MATHEMATICAL MODELING AND STABILITY ANALYSIS OF ENDEMIC EQUILIBRIUM POINT OF GAMING DISORDER

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Abstract. In this article, we propose a PEAR mathematical model that describes the dynamic of a population that reacts in the spread of the E-game infection. By using Routh-Hurwitz criteria and constructing Lyapunov functions, the local stability and the global stability of endimec equilibrium point are obtained. Since there are usually errors in data collection and assumed parameter values, we also study the sensitivity analysis of the model parameters to know the parameters that have a high impact on the reproduction number $\mathcal{R}_0$. The stability analysis of the model that they proposed shows that the system is locally and globally asymptotically stable at endemic equilibrium point when $\mathcal{R}_0 > 1$. Finally, some numerical simulations are performed to verify the theoretical analysis using Matlab software.

Keywords: electronic games; gaming disorder; mathematical model; global stability; local stability; sensitivity analysis.

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1. INTRODUCTION

Electronic games are widespread among children and youth and are now a lucrative business and are an important part of modern digital culture that affects people in many different ways. Electronic games can be a big problem if used frequently by a child or teenager, they can become a cause of neglect of personal, family or educational responsibilities. Electronic games can become addictive. trying to prohibit the child or teenager from using these games, will make them either sad or angry, and they may want to spend more time playing these games, it should be noted that problems with electronic games are very common, as the start of addiction statistics for these games was underway. In the early eighties, these statistics increased with the increase of the distribution of these games and their connection to the Internet. The problem of addiction to electronic games has been classified as a major problem that can lead to destruction of public health. The concept of electronic games has been linked to many health issues such as obesity, laziness, convergence and social illnesses, as it also affects physical and mental health, which occurs due to frequent use of these games. It also affects children's behaviour, as a study was conducted by a group of children aged 8 to 18 over a period of three years, and the results of this study indicated that violent electronic games stimulate aggressive behaviour, in children due to the large number of children witnessing violence [1]. Dr. Douglas Gentili, an expert on violence in video games and doctor of evolutionary psychology at Iowa State University, says that electronic games affect everybody, whether they are aggressive in nature or not [2]. The World Health Organization has officially classified (International Classification of Diseases ICD-11; WHO, 2018) continuing to play video or electronic games as an addiction leading to a mania, and has announced that people with this mania have certain characteristics, as the inability to 'stop gambling on winning, and according to the organization, a person is classified as having this disease if their addictive behaviour persists for 12 months. However, the diagnosis can be confirmed in a shorter period of time if it is certain that all symptoms are present, the warning about the severity of electronic gaming addiction is not new, but the formal classification of this addiction as a pathological obsession by the World Health Organization can be a major impetus to raise awareness of this disease and take serious action in families and societies to counter it [3] [4] . For years, research has been going on to study and understand the effect

of electronic games on the behavior and health of children, for example, we mention. In 2000, Jeanne B. Funk et Al [5] , this study examined relationships among time commitment, gender, prefirence for violent games, and self-concept in 364 fourth and fifth graders. Jeanne B. Funk et Al (2002) [6], correlations were examined between the preference for violent electronic gaming and adolescents' self-perception of emotional behaviors and feelings. In 2003, Barbara Krahé et Al [7], study to sample widely used electronic games varying in violent content,231 adolescents in Germany reported their use of and attraction to violent electronic games. Significant relationships were found between attraction to violent electronic games and the acceptance of norms condoning physical aggression. In 2007, L. Rowell Huesmann [8], the impact of media violence was compared to other known threats to society in order to estimate the importance of the threat to consider. Tom Baranowski et Al (2012) [9], This article explains the basic characteristics of a group of different technological methods; It shows the strengths and weaknesses of each of them in meeting the needs of children of all ages. Leon Straker et Al (2014) [10], proposes a model for factors affecting children's interaction with electronic games and discusses available guidelines and their role in the wise use of electronic games by children. These guidelines provide an accessible combination of available knowledge and practical evidence-based guidelines for electronic game and related research. Duven,E.C. et Al (2015) [11], diagnostic specificity to distinguish between gaming addiction and high engagement. Lemmens, J.S et Al (2016)[12], Examining the relationship between game genres and Internet Gaming Disorder.

In this work, and primarily based on the research done on the topic, we present a new approach in addressing the phenomenon by defining the degrees of the impact of electronic games on children and adolescents and satisfying them to a potential category that interests all children and adolescents between the ages of 0 and 16 years, including a group interested and plays less than 4 hours a day, and a class that has reached addiction level. Mathematical modeling is one of the most necessary functions that contribute to the representation and simulation of ecological, social and economic phenomena, and convert them into mathematical equations formulated, studied, analyzed, and interpreted their results as examples; Juan Gabriel Brida et Al [13], generalizes the classical model of determination of production prices for two commodities by introducing a dynamics generated by the possibility that the prot rate can be computed

using prices of different stages. Zeyang Wang et Al [14], considers and describes the class of cooperative differential games with non-transferable utility and the process of construction of the optimal Pareto strategy with continuous updating. Many studies and research in the social sciences have focused on this subject ([15-17]). However, mathematical studies and research on this subject are still limited and most have focused on the statistical aspect of the phenomenon, for example the study completed by F. A. E tindele Sosso et Al (2020) [18] on insomnia, sleepiness, anxiety and depression among different types of players in African countries.

Country	Total	Age	Men	Women	Mean Hours of Gaming/ week	Mean Months of Gaming/ gamer	Addicted Gamers	Problematic gamers	Engaged Gamers	Non problematic Gamers	Gamers using smartphones	Gamers using tablets	Gamers using computers	Gamers using consoles
South Africa	2580	21±3	2194 (85.04%)	386 (14.96%)	17±2.5	14±2	820 (31.78%)	805 (31.2%)	105 (4.07%)	850 (32.95%)	1500 (58.14%)	94 (3.64%)	800 (31%)	186 (7.22%)
Cameroon	1456	19±3	1300 (89.28%)	156 (10.72%)	10±1.75	19±1	456 (31.32%)	435 (29.87%)	85 (5.84%)	480 (32.97%)	700 (48.08%)	63 (4.33%)	286 (19.64%)	407 (27.95%)
Ivory Coast	908	23±2	785 (86.45%)	123 (13.55%)	12±1.25	13±1	278 (30.62%)	227 (25%)	53 (5.84%)	350 (38.54%)	230 (25.33%)	58 (6.39%)	520 (57.27%)	100 (11.01%)
Gabon	449	20±2	414 (92.2%)	35 (7.8%)	6±0.5	15±1	134 (29.84%)	123 (27.4%)	46 (10.24%)	146 (32.52%)	153 (34.08%)	217 (48.33%)	62 (13.8%)	17 (3.79%)
Morocco	1831	20±2	1662 90.77%	169 (9.23%)	18±2.5	18±2	504 (27.53%)	497 (27.14%)	157 (8.57%)	673 (36.76%)	950 (51.9%)	41 (2.24%)	689 (37.62%)	151 (8.24%)
Nigeria	1298	25±2	1147 (88.37%)	151 (11.63%)	12±2.25	13±1.75	350 (26.96%)	350 (26.96%)	126 (9.71%)	472 (36.37%)	456 (35.13%)	83 (6.4%)	605 (46.61%)	154 (11.86%)
Rwanda	897	25±1	809 (90.19%)	88 (9.81%)	8±2	10±2	260 (29%)	230 (25.64%)	146 (16.27%)	261 (29.09%)	371 (41.36%)	18 (2.01%)	316 (35.23%)	192 (21.4%)
Sénégal	846	22±2	770 (91%)	76 (9%)	6±2.5	11±0.25	211 (24.94%)	320 (37.83%)	54 (6.38%)	261 (30.85%)	387 (45.74%)	7 (0.83%)	366 (43.26%)	86 (10.17%)
Tunisia	301	26±4	285 (94.68%)	16 (5.32%)	20±2.5	26±2	57 (18.94%)	83 (27.58%)	73 (24.25%)	88 (29.23%)	100 (33.22%)	26 (8.64%)	94 (31.23%)	81 (26.91%)

FIGURE 1. Descriptive epidemiology of gaming among the nine African countries where prevalence of gaming, mean hours of gaming per week, period from when participant considered himself a gamer and type of device used for gaming purposes are described with age and sex[18].

Bouchaib Khajji et Al [19] studied the dynamical behavior of continuous mathematical model of alcohol drinking with the influence of private and public addiction treatment centers and discussed the basic properties of the system and determine its basic reproduction number $\mathscr{R}_0$. Omar Balatif et Al [20] this work, proposes a fractional-order model that describes the dynamics of citizens who have the right to register on the electoral lists and the negative influence of abstainers on the potential electors (the local and the global stability of abstaining-free equilibrium and abstaining equilibrium are obtained).

In this work, we propose a new model that describes the dynamics of electronic game addiction. By using Routh-Hurwitz criteria and constructing Lyapunov functions, the local stability and the global stability of endimec equilibrium point are obtained. Since there are usually errors in data collection and assumed parameter values, we also study the sensitivity analysis of the model parameters to know the parameters that have a high impact on the reproduction number $\mathcal{R}_0$. The stability analysis of the model that they proposed shows that the system is locally and globally asymptotically stable at endemic equilibrium point when $\mathcal{R}_0 > 1$.

The paper is organized as follows. in Section 2, we propose a PEAR mathematical model that describes the dynamic of a population that reacts in the spread of the E-game infection. In Section 3, we give some basic properties of the model. In Section 4, we analyze the local stability and global stability and the problem of parameters' sensitivity and some numerical simulations through Matlab software are given. Finally, we conclude the paper in Section 5.

2. A MATHEMATICAL MODEL OF GAMING DISORDER

2.1. Description of the Model: This section discusses the transmission dynamics of Gaming Disorder. we consider the compartments:

Potential Gamers (P), represent children and youth who are vulnerable to infection or who are more likely to become addicted to electronic games. This compartment increasing by the recruitment rate denoted Λ. It is decreasing by μ the rate of people who are older than the age limit set for people concerned with the study. Also It is decreasing by an effective contact with engaged gamers at rate β_1(the rate of patients who become engaged gamers because of the bad contact with the other engaged gamers) and with addicted gamers at rate β_2(the rate of patients who become engaged gamers because of the bad contact with the addicted gamers).

Engaged Gamers (E), represent children and youth interested in electronic games and plays more than four hours a day. This compartment is increasing by β_1 and β_2, and decreasing by the rate μ and by α_1 that represent a rate of the engaged gamers who have become addicted gamers. Also it is decreasing by γ the rate of the engaged gamers who have become the recovered gamers.

Addicted Gamers (A), represent children and youth who are addicted to electronic games and who suffer from gaming disorders and who have no control over their gaming habits, prioritize gaming over other interests and activities, and continue to game despite its negative consequences. This compartment increasing by α_1. The compartment of addicted gamers A is decreasing by the rate μ . Also it is decreasing by the rate of the addicted gamers who have become the recovered gamers denoted α_2.

Recovered Gamers (R), represent children and youth recovering from their addiction to electronic games. This compartment increasing by γ and α_2.

As a deterministic model, we suppose that, the compartments depend on time. The total population of individuals, N(t) at time t is given as; N(t) = P(t) + E(t) + A(t) + R(t). The following illustration will present disease trends in the compartments (Figure 2).
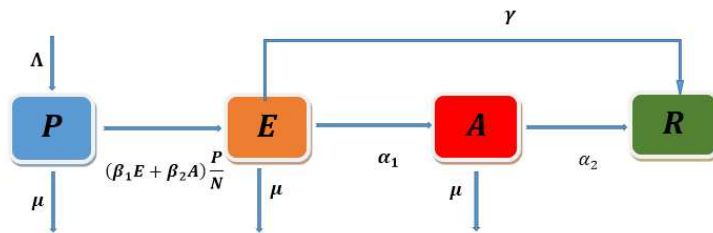

FIGURE 2. Description diagram of the gaming disorder dynamics

2.2. Model Equations. By adding the rates at which the compartment and also by subtracting the rates at which people leave compartment, we obtain a differential equations for the rate at which patients change in each compartmeent during separate times. Therefore, we present the gaming disorder model with the following system of differential equations:

(1)
$$\begin{cases} \frac{dP(t)}{dt} = \Lambda - \mu P(t) - \beta_1 \frac{P(t)E(t)}{N} - \beta_2 \frac{P(t)A(t)}{N} \\ \frac{dE(t)}{dt} = \beta_1 \frac{P(t)E(t)}{N} + \beta_2 \frac{P(t)A(t)}{N} - (\mu + \alpha_1 + \gamma)E(t) \\ \frac{dA(t)}{dt} = \alpha_1 E(t) - (\mu + \alpha_2)A(t) \\ \frac{dR(t)}{dt} = \alpha_2 A(t) + \gamma E(t) - \mu R(t) \end{cases}$$

where $P(0) \geq 0, E(0) \geq 0, A(0) \geq 0$, and $R(0) \geq 0$ are given initial states.

3. THE MODEL ANALYSIS BASIC PROPERTIES

3.1. Positivity of the model's solutions.

Theorem1.: If $P(0) \geq 0, E(0) \geq 0, A(0) \geq 0$, and $R(0) \geq 0$ then solutions $P(t), E(t), A(t)$, and $R(t)$ of system(1) are positive of all $t \geq 0$.

Proof.:

$$\frac{dP(t)}{dt} = \Lambda - \mu P(t) - \beta_1 \frac{P(t)E(t)}{N} - \beta_2 \frac{P(t)A(t)}{N} \geq -\mu P(t) - \beta_1 \frac{P(t)E(t)}{N} - \beta_2 \frac{P(t)A(t)}{N}$$

Then

$$\frac{dP(t)}{dt} + F(t)P(t) \geq 0$$

where

$$F(t) = \mu + \beta_1 \frac{E(t)}{N} + \beta_2 \frac{A(t)}{N}$$

The both sides in the last inequality are multiplied by $\exp(\int_0^t F(s)ds)$.

We obtain

$$\exp\left(\int_0^t F(s)ds\right)\frac{dP(t)}{dt} + F(t)\exp\left(\int_0^t F(s)ds\right)P(t) \geq 0$$

then

$$\frac{d(\exp(\int_0^t F(s)ds)P(t))}{dt} \geq 0$$

integrating this inequality from 0 to t gives :

$$P(t) \geq P(0)\exp\left(-\int_0^t (\mu + \beta_1 \frac{E(s)}{N} + \beta_2 \frac{A(s)}{N})ds\right)$$

So the solution $P(t)$ is positive.

For the positivity of E(t) and A(t) we have:

If $A(t) \geq E(t)$ then

$$\frac{dE(t)}{dt} + F(t)E(t) \geq 0$$

Where

$$F(t) = (\mu + \alpha_1 + \gamma - (\beta_1 + \beta_2)\frac{P(t)}{N})$$

So

$$E(t) \geq E(0)\exp(-\int_0^t (\mu + \alpha_1 + \gamma - (\beta_1 + \beta_2)\frac{P(s)}{N})ds)$$

If $E(t) \geq A(t)$

$$then \frac{dA(t)}{dt} + F(t)A(t) \geq 0$$

Where

$$F(t) = (\mu + \alpha_2 - \alpha_1)$$

So

$$A(t) \geq A(0)\exp(-\int_0^t (\mu + \alpha_2 - \alpha_1)ds)$$

We conclude that $E(t) \geq 0$ and $A(t) \geq 0$

Similarly, from the fourth equation of system(1), we have

$$R(t) \geq R(0)\exp(-\mu t) \geq 0$$

Therefore,we can see that $P(t) \geq 0, E(t) \geq 0, A(t) \geq 0$ and $R(t) \geq 0 \ \forall t \geq 0$.

3.2. Invariant Region. It is necessary to prove that all solutions of system(1) with positive initial data will remain positive for all times t. we obtained the invariant region, in whith the model solution is bounded. This will be established by the following lemma.

Lemma 1: The set defined by

$$\Omega = \{(P,E,A,R) \in \mathbb{R}_+^4; 0 \leq P+E+A+R \leq \frac{\Lambda}{\mu}\}$$

with initial condition $P(0) \geq 0, E(0) \geq 0, A(0) \geq 0$, and $R(0) \geq 0$ are positive invariants for system(1) .

Proof: By adding all equations in system (1), one has

$$\frac{dN}{dt} = \Lambda - \mu N$$

implies that

$$\Rightarrow N(t) = N_0 \exp(-\mu t) + \frac{\Lambda}{\mu}$$

where

$$N_0 = P(0) + E(0) + A(0) + R(0)$$

thus

$$0 \leq \limsup_{t \longrightarrow +\infty} N(t) = \frac{\Lambda}{\mu}$$

Then all possible solutions of the system(1) enter the region Ω. It inplies that Ω is a positively invariant set for the system(1). Therefore the basic model is well posed epidemiologically and mathematically. Hence, it is sufficient to study the dynamics of the basic model in Ω.

4. STABILITY ANALYSIS AND SENSITIVITY OF THE MODEL PARAMETERS

In this section, we will study the stability behavior of system(1) at an Disease an Endemic Equilibrium point. The first three equations in system(1) are independent of the variable R. Hence, the dynamics of equation system(1) is equivalent to the dynamics of the equation system:

$$(2) \quad \begin{cases} \frac{dP(t)}{dt} = \Lambda - \mu P(t) - \beta_1 \dfrac{P(t)E(t)}{N} - \beta_2 \dfrac{P(t)A(t)}{N} \\ \frac{dE(t)}{dt} = \beta_1 \dfrac{P(t)E(t)}{N} + \beta_2 \dfrac{P(t)A(t)}{N} - (\mu + \alpha_1 + \gamma)E(t) \\ \frac{dA(t)}{dt} = \alpha_1 E(t) - (\mu + \alpha_2)A(t) \end{cases}$$

4.1. Equilibrium Point.

4.1.1. *The Free Equilibrium Point.* To find the free equilibrium point (P_0, E_0, A_0, R_0), we equated the right hand side of model(1) to zero, evaluating it at $E=A=0$ and solving for the noninfected and noncarrier state variables. Therefore, the free equilibrium point $(\frac{\Lambda}{\mu}, 0, 0, 0)$.

4.1.2. *The Endemic Equilibrium Point.* The endemic equilibrium point (P^*, E^*, A^*, R^*) it occurs when the disease persists in the community. To obtain it, we equate all the model equations(1) to zero. then we obtain

$$P^* = \frac{\Lambda}{\mu \mathscr{R}_0}$$

$$E^* = \frac{\Lambda(\mathscr{R}_0 - 1)}{(\mu + \alpha_1 + \gamma)\mathscr{R}_0}$$

$$A^* = \frac{\alpha_1 \Lambda(\mathscr{R}_0 - 1)}{(\mu + \alpha_2)(\mu + \alpha_1 + \gamma)\mathscr{R}_0}$$

$$R^* = \frac{\Lambda(\mathscr{R}_0 - 1)[\gamma(\mu + \alpha_2) + \alpha_1 \alpha_2]}{\mu(\mu + \alpha_2)(\mu + \alpha_1 + \gamma)\mathscr{R}_0}$$

where $\mathscr{R}_0$ is the basic reproduction number given by:

$$\mathscr{R}_0 = \frac{\beta_1(\mu + \alpha_2) + \beta_2 \alpha_1}{(\mu + \alpha_2)(\mu + \alpha_1 + \gamma)}$$

4.2. The Basic Reproductive Number. In our work, the basic reproduction number $\mathscr{R}_0$ is defined as the average number of secondary infections produced by an infected individual in a completely potential gamers. To obtain the basic reproduction number, we used the next-generation matrix method formulated in [21,22].

Through the model equations system(2),then by the principle of next generation matrix, we obtained:

$$f = \begin{pmatrix} -\beta_1 \dfrac{P(t)E(t)}{N} - \beta_2 \dfrac{P(t)A(t)}{N} \\ \beta_1 \dfrac{P(t)E(t)}{N} + \beta_2 \dfrac{P(t)A(t)}{N} \\ 0 \end{pmatrix}$$

$$v = \begin{pmatrix} -\Lambda + \mu P(t) \\ (\mu + \alpha_1 + \gamma)E(t) \\ (\mu + \alpha_2)A(t) - \alpha_1 E(t) \end{pmatrix}$$

where $F = \begin{pmatrix} 0 & -\beta_1 & -\beta_2 \\ 0 & \beta_1 & \beta_2 \\ 0 & 0 & 0 \end{pmatrix}$

$$V = \begin{pmatrix} \mu & 0 & 0 \\ 0 & (\mu + \alpha_1 + \gamma) & 0 \\ 0 & -\alpha_1 & (\mu + \alpha_2) \end{pmatrix}$$

The inverse of V is given by

$$V^{-1} = \begin{pmatrix} \dfrac{1}{\mu} & 0 & 0 \\ 0 & \dfrac{1}{(\mu + \alpha_1 + \gamma)} & 0 \\ 0 & \dfrac{\alpha_1}{(\mu + \alpha_1 + \gamma)(\mu + \alpha_2)} & \dfrac{1}{(\mu + \alpha_2)} \end{pmatrix}$$

then

$$FV^{-1} = \begin{pmatrix} 0 & -\left(\dfrac{\beta_1(\mu + \alpha_2) + \beta_2\alpha_1}{(\mu + \alpha_1 + \gamma)(\mu + \alpha_2)}\right) & \dfrac{-\beta_2}{\mu + \alpha_2} \\ 0 & \left(\dfrac{\beta_1(\mu + \alpha_2) + \beta_2\alpha_1}{(\mu + \alpha_1 + \gamma)(\mu + \alpha_2)}\right) & \dfrac{\beta_2}{\mu + \alpha_2} \\ 0 & 0 & 0 \end{pmatrix}$$

Finally, we have

$$\mathscr{R}_0 = \rho(FV^{-1}) = \frac{\beta_1(\mu + \alpha_2) + \beta_2\alpha_1}{(\mu + \alpha_1 + \gamma)(\mu + \alpha_2)}$$

4.3. Local Stability of The Endemic Equilibrium point.

Theorem 2: The endemic equilibrium point is locally asymptotically stable if $\mathscr{R}_0 > 1$ and unstable otherwise.

Proof: The Jacobian matrix of system(1) at the endemic equilibrium point as follows:

$$J^* = \begin{pmatrix} -\mu\mathscr{R}_0 & -\dfrac{\beta_1}{\mathscr{R}_0} & -\dfrac{\beta_2}{\mathscr{R}_0} & 0 \\ \mu(\mathscr{R}_0 - 1) & \dfrac{\beta_1}{\mathscr{R}_0} - k_1 & \dfrac{\beta_2}{\mathscr{R}_0} & 0 \\ 0 & \alpha_1 & -k_2 & 0 \\ 0 & \gamma & \alpha_2 & -\mu \end{pmatrix}$$

Where $k_1 = (\mu + \alpha_1 + \gamma)$ and $k_2 = (\mu + \alpha_2)$

From the jacobian matrix J^* we obtained a characteristic polynomial:

$$p(\lambda) = (\mu + \lambda)(\lambda^3 + a\lambda^2 + b\lambda + c) = 0$$

with

$$a = \mu\mathscr{R}_0 + k_2 + \frac{\alpha_1\beta_2}{k_2\mathscr{R}_0} > 0$$

$$b = \mu(\mathscr{R}_0 k_2 + (\mathscr{R}_0 - 1)k_1 + \frac{k_1\beta_2}{k_2\mathscr{R}_0}) > 0$$

$$c = \mu k_1 k_2(\mathscr{R}_0 - 1) > 0$$

and

$$ab - c = \mu[(\mu \mathscr{R}_0 + \tfrac{\alpha_1 \beta_2}{k_2 \mathscr{R}_0})(\mathscr{R}_0 k_2 + (\mathscr{R}_0 - 1)k_1 + \tfrac{k_1 \beta_2}{k_2 \mathscr{R}_0}) + k_2(\mathscr{R}_0 k_2 + \tfrac{k_1 \beta_2}{k_2 \mathscr{R}_0})] > 0$$

we see that the characteristic equation $p(\lambda)$ of J^* has an eigenvalue $\lambda_1 = -\mu$ is negative.

So, in order to determine the stability of the endemic equilibrium point, we discuss the roots of the following equation $\lambda^3 + a\lambda^2 + b\lambda + c = 0$.

By Routh-Hurwitz criterion, system(1) is locally asymptotically stable if $a > 0$, $b > 0$, $c > 0$ and $ab > c$.

Obviously we see that a, b, c and ab-c to be positive, $(\mathscr{R}_0 - 1)$ must be positive, which leads to $\mathscr{R}_0 > 1$.

So the endemic equilibrium point is locally asymptotically stable if $\mathscr{R}_0 > 1$ and unstable if $\mathscr{R}_0 < 1$.

4.4. Global Stability of The Endemic Equilibrium point.

Theorem 3: If $\mathscr{R}_0 > 1$ then The endemic equilibrium point of the system is globally asymptotically stable on Ω.

Proof: Consider the lyapunov fonction $L_{v^*} : \Omega \to \mathbb{R}$ given by:

$$L_{v^*} = c_1(P - P^* ln(\tfrac{P}{P^*})) + c_2(E - E^* ln(\tfrac{E}{E^*}))$$

Where c_1 and c_2 are positive constants to be chosen latter. Then the time derivative of the lyapunov fonction is given by :

$$\frac{dL_{v^*}}{dt} = (\beta_1 + \beta_2 \tfrac{A}{E})\frac{(c_2 - c_1)}{N}(P - P^*)((E - E^*) - \frac{\Lambda c_1 (P - P^*)^2}{PP^*}$$

For $c_2 = c_1 = 1$ we have $\frac{dL_{v^*}}{dt} = -\frac{\Lambda(P - P^*)^2}{PP^*} \leq 0$

Also we obtain $\frac{dL_{v^*}}{dt} = 0 \Longleftrightarrow P = P^*$

Hence by la salle's invariance principle [23], the endemic equilibrium point is globally asymptotically stable on Ω.

4.5. Numerical Simulation.
In this section, we present some numerical solutions of system (1) for different values of the parameters. The resolution of system (1) was created using the Gauss-Seidel-like implicit finite-difference method developed by Gumel and al. [24], presented in [25] and denoted the GSS1 method. the different initial values for each variable of state,

and the following parameters: $\Lambda = 20000$, $\mu = 0.01$, $\alpha_1 = 0.3$, $\alpha_2 = 0.2$, $\beta_1 = 0.5$, $\beta_2 = 0.7$, $\gamma = 0.4$, we have the Endemic Equilibrium point $(1.93 * 10^6, 2.1 * 10^4, 3.062 * 10^4, 1.838 * 10^4)$ and $\mathscr{R}_0 = 2.1127 > 1$. In this case, we obtained the following remarks: Over time, we notice that the number of potential gamers is close to $1.93 * 10^6$. We also note that the number of the engaged gamers is close to $2.1 * 10^4$, and the number of the addicted gamers are close to $3.062 * 10^4$ (see Figure 3).

Therefore the solution curves to the endemic equilibruim point when $\mathscr{R}_0 > 1$.
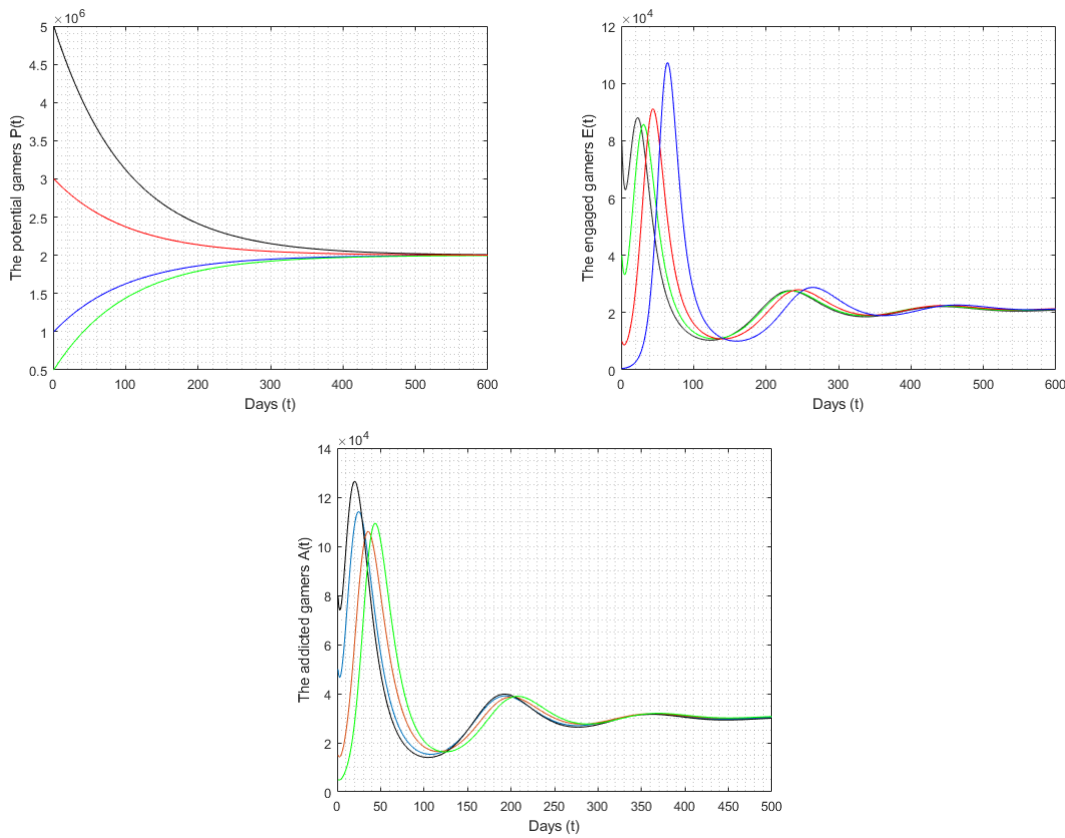

FIGURE 3. Trajectory of the solutions $P(t), E(t)$ and $A(t)$

According to theorem (3), the endemic equilibrium point of system (1) is globally asymptotically stable on Ω.

4.6. Sensitivity Analysis of Model Parameters. we performed a sensitivity analysis, On the basic parameters. to help us to know the parameters that have a high impact on the breeding number of base $\mathscr{R}_0$.

To achieve a sensitivity analysis of the model (1) , we followed the technique described by [26]. This technique develops a formula to obtain the sensitivity index of all the basic parameters, defined as

$\triangle_x^{\mathscr{R}_0} = \frac{x}{\mathscr{R}_0} * \frac{\partial \mathscr{R}_0}{\partial x}$, for x represents all the basic parameters. The sensitivity index of $\mathscr{R}_0$ with respect to the remaining parameters are computed as follows:

$$\triangle_\mu^{\mathscr{R}_0} = \frac{\mu}{\mathscr{R}_0} * \frac{\partial \mathscr{R}_0}{\partial \mu} = -\mu \Big[\frac{1}{(\mu + \alpha_1 + \gamma)} + \frac{\beta_2 \alpha_1}{(\mu + \alpha_2)(\beta_1(\mu + \alpha_2) + \beta_2 \alpha_1)} \Big]$$

$$\triangle_{\alpha_1}^{\mathscr{R}_0} = \frac{\alpha_1}{\mathscr{R}_0} * \frac{\partial \mathscr{R}_0}{\partial \alpha_1} = \frac{\alpha_1 [\beta_2(\mu + \gamma) - \beta_1(\mu + \alpha_2)^2]}{(\mu + \alpha_1 + \gamma)(\mu + \alpha_2)[\beta_1(\mu + \alpha_2) + \beta_2 \alpha_1]}$$

$$\triangle_{\alpha_2}^{\mathscr{R}_0} = \frac{\alpha_2}{R_0} * \frac{\partial \mathscr{R}_0}{\partial \alpha_2} = \frac{-\alpha_1 \alpha_2 \beta_2 (\mu + \alpha_1 + \gamma)}{(\mu + \alpha_1 + \gamma)(\mu + \alpha_2)[\beta_1(\mu + \alpha_2) + \beta_2 \alpha_1]}$$

$$\triangle_{\beta_1}^{\mathscr{R}_0} = \frac{\beta_1}{\mathscr{R}_0} * \frac{\partial \mathscr{R}_0}{\partial \beta_1} = \frac{\beta_1(\mu + \alpha_2)}{\beta_1(\mu + \alpha_2) + \beta_2 \alpha_1}$$

$$\triangle_{\beta_2}^{\mathscr{R}_0} = \frac{\beta_2}{\mathscr{R}_0} * \frac{\partial \mathscr{R}_0}{\partial \beta_2} = \frac{\beta_2 \alpha_1}{\beta_1(\mu + \alpha_2) + \beta_2 \alpha_1}$$

$$\triangle_\gamma^{\mathscr{R}_0} = \frac{\gamma}{\mathscr{R}_0} * \frac{\partial \mathscr{R}_0}{\partial \gamma} = \frac{-\gamma}{\mu + \alpha_1 + \gamma}$$

Their sensitivity indices obtained and evaluated are in Table 1:

Table 1: sensitivity indices of $\mathscr{R}_0$

Parameter Symbol	Value	Sensitivity indexes
μ	0.01	-0.0458305387882853
α_1	0.3	1.69237648110888
α_2	0.2	-0.634920634920635
β_1	0.5	0.333333333333333
β_2	0.7	0.666666666666667
γ	0.4	-0.563380281690141

Interpretation of Sensitivity indexes : The sensitivity indexes of the basic reproductive number with regard to the main parameters are arranged in order in Table 1. These parameters which

have positive indices show that they have a great impact on the expansion of the gaming disorder in the community if their values increase. the reason that the base reproduction number increases their values increase, it means that the average number of secondary cases of infection increase in the community. And also the parameters in which their sensitivity indices are negative have an influence of reducing at least the burden of the gaming disorder in the community their values increase while the others are left constant. And also as their values increase, the number of basic reproductions decreases, which leads to minimizing the endemicity of the gaming disorder in the community. Therefore, with sensitivity analysis, one can gain insight into the appropriate intervention strategies to prevent and control the spread and behavior of the gaming disorder described by model (1).

5. CONCLUSION

In this research a mathematical epidemic model for the human population of the gaming disorder is considered. In the pattern of transmission of infection spreads from the engaged or addicted gamers to the sensitive individual population, In the article we proceeded as , after the introduction and related literature in the first, we form a mathematical model that describes the dynamics of gaming disorder. The qualitative analysis of the model shows that the solution of the model is bounded and positive and also by using Routh–Hurwitz criterian and constructing Lyapunov functions, the local stability and the global stability of endemic equilibrium point are obtained. We have found $\mathscr{R}_0 = \dfrac{\beta_1(\mu + \alpha_2) + \beta_2 \alpha_1}{(\mu + \alpha_2)(\mu + \alpha_1 + \gamma)}$ as basic reproduction number of system (1), which helps us to determine the dynamical behavior of the system. We also studied the sensitivity analysis of the model parameters to know the parameters that have a high impact on the reproduction number $\mathscr{R}_0$. In our future work, we expect to obtain some more results based on real data from known gaming disorder disease to illustrate the validity of our theoretical results, such as how to predict the occurrence of gaming disorder disease, in which way do bifurcations, chaos and strange attractors impact on the dynamics of disease. In addition gain insight into the appropriate intervention strategies to prevent and control the spread and behavior of the gaming disorder described by model.

DATA AVAILABILITY

No data were used to support this study.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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