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PAIR DIFFERENCE CORDIALITY OF SOME GRAPHS DERIVED FROM LADDER GRAPH

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Abstract. Let $G=(V,E)$ be a (p,q) graph.

Define

$$\rho = \begin{cases} \frac{p}{2}, & \text{if } p \text{ is even} \\ \frac{p-1}{2}, & \text{if } p \text{ is odd} \end{cases}$$

and $L=\{\pm1,\pm2,\pm3,\cdots,\pm\rho\}$ called the set of labels.

Consider a mapping $f:V\longrightarrow L$ by assigning different labels in L to the different elements of V when p is even and different labels in L to p-1 elements of V and repeating a label for the remaining one vertex when p is odd. The labeling as defined above is said to be a pair difference cordial labeling if for each edge uv of G there exists a labeling $|f(u)-f(v)|$ such that $\left|\Delta_{f_1}-\Delta_{f_1^c}\right|\leq 1$, where Δ_{f_1} and $\Delta_{f_1^c}$ respectively denote the number of edges labeled with 1 and number of edges not labeled with 1. A graph G for which there exists a pair difference cordial labeling is called a pair difference cordial graph. In this paper we investigate the pair difference cordial labeling behaviour of some graphs like slanting ladder SL_n , mobius ladder M_n , triangular ladder TL_n.

Keywords: ladder; mobius ladder; slanting ladder; triangular ladder.

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1. Introduction

In this paper we consider only finite, undirected and simple graphs.The concept of cordial labeling was introduced by Cachit[1]. In the similar line the notion of pair diference cordial labeling of a graph was introducedin [4]. The pair difference cordial labeling behavior of several graphs like path, cycle, star, wheel,triangular snake,alternate triangular snake, butterfly etc have been investigated in [4,5]. In this paper we investigate the pair difference cordial labeling behavior of some graphs like slanting ladder SL_n , mobius ladder M_n , triangular ladder TL_n , $M_n \odot 2K_1$, $SL_n \odot 2K_1$, $TL_n \odot 2K_1$. Terms not defined here are follow from Gallian[2] and Harary[3].

2. Preliminaries

Definition 2.1. [4]. *Let $G = (V,E)$ be a (p,q) graph.*

Define

$$\rho = \begin{cases} \frac{p}{2}, & \text{if } p \text{ is even} \\ \frac{p-1}{2}, & \text{if } p \text{ is odd} \end{cases}$$

and $L = \{\pm 1, \pm 2, \pm 3, \cdots, \pm \rho\}$ called the set of labels.

Consider a mapping $f : V \longrightarrow L$ by assigning different labels in L to the different elements of V when p is even and different labels in L to p-1 elements of V and repeating a label for the remaining one vertex when p is odd. The labeling as defined above is said to be a pair difference cordial labeling if for each edge uv of G there exists a labeling $|f(u) - f(v)|$ such that $\left|\Delta_{f_1} - \Delta_{f_1^c}\right| \leq 1$, where Δ_{f_1} and $\Delta_{f_1^c}$ respectively denote the number of edges labeled with 1 *and number of edges not labeled with* 1. *A graph G for which there exists a pair difference cordial labeling is called a pair difference cordial graph.*

Definition 2.2. [3]. *The subdivision graph $S(G)$ of a graph G is obtained by replacing each edge uv by a path uvw.*

Definition 2.3. [2]. *The graph* $G_1 \odot G_2$ *is the graph obtained by taking one copy of* G_1 *and* n *copies of* G_2 *and joining the* i^{th} *vertex of* G_1 *with an edge to every vertex in the* i^{th} *copy* G_2, *where* G_1 *is graph of order* n.

Definition 2.4. [3]. *The ladder* L_n *is obtained from two copies of the paths* $a_1a_2\cdots a_n$ *and* $b_1b_2\cdots b_n$ *by joining each* a_i *with* $b_i, 1 \le i \le n$.*Clearly* L_n *has* $2n$ *vertices and* $3n-2$ *edges.*
That is $V(L_n) = \{a_i, b_i : 1 \le i \le n\}$ *and* $E(L_n) = \{a_ib_i : 1 \le i \le n\} \cup \{a_ia_{i+1}, b_ib_{i+1} : 1 \le i \le n-1\}$.

Definition 2.5. [2]. *The slanting ladder* SL_n *is obtained from two copies of the paths* $a_1a_2a_3\cdots a_n$ *and* $b_1b_2b_3\cdots b_n$ *by joining each* a_i *with* $b_{i+1}, 1 \le i \le n$.
That is $V(SL_n) = \{a_i, b_i : 1 \le i \le n\}$, $E(SL_n) = \{a_ia_{i+1}, b_ib_{i+1} : 1 \le i \le n-1\} \cup \{a_ib_{i+1} : 1 \le i \le n-1\}$. *It is easy to ver that* SL_n *has* $2n$ *vertices and* $3n-3$ *edges.*

Definition 2.6. [2]. *The mobius ladder* M_n *is obtained from the ladder* L_n *by joining the vertices* a_1 *with* b_n *and* a_n *with* b_1.
That is $V(M_n) = \{a_i, b_i : 1 \le i \le n\}$, $E(M_n) = \{a_ia_{i+1}, b_ib_{i+1} : 1 \le i \le n-1\} \cup \{a_ib_i : 1 \le i \le n\} \cup \{a_1b_n, a_nb_1\}$. *Note that* M_n *has* $2n$ *vertices and* $3n$ *edges.*

Definition 2.7. [2]. *The triangular ladder* TL_n *is obtained from the ladder* L_n *by adding the edges* a_ib_{i+1} *for* $1 \le i \le n-1$.
Obviously TL_n *has* $2n$ *vertices and* $4n-3$ *edges.*
That is $V(TL_n) = \{a_i, b_i : 1 \le i \le n\}$, $E(TL_n) = \{a_ia_{i+1}, b_ib_{i+1} : 1 \le i \le n-1\} \cup \{a_ib_{i+1} : 1 \le i \le n-1\}$.

3. Main Results

Theorem 3.1. The slating ladder SL_n is pair difference cordial for all values of $n \ge 2$.

Proof. Take the vertex set and edge set from definition 2.5. There are two cases arises.

Case 1. n is even.

Assign the labels $1,2$ to the vertices a_1, a_2 respectively and assign the labels $-3,-4$ respectively to the vertices a_3, a_4. Next assign the labels $5,6$ to the vertices a_5, a_6 respectively and assign the labels $-7,-8$ respectively to the vertices a_7, a_8.

Proceeding like this until we reach to the vertex a_n.Secondly assign the labels $-1,-2$ to the vertices b_1, b_2 respectively and assign the labels $3,4$ respectively to the vertices b_3, b_4. Next assign the labels $-5,-6$ to the vertices b_5, b_6 respectively and assign the labels $7,8$ respectively to the vertices b_7, b_8. Proceeding like this until we reach to the vertex b_n.

Case 2. n is odd.

As in case 1, assign the labels to the vertices $a_i, b_i, 1 \leq i \leq n-1$. Next assign the labels $n, -n$ respectively to the vertices a_n, b_n when $n \equiv 3 \pmod 4$ and assign the labels $-n, n$ to the vertices a_n, b_n when $n \equiv 1 \pmod 4$.

The Table 1 given below establish that this vertex labeling f is a pair difference cordial labeling of $SL_n, n \geq 2$.

Nature of n	$\Delta_{f_1^c}$	Δ_{f_1}
n is even	$\frac{3n-3}{2}$	$\frac{3n-1}{2}$
n is odd	$\frac{3n-2}{2}$	$\frac{3n-2}{2}$

TABLE 1

□

Theorem 3.2. The mobius ladder M_n is pair difference cordial for all values of $n \geq 2$.

Proof. Take the vertex set and edge set from definition 2.6.There are four cases arises.

Case 1. $n \equiv 0 \pmod 4$.

Subcase 1. $n = 4$.

Assign the labels $-1,-2,-3,-4$ to the vertices a_1, a_2, a_3, a_4 respectively and assign the labels

$1,2,3,4$ respectively to the vertices b_1,b_2,b_3,b_4.

Subcase 2. $n>4$.

Assign the labels $-1,-2,-3,\cdots,-n$ to the vertices $a_1,a_2,a_3,\cdots,a_n$ and assign the labels $1,2,3,\cdots,\frac{n+2}{2}$ respectively to the vertices $b_1,b_2,b_3,\cdots,b_{\frac{n+2}{2}}$. Now assign the labels $\frac{n+6}{2},\frac{n+10}{2},\frac{n+14}{2},\cdots,n-1$ respectively to the vertices $b_{\frac{n+4}{2}},b_{\frac{n+6}{2}},b_{\frac{n+8}{2}},\cdots,b_{\frac{3n}{4}}$ and $n,n-2,n-4,\cdots,\frac{n+8}{2},\frac{n+4}{2}$ respectively to the vertices $b_{\frac{3n+4}{4}},b_{\frac{3n+8}{4}},b_{\frac{3n+12}{4}},\cdots,b_{n-1},b_n$.

Case 2. $n\equiv 1 \pmod 4$.

Subcase 1. $n=5$.

Assign the labels $-1,-2,-3,-4,-5$ respectively to the vertices a_1,a_2,a_3,a_4,a_5 and assign the labels $1,2,3,4,5$ to the vertices b_1,b_2,b_3,b_4,b_5 respectively.

Subcase 2. $n>5$.

Assign the labels $-1,-2,-3,\cdots,-n$ respectively to the vertices $a_1,a_2,a_3,\cdots,a_n$ and assign the labels $1,2,3,\cdots,\frac{n+1}{2}$ to the vertices $b_1,b_2,b_3,\cdots,b_{\frac{n+1}{2}}$ respectively. Now assign the labels $\frac{n+5}{2},\frac{n+9}{2},\frac{n+13}{2},\cdots,n$ respectively to the vertices $b_{\frac{n+3}{2}},b_{\frac{n+5}{2}},b_{\frac{n+7}{2}},\cdots,b_{\frac{3n+1}{4}}$ and $n-1,n-3,n-5,\cdots,\frac{n+7}{2},\frac{n+3}{2}$ respectively to the vertices $b_{\frac{3n+5}{4}},b_{\frac{3n+9}{4}},b_{\frac{3n+13}{4}},\cdots,b_{n-1},b_n$.

Case 3. $n\equiv 2 \pmod 4$.

Assign the labels $-1,-2,-3,\cdots,-n$ to the vertices $a_1,a_2,a_3,\cdots,a_n$ respectively and assign the labels $1,2,3,\cdots,\frac{n+2}{2}$ respectively to the vertices $b_1,b_2,b_3,\cdots,b_{\frac{n+2}{2}}$. Next assign the labels $\frac{n+6}{2},\frac{n+10}{2},\frac{n+14}{2},\cdots,n$ respectively to the vertices $b_{\frac{n+3}{2}},b_{\frac{n+5}{2}},b_{\frac{n+7}{2}},\cdots,b_{\frac{3n+2}{4}}$ and $n-1,n-3,n-5,\cdots,\frac{n+8}{2},\frac{n+4}{2}$ respectively to the vertices $b_{\frac{3n+6}{4}},b_{\frac{3n+10}{4}},b_{\frac{3n+14}{4}},\cdots,b_{n-1},b_n$.

Case 4. $n\equiv 3 \pmod 4$.

Assign the labels $-1,-2,-3,\cdots,-n$ to the vertices $a_1,a_2,a_3,\cdots,a_n$ respectively and assign the labels $1,2,3,\cdots,\frac{n+1}{2}$ respectively to the vertices $b_1,b_2,b_3,\cdots,b_{\frac{n+1}{2}}$. Lastly assign the labels $\frac{n+5}{2},\frac{n+9}{2},\frac{n+13}{2},\cdots,n-1$ respectively to the vertices $b_{\frac{n+3}{2}},b_{\frac{n+5}{2}},b_{\frac{n+7}{2}},\cdots,b_{\frac{3n+3}{4}}$ and

$n, n-2, n-4, \cdots, \frac{n+7}{2}, \frac{n+3}{2}$ respectively to the vertices $b_{\frac{3n+7}{4}}, b_{\frac{3n+11}{4}}, b_{\frac{3n+15}{4}}, \cdots, b_{n-1}, b_n$.

The Table 2 given below establish that this vertex labeling is a pair difference cordial labeling of M_n for all values of $n \geq 2$.

Nature of n	Δ_{f_1}	$\Delta_{f_1^c}$
$n \equiv 0 \pmod 4$	$\frac{3n}{2}$	$\frac{3n}{2}$
$n \equiv 1 \pmod 4$	$\frac{3n-1}{2}$	$\frac{3n+1}{2}$
$n \equiv 2 \pmod 4$	$\frac{3n}{2}$	$\frac{3n}{2}$
$n \equiv 3 \pmod 4$	$\frac{3n-1}{2}$	$\frac{3n+1}{2}$

TABLE 2

□

Theorem 3.3. The triangular ladder TL_n is pair difference cordial for all values of $n \geq 2$.

Proof. Take the vertex set and edge set from definition 2.7. Assign the labels $1, 2, 3, \cdots, n$ to the vertices $a_1, a_2, a_3, \cdots, a_n$. Finally assign the labels $-1, -2, -3, \cdots, -n$ to the vertices $b_1, b_2, b_3, \cdots, b_n$. This vertex labeling gives that TL_n is pair difference cordial,since $\Delta f_1 = 2n-2, \Delta {f_1}^c = 2n-1$.

□

Theorem 3.4. $M_n \odot 2K_1$ is pair difference cordial for all values of $n \geq 2$.

Proof. We use the vertex set and edge set of the mobius ladder M_n from the definition 2.6. Let $V(M_n \odot 2K_1) = V(M_n) \cup \{x_i, y_i, u_i, v_i : 1 \leq i \leq n\}, E(M_n \odot 2K_1) = E(M_n) \cup \{a_i x_i, a_i u_i, b_i v_i, b_i y_i : 1 \leq i \leq n\}$.Clearly $M_n \odot 2K_1$ has $6n$ vertices and $7n$ edges.

Case 1. $n \equiv 0 \pmod 4$.

Define a map $f : V(M_n \odot 2K_1) \to \{\pm 1, \pm 2, \cdots, \pm 3n\}$ by

$$
\begin{aligned}
& f(a_1) = 2, && f(x_1) = 1, \\
& f(y_1) = 3, && \\
& f(a_i) = f(a_{i-1}) + 3, && 2 \le i \le \frac{3n}{4}, \\
& f(x_i) = f(x_{i-1}) + 3, && 2 \le i \le \frac{3n}{4}, \\
& f(y_i) = f(y_{i-1}) + 3, && 2 \le i \le \frac{3n}{4}, \\
& f(a_{\frac{3n+4}{4}}) = f(a_{\frac{3n}{4}}) + 2, && \\
& f(x_{\frac{3n+4}{4}}) = f(x_{\frac{3n}{4}}) + 4, && \\
& f(y_{\frac{3n+4}{4}}) = f(y_{\frac{3n}{4}}) + 3, f(a_{\frac{3n+4i+4}{4}}) && = f(a_{\frac{3n+4}{4}}) + 3i, 1 \le i \le \frac{n-4}{4}, \\
& f(x_{\frac{3n+4i+4}{4}}) = f(x_{\frac{3n+4}{4}}) + 3i, && 1 \le i \le \frac{n-4}{4}, \\
& f(y_{\frac{3n+4i+4}{4}}) = f(y_{\frac{3n+4}{4}}) + 3i, && 1 \le i \le \frac{n-4}{4}, \\
& f(b_i) = -f(a_i), && 1 \le i \le n, \\
& f(u_i) = -f(x_i), && 1 \le i \le n, \\
& f(v_i) = -f(y_i), && 1 \le i \le n.
\end{aligned}
$$

Case 2. $n \equiv 1 \pmod 4$.

Define a map $f : V(M_n \odot 2K_1) \to \{\pm 1, \pm 2, \cdots, \pm 3n\}$ by

$$
\begin{aligned}
& f(a_1) = 2, && f(x_1) = 1, \\
& f(y_1) = 3, && \\
& f(a_i) = f(a_{i-1}) + 3, && 2 \le i \le \frac{3n+1}{4}, \\
& f(x_i) = f(x_{i-1}) + 3, && 2 \le i \le \frac{3n+1}{4}, \\
& f(y_i) = f(y_{i-1}) + 3, && 2 \le i \le \frac{3n+1}{4},
\end{aligned}
$$

$$f(a_{\frac{3n+5}{4}}) = f(a_{\frac{3n+1}{4}}) + 2,$$

$$f(x_{\frac{3n+5}{4}}) = f(x_{\frac{3n+1}{4}}) + 4,$$

$$f(y_{\frac{3n+5}{4}}) = f(y_{\frac{3n+1}{4}}) + 3,$$

$$f(a_{\frac{3n+4i+5}{4}}) = f(a_{\frac{3n+1}{4}}) + 3i, \qquad 1 \le i \le \frac{n-1}{4},$$

$$f(x_{\frac{3n+4i+5}{4}}) = f(x_{\frac{3n+1}{4}}) + 3i, \qquad 1 \le i \le \frac{n-1}{4},$$

$$f(y_{\frac{3n+4i+1}{4}}) = f(y_{\frac{3n+1}{4}}) + 3i, \qquad 1 \le i \le \frac{n-1}{4},$$

$$f(b_i) = -f(a_i), \qquad 1 \le i \le n,$$

$$f(u_i) = -f(x_i), \qquad 1 \le i \le n,$$

$$f(v_i) = -f(y_i), \qquad 1 \le i \le n.$$

Case 3. $n \equiv 2 \pmod 4$.

Define a map $f : V(M_n \odot 2K_1) \to \{\pm 1, \pm 2, \cdots, \pm 3n\}$ by

$$f(a_1) = 2, \qquad f(x_1) = 1,$$

$$f(y_1) = 3,$$

$$f(a_i) = f(a_{i-1}) + 3, \qquad 2 \le i \le \frac{3n-2}{4},$$

$$f(x_i) = f(x_{i-1}) + 3, \qquad 2 \le i \le \frac{3n-2}{4},$$

$$f(y_i) = f(y_{i-1}) + 3, \qquad 2 \le i \le \frac{3n-2}{4},$$

$$f(a_{\frac{3n+2}{4}}) = f(a_{\frac{3n-2}{4}}) + 2,$$

$$f(x_{\frac{3n+2}{4}}) = f(x_{\frac{3n-2}{4}}) + 4,$$

$$f(y_{\frac{3n+2}{4}}) = f(y_{\frac{3n-2}{4}}) + 3,$$

$$f(a_{\frac{3n+4i+2}{4}}) = f(a_{\frac{3n+2}{4}}) + 3i, \qquad 1 \le i \le \frac{n-2}{4},$$

$$f(x_{\frac{3n+4i+2}{4}}) = f(x_{\frac{3n+2}{4}}) + 3i, \qquad 1 \le i \le \frac{n-2}{4},$$

$$f(y_{\frac{3n+4i+2}{4}}) = f(y_{\frac{3n+2}{4}}) + 3i, \qquad 1 \le i \le \frac{n-2}{4},$$

$$f(b_i) = -f(a_i), \quad 1 \le i \le n-1,$$

$$f(u_i) = -f(x_i), \quad 1 \le i \le n-1,$$

$$f(v_i) = -f(y_i), \quad 1 \le i \le n-1.$$

$$f(b_n) = -f(x_n) \quad f(u_n) = -f(a_n),$$

$$f(v_n) = -f(y_n).$$

Case 4. $n \equiv 3 \pmod 4$.

Define a map $f : V(M_n \odot 2K_1) \to \{\pm 1, \pm 2, \cdots, \pm 3n\}$ by

$$f(a_1) = 2, \quad f(x_1) = 1,$$

$$f(y_1) = 3,$$

$$f(a_i) = f(a_{i-1}) + 3, \quad 2 \le i \le \frac{3n-1}{4},$$

$$f(x_i) = f(x_{i-1}) + 3, \quad 2 \le i \le \frac{3n-1}{4},$$

$$f(y_i) = f(y_{i-1}) + 3, \quad 2 \le i \le \frac{3n-1}{4},$$

$$f(a_{\frac{3n+3}{4}}) = f(a_{\frac{3n-1}{4}}) + 2,$$

$$f(x_{\frac{3n+3}{4}}) = f(x_{\frac{3n-1}{4}}) + 4,$$

$$f(y_{\frac{3n+3}{4}}) = f(y_{\frac{3n-1}{4}}) + 3,$$

$$f(a_{\frac{3n+4i+3}{4}}) = f(a_{\frac{3n-1}{4}}) + 3i, \quad 1 \le i \le \frac{n+1}{4},$$

$$f(x_{\frac{3n+4i+3}{4}}) = f(x_{\frac{3n-1}{4}}) + 3i, \quad 1 \le i \le \frac{n+1}{4},$$

$$f(y_{\frac{3n+4i+3}{4}}) = f(y_{\frac{3n-1}{4}}) + 3i, \quad 1 \le i \le \frac{n+1}{4},$$

$$f(b_i) = -f(a_i), \quad 1 \le i \le n,$$

$$f(u_i) = -f(x_i), \quad 1 \le i \le n,$$

$$f(v_i) = -f(y_i), \quad 1 \le i \le n.$$

The Table 3 given below establish that this vertex labeling f is a pair difference cordial labeling of $M_n \odot 2K_1$ for all values of $n \geq 2$.

Nature of n	Δ_{f_1}	$\Delta_{f_1^c}$
$n \equiv 0 \pmod 4$	$\frac{7n}{2}$	$\frac{7n}{2}$
$n \equiv 1 \pmod 4$	$\frac{7n+1}{2}$	$\frac{7n-1}{2}$
$n \equiv 2 \pmod 4$	$\frac{7n}{2}$	$\frac{7n}{2}$
$n \equiv 3 \pmod 4$	$\frac{7n+1}{2}$	$\frac{7n-1}{2}$

TABLE 3

□

Theorem 3.5. $SL_n \odot 2K_1$ is pair difference cordial for all values of $n \geq 2$.

Proof. Take the vertex set and edge set of the slanting ladder SL_n from the definition 2.5. Let $V(SL_n \odot 2K_1) = V(SL_n) \cup \{x_i, y_i, u_i, v_i : 1 \leq i \leq n\}, E(SL_n \odot 2K_1) = E(SL_n) \cup \{a_i x_i, a_i u_i, b_i v_i, b_i y_i : 1 \leq i \leq n\}$. Clearly $SL_n \odot 2K_1$ has $6n$ vertices and $7n-3$ edges.

Case 1. $n \equiv 0 \pmod 4$.

Define a map $f : V(SL_n \odot 2K_1) \to \{\pm 1, \pm 2, \cdots, \pm 3n\}$ by

$$f(a_1) = 2, \qquad f(x_1) = 1,$$

$$f(y_1) = 3,$$

$$f(a_i) = f(a_{i-1}) + 3, \qquad 2 \leq i \leq \frac{3n-4}{4},$$

$$f(x_i) = f(x_{i-1}) + 3, \qquad 2 \leq i \leq \frac{3n-4}{4},$$

$$f(y_i) = f(y_{i-1}) + 3, \qquad 2 \leq i \leq \frac{3n-4}{4},$$

$$f(a_{\frac{3n}{4}}) = f(a_{\frac{3n-4}{4}}) + 2,$$

$$f(x_{\frac{3n}{4}}) = f(x_{\frac{3n-4}{4}}) + 4,$$

$$f(y_{\frac{3n}{4}}) = f(y_{\frac{3n-4}{4}}) + 3,$$

$$f(a_{\frac{3n+4i}{4}}) = f(a_{\frac{3n}{4}}) + 3i, \qquad 1 \le i \le \frac{n}{4},$$

$$f(x_{\frac{3n+4i}{4}}) = f(x_{\frac{3n}{4}}) + 3i, \qquad 1 \le i \le \frac{n}{4},$$

$$f(y_{\frac{3n+4i}{4}}) = f(y_{\frac{3n}{4}}) + 3i, \qquad 1 \le i \le \frac{n}{4},$$

$$f(b_i) = -f(a_i), \qquad 1 \le i \le n,$$

$$f(u_i) = -f(x_i), \qquad 1 \le i \le n,$$

$$f(v_i) = -f(y_i), \qquad 1 \le i \le n.$$

Case 2. $n \equiv 1 \pmod 4$.

Define a map $f : V(SL_n \odot 2K_1) \to \{\pm 1, \pm 2, \cdots, \pm 3n\}$ by

$$f(a_1) = 2, \qquad f(x_1) = 1,$$

$$f(y_1) = 3,$$

$$f(a_i) = f(a_{i-1}) + 3, \qquad 2 \le i \le \frac{3n-3}{4},$$

$$f(x_i) = f(x_{i-1}) + 3, \qquad 2 \le i \le \frac{3n-3}{4},$$

$$f(y_i) = f(y_{i-1}) + 3, \qquad 2 \le i \le \frac{3n-3}{4},$$

$$f(a_{\frac{3n+1}{4}}) = f(a_{\frac{3n-3}{4}}) + 2,$$

$$f(x_{\frac{3n+1}{4}}) = f(x_{\frac{3n-3}{4}}) + 4,$$

$$f(y_{\frac{3n+1}{4}}) = f(y_{\frac{3n-3}{4}}) + 3,$$

$$f(a_{\frac{3n+4i+1}{4}}) = f(a_{\frac{3n+1}{4}}) + 3i, \qquad 1 \le i \le \frac{n-1}{4},$$

$$f(x_{\frac{3n+4i+1}{4}}) = f(x_{\frac{3n+1}{4}}) + 3i, \qquad 1 \le i \le \frac{n-1}{4},$$

$$f(y_{\frac{3n+4i+1}{4}}) = f(y_{\frac{3n+1}{4}}) + 3i, \qquad 1 \le i \le \frac{n-1}{4},$$

$$f(b_i) = -f(a_i), \qquad 1 \le i \le n,$$

$$f(u_i) = -f(x_i), \qquad 1 \le i \le n,$$

$$f(v_i) = -f(y_i), \qquad 1 \le i \le n.$$

Case 3. $n \equiv 2 \pmod 4$.

Define a map $f : V(SL_n \odot 2K_1) \to \{\pm 1, \pm 2, \cdots, \pm 3n\}$ by

$$
\begin{aligned}
f(a_1) &= 2, & f(x_1) &= 1,\\
f(y_1) &= 3, & &\\
f(a_i) &= f(a_{i-1}) + 3, & 2 &\le i \le \frac{3n-2}{4},\\
f(x_i) &= f(x_{i-1}) + 3, & 2 &\le i \le \frac{3n-2}{4},\\
f(y_i) &= f(y_{i-1}) + 3, & 2 &\le i \le \frac{3n-2}{4},\\
f(a_{\frac{3n+2}{4}}) &= f(a_{\frac{3n-2}{4}}) + 2, & &\\
f(x_{\frac{3n+2}{4}}) &= f(x_{\frac{3n-2}{4}}) + 4, & &\\
f(y_{\frac{3n+2}{4}}) &= f(y_{\frac{3n-2}{4}}) + 3, & &\\
f(a_{\frac{3n+4i+2}{4}}) &= f(a_{\frac{3n+2}{4}}) + 3i, & 1 &\le i \le \frac{n-2}{4},\\
f(x_{\frac{3n+4i+2}{4}}) &= f(x_{\frac{3n+2}{4}}) + 3i, & 1 &\le i \le \frac{n-2}{4},\\
f(y_{\frac{3n+4i+2}{4}}) &= f(y_{\frac{3n+2}{4}}) + 3i, & 1 &\le i \le \frac{n-2}{4},\\
f(b_i) &= -f(a_i), & 1 &\le i \le n,\\
f(u_i) &= -f(x_i), & 1 &\le i \le n,\\
f(v_i) &= -f(y_i), & 1 &\le i \le n.
\end{aligned}
$$

Case 4. $n \equiv 3 \pmod 4$.

Define a map $f : V(SL_n \odot 2K_1) \to \{\pm 1, \pm 2, \cdots, \pm 3n\}$ by

$$
\begin{aligned}
f(a_1) &= 2, & f(x_1) &= 1,\\
f(y_1) &= 3,
\end{aligned}
$$

$$f(a_i) = f(a_{i-1}) + 3, \qquad 2 \le i \le \frac{3n+3}{4},$$

$$f(x_i) = f(x_{i-1}) + 3, \qquad 2 \le i \le \frac{3n+3}{4},$$

$$f(y_i) = f(y_{i-1}) + 3, \qquad 2 \le i \le \frac{3n+3}{4},$$

$$f(a_{\frac{3n+7}{4}}) = f(a_{\frac{3n+3}{4}}) + 2,$$

$$f(x_{\frac{3n+7}{4}}) = f(x_{\frac{3n+3}{4}}) + 4,$$

$$f(y_{\frac{3n+7}{4}}) = f(y_{\frac{3n+3}{4}}) + 3,$$

$$f(a_{\frac{3n+4i+7}{4}}) = f(a_{\frac{3n+3}{4}}) + 3i, \qquad 1 \le i \le \frac{n-7}{4},$$

$$f(x_{\frac{3n+4i+7}{4}}) = f(x_{\frac{3n+3}{4}}) + 3i, \qquad 1 \le i \le \frac{n-7}{4},$$

$$f(y_{\frac{3n+4i+7}{4}}) = f(y_{\frac{3n+3}{4}}) + 3i, \qquad 1 \le i \le \frac{n-7}{4},$$

$$f(b_i) = -f(a_i), \qquad 1 \le i \le n-3,$$

$$f(u_i) = -f(x_i), \qquad 1 \le i \le n,$$

$$f(v_i) = -f(y_i), \qquad 1 \le i \le n,$$

$$f(b_{n-2}) = -f(a_n) \qquad f(b_{n-1}) = -f(a_{n-1}),$$

$$f(b_n) = -f(a_{n-2}).$$

The Table 4 given below establish that this vertex labeling f is a pair difference cordial labeling of $SL_n \odot 2K_1$ for all values of $n \ge 2$.

Nature of n	Δ_{f_1}	$\Delta_{f_1^c}$
$n \equiv 0 \pmod 4$	$\frac{7n-4}{2}$	$\frac{7n-2}{2}$
$n \equiv 1 \pmod 4$	$\frac{7n-3}{2}$	$\frac{7n-3}{2}$
$n \equiv 2 \pmod 4$	$\frac{7n-2}{2}$	$\frac{7n-4}{2}$
$n \equiv 3 \pmod 4$	$\frac{7n-3}{2}$	$\frac{7n-3}{2}$

TABLE 4

□

Theorem 3.6. $TL_n \odot 2K_1$ is pair difference cordial for all values of $n \geq 2$.

Proof. We use the vertex set and edge set of the triangular ladder TL_n from the definition 2.7. Let $V(TL_n \odot 2K_1) = V(TL_n) \cup \{x_i, y_i, u_i, v_i : 1 \leq i \leq n\}, E(TL_n \odot 2K_1) = E(SL_n) \cup \{a_ix_i, a_iu_i, b_iv_i, b_iy_i : 1 \leq i \leq n\}$. Clearly $TL_n \odot 2K_1$ has $6n$ vertices and $8n-3$ edges.

Define a map $f : V(TL_n \odot 2K_1) \rightarrow \{\pm 1, \pm 2, \cdots, \pm 3n\}$ by

$$
\begin{aligned}
&f(a_1) = 2, && f(x_1) = 1,\\
&f(y_1) = 3, &&\\
&f(a_i) = f(a_{i-1}) + 3, && 2 \leq i \leq n-1,\\
&f(x_i) = f(x_{i-1}) + 3, && 2 \leq i \leq n-1,\\
&f(y_i) = f(y_{i-1}) + 3, && 2 \leq i \leq n-1,\\
&f(a_n) = f(a_{n-1}) + 2, &&\\
&f(x_n) = f(x_{n-1}) + 4, &&\\
&f(y_n) = f(y_{n-1}) + 3 . f(b_i) && = -f(a_i), 1 \leq i \leq n,\\
&f(u_i) = -f(x_i), && 1 \leq i \leq n,\\
&f(v_i) = -f(y_i), && 1 \leq i \leq n.
\end{aligned}
$$

Clearly $\Delta f_1 = 4n-2, \Delta {f_1}^c = 4n-1$, this vertex labeling gives that $TL_n \odot 2K_1$ is pair difference cordial for all values of $n \geq 2$.

□

Theorem 3.7. The subdivision of triangular ladder TL_n,$S(TL_n)$ is pair difference cordial for $n \geq 2$.

Proof. Let $V(S(TL_n)) = \{a_i, b_i, u_i : 1 \leq i \leq n\} \cup \{v_i, x_i, y_i : 1 \leq i \leq n-1\}$ and $E(S(TL_n)) = \{a_ix_i, b_iy_i, v_iy_i : 1 \leq i \leq n-1\} \cup \{u_ia_i, u_ib_i : 1 \leq i \leq n\} \cup \{x_ia_{i+1}, y_ib_{i+1}, v_ib_{i+1} : 1 \leq i \leq n-1\}$.

Case 1. n is odd.

Define a map $f : V(S(TL_n)) \to \{\pm 1, \pm 2, \cdots, \pm 3n-1\}$ by

$$
\begin{aligned}
f(a_i) &= 2i-1, & 1 \le i \le n,\\
f(x_i) &= 2i, & 1 \le i \le n-1,\\
f(u_i) &= f(a_n)+2i, & 1 \le i \le \frac{n-1}{2},\\
f(u_{\frac{n-2i-1}{2}}) &= -f(u_i), & 1 \le i \le \frac{n-1}{2},\\
f(v_i) &= 2n+2i, & 1 \le i \le \frac{n-3}{2},\\
f(v_{\frac{n+2i-3}{2}}) &= -(2n+2i-2), & 1 \le i \le \frac{n-1}{2},\\
f(b_i) &= -f(a_i), & 1 \le i \le n,\\
f(y_i) &= -f(x_i), & 1 \le i \le n,\\
f(u_{n-1}) &= f(v_{n-1}) & = -3n+2,\\
f(u_n) &= 2n.
\end{aligned}
$$

Clearly $\Delta f_1 = \Delta {f_1}^c = 4n-3$, this vertex labeling yields that $S(TL_n)$ is pair difference cordial for all odd values of $n \ge 2$.

Case 2. n is even.

Define a map $f : V(S(TL_n)) \to \{\pm 1, \pm 2, \cdots, \pm 3n-2\}$ by

$$
\begin{aligned}
f(a_i) &= 2i-1, & 1 \le i \le n,\\
f(x_i) &= 2i, & 1 \le i \le n-1,\\
f(u_i) &= f(a_n)+2i, & 1 \le i \le \frac{n-2}{2},\\
f(u_{\frac{n-2i-2}{2}}) &= -f(u_i), & 1 \le i \le \frac{n-2}{2},\\
f(v_i) &= 2n+2i, & 1 \le i \le \frac{n-2}{2},
\end{aligned}
$$

$$f(v_{\frac{n+2i-2}{2}}) = -(2n+2i-2), \qquad 1 \le i \le \frac{n-2}{2},$$
$$f(b_i) = -f(a_i), \qquad 1 \le i \le n,$$
$$f(y_i) = -f(x_i), \qquad 1 \le i \le n,$$
$$f(v_{n-1}) = f(v_{n-2}) \qquad f(u_n) = 2n.$$

Clearly $\Delta f_1 = \Delta {f_1}^c = 4n-3$, this vertex labeling yields that $S(TL_n)$ is pair difference cordial for all odd values of $n \ge 2$.

□

Theorem 3.8. The subdivision of slanting ladder SL_n, $S(SL_n)$ is pair difference cordial for $n \ge 2$.

Proof. Let $V(S(SL_n)) = \{u_i, v_i, x_i : 1 \le i \le n\} \cup \{a_i, b_i : 1 \le i \le n-1\}$ and $E(S(SL_n)) = \{a_iu_i, a_ix_i, b_iv_i : 1 \le i \le n-1\} \cup \{u_ia_{i+1}, v_ib_{i+1}, x_ib_{i+1} : 1 \le i \le n-1\}$.

Case 1. $n \equiv 0 \pmod 4$.

Define a map $f : V(S(SL_n)) \to \{\pm 1, \pm 2, \cdots, \pm \frac{5n-4}{2}\}$ by

$$f(a_i) = 2i-1, \qquad 1 \le i \le \frac{n}{2},$$
$$f(u_i) = 2i, \qquad 1 \le i \le \frac{n}{2},$$
$$f(a_{\frac{n+2i-2}{2}}) = n+4i-1, \qquad 1 \le i \le \frac{n}{4},$$
$$f(a_{\frac{n+2i+2}{2}}) = n+4i+1, \qquad 1 \le i \le \frac{n}{4},$$
$$f(u_{\frac{n+2i-2}{2}}) = n+4i-2, \qquad 1 \le i \le \frac{n}{4},$$
$$f(u_{\frac{n+2i+2}{2}}) = n+4i+2, \qquad 1 \le i \le \frac{n}{4},$$
$$f(x_i) = 2n+i-1, \qquad 1 \le i \le \frac{n-2}{2},$$
$$f(x_{\frac{n-2+2i}{2}}) = -f(x_i), \qquad 2 \le i \le \frac{n-2}{2},$$
$$f(x_n) = 2n.$$

Case 2. $n \equiv 1 \pmod 4$.

Define a map $f : V(S(SL_n)) \to \{\pm 1, \pm 2, \cdots, \pm \frac{5n-3}{2}\}$ by

$$
\begin{aligned}
f(a_i) &= 2i-1, && 1 \le i \le \frac{n+1}{2},\\
f(u_i) &= 2i, && 1 \le i \le \frac{n+1}{2},\\
f(a_{\frac{n+2i-1}{2}}) &= n+4i-1, && 1 \le i \le \frac{n-1}{4},\\
f(a_{\frac{n+2i+1}{2}}) &= n+4i+1, && 1 \le i \le \frac{n-1}{4},\\
f(u_{\frac{n+2i-1}{2}}) &= n+4i-2, && 1 \le i \le \frac{n-1}{4},\\
f(u_{\frac{n+2i+1}{2}}) &= n+4i+2, && 1 \le i \le \frac{n-5}{4},\\
f(x_i) &= 2n+i-1, && 1 \le i \le \frac{n-1}{2},\\
f(x_{\frac{n-1+2i}{2}}) &= -f(x_i), && 1 \le i \le \frac{n-1}{2}.
\end{aligned}
$$

Case 3. $n \equiv 2 \pmod 4$.

Define a map $f : V(S(SL_n)) \to \{\pm 1, \pm 2, \cdots, \pm \frac{5n-4}{2}\}$ by

$$
\begin{aligned}
f(a_i) &= 2i-1, && 1 \le i \le \frac{n}{2},\\
f(u_i) &= 2i, && 1 \le i \le \frac{n}{2},\\
f(a_{\frac{n+2i-2}{2}}) &= n+4i-1, && 1 \le i \le \frac{n-2}{4},\\
f(a_{\frac{n+2i+2}{2}}) &= n+4i+1, && 1 \le i \le \frac{n-2}{4},\\
f(u_{\frac{n+2i-2}{2}}) &= n+4i-2, && 1 \le i \le \frac{n-2}{4},\\
f(u_{\frac{n+2i+2}{2}}) &= n+4i+2, && 1 \le i \le \frac{n-6}{4},\\
f(x_i) &= 2n+i-1, && 1 \le i \le \frac{n-2}{2},\\
f(x_{\frac{n-2+2i}{2}}) &= -f(x_i), && 2 \le i \le \frac{n-2}{2},\\
f(x_n) &= 2n.
\end{aligned}
$$

Case 4. $n \equiv 3 \pmod 4$.

Define a map $f : V(S(SL_n)) \to \{\pm1, \pm2, \cdots, \pm\frac{5n-3}{2}\}$ by

$$f(a_i) = 2i-1, \qquad 1 \le i \le \frac{n+1}{2},$$
$$f(u_i) = 2i, \qquad 1 \le i \le \frac{n+1}{2},$$
$$f(a_{\frac{n+2i-1}{2}}) = n+4i-1, \qquad 1 \le i \le \frac{n-3}{4},$$
$$f(a_{\frac{n+2i+1}{2}}) = n+4i+1, \qquad 1 \le i \le \frac{n-3}{4},$$
$$f(u_{\frac{n+2i-1}{2}}) = n+4i-2, \qquad 1 \le i \le \frac{n-3}{4},$$
$$f(u_{\frac{n+2i+1}{2}}) = n+4i+2, \qquad 1 \le i \le \frac{n-3}{4},$$
$$f(x_i) = 2n+i-1, \qquad 1 \le i \le \frac{n-1}{2},$$
$$f(x_{\frac{n-1+2i}{2}}) = -f(x_i), \qquad 1 \le i \le \frac{n-1}{2}.$$

From the above four cases ,clearly $\Delta f_1 = \Delta {f_1}^c = 3n-3$. This vertex labeling yields that $S(SL_n)$ is pair difference cordial for all values of $n \ge 2$.

□

Theorem 3.9. The subdivision of mobius ladder M_n, $S(M_n)$ is pair difference cordial for $n \ge 2$

.

Proof. Let $V(S(M_n)) = \{u_i, v_i, z_i : 1 \le i \le n\} \cup \{x_i, y_i : 1 \le i \le n-1\} \cup \{a_1, a_2\}1$ and $E(S(M_n)) = \{u_ix_i, x_iu_{i+1}, v_iy_i, y_iv_{i+1} : 1 \le i \le n-1\} \cup \{u_iz_i, v_iz_i : 1 \le i \le n\} \cup \{u_1a_1, a_1v_n, a_2v_1, a_2u_n\}$.Clearly $S(M_n)$ has $5n$ vertices and $6n$ edges. There are two cases arises.

Case 1. n is odd.

Define a map $f : V(S(M_n)) \to \{\pm1, \pm2, \cdots, \pm\frac{5n-1}{2}\}$ by

$$f(u_1) = 1, \qquad f(u_2) = 3,$$
$$f(u_3) = 4, \qquad f(y_1) = -2,$$

$$f(v_1) = -1, \qquad f(x_1) = 2,$$

$$f(x_2) = 5, \qquad f(z_n) = 2n-1,$$

$$f(a_1) = -2n, \qquad f(a_2) = 2n,$$

$$f(u_{2i+1}) = f(u_{2i-1})+4, \qquad 2 \leq i \leq \frac{n-3}{2},$$

$$f(u_{2i+2}) = f(u_{2i})+4, \qquad 1 \leq i \leq \frac{n-3}{2},$$

$$f(x_{2i+1}) = f(x_{2i-1})+4, \qquad 1 \leq i \leq \frac{n-3}{2},$$

$$f(x_{2i+2}) = f(x_{2i})+4, \qquad 1 \leq i \leq \frac{n-3}{2},$$

$$f(v_i) = f(v_{i-1})-2, \qquad 2 \leq i \leq n,$$

$$f(y_i) = f(y_{i-1})-2, \qquad 2 \leq i \leq n-1,$$

$$f(z_i) = 2n+1+i, \qquad 1 \leq i \leq \frac{n-1}{2},$$

$$f(z_{\frac{n-1+2i}{2}}) = -f(z_i), \qquad 1 \leq i \leq \frac{n-1}{2}.$$

Clearly $\Delta f_1 = \Delta {f_1}^c = 3n$, this vertex labeling yields that $S(M_n)$ is pair difference cordial for all odd values of $n \geq 2$.

Case 2. n is even.

Define a map $f : V(S(M_n)) \to \{\pm 1, \pm 2, \cdots, \pm \frac{5n}{2}\}$ by

$$f(u_1) = 1, \qquad f(u_2) = 3,$$

$$f(u_3) = 4, \qquad f(y_1) = -2,$$

$$f(v_1) = -1, \qquad f(x_1) = 2,$$

$$f(x_2) = 5,$$

$$f(a_1) = -2n, \qquad f(a_2) = 2n,$$

$$
\begin{aligned}
f(u_{2i+1}) &= f(u_{2i-1})+4, && 2 \le i \le \frac{n-4}{2},\\
f(u_{2i+2}) &= f(u_{2i})+4, && 1 \le i \le \frac{n-2}{2},\\
f(x_{2i+1}) &= f(x_{2i-1})+4, && 1 \le i \le \frac{n-2}{2},\\
f(x_{2i+2}) &= f(x_{2i})+4, && 1 \le i \le \frac{n-4}{2},\\
f(v_i) &= f(v_{i-1})-2, && 2 \le i \le n,\\
f(y_i) &= f(y_{i-1})-2, && 2 \le i \le n-1,\\
f(z_i) &= 2n+1+i, && 1 \le i \le \frac{n}{2},\\
f(z_{\frac{n+2i}{2}}) &= -f(z_i), && 1 \le i \le \frac{n}{2}.
\end{aligned}
$$

Clearly $\Delta f_1 = \Delta {f_1}^c = 3n$, this vertex labeling yields that $S(M_n)$ is pair difference cordial for all even values of $n \ge 2$.

□

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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