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HERMITE WAVELET METHOD FOR SOLVING OSCILLATORY ELECTRICAL CIRCUIT EQUATIONS

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Abstract: In this research, numerical solutions based on Hermite wavelets have been presented for solving oscillatory electrical circuit equations. The proposed numerical technique is based on Hermite wavelet basis functions, operational matrices of integration and collocation points. Numerical experiments have been performed to illustrate the accuracy and efficiency of the proposed scheme.

Keywords: Hermite wavelets; oscillatory electrical circuit equations; collocation points; operational matrices of integration; numerical examples.

2010 AMS Subject Classification: 65N99.

1. INTRODUCTION

The linear differential equations with constant coefficients find their most important applications in the study of electrical, mechanical and other linear systems. In fact such equations play a dominant role in unifying the theory of electrical and mechanical oscillatory systems. An oscillatory circuit is an electronic circuit which originates a periodic, oscillating electronic signal

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in the form of sine wave or a square wave and many numerical schemes have been developed for solving these mathematical models. A simple oscillatory electrical circuit consists of the following elements connected in series with a key K: (a) an inductor of inductance L (b) a capacitor of capacitance C (c) a resistor of resistance R (d) a battery which supplies an electromotive force $(e.m.f)$

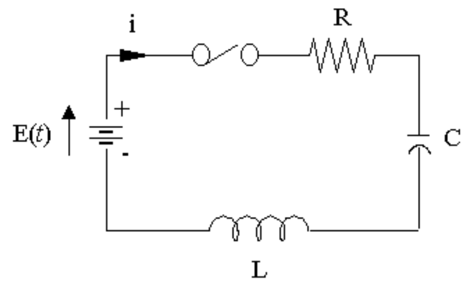

Figure 1: Simple L-C-R circuit

Figure 1 shows the simple series L-C-R circuit. The differential equation of the above circuit is

$$L\frac{d^2Q}{dt^2}+R\frac{dQ}{dt}+\frac{Q}{C}=E, \qquad Q(0)=0,\ Q'(0)=0$$

where L is the inductance of an inductor, C is the capacitance of the capacitor, R is the resistance of the resistor, Q is the charge and E is the electromotive force.

Hermite wavelets based collocation method has been presented for solving boundary value problems in [1]. Haar wavelets based numerical techniques have been developed for solving differential equations in [2, 3]. In [4], Haar wavelet method has been presented for solving lumped and distributed–parameter systems. For optimizing dynamic system, wavelet based technique has been developed in [5]. Hermite wavelet based technique has been developed in [6] for solving Jaulent-Miodek equation associated with energy-dependent Schrdinger potential. In [7], Hermite wavelet based numerical approach has been presented to estimate the solution for Bratu's problem. Haar wavelet based modified numerical techniques have been developed for solving differential and integral equations in [8, 9]. Hermite wavelet based numerical scheme has been developed for solving two- dimensional hyperbolic telegraph equation in [10]. For the solution of fractional order differential equations, Hermite wavelet based numerical technique has been presented in [11]. Haar wavelet based collocation method have been presented for solving partial differential equations in

[12, 13]. Haar wavelet method has been presented for solving nonlinear Volterra integral equations in [14]. In [15], Hermite wavelet based numerical scheme has been presented for solving nonlinear singular initial value problems. Numerical integration has been evaluated with the help of Hermite wavelets in [16].

2. HERMITE WAVELETS AND ITS PROPERTIES

Wavelets constitute a family of mathematical functions $\psi_{a,b}$ derived from dilation (change of scale) and translation (change of position) of a single function ψ called the mother wavelet. If the dilation parameter $'a'$ and translation parameter $'b'$ are considered to vary continuously, the family of continuous wavelets can be written as

$$\psi_{a,b,}(x) = \frac{1}{\sqrt{a}}\psi\left(\frac{x-b}{a}\right), \begin{matrix} a > 0, \\ b \in R \end{matrix} \tag{1}$$

By restricting the parameters a and b to discrete values as

$$a = a_0^{-k}, b = nb_0a_0^{-k}, a_0 > 1, b_0 > 0,$$

we obtain the following family of discrete wavelets:

$$\psi_{k,n}(x) = |a|^{-1/2}\psi\left(a_0^k x - nb_0\right), \forall a, b \in R, a \neq 0,$$

where $\psi_{k,n}$ form a wavelet basis for $L^2(R)$.

Particularly, when we choose $a_0 = 2$ and $b_0 = 1$, then $\psi_{k,n}(x)$ form an orthonormal basis. Hermite wavelets are defined as

$$\psi_{n,m}(x) = \begin{cases} \frac{2^{\frac{k+1}{2}}}{\sqrt{\pi}} H_m(2^k x - 2n + 1), & \frac{n-1}{2^{k-1}} \leq x < \frac{n}{2^{k-1}} \\ 0 & ,\text{Otherwise} \end{cases} \tag{2}$$

where $m = 0,1 \cdots, M-1$, and $H_m(x)$ is Hermite polynomial of degree m. Hermite polynomials $H_n(x)$, are the solutions of Hermite's differential equation given by

$$y'' - 2xy' + 2ny = 0, \quad n = 0, 1, 2, 3\ldots\ldots$$

These polynomials are given by the Rodrigue's formula

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}(e^{-x^2})$$

and are defined in the interval $(-\infty, \infty)$.

Take $k = 1$ and $M = 6$ $(m = 0, 1, 2, 3, 4, 5)$. The six basis functions on $[0, 1)$ are given by

$$\psi_{1,0}(x) = \frac{2}{\sqrt{\pi}},$$

$$\psi_{1,1}(x) = \frac{2}{\sqrt{\pi}}(4x - 2),$$

$$\psi_{1,2}(x) = \frac{2}{\sqrt{\pi}}(16x^2 - 16x + 2),$$

$$\psi_{1,3}(x) = \frac{2}{\sqrt{\pi}}(64x^3 - 96x^2 + 36x - 2),$$

$$\psi_{1,4}(x) = \frac{2}{\sqrt{\pi}}(256x^4 - 512x^3 + 320x^2 - 64x + 2),$$

$$\psi_{1,5}(x) = \frac{2}{\sqrt{\pi}}(1024x^5 - 2560x^4 + 2240x^3 - 800x^2 + 100x - 2).$$

Let

$$\psi_6(x) = (\psi_{1,0}(x), \psi_{1,1}(x), \psi_{1,2}(x), \psi_{1,3}(x), \psi_{1,4}(x), \psi_{1,5}(x))^T$$

Integrating the above basis functions with respect to x from 0 to x and expressing in matrix form, we obtain

$$\int_0^x \psi_{1,0}(x)dx = \frac{2}{\sqrt{\pi}}x = \left[\frac{1}{2}, \frac{1}{4}, 0,0,0,0\right] \psi_6(x)$$

$$\int_0^x \psi_{1,1}(x)dx = \frac{2}{\sqrt{\pi}}(2x^2 - 2x) = \left[\frac{-1}{4}, 0, \frac{1}{8}, 0,0,0\right] \psi_6(x)$$

$$\int_0^x \psi_{1,2}(x)dx = \frac{2}{\sqrt{\pi}}\left(\frac{16}{3}x^3 - 8x^2 + 2x\right) = \left[\frac{-1}{3}, 0,0, \frac{1}{12}, 0,0\right] \psi_6(x)$$

$$\int_0^x \psi_{1,3}(x)dx = \frac{2}{\sqrt{\pi}}(16x^4 - 32x^3 + 12x^2 + 4x) = \left[\frac{-5}{4}, 0,0,0, \frac{1}{16}, 0\right] \psi_6(x)$$

$$\int_0^x \psi_{1,4}(x)dx = \frac{2}{\sqrt{\pi}}\left(\frac{256}{5}x^5 - 128x^4 + 64x^3 + 32x^2 - 20x\right) = \left[\frac{-2}{5}, 0,0,0,0, \frac{1}{20}\right] \psi_6(x)$$

$$\int_0^x \psi_{1,5}(x)dx = \frac{2}{\sqrt{\pi}}\left(\frac{512}{3}x^6 - 512x^5 + 560x^4 - \frac{800}{3}x^3 + 50x^2 - 2x\right)$$

$$= \left[\frac{-23}{3}, 0,0,0,0,0\right] \psi_6(x) + \frac{1}{24}\psi_{1,6}(x)$$

Therefore,

$$\int_0^x \psi_6(x)dx = P_{6\times 6}\,\psi_6(x) + \overline{\psi}_6(x)$$

where

$$P_{6\times 6} = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{-1}{4} & 0 & \frac{1}{8} & 0 & 0 & 0 \\ \frac{-1}{3} & 0 & 0 & \frac{1}{12} & 0 & 0 \\ \frac{-5}{4} & 0 & 0 & 0 & \frac{1}{16} & 0 \\ \frac{-2}{5} & 0 & 0 & 0 & 0 & \frac{1}{20} \\ \frac{-23}{3} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \overline{\psi}_6(x) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{24}\,\psi_{1,6}(x) \end{bmatrix}$$

The double integration of above six basis is given by,

$$\int_0^x\int_0^x \psi_{1,0}(x)dx\,dx = \frac{2}{\sqrt{\pi}}\frac{x^2}{2} = \left[\frac{3}{16},\frac{1}{8},\frac{1}{32},0,0,0\right]\psi_6(x)$$

$$\int_0^x\int_0^x \psi_{1,1}(x)dx\,dx = \frac{2}{\sqrt{\pi}}\left(\frac{2}{3}x^3 - x^2\right) = \left[\frac{-1}{6},\frac{-1}{16},0,\frac{1}{96},0,0,\right]\psi_6(x)$$

$$\int_0^x\int_0^x \psi_{1,2}(x)dx\,dx = \frac{2}{\sqrt{\pi}}\left(\frac{4}{3}x^4 - \frac{8}{3}x^3 + x^2\right) = \left[\frac{-1}{16},\frac{-1}{12},0,\frac{1}{192},0\right]\psi_6(x)$$

$$\int_0^x\int_0^x \psi_{1,3}(x)dx\,dx = \frac{2}{\sqrt{\pi}}\left(\frac{16}{5}x^5 - 8x^4 + 4x^3 + 2x^2\right) = \left[\frac{3}{5},\frac{5}{16},0,0,0,\frac{1}{320},0\right]\psi_6(x)$$

$$\int_0^x\int_0^x \psi_{1,4}(x)dx\,dx = \frac{2}{\sqrt{\pi}}\left(\frac{128}{15}x^6 - \frac{128}{5}x^5 + 16x^4 + \frac{32}{3}x^3 - 10x^2\right)$$

$$= \left[\frac{-7}{12},\frac{-1}{10},0,0,0,0\right]\psi_6(x) + \frac{1}{480}\psi_{1,6}(x)$$

$$\int_0^x\int_0^x \psi_{1,5}(x)dx\,dx = \frac{2}{\sqrt{\pi}}\left(\frac{512}{21}x^7 - \frac{256}{3}x^6 + 64x^5 + \frac{160}{3}x^4 - \frac{200}{3}x^3 + 4x^2\right)$$

$$= \left[\frac{-22}{7},\frac{-23}{12},0,0,0,0\right]\psi_6(x) + \frac{1}{672}\psi_{1,7}(x)$$

Hence

$$\int_0^x\int_0^x \psi_6(x)dx\,dx = P'_{6\times 6}\,\psi_6(x) + \overline{\psi}'_6(x)$$

where

$$P'_{6\times 6}=\begin{bmatrix} \frac{3}{16} & \frac{1}{8} & \frac{1}{32} & 0 & 0 & 0 \\ \frac{-1}{6} & \frac{-1}{16} & 0 & \frac{1}{96} & 0 & 0 \\ \frac{-1}{16} & \frac{-1}{12} & 0 & 0 & \frac{1}{192} & 0 \\ \frac{3}{5} & \frac{5}{16} & 0 & 0 & 0 & \frac{1}{320} \\ \frac{-7}{12} & \frac{-1}{10} & 0 & 0 & 0 & 0 \\ \frac{-22}{7} & \frac{-23}{12} & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \overline{\psi}'_6(x)=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{480}\psi_{1,6}(x) \\ \frac{1}{672}\psi_{1,7}(x) \end{bmatrix}$$

Similarly we can take any number of basis functions to find the corresponding operational matrices of integration.

3. Function Approximation

Consider any square integrable function $y(x)$ can be expanded in terms of infinite series of Hermite basis functions as:

$$y(x)=\sum_{n=1}^{\infty}\sum_{m=0}^{\infty} c_{n,m}\psi_{n,m}(x)$$

where $c_{n,m}$ are the constants of this infinite series, known as wavelet coefficients. For the numerical approximation, the above infinite series is truncated upto finite number of terms as follow:

$$y(x)=\sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1} c_{n,m}\psi_{n,m}(x)=C^T\,\boldsymbol{\psi}(x), \tag{3}$$

where C and $\boldsymbol{\psi}$ are matrices of order $2^{k-1}M\times 1$ and are given by

$$C^T=[c_{1,0},\dots,c_{1,M-1},\dots,c_{2^{k-1},0},\dots,c_{2^{k-1},M-1}]$$

and

$$\boldsymbol{\psi}=[\psi_{1,0},\dots,\psi_{1,M-1},\dots,\psi_{2^{k-1},0},\dots,\psi_{2^{k-1},M-1}]^T$$

where T represents the transpose of the matrix.

4. PROPOSED METHOD FOR SOLVING LCR CIRCUIT EQUATIONS

Consider the LCR circuit equations of the form

$$LQ''(t) + RQ'(t) + \frac{Q(t)}{C} = E \tag{4}$$

where L is inductance, R is resistance, C is capacitance, Q is charge,

Consider the approximation

$$Q''(t) = \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m}\psi_{n,m}(t)\,, \tag{5}$$

Integrating (5), twice w.r.t t, from 0 to t, we obtain

$$Q'(t) = Q'(0) + \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m} \int_0^t \psi_{n,m}(t)dt\,, \tag{6}$$

$$Q(t) = Q(0) + t.Q'(0) + \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m} \int_0^t \int_0^t \psi_{n,m}(t)dt\,dt\,, \tag{7}$$

Substituting the values of Q, $Q'$and Q'' in (4), we obtain an algebraic system of equations. After solving such system of algebraic equations, we obtain wavelet coefficients. The numerical solution is obtained by substituting wavelet coefficients into (7).

5. NUMERICAL EXPERIMENTS

In this section, we perform some experiments to illustrate the accuracy of the proposed numerical scheme. The accuracy of the numerical results are obtained by using the following relation

$$Absolute\ Error = |Q_{Exact} - Q_{Approximate}|$$

Example 1: In LCR circuit equation, if an alternating $EMF = Esin(wt)$ is applied and letting $L = 1H,\ R = 2ohm,\ C = 1F,\ w = 1$ and $E = 1v$, we obtain

$$\frac{d^2Q}{dt^2} + 2\frac{dQ}{dt} + Q = \sin t\,, \qquad Q(0) = Q'(0) = 0$$

The exact solution in this case is:

$$Q(t) = \frac{1}{2}(1+t)e^{-t} - \frac{1}{2}cost$$

Table 1 shows the comparison of exact solution and Hermite wavelet solution ($k = 1,\ M = 8$) of Example 1. Figure 2 represents the accuracy of the proposed method.

t	**Exact Solution**	**Hermite Wavelet Solution**	**Absolute Error**
1/16	3.9434e-005	3.9434e-005	2.3557e-011
3/16	9.9938e-004	9.9938e-004	6.8601e-011
5/16	4.3388e-003	4.3388e-003	9.8535e-011
7/16	1.1153e-002	1.1153e-002	1.2105e-010
9/16	2.2181e-002	2.2181e-002	1.3672e-010
11/16	3.7847e-002	3.7847e-002	1.4658e-010
13/16	5.8303e-002	5.8303e-002	1.5281e-010
15/16	8.3465e-002	8.3465e-002	1.4949e-010

Table 1: Comparison of Exact and Hermite wavelet solutions of Example 1

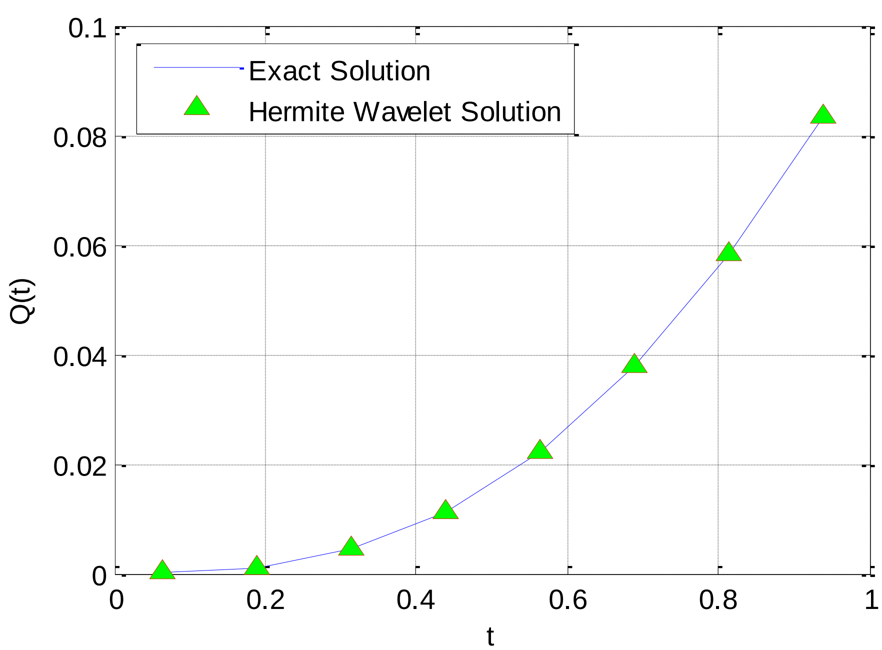

Figure 2: Comparison of Exact and Hermite wavelet solutions ($k = 1,\ M = 8$) of Example 1

Example 2: In LCR circuit equation, if an alternating $EMF = Esin(wt)$ is applied to an inductance L and a capacitor C in series, then the differential equation will be (where $R = 0$)

$$L\frac{d^2Q}{dt^2} + \frac{Q}{C} = Esin(wt), \quad Q(0) = 0, \quad Q'(0) = 0$$

The exact solution in this case is

$$Q(t) = -\frac{ECw}{(1 - LCw^2)}\sqrt{LC}\sin\left(\frac{t}{\sqrt{LC}}\right) + \frac{EC}{(1 - LCw^2)}\sin(wt)$$

Case 1: Taking $E = 2v,\ w = 2radian,\ C = 2\mu F,\ L = 1H$

Table 2 shows the comparison of exact solution and Hermite wavelet solution ($k = 1,\ M = 8$) for Case 1. Figure 3 represents the accuracy of the proposed method for Case 1.

t	**Exact Solution**	**Hermite Wavelet Solution**	**Absolute Error**
1/16	4.9632e-007	4.9875e-007	2.4276e-009
3/16	1.4584e-006	1.4651e-006	6.6894e-009
5/16	2.3305e-006	2.3404e-006	9.8737e-009
7/16	3.0589e-006	3.0702e-006	1.1271e-008
9/16	3.5984e-006	3.6091e-006	1.0671e-008
11/16	3.9154e-006	3.9236e-006	8.2431e-009
13/16	3.9899e-006	3.9939e-006	4.0290e-009
15/16	3.8168e-006	3.8197e-006	2.9154e-009

Table 2: Comparison of exact and Hermite wavelet solutions for Case I

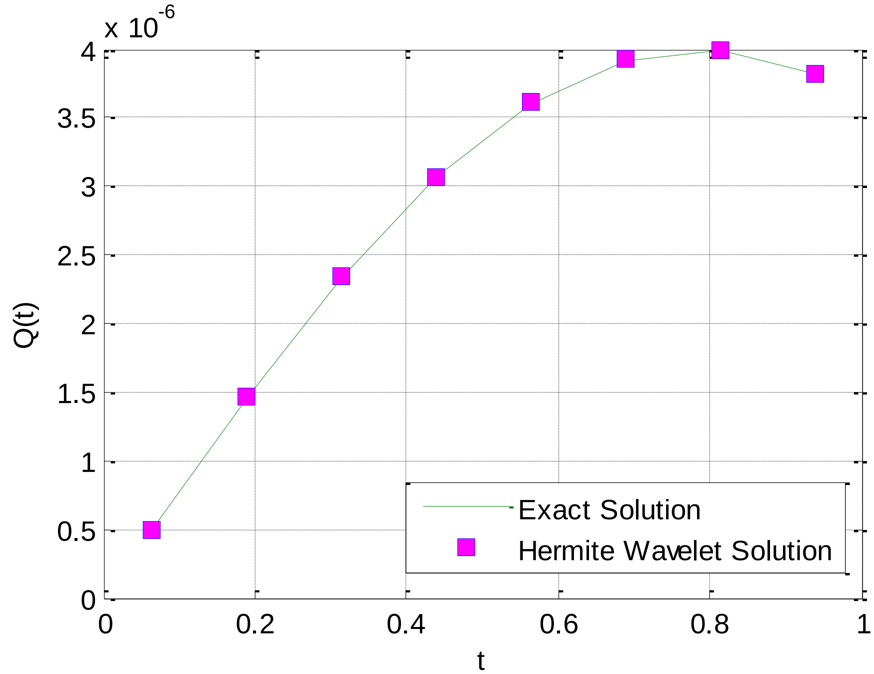

Figure 3: Comparison of exact and Hermite wavelet solutions ($k = 1,\ M = 8$) for Case I

Case II: Taking $E = 4v, w = 1\, radian, C = 0.3\mu F, L = 2H$

Table 3 shows the comparison of exact solution and Hermite wavelet solution ($k = 1,\ M = 8$) for Case II. Figure 4 represents the accuracy of the proposed method for Case II.

t	**Exact Solution**	**Hermite Wavelet Solution**	**Absolute Error**
1/16	7.5730e-008	7.4952e-008	7.7820e-010
3/16	2.2383e-007	2.2368e-007	1.4769e-010
5/16	3.6803e-007	3.6893e-007	8.9860e-010
7/16	5.0899e-007	5.0841e-007	5.8216e-010
9/16	6.4039e-007	6.3996e-007	4.2674e-010
11/16	7.6060e-007	7.6153e-007	9.2861e-010
13/16	8.7154e-007	8.7121e-007	3.3226e-010
15/16	9.6796e-007	9.6737e-007	5.8831e-010

Table 3: Comparison of exact and Hermite wavelet solutions for Case II

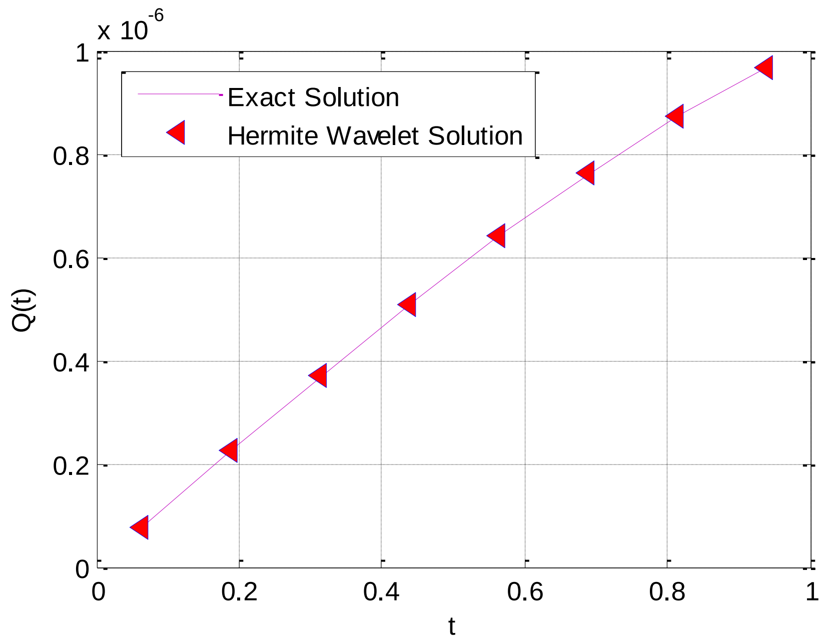

Figure 4: Comparison of exact and Hermite wavelet solutions ($k = 1,\ M = 8$) for Case II.

CONCLUSION

From the above numerical experiments, it is concluded that Hermite wavelet based collocation method is an accurate numerical technique for solving LCR circuit equations. The accuracy is improved by increasing the number of collocation points. For future scope, it will be applicable for two- and three- dimensional mathematical models arising in different branches of sciences and engineering.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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