MARKOVIAN TWO SERVER SINGLE VACATION QUEUE WITH BALKING AND HETEROGENEOUS SERVICE RATES

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Abstract. In this paper, we consider a queue with two heterogeneous server. In which, the servers takes a single vacation, if there are no customers in the system at a service completion point. The negative exponential distributions follows to the inter-arrival time and the service times. Every arrival of customer requires accurately single server for its service. The customers choose the servers on random selection basis, here the queue discipline is FCFS. The vacation period follows negative exponential distribution. In addition an arriving customer who finds the servers are on vacation, may balk with probability $1 - \beta$ or may join with probability β. The steady state results have been obtained to this model. Some performance measure and some numerical models are obtained.

Keywords: heterogeneous server; vacation; steady state; performance measures.

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1. INTRODUCTION

Krishnamoorthy and Sreenivasan [6] examines two heterogeneous servers with a queueing system, one of which is invariably obtainable except the another one goes on vacation in the non-appear of customers stand by service. Bhupender Som et. al [2], Considered to stay forward in

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today's modest occupation situation, comprehension the behaviour of the customer in advance is of most importance for each organisation. Mathematical model established in this paper can help organisations to hypothetically derive the system behaviour. Rakesh Kumar and Sumeet Sharma [8] studied the thought of customer reneging and balking have been demoralized to an excessive extent in recent past by the queuing modelers. Al-Seedy et. al [1] can be used to get the busy state density function of an $M/M/1$ queue with reneging and balking. Swathi et. al [9] deals with customer's reaction (reneging and balking) and studied an $M/M/1$ queueing system. Customer's reactions (reneging and balking) are assumed to arise due to non-accessibility of the server during the period of breakdown and vacation. Monita Baruah et. al [7] Studied about the customers with a single server queue or units arriving in batches in a system delivering the two types of common heterogeneous service. The arrival of customers in bunch or group, in a single server queueing system is studied by Uma and Manikandan [10] and also which provides different types of general services in bunch of limited size M in FCFS in Poisson distribution. Kalyanaraman and Senthilkumar [4] studied change of service mode of initial server behind a threshold with two different server Markovian queue. The solution of steady state have been obtained in this model. A Poisson queue with two heterogeneous servers are considered and Some oppositions to the use of classical queue discine are raised and Since the heterogeneity of servers, chief among them being the destruction of the "first-come first-serve" principle. Considered the two alternative queue disciplines, one with a slight and the another with a larger alteration of the standard one is examined by Krishnamoorthi [5]. Jau ke et. al [3] studied a retrial feedback queue with subject to failure and customer balking are inspired by some real stochastic service systems.

2. MODEL DESCRIPTION AND ANALYSIS

We consider a waiting line of infinite waiting capacity of Markovian two server queueing system. The arrival of customers at the system according to a Poisson process with rate λ. The customers service time follows to be an exponential distribution. If no customer in the system at a service completion epoch the servers take vacation of random period, follows negative exponential distribution with rate θ. After completion of a single vacation period the servers return to the system independent of the number of customers in the queue. The customer's

arriving who finds the servers are on vacation, may join with probability β. or may balk with probability $1 - \beta$. Thus $\beta\lambda$ is the arrival rate when the servers are in vacation. The service rates are defined as follows:

- Service rate of first server is μ_1
- Service rate of second server is μ_2, $\mu_1 > \mu_2$

For the analysis the following probabilities have been defined in steady state:

$p_{0,1} = \Pr\{\text{the servers are idle and there are no customers}\}$;

$p_{n,0} = \Pr\{\text{the severs are in vacation and there are } n \text{ customers in the system}\}$;

$p_{n,1} = \Pr\{\text{the severs are busy and } n \text{ customers are there in the system}\}$; $n \geq 0$

Using birth-death arguments the difference equations have been derived as follows.

(1)
$$\lambda p_{0,1} = \theta p_{0,0}$$

(2)
$$(\lambda + \mu_1 + \mu_2)p_{n,1} = \lambda p_{n-1,1} + (\mu_1 + \mu_2)p_{n+1,1} + \theta p_{n,0}; \ n \geq 1$$

(3)
$$(\lambda\beta + \theta)p_{0,0} = (\mu_1 + \mu_2)p_{1,1}$$

(4)
$$(\lambda\beta + \theta)p_{n,0} = \beta\lambda p_{n-1,0}; \ n \geq 1$$

and the normalizing condition

$$\sum_{n=0}^{\infty} p_{n,0} + \sum_{n=0}^{\infty} p_{n,1} = 1.$$

The following probability generating functions (P.G.F.'s) have been defined for the analysis

$$P_0(z) = \sum_{n=0}^{\infty} p_{n,0}z^n, P_1(z) = \sum_{n=0}^{\infty} p_{n,1}z^n$$

Then, multiplying equation (2) by z^n, and then adding equation (1) and sum of all possible values of n, and we get

(5)
$$P_1(z) = \frac{[\theta(\mu_1 + \mu_2)(1 - z) + \lambda z(\lambda\beta + \theta)] p_{0,0} - \lambda\theta z P_0(z)}{\lambda[\lambda z^2 - (\lambda + \mu_1 + \mu_2)z + \mu_1 + \mu_2]}$$

In a similar way, we get from equations (3) and (4)

(6)
$$P_0(z) = \frac{(\lambda\beta + \theta)p_{0,0}}{\lambda\beta(1 - z) + \theta}$$

Substituting (6) in (5), we get

(7) $$P_1(z) = \frac{\left[\theta(\mu_1 + \mu_2)(1 - z)[\lambda\beta(1 - z) + \theta] + \lambda^2\beta(\lambda\beta + \theta)z(1 - z)\right]p_{0,0}}{\lambda\left[\lambda z^2 - (\lambda_1 + \mu_1 + \mu_2)z + \mu_1 + \mu_2\right][\lambda\beta(1 - z) + \theta]}$$

where $P_0(z)$ and $P_1(z)$ is the P.G.F. of number of customers in the system when the servers are in vacation and busy respectively.

To find $p_{0,0}$

By applying L'Hospital rule to (7) and $z = 1$, we get

(8) $$P_1(1) = \frac{[\theta^2(\mu_1 + \mu_2) + \lambda^2\beta(\lambda\beta + \theta)]p_{0,0}}{\lambda\theta(\mu_1 + \mu_2 - \lambda)}$$

Similarly $z = 1$ in (6), we get

(9) $$P_0(1) = \frac{(\lambda\beta + \theta)p_{0,0}}{\theta}$$

Using normalizing condition, we get from (8)and (9)

$$p_{0,0} = \frac{\lambda\theta(\mu_1 + \mu_2 - \lambda)}{\theta^2(\mu_1 + \mu_2) + \lambda(\lambda\beta + \theta)(\mu_1 + \mu_2 + \lambda\beta - \lambda)}$$

3. PERFORMANCE MEASURES

In this section, we present some performance measures related to the model discussed in this article. The performance measures are obtained using straight forward calculations.

(i). Mean number of customers in the system when the servers are in vacation

$$L_{sv} = \left(\frac{d}{dz}P_0(z)\right)_{z \to 1} = \frac{\lambda^2\beta(\mu_1 + \mu_2 - \lambda)(\lambda\beta + \theta)}{\theta[\theta^2(\mu_1 + \mu_2) + \lambda(\lambda\beta + \theta)(\mu_1 + \mu_2 + \lambda\beta - \lambda)]}$$

(ii). Mean number of customers in the system when the servers are busy

$$L_{sb} = \left(\frac{d}{dz}P_1(z)\right)_{z \to 1} = \frac{\lambda\theta^3(\mu_1 + \mu_2) + \lambda^2\beta(\lambda\beta + \theta)[(\mu_1 + \mu_2)(\lambda\beta + \theta) - \beta\lambda^2]}{\theta(\lambda - \mu_1 - \mu_2)[\theta^2(\mu_1 + \mu_2) + \lambda(\lambda\beta + \theta)(\mu_1 + \mu_2 + \lambda\beta - \lambda)]}$$

(iii). Mean number of customers in the system

$$L_s = L_{sv} + L_{sb}$$

(iv) Probability that the server is on vacation

$$P_v = \frac{\lambda(\lambda\beta + \theta)(\mu_1 + \mu_2 - \lambda)}{\theta^2(\mu_1 + \mu_2) + \lambda(\lambda\beta + \theta)(\mu_1 + \mu_2 + \lambda\beta - \lambda)}$$

(v) Probability that the server is busy

$$P_b = \frac{\theta^2(\mu_1 + \mu_2) + \lambda^2 \beta(\lambda\beta + \theta)}{\theta^2(\mu_1 + \mu_2) + \lambda(\lambda\beta + \theta)(\mu_1 + \mu_2 + \lambda_\beta - \lambda)}$$

4. A NUMERICAL STUDY

In this section, we calculate numerically, the performance measures using the expression obtained in the above section. The numerical results are presented in figures. We fixed the parameters $\lambda = 5$, $\theta = 3$.

Figure 1, explains the effect of β on mean number of customers in the system when the server is busy. The graphs are drawn for both homogeneous and heterogeneous models. The graphs shown an upward trend initially and then steeply increases for both models. Also for higher service rates the queue length comparatively decreases, as expected.

Figure 2, analyzes the effect of β on mean number of customers in the system when the server is on vacation. The graphs are drawn for both homogeneous and heterogeneous models. The graphs shown an upward trend initially and then reverse the direction.

Figure 3, shows the effect of β on mean number of customers in the system, irrespective of server state. The trend is same as in figure 1. Figure 4 and 5, presents the Probability that the server is busy and Probability that the server is on vacation, for larger values of β the probability is low for homogeneous model compare to heterogeneous model. In the case of vacation probability we experiences the converse trend.
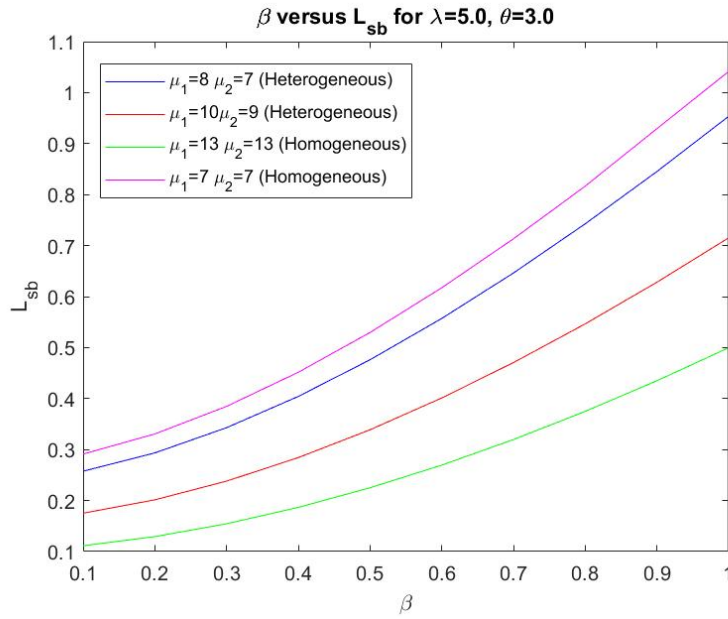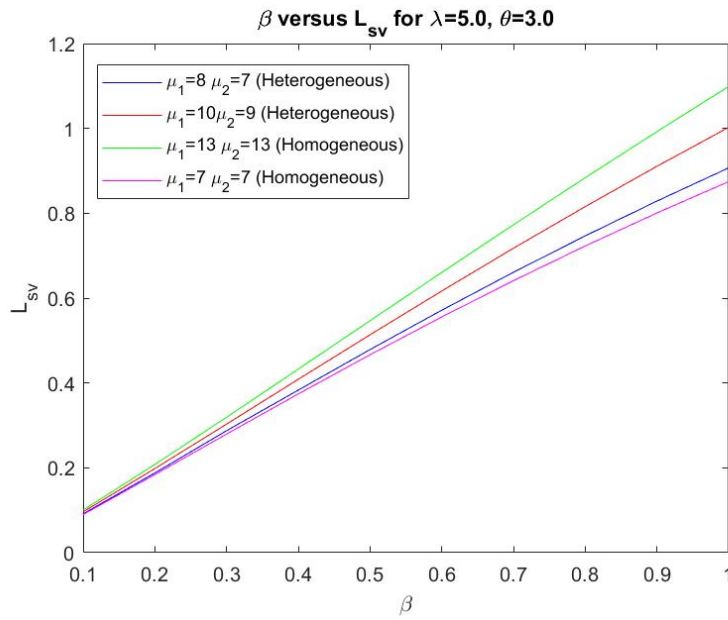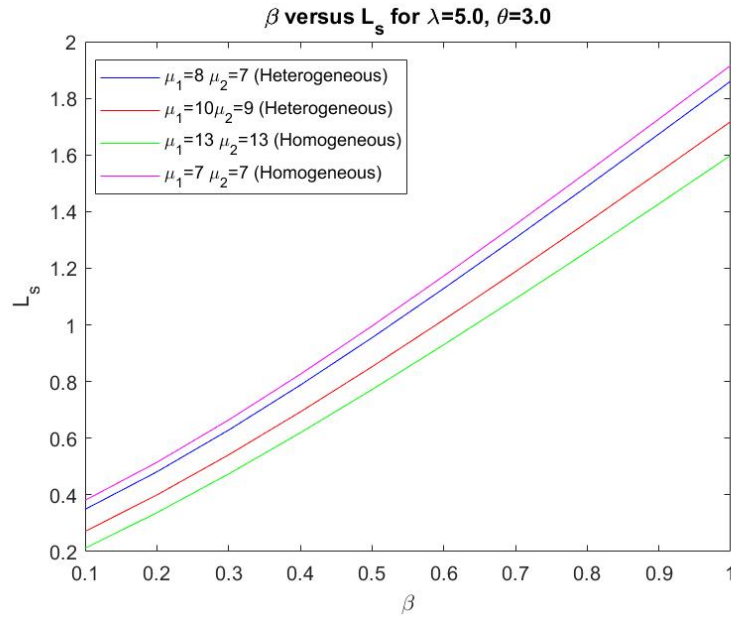
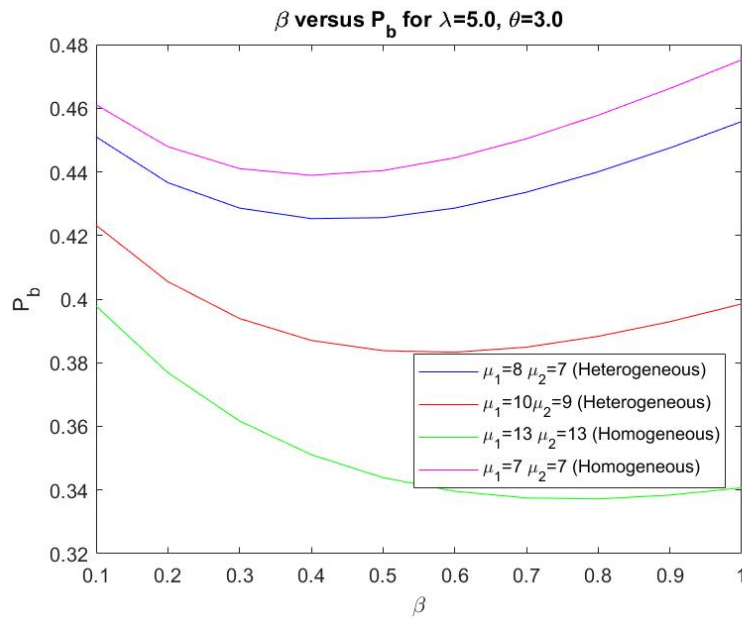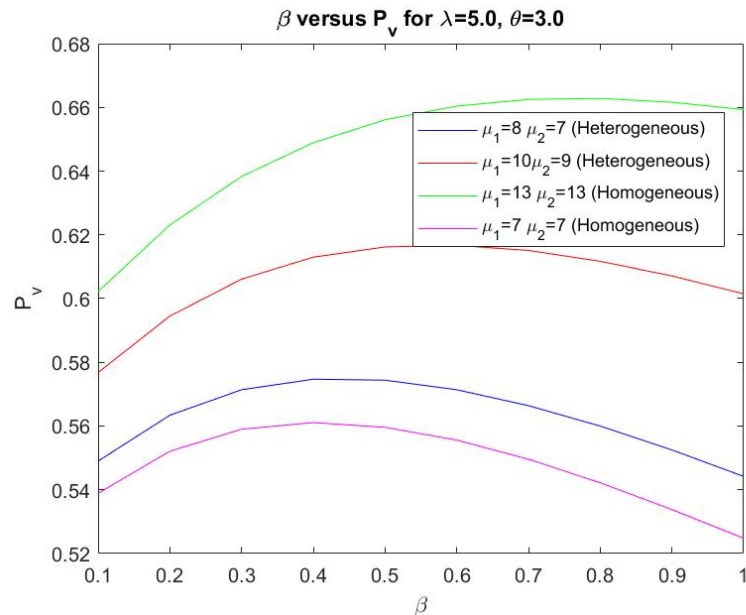FIGURE 1. L_{sb}

FIGURE 2. L_{sv}

FIGURE 3. L_s

FIGURE 4. P_b

FIGURE 5. P_v

5. CONCLUSION

The graphs of and expresses the queue length comparatively decreases as expected, shown an upward trend initially and reverse the direction for both homogeneous and heterogeneous models, and then and represents the probability is low for homogeneous model compare to heterogeneous model.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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