3-Gm\operatorname{Re}c_{39}}{f_5\left(t_{19}\right)}\ ,\quad c_{60}=\frac{\operatorname{Re}c_{16}c_{12}t_4-Gm\operatorname{Re}c_{40}}{f_5\left(t_{20}\right)}\ ,$$

$$c_{61}=\frac{-Gr\operatorname{Re}c_{21}-Gm\operatorname{Re}c_{41}}{f_5\left(t_{21}\right)},\ c_{62}=\frac{-Gr\operatorname{Re}c_{22}-Gm\operatorname{Re}c_{42}}{f_5\left(t_{22}\right)},\ c_{63}=\frac{\operatorname{Re}c_{13}c_9t_1-Gr\operatorname{Re}c_{23}-Gm\operatorname{Re}c_{43}}{f_5\left(t_{23}\right)},$$

$$c_{64}=\frac{\operatorname{Re}c_{13}c_{10}t_2-Gr\operatorname{Re}c_{24}-Gm\operatorname{Re}c_{44}}{f_5\left(t_{24}\right)}\ ,\quad c_{65}=\frac{\operatorname{Re}c_{14}c_9t_1-Gr\operatorname{Re}c_{25}-Gm\operatorname{Re}c_{44}}{f_5\left(t_{25}\right)}\ ,$$

$$c_{66}=\frac{\operatorname{Re}c_{14}c_{10}t_2-Gr\operatorname{Re}c_{26}-Gm\operatorname{Re}c_{46}}{f_5\left(t_{26}\right)}\ ,\quad c_{67}=\frac{\operatorname{Re}c_{15}c_9t_1-Gr\operatorname{Re}c_{27}-Gm\operatorname{Re}c_{47}}{f_5\left(t_{27}\right)}\ ,$$

$$c_{68}=\frac{\operatorname{Re}c_{15}c_{10}t_2-Gr\operatorname{Re}c_{28}-Gm\operatorname{Re}c_{48}}{f_5\left(t_{28}\right)}\ ,\quad c_{69}=\frac{\operatorname{Re}c_{16}c_9t_1-Gr\operatorname{Re}c_{29}-Gm\operatorname{Re}c_{49}}{f_5\left(t_{29}\right)}\ ,$$

$$c_{70}=\frac{\operatorname{Re}c_{16}c_{10}t_2-Gr\operatorname{Re}c_{30}-Gm\operatorname{Re}c_{50}}{f_5\left(t_{30}\right)}\ ,\quad c_{71}=\frac{\operatorname{Re}c_{13}c_7t_5}{f_5\left(t_{31}\right)}\ ,\quad c_{72}=\frac{\operatorname{Re}c_{13}c_8t_6}{f_5\left(t_{32}\right)}\ ,\quad c_{73}=\frac{\operatorname{Re}c_{14}c_7t_5}{f_5\left(t_{33}\right)}\ ,$$

$$c_{74}=\frac{\operatorname{Re}c_{14}c_8t_6}{f_5\left(t_{34}\right)},\ c_{75}=\frac{\operatorname{Re}c_{15}c_7t_5}{f_5\left(t_{35}\right)},\ c_{76}=\frac{\operatorname{Re}c_{15}c_8t_6}{f_5\left(t_{36}\right)},\ c_{77}=\frac{\operatorname{Re}c_{16}c_7t_5}{f_5\left(t_{37}\right)},\ c_{78}=\frac{\operatorname{Re}c_{16}c_8t_6}{f_5\left(t_{38}\right)},$$

$$\begin{aligned}R_1={}&c_{51}+c_{52}+c_{53}+c_{54}+c_{55}+c_{56}+c_{57}+c_{58}+c_{59}+c_{60}+c_{61}+c_{62}+c_{63}+c_{64}+c_{65}+c_{66}+c_{67}+c_{68}\\&+c_{69}+c_{70}+c_{71}+c_{72}+c_{73}+c_{74}+c_{75}+c_{76}+c_{77}+c_{78},\end{aligned}$$

$$\begin{aligned}R_2={}&c_{51}e^{t_{11}}\left(1-ht_{11}\right)+c_{52}e^{t_{12}}\left(1-ht_{12}\right)+c_{53}e^{t_{13}}\left(1-ht_{13}\right)+c_{54}e^{t_{14}}\left(1-ht_{14}\right)+c_{55}e^{t_{15}}\left(1-ht_{15}\right)\\&+c_{56}e^{t_{16}}\left(1-ht_{16}\right)+c_{57}e^{t_{17}}\left(1-ht_{17}\right)+c_{58}e^{t_{18}}\left(1-ht_{18}\right)+c_{59}e^{t_{19}}\left(1-ht_{19}\right)+c_{60}e^{t_{20}}\left(1-ht_{20}\right)\\&+c_{61}e^{t_{21}}\left(1-ht_{21}\right)+c_{62}e^{t_{22}}\left(1-ht_{22}\right)+c_{63}e^{t_{23}}\left(1-ht_{23}\right)+c_{64}e^{t_{24}}\left(1-ht_{24}\right)+c_{65}e^{t_{25}}\left(1-ht_{25}\right)\\&+c_{66}e^{t_{26}}\left(1-ht_{26}\right)+c_{67}e^{t_{27}}\left(1-ht_{27}\right)+c_{68}e^{t_{28}}\left(1-ht_{28}\right)+c_{69}e^{t_{29}}\left(1-ht_{29}\right)+c_{70}e^{t_{30}}\left(1-ht_{11}\right)\\&+c_{71}e^{t_{31}}\left(1-ht_{31}\right)+c_{72}e^{t_{32}}\left(1-ht_{32}\right)+c_{73}e^{t_{33}}\left(1-ht_{33}\right)+c_{74}e^{t_{34}}\left(1-ht_{34}\right)+c_{75}e^{t_{35}}\left(1-ht_{35}\right)\\&+c_{76}e^{t_{36}}\left(1-ht_{36}\right)+c_{77}e^{t_{37}}\left(1-ht_{37}\right)+c_{78}e^{t_{38}}\left(1-ht_{38}\right),\end{aligned}$$

$$c_{79}=\frac{R_2-R_1e^{t_{40}}\left(1-ht_{40}\right)}{e^{t_{40}}\left(1-ht_{40}\right)-e^{t_{39}}\left(1-ht_{39}\right)},\ c_{80}=\frac{R_1e^{t_{39}}\left(1-ht_{39}\right)-R_2}{e^{t_{40}}\left(1-ht_{40}\right)-e^{t_{39}}\left(1-ht_{39}\right)}.$$

Conflict of Interests

The author(s) declare that there is no conflict of interests.

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