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## MATHEMATICAL ANALYSIS AND ADAPTIVE CONTROL SPREADING OF CORONAVIRUS DISEASE 2019 (COVID-19)

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**Abstract:** Coronavirus disease 2019 (COVID-19) is an infectious disease caused by severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2), which is spreading all over the world and the main causes of worldwide death. For this reason, the control analysis of this coronavirus disease 2019 has a significant importance prevent the spread of it. In this paper, we present the modeling, mathematical analysis, and adaptive control of spreading coronavirus disease 2019. The mathematical analysis shows that the two fixed points of the coronavirus disease 2019 are globally asymptotically stable and the basic reproduction ratio  $R_0$  is obtained, which characterizes the disease transmission. Moreover, an adaptive control is designed to control and treat coronavirus outbreak. The sufficient control conditions are derived for the existence of stable coronavirus disease 2019 free is presented.

**Keywords:** fixed point; stability; adaptive control; coronavirus disease 2019 (COVID-19).

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## 1. INTRODUCTION

Infectious diseases mainly caused by pathogenic microorganisms, such as bacteria, viruses, fungi, and parasites. The diseases can spread directly or indirectly from one person to another or from animals/birds to humans. These diseases are one of the main causes of worldwide death.

Coronavirus disease 2019 (COVID-19) is an infectious disease caused by severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2). The first known case was identified in Wuhan, China in December 2019 [24]. The disease has since spread worldwide more than 150 countries across the world. On 8 March 2020, the WHO announced COVID-19 as a global pandemic [34]. As of 29 May 2021, there are over 170 million reported cases, and a death toll exceeding 3 million.

Transmission of COVID-19 occurs when people are exposed to virus-containing respiratory droplets and airborne particles exhaled by an infected person. Those particles may be inhaled or may reach the mouth, nose, or eyes of a person through touching or direct deposition such as being coughed. The risk of infection is highest when people are in proximity for a long time, but particles can be inhaled over longer distances, particularly indoors in poorly ventilated and crowded spaces. In those conditions small particles can remain suspended in the air for minutes to hours. Touching a contaminated surface or object may lead to infection. Symptoms of COVID-19 are including fever, cough, headache, fatigue, breathing difficulties, and loss of smell and taste [1, 10, 16, 26, 27]. Symptoms may begin one to fourteen days after exposure to the virus. The most 81% develop mild to moderate symptoms, while 14% develop severe symptoms, and 5% suffer critical symptoms.

Mathematical models have been developed to study the transmission dynamics of COVID-19 [5, 10, 22, 28]. Several mathematical models have already been formulated to analyze the complex transmission pattern of the COVID-19 pandemic, using ordinary differential equations [18, 27, 31], delay differential equations [11], stochastic differential equations [30]. At present, the SIS [21, 32], SIR [7] and SEIR [2, 3] models provide another way for the simulation of epidemics. Lots of research works have been reported. It shows that SIS, SIR and SEIR models can reflect the dynamics of different epidemics well, these models have been used to model the COVID-19 [9,

29]. For instance, the SIR model is commonly used for disease modeling for the COVID-19 analysis [4, 8, 23], Tang et al. [29] investigated a general SEIR type epidemiological model where quarantine, isolation and treatment are considered. Moreover, there are also other methods for modeling of the COVID-19 [35]. Wang et al. [33] applied the phase-adjusted estimation for the number of coronavirus disease 2019 cases in Wuhan.

Recently, many scholars have studied the optimal control of COVID-19 introducing different control variables and given the corresponding control strategies [12, 17, 25]. In fact, the optimal control theory can only aim at the systems with known parameters and obtain the system inputs by minimizing the cost function. It is worth noting that there are various uncertainties in the transmission of COVID-19. If there are uncertain parameters in the system, it is impossible for optimal control theory to obtain the desired results, which need to identify the system parameters in advance. Fortunately, adaptive control can update the system parameters by using adaptive laws and guarantee the stability of closed loop system [6].

In this paper, we present the modeling, mathematical analysis, and adaptive control of spreading coronavirus disease. First, we discuss the description of the proposed model. Then, we present its mathematical analysis, the two fixed points of the system are globally asymptotically stable by using Lasalle's theorem and the basic reproduction ratio  $R_0$  is obtained, which characterizes the disease transmission. Next, we design adaptive control for globally stabilize a general coronavirus disease 2019 model by using Lyapunov stability theory. Finally, the conclusion this paper is presented.

## 2. PRELIMINARIES

Consider a dynamical system which satisfies,

$$\dot{x} = f(x, t), \quad x(t_0) = x_0; \quad x \in R^n \quad (1)$$

where  $f(x, t): R^n \times R^+ \rightarrow R^n$ . A point  $x_e \in R^n$  is a fixed point of the system if  $f(x_e, t) = 0$ .

**Definition 2.1** The fixed point  $x_e$  is stable if for  $x_e > 0$  there exists  $\delta > 0$  such that,

$$\|x(0) - x_e\| < \delta \Rightarrow \|x(t) - x_e\| < \varepsilon$$

where  $x(0)$  is unique solution. The fixed point  $x_e$  is unstable if it is not stable. The fixed point  $x_e$  is asymptotically stable if it is stable and there exists  $\delta > 0$  such that,

$$\|x(0) - x_e\| < \delta \Rightarrow x(t) \rightarrow x_e \quad \text{as } t \rightarrow \infty$$

The fixed point  $x_e$  is globally asymptotically stable if it is stable and

$$x(t) \rightarrow x_e \quad \text{as } t \rightarrow \infty$$

The following result is the fundamental importance of a Lyapunov stability theorem [13].

**Theorem 2.2** (Lyapunov global asymptotically stability theorem or G.A.S)

Suppose there is a function  $V: R^n \rightarrow R$  such that,

- (1)  $V$  is positive definite,
- (2)  $\dot{V}(x) < 0$  for all  $x \neq 0, \dot{V}(0) = 0$

then, every trajectory of  $\dot{x} = f(x)$  converges to zero as  $t \rightarrow \infty$ , that is the system is globally asymptotically stable.

**Theorem 2.3** (Lasalle's theorem [20])

Lasalle's theorem allows us to conclude G.A.S. of a system with only  $\dot{V}(x) \leq 0$ , along with an observability type condition. We consider  $\dot{x} = f(x)$ . Suppose there is a function  $V: R^n \rightarrow R$  such that,

- (1)  $V$  is positive definite,
- (2)  $\dot{V}(x) \leq 0$ ,
- (3) the only solution  $\dot{\omega} = f(\omega), \dot{V}(\omega) = 0$  is  $\omega(t) = 0$  for all  $t$

then, the system  $\dot{x} = f(x)$  is globally asymptotically stable or G.A.S.

Next, we consider a nonlinear nonautonomous differential equation of the general form,

$$\begin{aligned} \dot{x}(t) &= f(t, x(t)); t \geq t_0 \in R \\ x(t_0) &= x_0 \end{aligned} \tag{2}$$

where the state  $x(t)$  take values in  $X$ ,  $f(t, x): R \times X \rightarrow X$  is a given nonlinear function and  $f(t, 0) = 0$ , for all  $t \in R$ .

An adaptive control is an active field in the design of control systems to deal with uncertainties. To design control laws that stabilize of the chaotic system. The control system can

be written as

$$\dot{x}(t) = f(t, x(t), u(t)); t \geq 0 \quad (3)$$

where  $u(t)$  is the external control.

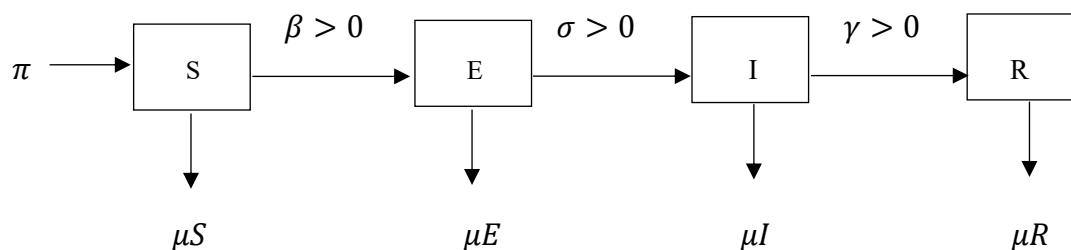
**Definition 2.4** The control system (3) is stabilizable if there exist the control  $u(t) = k(x(t))$  such that the system,

$$\dot{x}(t) = f(t, x(t), k(x(t))); t \geq 0 \quad (4)$$

is asymptotically stable.

### 3. MODEL DESCRIPTION

In this section, we formulate a general SEIR model for the spreading of coronavirus disease 2019 (COVID-19). We separate the total population  $N(t)$  into four distinct subgroups which are Susceptible  $S(t)$ , Exposed  $E(t)$ , Infectious  $I(t)$ , and recovered  $R(t)$ . Group of susceptible individuals  $S(t)$  which will be increased based on the birth  $\pi$  and decrease because the death  $\mu$  and direct contact with an infected individual group  $\beta$ , group of the exposed  $E(t)$ , which will increase with transmission rate  $\beta$ , decreased due to natural mortality  $\mu$  and have been the incubation period  $\sigma$ , group of individuals who are infected with coronavirus disease  $I(t)$ , which will increase with the incubation period  $\sigma$ , decreased due to natural mortality  $\mu$  and have been recovery with the rate  $\gamma$ , and group of individuals who recover  $R(t)$  who increased because of there was the recovery with rate  $\gamma$  and decreased because of natural mortality  $\mu$ .



Based on the assumptions, the mathematical model for spread of coronavirus disease 2019 can be represented by the following system of differential equations

$$\begin{aligned}
 \dot{S}(t) &= \pi - \beta SI - \mu S \\
 \dot{E}(t) &= \beta SI - \lambda_1 E \\
 \dot{I}(t) &= \sigma E - \lambda_2 I \\
 \dot{R}(t) &= \gamma I - \mu R
 \end{aligned} \tag{5}$$

where  $\lambda_1 = \sigma + \mu$ ,  $\lambda_2 = \gamma + \mu$  with  $N(t) = S(t) + E(t) + I(t) + R(t)$  and the initial condition

$$S(0) \geq 0, E(0) \geq 0, I(0) \geq 0, R(0) \geq 0$$

Considering the total population density is a constant value,

$$\frac{dN(t)}{dt} = 0$$

$$\frac{dS(t)}{dt} + \frac{dE(t)}{dt} + \frac{dI(t)}{dt} + \frac{dR(t)}{dt} = 0$$

We have  $\pi = \mu$ . By the total population density, we have  $S(t) + E(t) + I(t) + R(t) = 1$ .

Therefore, it is enough to consider,

$$\begin{aligned}
 \dot{S}(t) &= \pi - \beta SI - \mu S \\
 \dot{E}(t) &= \beta SI - \lambda_1 E \\
 \dot{I}(t) &= \sigma E - \lambda_2 I
 \end{aligned} \tag{6}$$

which the region for the above system as

$$\Omega = \{S(t), E(t), I(t) \in R_+^3, S(t) + E(t) + I(t) \leq 1\}$$

## 4. MATHEMATICAL ANALYSIS

### 4.1 FIXED POINT AND BASIC REPRODUCTION RATIO

In this section, we will discuss about the fixed point and basic reproduction ratio  $R_0$  of SEIR model.

The two fixed points are obtained as follows:

1. The disease-free fixed point of the proposed SEIR model is acquired by setting,

$E = I = 0$  in the system (6). Hence, we obtain the disease-free fixed point in the form

$$E_0 = \left(\frac{\pi}{\mu}, 0, 0\right).$$

2. The epidemic fixed point of the proposed SEIR model is acquired by set all the derivatives equal to zero and solved the system (6) as follows,

$$\dot{S}(t) = \dot{E}(t) = \dot{I}(t) = 0$$

Then, the system (6) gives,

$$\pi - \beta SI - \mu S = 0 \quad (7)$$

$$\beta SI - \lambda_1 E = 0 \quad (8)$$

$$\sigma E - \lambda_2 I = 0 \quad (9)$$

From equation (9), we have

$$E = \frac{\lambda_2}{\sigma} I \quad (10)$$

From equation (8), we have

$$S = \frac{\lambda_1 \lambda_2}{\sigma \beta} \quad (11)$$

Substituting equation (10) and (11) into equation (7), we get

$$I = \frac{\pi \sigma}{\lambda_1 \lambda_2} - \frac{\mu}{\beta} \quad (12)$$

or

$$I = \frac{\mu}{\beta} \left( \frac{\beta \pi \sigma}{\mu \lambda_1 \lambda_2} - 1 \right) \quad (13)$$

where  $R_0 = \frac{\beta \pi \sigma}{\mu \lambda_1 \lambda_2} = \frac{\beta \pi \sigma}{\mu(\sigma + \mu)(\gamma + \mu)}$  is the basic reproduction ratio. Hence, we obtain the epidemic

fixed point in the form  $E_1 = \left( \frac{\lambda_1 \lambda_2}{\sigma \beta}, \frac{\lambda_2}{\sigma} I, \frac{\mu}{\beta} \left( \frac{\beta \pi \sigma}{\mu \lambda_1 \lambda_2} - 1 \right) \right)$ . The basic reproduction ratio  $R_0$  can be

used to measure the rate of spreading of a disease,

1. For  $R_0 \leq 1$ , the patient could transmit the disease to a person and eventually the disease will disappear, this mean that the epidemic will not happened.

2. For  $R_0 > 1$ , the patient could infect in more a new patient and eventually the disease will epidemic, this mean that epidemic is happened.

## 4.2 STABILITY ANALYSIS OF FIXED POINT

In this section, we have discussed stability analysis of both fixed points. We used Lasalle's Theorem for both the disease-free fixed point and the endemic fixed point of the proposed model. First, we present the globally asymptotically stable of the disease-free fixed point.

**Theorem 4.1** If  $R_0 \leq 1$ , then the disease-free fixed point  $E_0 = (\frac{\pi}{\mu}, 0, 0)$  of the system is globally asymptotically stable on  $\Omega$ .

**Proof** To establish the globally asymptotically stable of the disease-free fixed point  $E_0$ , we construct the Lyapunov function  $V$ . Let  $V: \Omega \rightarrow R$  define by

$$V(S, E, I) = \left[ S - S^* \ln \left( \frac{S}{S^*} \right) \right] + \frac{E}{\lambda_1} + \frac{I}{\sigma} \quad (14)$$

The time derivative of  $V$  along the solution of the system, we obtain

$$\dot{V} = \left( 1 - \frac{S^*}{S} \right) \dot{S} + \frac{\dot{E}}{\lambda_1} + \frac{\dot{I}}{\sigma} \quad (15)$$

Substituting  $\dot{S}(t)$ ,  $\dot{E}(t)$ ,  $\dot{I}(t)$  in the equation (15) and let  $S^* = \frac{\pi}{\mu}$ , we get

$$\begin{aligned} \dot{V} &= \left( 1 - \frac{S^*}{S} \right) (\pi - \beta SI - \mu S) + \frac{1}{\lambda_1} (\beta SI - \lambda_1 E) + \frac{1}{\sigma} (\sigma E - \lambda_2 I) \\ &= \left( 1 - \frac{\pi}{\mu S} \right) (\pi - \beta SI - \mu S) + \frac{\beta SI}{\lambda_1} - \frac{\lambda_2 I}{\sigma} \\ &= 2\pi - \beta SI - \mu S - \frac{\pi^2}{\mu S} + \frac{\pi}{\mu} \beta I + \frac{\beta}{\lambda_1} SI - \frac{\lambda_2 I}{\sigma} \\ &= -\pi \left( \frac{\mu S}{\pi} + \frac{\pi}{\mu S} - 2 \right) + I \left( \frac{\pi}{\mu} \beta - \beta S + \frac{\beta}{\lambda_1} S - \frac{\lambda_2}{\sigma} \right) \\ &= -\pi \left( \frac{\mu S}{\pi} + \frac{\pi}{\mu S} - 2 \right) + I \frac{\lambda_2}{\sigma} \left( \frac{\beta \pi \sigma}{\mu \lambda_1 \lambda_2} - 1 \right) \\ &= -\pi \left( \frac{\mu S}{\pi} + \frac{\pi}{\mu S} - 2 \right) + I \frac{\lambda_2}{\sigma} (R_0 - 1) \end{aligned}$$

We can see that  $\dot{V} \leq 0$  for  $R_0 \leq 1$ . Therefore, by the Lasalle's Theorem [20], the disease-free fixed point  $E_0$  of the system is globally asymptotically stable on  $\Omega$ .



**Theorem 4.2** The endemic fixed point  $E_1 = \left( \frac{\lambda_1 \lambda_2}{\sigma \beta}, \frac{\lambda_2}{\sigma} I, \frac{\mu}{\beta} \left( \frac{\beta \pi \sigma}{\mu \lambda_1 \lambda_2} - 1 \right) \right)$  of the system is globally asymptotically stable on  $\Omega$ .

**Proof** To establish the globally asymptotically stable of the endemic fixed point  $E_1 = (S^*, E^*, I^*)$  where

$$S^* = \frac{\lambda_1 \lambda_2}{\sigma \beta}, \quad E^* = \frac{\lambda_2}{\sigma} I, \quad I^* = \frac{\mu}{\beta} \left( \frac{\beta \pi \sigma}{\mu \lambda_1 \lambda_2} - 1 \right)$$

We can construct the Lyapunov function  $V: \Omega_+ \rightarrow R$ , where

$$\Omega_+ = \{(S(t), E(t), I(t)) \in \Omega / S(t) > 0, E(t) > 0, I(t) > 0\}$$

is given by

$$V(S, E, I) = V_1 \left[ S - S^* \ln \left( \frac{S}{S^*} \right) \right] - V_2 \left[ E - E^* \ln \left( \frac{E}{E^*} \right) \right] - V_3 \left[ I - I^* \ln \left( \frac{I}{I^*} \right) \right] \quad (16)$$

where  $V_1, V_2$  and  $V_3$  are positive constants to be chosen. By taking the derivative of the above function, we obtain

$$\begin{aligned} \dot{V} &= \left[ \frac{\partial}{\partial S} V_1 S - \frac{V_1 S^*}{S} \right] (\pi - \beta SI - \mu S) - \left[ \frac{\partial}{\partial E} V_2 E - \frac{V_2 E^*}{E} \right] (\beta SI - \lambda_1 E) \\ &\quad - \left[ \frac{\partial}{\partial I} V_3 I - \frac{V_3 I^*}{I} \right] (\sigma E - \lambda_2 I) \\ &= \left[ V_1 - \frac{V_1 S^*}{S} \right] (\pi - \beta SI - \mu S) - \left[ V_2 - \frac{V_2 E^*}{E} \right] (\beta SI - \lambda_1 E) \\ &\quad - \left[ V_3 - \frac{V_3 I^*}{I} \right] (\sigma E - \lambda_2 I) \\ &= [V_1 S - V_1 S^*] \left[ \frac{\pi}{S} - \beta I - \mu \right] - [V_2 E - V_2 E^*] \left[ \frac{\beta SI}{E} - \lambda_1 \right] \\ &\quad - [V_3 I - V_3 I^*] \left[ \frac{\sigma E}{I} - \lambda_2 \right] \end{aligned} \quad (17)$$

From the fixed point  $E_1 = (S^*, E^*, I^*) = \left( \frac{\lambda_1 \lambda_2}{\sigma \beta}, \frac{\lambda_2}{\sigma} I, \frac{\mu}{\beta} \left( \frac{\beta \pi \sigma}{\mu \lambda_1 \lambda_2} - 1 \right) \right)$ , we have

$$\begin{aligned} -\mu &= \beta I^* - \frac{\beta \pi \sigma}{\lambda_1 \lambda_2} \\ -\lambda_1 &= -\frac{\beta I S^*}{E^*} \\ -\lambda_2 &= -\frac{\sigma E^*}{I} \end{aligned} \quad (18)$$

Substituting (18) into the equation (17) become

$$\dot{V} = -V_1\beta[S - S^*][I - I^*] - V_2\beta I[E - E^*] \left[ \frac{S}{E} - \frac{S^*}{E^*} \right] - V_3 \frac{\sigma}{I} [E - E^*]$$

We can see that  $\dot{V} \leq 0$ . Therefore, by the Lasalle's Theorem [20], the endemic fixed point  $E_1$  of the system is globally asymptotically stable on  $\Omega$ .

### 4.3 ADAPTIVE CONTROL OF CORONAVIRUS DISEASE 2019 (COVID-19)

In this section, we design the adaptive control for global stabilized the coronavirus disease 2019 (COVID-19). The sufficient control conditions are derived using by Lyapunov stability theorem.

**Theorem 4.3** The coronavirus disease 2019 is global stabilized for the initial conditions  $S(t) > 0$ ,  $E(t) > 0$ ,  $I(t) > 0$ ,  $R(t) > 0$  by the adaptive control  $u_1$ ,  $u_2$  and  $u_3$  where the estimate parameters are given by  $\hat{\pi} = S + K_4 e_\pi$ ,  $\hat{\beta} = SEI + K_5 e_\beta$ ,  $\hat{\mu} = -S^2 + K_6 e_\mu$ ,  $\hat{\sigma} = EI + K_7 e_\sigma$ ,  $\hat{\lambda}_1 = -E^2 + K_8 e_{\lambda_1}$ , and  $\hat{\lambda}_2 = -I^2 + K_9 e_{\lambda_2}$ .

**Proof** We design adaptive control for the coronavirus disease as follows,

$$\begin{aligned} \dot{S}(t) &= \pi - \beta SI - \mu S + u_1 \\ \dot{E}(t) &= \beta SI - \lambda_1 E + u_2 \\ \dot{I}(t) &= \sigma E - \lambda_2 I + u_3 \end{aligned} \quad (19)$$

where  $u_1, u_2$  and  $u_3$  are controllers to be designed using the states and estimates of the parameters of the system. We consider the adaptive control functions,

$$\begin{aligned} u_1 &= -\hat{\pi} + \hat{\beta} SI + \hat{\mu} S - K_1 S \\ u_2 &= -\hat{\beta} SI + \hat{\lambda}_1 E - K_2 E \\ u_3 &= -\hat{\sigma} E + \hat{\lambda}_2 I - K_3 I \end{aligned} \quad (20)$$

where  $\hat{\pi}$ ,  $\hat{\beta}$ ,  $\hat{\mu}$ ,  $\hat{\sigma}$ ,  $\hat{\lambda}_1$ , and  $\hat{\lambda}_2$  are estimates of the parameters and  $K_1, K_2$ , and  $K_3$  are positive constants. Substituting the controllers into the coronavirus disease, we have

$$\dot{S}(t) = (\pi - \hat{\pi}) - (\beta - \hat{\beta})SI - (\mu - \hat{\mu})S - K_1 S$$

$$\dot{E}(t) = (\beta - \hat{\beta})SI - (\lambda_1 - \hat{\lambda}_1)E - K_2E \quad (21)$$

$$\dot{I}(t) = (\sigma - \hat{\sigma})E - (\lambda_2 - \hat{\lambda}_2)I - K_3I$$

Define the parameter errors as

$$e_\pi = \pi - \hat{\pi}, e_\beta = \beta - \hat{\beta}, e_\mu = \mu - \hat{\mu}, e_\sigma = \sigma - \hat{\sigma}, e_{\lambda_1} = \lambda_1 - \hat{\lambda}_1, e_{\lambda_2} = \lambda_2 - \hat{\lambda}_2 \quad (22)$$

Thus, the equation (21) can be rewritten as

$$\dot{S}(t) = e_\pi - e_\beta SI - e_\mu S - K_1S$$

$$\dot{E}(t) = e_\beta SI - e_{\lambda_1}E - K_2E \quad (23)$$

$$\dot{I}(t) = e_\sigma E - e_{\lambda_2}I - K_3I$$

Consider the quadratic Lyapunov function,

$$V = \frac{1}{2}(S^2 + E^2 + I^2 + e_\pi^2 + e_\beta^2 + e_\mu^2 + e_\sigma^2 + e_{\lambda_1}^2 + e_{\lambda_2}^2) \in R^9 \quad (24)$$

which is a positive definite function on  $R^9$ . So that

$$\dot{e}_\pi = -\dot{\hat{\pi}}, \dot{e}_\beta = -\dot{\hat{\beta}}, \dot{e}_\mu = -\dot{\hat{\mu}}, \dot{e}_\sigma = -\dot{\hat{\sigma}}, \dot{e}_{\lambda_1} = -\dot{\hat{\lambda}}_1, \dot{e}_{\lambda_2} = -\dot{\hat{\lambda}}_2 \quad (25)$$

Differentiating V along the equation (23) and using equation (25), we get

$$\begin{aligned} \dot{V} = & Se_\pi - e_\beta S^2 I - e_\mu S^2 - K_1 S^2 + e_\beta SEI - e_{\lambda_1} E^2 - K_2 E^2 \\ & + e_\sigma EI - e_{\lambda_2} I^2 - K_3 I^2 - e_\pi \dot{\hat{\pi}} - e_\beta \dot{\hat{\beta}} - e_\mu \dot{\hat{\mu}} - e_\sigma \dot{\hat{\sigma}} - e_{\lambda_1} \dot{\hat{\lambda}}_1 - e_{\lambda_2} \dot{\hat{\lambda}}_2 \end{aligned} \quad (26)$$

The estimated parameters of equation (26) are update by the following,

$$\dot{\hat{\pi}} = S + K_4 e_\pi,$$

$$\dot{\hat{\beta}} = SEI + K_5 e_\beta,$$

$$\dot{\hat{\mu}} = -S^2 + K_6 e_\mu, \quad (27)$$

$$\dot{\hat{\sigma}} = EI + K_7 e_\sigma,$$

$$\dot{\hat{\lambda}}_1 = -E^2 + K_8 e_{\lambda_1},$$

$$\dot{\lambda}_2 = -I^2 + K_9 e_{\lambda_2}$$

where  $K_4, K_5, K_6, K_7, K_8$  and  $K_9$  are positive constants. Substituting equation (27) into equation (26), we obtain

$$\begin{aligned} \dot{V} &= S e_{\pi} - e_{\beta} S^2 I - e_{\mu} S^2 - K_1 S^2 + e_{\beta} S E I - e_{\lambda_1} E^2 - K_2 E^2 \\ &\quad + e_{\sigma} E I - e_{\lambda_2} I^2 - K_3 I^2 - e_{\pi} S - K_4 e_{\pi}^2 - e_{\beta} S E I - K_5 e_{\beta}^2 \\ &\quad + e_{\mu} S^2 - K_6 e_{\mu}^2 - e_{\sigma} E I - K_7 e_{\sigma}^2 + e_{\lambda_1} E^2 - K_8 e_{\lambda_1}^2 + e_{\lambda_2} I^2 - K_9 e_{\lambda_2}^2 \\ &= -e_{\beta} S^2 I - K_1 S^2 - K_2 E^2 - K_3 I^2 - K_4 e_{\pi}^2 - K_5 e_{\beta}^2 - K_6 e_{\mu}^2 - K_7 e_{\sigma}^2 \\ &\quad - K_8 e_{\lambda_1}^2 - K_9 e_{\lambda_2}^2 \end{aligned}$$

which is a negative definite function on  $R^9$ . Therefore, by Lyapunov stability theorem [13], we obtain the coronavirus disease 2019 is global stabilized for initial conditions  $S(t) > 0$ ,  $E(t) > 0$ ,  $I(t) > 0$ ,  $R(t) > 0$  by the adaptive control  $u_1, u_2$  and  $u_3$  where the estimate parameters are given by  $\hat{\pi} = S + K_4 e_{\pi}$ ,  $\hat{\beta} = S E I + K_5 e_{\beta}$ ,  $\hat{\mu} = -S^2 + K_6 e_{\mu}$ ,  $\hat{\sigma} = E I + K_7 e_{\sigma}$ ,  $\hat{\lambda}_1 = -E^2 + K_8 e_{\lambda_1}$ , and  $\hat{\lambda}_2 = -I^2 + K_9 e_{\lambda_2}$ .

## 5. CONCLUSIONS

In this paper, we present the modeling, mathematical analysis, and adaptive control of spreading coronavirus disease 2019. The mathematical analysis of the model showed that the proposed model has two fixed points: the disease-free fixed point  $E_0$  and the endemic fixed point  $E_1$ . The proposed model is determined by the basic reproduction ratio  $R_0$ , which depends on the parameter values, if  $R_0 \leq 1$ , then the patient could transmit the disease to a person and eventually the disease will disappear or disease free, while  $R_0 > 1$  the patient could infect in more a new patient and eventually the disease will epidemic. We also presented that the stability analysis of the disease-free fixed point  $E_0$  is globally asymptotically stable if  $R_0 \leq 1$ . On the other hand, the

global asymptotic stability of the endemic fixed point  $E_1$  occurs if  $R_0 > 1$ . Moreover, an adaptive control is designed to control and treat Coronavirus disease 2019 outbreak. The sufficient control conditions are derived for the existence of stable coronavirus disease 2019 free is presented.

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## CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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