Available online at http://scik.org
J. Math. Comput. Sci. 11 (2021), No. 5, 6403-6419
https://doi.org/10.28919/jmcs/6256
ISSN: 1927-5307

BIPOLAR INTUITIONISTIC FUZZY GRAPH OVER CAYLEY GROUPS

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Abstract. In this article, we discuss the notion of Cayley bipolar intuitionistic fuzzy graph and proved some results and properties on it. We study and discuss many results and properties of bipolar intuitionistic fuzzy graph with respect of algebraic structures. On the other hand we study Cayley bipolar intuitionistic fuzzy graph connectedness.

Keywords: bipolar intuitionistic fuzzy set's; Cayley bipolar intuitionistic fuzzy graph; bipolar intuitionistic fuzzy subsemigroups; connectedness.

2010 AMS Subject Classification: 05C25, 03E72, 05C99, 03F55.

1. Introduction

In 1965, Zadeh [1] studied and introduced fuzzy set theory. Fuzzy set theory has many applications in different disciplines including enginearing, mathematics, medical, life sciences and so on. In 1983, Atanassov [2] gave the concept of intuitionistic fuzzy set's. Many researchers have used this concept and applied it to many articles, such that, in 2014, Yahya and Jahir [3] studied the concept of isomoomorphism and complements on irregular intuitionistic fuzzy graph. In 2015, Pathinan and Rosline [4] introduced and discussed the concept of vertex degree of cartesion product of intuitionistic fuzzy graph.

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Received June 08, 2021

Massa'deh et al used this concept in more than one field in the mathematics subjects see [5, 6, 7, 8]. Zhang [9], in 1994 discussed a bipolar fuzzy set concept so that a gerenalization of fuzzy sets. Recently, the bipolar fuzzy set have been discussed in more than one area see [10, 11, 12, 13, 14]. In 2015, Sankar and Ezhilmaran [15] have defined and used the concept of bipolar intuitionistic fuzzy set's, after that in 2020 Al naser et al [16] apply this concept to fuzzy graph and it's matrices. In 1878, Cayley graph concept introduced by Cayley for finite group. Max. D unpublished lectures on the theory of group from 1909-1910 were re-introduced Cayley graph through a new name and its Gruppenbild.

Shahzamarian et al [17] discussed roughness concepts in Cayley graphs on the other hand, Namboothiri et al [18] studied Cayley fuzzy graph and investigated several properties.

In this article, we gave and study a Cayley bipolar intuitionistic fuzzy graphs and mentioned many of their properties, also we introduce many results and properties of bipolar intuitionistic fuzzy graph with respect to Cayley group and we discuss and study connectedness over Cayley bipolar intuitionistic fuzzy graph's.

2. Preliminaries

Definition 2.1. [1] Given a non empty set Y. A bipolar fuzzy set μ over Y is an object of the form: $\mu = \{(a, \mu^+(a), \mu^-(a); a \in Y)\}$ such that $\mu^+ : Y \to [0,1], \mu^- : Y \to [-1,0]$ are mappings.

Definition 2.2. [2] Let Y be a non empty set. $\delta = \{a, \delta_\mu(a), \delta_\lambda(a); a \in Y\}$ is an intuitionistic fuzzy set where $\delta_\mu : Y \to [0,1]$ and $\delta_\lambda : Y \to [0,1]$ are mapping such that $0 \le \delta_\mu(a) + \delta_\lambda(a) \le 1$.

Definition 2.3. [15] Let Y be a non empty set. A bipolar intuitionistic fuzzy set $\delta = \{(a, \delta_\mu^+(a), \delta_\mu^-(a), \delta_\lambda^+(a), \delta_\lambda^-(a), a \in Y\}$ where $\delta_\mu^+ : Y \to [0,1], \delta_\mu^- : Y \to [-1,0], \delta_\lambda^+ : Y \to [0,1]$ and $\delta_\lambda^- : Y \to [-1,0]$ are the mappings such that $0 \le \delta^+(a) + \delta_\lambda^+(a) \le 1, -1 \le \delta_\mu^-(a) + \delta_\lambda^-(a) \le 0$, we use the degree of positive membership $\delta_\mu^+(a)$ to denote and to illustrate the degree of satisfaction of the a element to illustrate the degree of satisfaction of the a element to a bipolar intuitionistic fuzzy set δ and the negative membership degree $\delta_\mu^-(a)$ to illustrate the degree of satisfaction of the a element to some implicit counter to illustrate the degree of satisfaction of the a element to a bipolar intuitionistic fuzzy set. In similar way, we apply a positive degree of a non-membership $\delta_\lambda^+(a)$ to denote the degree of satisfaction of the a element to the property corressponding to a bipolar intuitionistic fuzzy set and the degree negative of a non-membership $\delta_{\lambda}^{-}(a)$ to represent the degree of satisfaction of an element a to some implicit counter property corressponding to a bipolar intuitionistic fuzzy set.

If $\delta_{\mu}^{+}(a) \neq 0, \delta_{\mu}^{-}(a) = 0$ & $\delta_{\lambda}^{+}(a) = 0, \delta_{\lambda}^{+}(a) = 0$, then it is the situation that a regerded as having only the positive membership property of a bipolar intuitionistic fuzzy set. If $\delta_{\mu}^{+}(a) = 0, \delta_{\mu}^{-}(a) \neq 0$ and $\delta_{\lambda}^{+}(a) = 0, \delta_{\lambda}^{+}(a) = 0$, it is the situation that a regerded as having only the negative membership property of a bipolar intuitionistic fuzzy set. If $\delta_{\mu}^{+}(a) = 0, \delta_{\mu}^{-}(a) = 0$ and $\delta_{\lambda}^{+}(a) \neq 0, \delta_{\lambda}^{+}(a) = 0$, it is the situation that a regerded as having only the positive non membership property of a bipolar intuitionistic fuzzy set. If $\delta_{\mu}^{+}(a) = 0, \delta_{\mu}^{-}(a) = 0$ and $\delta_{\lambda}^{+}(a) = 0, \delta_{\lambda}^{+}(a) \neq 0$, it is the situation that a regerded as having only the negative non membership property of a bipolar intuitionistic fuzzy set. It is possible for an element a to be such that $\delta_{\mu}^{+}(a) \neq 0, \delta_{\mu}^{-}(a) \neq 0$ and $\delta_{\lambda}^{+}(a) \neq 0, \delta_{\lambda}^{+}(a) \neq 0$ when membership and non membership function of the property overlaps with it's counter property over some position of Y.

Definition 2.4. [17] $G^{*} = (V,E)$ is a digraph such that V is a finite set, $E \subseteq V \times V$. If $G_1^{*} = (V_1,E_1)$ & $G_2^{*} = (V_2,E_2)$ are two digraphs, then the cartesian product of G_1^{*} and G_2^{*} gives a digraph $G_1^{*} \times G_1^{*} = (V,E)$ where $V = V_1 \times V_2$, $E = \{(v,v_2) \to (v,e_2); v \in V_1, v_2 \to e_2 \in E_2\} \cup \{(v_1,u) \to (z_1,u); v_1 \to z_1 \in E_1, u \in V_2\}$. We use $vu \in E$ to $v \to u \in E$ and if $e = vu \in E, v$ and u are adjacent v is a starting vertex & u is an ending vertex.

Definition 2.5. [18] If G is a finite graph and H is a set such that its a minimal generating of a graph G. A Cayley graph (G,H) has elements of G as it's vertices and $\{(e,e_h); g \in G, h \in H\}, v_1, v_2$ are adjacent if $v_2 = v_1 \cdot h; h \in H$. A generating set H is minimal if H generates G but no proper subset of H does.

Definition 2.6. [18] Let $(G,\star)$ be a group & X be any subset of G. Then the Cayley graph resulting from $(G,\star,X)$ is the graph $G = (G,R)$ such that $R = \{(v,u); v^{-1}u \in X\}$.

Definition 2.7. [15] If δ_1 and δ_2 are two bipolar intuitionistic fuzzy set. Then:

(1) $(\delta_1 \cap \delta_2)(i) = \{\min\{\delta_{1\mu}^{+}(i), \delta_{2\mu}^{+}(i)\}, \max\{\delta_{1\lambda}^{-}(i), \delta_{2\lambda}^{-}(i)\}\}$.
(2) $(\delta_1 \cup \delta_2)(i) = \{\max\{\delta_{1\mu}^{+}(i), \delta_{2\mu}^{+}(i)\}, \min\{\delta_{1\mu}^{-}(i), \delta_{2\mu}^{-}(i)\}\}$.
(3) $(\delta_1 \cap \delta_2)(i) = \{\max\{\delta_{1\lambda}^{+}(i), \delta_{2\lambda}^{+}(i)\}, \min\{\delta_{1\lambda}^{-}(i), \delta_{2\lambda}^{-}(i)\}\}$.

(4) $(\delta_1 \cup \delta_2)(i) = \{\min\{\delta_{1\lambda}^+(i), \delta_{2\lambda}^+(i)\}, \max\{\delta_{1\lambda}^-(i), \delta_{2\lambda}^-(i)\}\}$.

Definition 2.8. [15] If $S \neq \phi$ then a mapping $\delta : S \times S \to ([0,1] \times [-1,0] \times [0,1] \times [-1,0])$ a bipolar intuitionistic fuzzy relation on S where $\delta_\mu^+(v,u) \in [0,1], \delta_\mu^-(v,u) \in [-1,0], \delta_\lambda^+(v,u) \in [0,1]$ and $\delta_\lambda^+(v,u) \in [-1,0]$.

Definition 2.9. [15] If δ_1 and δ_2 are two bipolar intuitionistic fuzzy sets on a set G. If δ_1 is a bipolar intuitionistic fuzzy relation on δ_2 if:

$\delta_{1\mu}^+(v,u) \leq \min\{\delta_{2\mu}^+(v), \delta_{2\mu}^+(u)\}$,

$\delta_{1\mu}^-(v,u) \geq \max\{\delta_{2\mu}^-(v), \delta_{2\mu}^-(u)\}$,

$\delta_{1\lambda}^+(v,u) \geq \max\{\delta_{2\lambda}^+(v), \delta_{2\lambda}^+(u)\}$,

$\delta_{1\lambda}^-(v,u) \leq \min\{\delta_{2\lambda}^-(v), \delta_{2\lambda}^-(u)\} \forall v,u \in G$.

A bipolar intuitionistic fuzzy relation δ_1 on G is said to be:

(1) Reflexive, if $\delta_\mu^+(v,v) = (1,0), \delta_\mu^-(v,v) = (0,-1), \delta_\lambda^+(v,v) = (1,0), \delta_\lambda^-(v,v) = (0,-1)$.

(2) Symmetric, if $\delta_\mu^+(v,u) = \delta_\mu^+(u,v), \delta_\mu^-(v,u) = \delta_\mu^-(u,v)$ and $\delta_\lambda^+(v,u) = \delta_\lambda^+(u,v), \delta_\lambda^-(v,u) = \delta_\lambda^-(u,v), \forall v,u \in G$.

(3) Transitive, if $\delta(v,w) \geq V_u\{\min\{\delta(v,u), \delta(u,w)\}\}$.

(4) Antisymmetric, if $\delta_\mu^+(v,u) \neq \delta_\mu^+(u,v), \delta_\mu^-(v,u) \neq \delta_\mu^-(u,v), \delta_\lambda^+(v,u) \neq \delta_\lambda^+(u,v)$ and $\delta_\lambda^-(v,u) \neq \delta_\lambda^-(u,v)$.

Definition 2.10. If δ is a bipolar intuitionistic fuzzy relation on universe G. Then δ is said to be a bipolar intuitionistic fuzzy linear order relation on G if the following axioms are hold:

(1) δ is a bipolar intuitionistic fuzzy partial relation.

(2) $\min\{\delta_\mu^+(v,u), \delta_{\mu^{-1}}^+(v,u)\} > 0$

$\max\{\delta_\mu^-(v,u), \delta_{\mu^{-1}}^-(v,u)\} < 0$

$\max\{\delta_\lambda^+(v,u), \delta_{\lambda^{-1}}^+(v,u)\} > 0$

$\min\{\delta_\lambda^-(v,u), \delta_{\lambda^{-1}}^-(v,u)\} < 0$.

3. Main Results

Definition 3.1. A bipolar intuitionistic fuzzy digroup of a digraph $G^* = (V,E)$ is a pair $G = (\delta,\gamma)$ where $\delta = (\delta_\mu^+,\delta_\mu^-,\delta_\lambda^+,\delta_\lambda^+)$ is a bipolar intuitionistic fuzzy set in V & $\gamma = (\gamma_\mu^+,\gamma_\mu^-,\gamma_\lambda^+,\gamma_\lambda^-)$ is a bipolar intuitionistic fuzzy relation on E such that:

(1) $\gamma_\mu^+(vu) \leq \min\{\delta_\mu^+(v),\delta_\mu^+(u)\}, \forall vu \in V \times V$
(2) $\gamma_\mu^-(vu) \geq \max\{\delta_\mu^-(v),\delta_\mu^-(u)\}, \forall vu \in V \times V$
(3) $\gamma_\lambda^+(vu) \geq \max\{\delta_\lambda^+(v),\delta_\lambda^+(u)\}, \forall vu \in V \times V$
(4) $\gamma_\lambda^-(vu) \leq \min\{\delta_\lambda^-(v),\delta_\lambda^-(u)\}, \forall vu \in V \times V$
(5) $\gamma_\mu^+(vu) = \gamma_\mu^-(vu) = 0 \forall vu \in V \times V - E$
(6) $\gamma_\lambda^+(vu) = \gamma_\lambda^-(vu) = 0 \forall vu \in V \times V - E$

Definition 3.2. If G is a bipolar intuitionistic fuzzy digraph. A vertex indegree for v in G given by $InD(v) = (InD\delta_\mu^+(v), InD_\mu^-(v), InD\delta_\lambda^+(v), InD\delta_\lambda^-(v))$, where $InD_\mu^+(v) = \sum_{v\neq u}\delta_\mu^+(vu), InD\delta_\mu^-(v) = \sum_{v\neq u}\delta_\mu^-(vu), InD\delta_\lambda^+(v) = \sum_{v\neq u}\delta_\lambda^+(vu)$ and $InD\delta_\lambda^-(v) = \sum_{v\neq u}\delta_\lambda^-(vu)$. On the other hand, a vertex outdegree for v in G is defined by $OutD(v) = OutD\delta_\mu^+(v), OutD\delta_\mu^-,$
$OutD\delta_\lambda^+(v), OutD\delta_\lambda^-,$ such that, $OutD\delta_\mu^+(v) = \sum_{v\neq u}\gamma_\mu^+(vu), OutD\delta_\mu^-(v) = \sum_{v\neq u}\gamma_\mu^-(vu), OutD\delta_\lambda^+(v) = \sum_{v\neq u}\gamma_\lambda^+(vu)$ and $OutD\delta_\lambda^-(v) = \sum_{v\neq u}\gamma_\mu^-(vu)$.

Remark 3.3. A bipolar intuitionistic fuzzy digraph G such that vertex has the same out degree n is said to be an out regular digraph with out regularity index n. Similarly, a bipolar intuitionistic fuzzy digraph G such that each vertex has the same in degree m is said to be an in regular digraph with in regularity index m.

Example 3.4. Let G be a bipolar intuitionistic fuzzy digraph of $G^* = (V,E)$ where $V = \{a_1,a_2,a_3,a_4\}, E = \{e_1,e_2,e_3,e_4\}$. By routine computations, it is easy to see from the following graph that the bipolar intuitionistic fuzzy digraph is nether in-regular digraph nor out-regular digraph.

$$\begin{array}{ccc} a_1(0.6,-0.4,0.8,-0.2) & \xrightarrow{(0.3,-0.1,0.9,-0.8)} & a_2(0.5,-0.3,0.2,-0,7) \\ {\scriptstyle (0.5,-0.3,0.9,-0.6)}\uparrow & & \downarrow{\scriptstyle (0.2,-0.2,0.5,-0.1)} \\ a_4(0.7,-0.4,0.3,-0.5) & \xleftarrow[(0.2,-0.3,0.5,-0.6)]{} & a_3(0.3,-0.6,0.4,-0.1) \end{array}$$

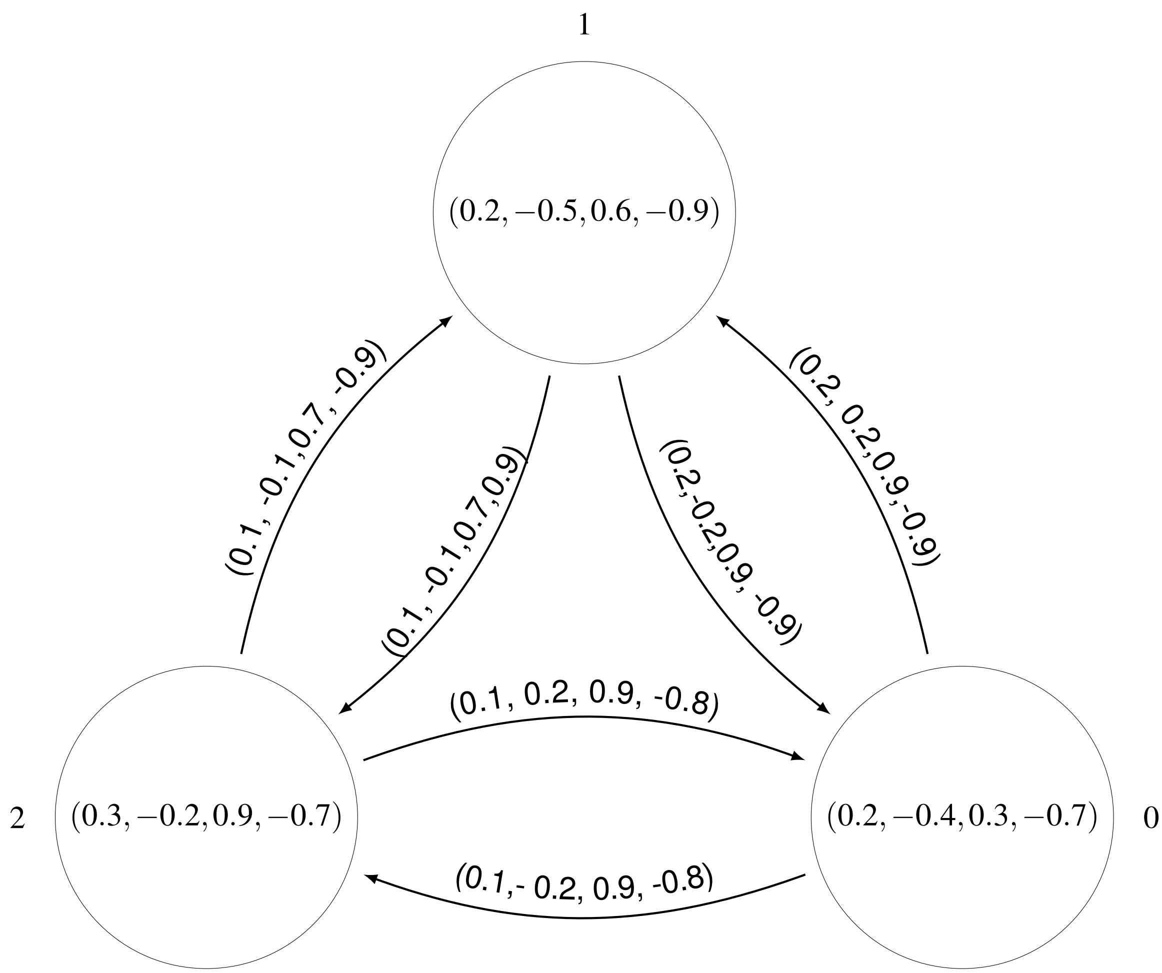
Definition 3.5. Let $(H,\star)$ be a group & $\delta = (\delta_\mu^+, \delta_\mu^-, \delta_\lambda^+, \delta_\lambda^-)$ be a bipolar intuitionistic fuzzy set of G. Then the bipolar intuitionistic fuzzy relation R gave on G as $R(v,u) = (\delta_{\mu R}^+(v^{-1}u), (\delta_{\mu R}^-(v^{-1}u),$
$(\delta_{\lambda R}^+(v^{-1}u), (\delta_{\lambda R}^-(v^{-1}u))$ for all $v,u \in H$.

Induces a bipolar intuitionistic fuzzy graph $G(H,R)$ said to be the Cayley bipolar intuitionistic fuzzy graph induced by the $(H,\star,\delta_\mu^+,\delta_\mu^-,\delta_\lambda^+,\delta_\lambda^-)$.

Definition 3.6. Let $(H,\star)$ be a group & if δ be a bipolar intuitionistic fuzzy subset of H. Then a bipolar intuitionistic fuzzy relation R on H denoted as $R(v,u) = \{\delta_{\mu R}^+(v^{-1}u),$
$(\delta_{\mu R}^-(v^{-1}u), (\delta_{\lambda R}^+(v^{-1}u), (\delta_{\lambda R}^-(v^{-1}u))$ for all $v,u \in H\}$ induces a bipolar intuitionistic fuzzy graph $G = (H,R)$ said to be Cayley bipolar intuitionistic fuzzy graph induced as $(H,\star,\delta)$.

Example 3.7. Let $H = (Z_3,+)$ be a group. Define $\delta_\mu^+ : H \to [0,1], \delta_\mu^- : H \to [-1,0], \delta_\lambda^+ : H \to [0,1]$ and $\delta_\lambda^- : H \to [-1,0]$ by $\delta_\mu^+(0) = 0.2, \delta_\mu^+(1) = 0.2, \delta_\mu^+(0) = 0.3, \delta_\mu^-(0) = -0.4, \delta_\mu^-(1) = 0.5, \delta_\mu^-(2) = -0.2, \delta_\lambda^+(0) = 0.3, \delta_\lambda^+(1) = 0.6, \delta_\lambda^+(2) = 0.4, \delta_\lambda^-(0) = -0.7, \delta_\lambda^-(1) = -0.9, \delta_\lambda^-(2) = -0.7$. Then the Cayley bipolar intuitionistic fuzzy graph $G = (H,R)$ induceed as $(Z_3,+,\delta)$ is given by the following table and graph:

x	y	(-x) + y	R(x,y)
0	0	0	(0.2, - 0.4, 0.3, - 0.7)
0	1	1	(0.2, - 0.2, 0.9, - 0.9)
0	2	2	(0.1, - 0.2, 0.9, - 0.8)
1	0	2	(0.1, - 0.2, 0.9, - 0.8)
1	1	0	(0.2, - 0.4, 0.3, - 0.7)
1	2	1	(0.3, - 0.1, 0.7, - 0.9)
2	0	1	(0.1, - 0.2, 0.9, - 0.8)
2	1	2	(0.3, - 0.1, 0.7, - 0.9)
2	2	0	(0.2, - 0.4, 0.3, - 0.7)



Theorem 3.8. The Cayley bipolar intuitionistic fuzzy graph G is vertex transitive.

Proof. If $u,v \in H$. Give $\psi : H \to H$ by $\psi(x) = uv^{-1}x$ for all $x \in H$. Since ψ is a bijective map, then for each $a,b \in H$:

$R(\psi(a),\psi(b)) = (R_{\delta_\mu^+}(\psi(a),\psi(b)), R_{\delta_\mu^-}(\psi(a),\psi(b)), R_{\delta_\lambda^+}(\psi(a),\psi(b)), R_{\delta_\lambda^-}(\psi(a),\psi(b)))$

$$\begin{aligned} R_{\delta_\mu^+}(\psi(a),\psi(b)) &= R_{\delta_\mu^+}(uv^{-1}a, uv^{-1}b) \\ &= R_{\delta_\mu^+}((uv^{-1}a)^{-1}(uv^{-1}b)) \\ &= R_{\delta_\mu^+}(a^{-1}b) \\ &= R_{\delta_\mu^+}(a,b). \end{aligned}$$

$$\begin{aligned} R_{\delta_\mu^-}(\psi(a),\psi(b)) &= R_{\delta_\mu^-}(uv^{-1}a, uv^{-1}b) \\ &= R_{\delta_\mu^-}((uv^{-1}a)^{-1}(uv^{-1}b)) \\ &= R_{\delta_\mu^-}(a^{-1}b) \\ &= R_{\delta_\mu^-}(a,b). \end{aligned}$$

$$\begin{aligned} R_{\delta_\lambda^+}(\psi(a),\psi(b)) &= R_{\delta_\lambda^+}(uv^{-1}a, uv^{-1}b) \\ &= R_{\delta_\lambda^+}((uv^{-1}a)^{-1}(uv^{-1}b)) \\ &= R_{\delta_\lambda^+}(a^{-1}b) \\ &= R_{\delta_\lambda^+}(a,b). \end{aligned}$$

$$\begin{aligned} R_{\delta_\lambda^-}(\psi(a),\psi(b)) &= R_{\delta_\lambda^-}(uv^{-1}a, uv^{-1}b) \\ &= R_{\delta_\lambda^-}((uv^{-1}a)^{-1}(uv^{-1}b)) \\ &= R_{\delta_\lambda^-}(a^{-1}b) \\ &= R_{\delta_\lambda^-}(a,b). \end{aligned}$$

Hence $R(\psi(a),\psi(b)) = R(a,b)$. Therefore ψ is an ontomorphism on G. And $\psi(v) = v$. Thus G is vertex transitive. □

Theorem 3.9. Every vertex transitive bipolar intuitionistic fuzzy graph is regular.

Proof. If $G=(H,R)$ is a bipolar intuitionistic fuzzy graph with vertex transitive, suppose that $i,j\in H$. Then there is an outomorphism ψ on G such that $\psi(i)=j$.

$$\begin{aligned}
InD(i) &= \textstyle\sum_{w\in H} R(w,i)\\
&= \textstyle\sum_{w\in H}(R_{\delta_\mu^+}(w,i),R_{\delta_\mu^-}(w,i),R_{\delta_\lambda^+}(w,i),R_{\delta_\lambda^-}(w,i))\\
&= \textstyle\sum_{w\in H}(R_{\delta_\mu^+}(\psi(w),\psi(i)),R_{\delta_\mu^-}(\psi(w),\psi(i)),R_{\delta_\lambda^+}(\psi(w),\psi(i)),R_{\delta_\lambda^-}(\psi(w),\psi(i)))\\
&= \textstyle\sum_{w\in H}(R_{\delta_\mu^+}(\psi(w),j),R_{\delta_\mu^-}(\psi(w),j),R_{\delta_\lambda^+}(\psi(w),j),R_{\delta_\lambda^-}(\psi(w),j))\\
&= \textstyle\sum_{w\in H}(R_{\delta_\mu^+}(y,j),R_{\delta_\mu^-}(y,j),R_{\delta_\lambda^+}(y,j),R_{\delta_\lambda^-}(y,j))\\
&= InD(j).
\end{aligned}$$

$$\begin{aligned}
OutD(i) &= \textstyle\sum_{w\in H} R(w,i)\\
&= \textstyle\sum_{w\in H}(R_{\delta_\mu^+}(i,w),R_{\delta_\mu^-}(i,w),R_{\delta_\lambda^+}(i,w),R_{\delta_\lambda^-}(i,w))\\
&= \textstyle\sum_{w\in H}(R_{\delta_\mu^+}(\psi(i),\psi(w)),R_{\delta_\mu^-}(\psi(i),\psi(w)),R_{\delta_\lambda^+}(\psi(i),\psi(w)),R_{\delta_\lambda^-}(\psi(i),\psi(w)))\\
&= \textstyle\sum_{w\in H}(R_{\delta_\mu^+}(j,\psi(w)),R_{\delta_\mu^-}(j,\psi(w)),R_{\delta_\lambda^+}(j,\psi(w)),R_{\delta_\lambda^-}(j,\psi(w)))\\
&= \textstyle\sum_{w\in H}(R_{\delta_\mu^+}(j,y),R_{\delta_\mu^-}(j,y),R_{\delta_\lambda^+}(j,y),R_{\delta_\lambda^-}(j,y)\\
&= OutD(j).
\end{aligned}$$

Therefore G is regular. □

Corollary 3.10. Cayley bipolar intuitionistic fuzzy graphs are regular.

Proof. straghtforword. □

Theorem 3.11. If $G=(H,R)$ is a bipolar intuitionistic fuzzy graph. Then bipolar intuitionistic fuzzy relation R is reflexive iff $\delta_\mu^+(1)=1,\delta_\mu^-(1)=0,\delta_\lambda^+(1)=0,\delta_\lambda^-(1)=-1$.

Proof. R is reflexive iff $R(a,a,a,a)=(1,0,0,-1)$ for all $a\in H$. Now $R(a,a,a,a)=(\delta_\mu^+(a^{-1}a),\delta_\mu^-(a^{-1}a),$
$\delta_\lambda^+(a^{-1}a),\delta_\lambda^-(a^{-1}a))=(\delta_\mu^+(1),\delta_\mu^-(1),\delta_\lambda^+(1),\delta_\lambda^-(1)),\forall a\in H$. Therefore R is reflexive iff $\delta_\mu^+(1)=1,\delta_\mu^-(1)=0,\delta_\lambda^+(1)=0,\delta_\lambda^-(1)=-1$. □

Theorem 3.12. If $G=(H,R)$ is a bipolar intuitionistic fuzzy graph. Then the bipolar intuitionistic fuzzy relation R is symmetric iff $(\delta_\mu^+(i),\delta_\mu^-(i),\delta_\lambda^+(i),\delta_\lambda^-(i))=(\delta_\mu^+(i^{-1}),\delta_\mu^-(i^{-1}),$
$\delta_\lambda^+(i^{-1}),\delta_\lambda^-(i^{-1})),\forall i\in H$.

Proof. ($\Rightarrow$) R is symmetric, then for any $i \in H, (\delta_\mu^+(i), \delta_\mu^-(i), \delta_\lambda^+(i), \delta_\lambda^-(i)) = (\delta_\mu^+(i^{-1}i^2),$ $\delta_\mu^-(i^{-1}i^2), \delta_\lambda^+(i^{-1}i^2), \delta_\lambda^-(i^{-1}i^2)) = R(i,i^2) = R(i^2,i)$. ($R$ is symmetric) $= (\delta_\mu^+((i^2)^{-1}i),$ $\delta_\mu^-((i^2)^{-1}i), \delta_\lambda^+((i^2)^{-1}i), \delta_\lambda^-((i^2)^{-1}i)) = (\delta_\mu^+(i^{-2}i), \delta_\mu^-(i^{-2}i), \delta_\lambda^+(i^{-2}i), \delta_\lambda^-(i^{-2}i))$ $= (\delta_\mu^+(i^{-1}), \delta_\mu^-(i^{-1}), \delta_\lambda^+(i^{-1}), \delta_\lambda^-(i^{-1}))$.
$\Leftarrow$ Since $(\delta_\mu^+(i), \delta_\mu^-(i), \delta_\lambda^+(i), \delta_\lambda^-(i)) = (\delta_\mu^+(i^{-1}), \delta_\mu^-(i^{-1}), \delta_\lambda^+(i^{-1}), \delta_\lambda^-(i^{-1})), \forall i \in H$. Hence for all $i,j \in H, R(i,j) = (\delta_\mu^+(i^{-1}j), \delta_\mu^-(i^{-1}j), \delta_\lambda^+(i^{-1}j), \delta_\lambda^-(i^{-1}j)) = (\delta_\mu^+(j^{-1}i),$ $\delta_\mu^-(j^{-1}i), \delta_\lambda^+(j^{-1}i), \delta_\lambda^-(j^{-1}i)) = R(j,i)$, thus R is symmetric. □

Definition 3.13. Let $G = (H, \star)$ be a semigroup & $\delta = (\delta_\mu^+, \delta_\mu^-, \delta_\lambda^+, \delta_\lambda^-)$ be a bipolar intuitionistic fuzzy subset of H. Then δ is said to be a bipolar intuitionistic fuzzy subsemigroup of H when:

(1) $\delta_\mu^+(ij) \geq \min\{\delta_\mu^+(i), \delta_\mu^+(j)\}$
(2) $\delta_\mu^-(ij) \leq \max\{\delta_\mu^-(i), \delta_\mu^-(j)\}$
(3) $\delta_\lambda^+(ij) \leq \max\{\delta_\lambda^+(i), \delta_\lambda^+(j)\}$
(4) $\delta_\lambda^-(ij) \geq \min\{\delta_\lambda^-(i), \delta_\lambda^-(j)\}$, for all $i,j \in H$.

Theorem 3.14. A bipolar intuitionistic fuzzy relation R is transitive iff δ is a bipolar intuitionistic fuzzy subsemigroup of $(H, \star)$.

Proof. $\Rightarrow$ Let $a,b \in H$, thus $R^2 \leq R$. Now for any $a \in H$, we have $R(1,a,a,a) = (\delta_\mu^+(a), \delta_\mu^-(a), \delta_\lambda^+(a), \delta_\lambda^-(a))$. Thus $\min\{R(1,c), R(c,ab); c \in H\} = R^2(1,ab) \leq R(1,ab)$, that is, $\max\{\min\{\delta_\mu^+(c), \delta_\mu^+(c^{-1}ab), c \in H\}\} \leq \delta_\mu^+(ab)$,
$\min\{\max\{\delta_\mu^-(c), \delta_\mu^-(c^{-1}ab), c \in H\}\} \geq \delta_\mu^-(ab)$,
$\min\{\max\{\delta_\lambda^+(c), \delta_\lambda^+(c^{-1}ab), c \in H\}\} \geq \delta_\lambda^+(ab)$ and
$\max\{\min\{\delta_\lambda^-(c), \delta_\lambda^-(c^{-1}ab), c \in H\}\} \leq \delta_\lambda^-(ab)$.
Therefore $\delta_\mu^+(ab) \geq \min\{\delta_\mu^+(a), \delta_\mu^+(b)\}$, $\delta_\mu^-(ab) \leq \max\{\delta_\mu^-(a), \delta_\mu^-(b)\}$, $\delta_\lambda^+(ab) \leq \max\{\delta_\lambda^+(a), \delta_\lambda^+(b)\}$ and $\delta_\lambda^-(ab) \geq \min\{\delta_\lambda^-(a), \delta_\lambda^-(b)\}$. Thus δ is a bipolar intuitionistic fuzzy subsemigroup of $(H, \star)$.
$\Leftarrow$ For any $a,b \in H, R^2(a,b) = (R^2_{\delta_\mu^+}(a,b), R^2_{\delta_\mu^-}(a,b), R^2_{\delta_\lambda^+}(a,b), R^2_{\delta_\lambda^-}(a,b))$. So

$$\begin{aligned}
R^2_{\delta^+_\mu}(a,b) &= \max\{\min\{R_{\delta^+_\mu}(a,c), R_{\delta^+_\mu}(c,b), c\in H\}\}\\
&= \max\{\min\{\delta^+_\mu(a^{-1}c), \delta^+_\mu(c^{-1}b), c\in H\}\}\\
&\le \delta^+_\mu(a^{-1}b)\\
&= R_{\delta^+_\mu}(a,b)\\
R^2_{\delta^-_\mu}(a,b) &= \min\{\max\{R_{\delta^-_\mu}(a,c), R_{\delta^-_\mu}(c,b), c\in H\}\}\\
&= \min\{\max\{\delta^-_\mu(a^{-1}c), \delta^-_\mu(c^{-1}b), c\in H\}\}\\
&\ge \delta^-_\mu(a^{-1}b)\\
&= R_{\delta^-_\mu}(a,b)\\
R^2_{\delta^+_\lambda}(a,b) &= \min\{\max\{R_{\delta^+_\lambda}(a,c), R_{\delta^+_\lambda}(c,b), c\in H\}\}\\
&= \min\{\max\{\delta^+_\lambda(a^{-1}c), \delta^+_\lambda(c^{-1}b), c\in H\}\}\\
&\ge \delta^+_\lambda(a^{-1}b)\\
&= R_{\delta^+_\lambda}(a,b)\\
R^2_{\delta^-_\lambda}(a,b) &= \max\{\min\{R_{\delta^-_\lambda}(a,c), R_{\delta^-_\mu}(c,b), c\in H\}\}\\
&= \max\{\min\{\delta^-_\lambda(a^{-1}c), \delta^-_\lambda(c^{-1}b), c\in H\}\}\\
&\le \delta^-_\lambda(a^{-1}b)\\
&= R_{\delta^-_\lambda}(a,b).
\end{aligned}$$

Therefore R transitive. □

Corollary 3.15. A bipolar intuitionistic fuzzy relation R is a partial order iff $\delta = (\delta^+_\mu, \delta^-_\mu, \delta^+_\lambda, \delta^-_\lambda)$ is a bipolar intuitionistic fuzzy subsemigroup of $(H,\star)$ satisfying:

(1) $\delta^+_\mu(1) = 1, \delta^-_\mu(1) = 0, \delta^+_\lambda(1) = 0, \delta^-_\lambda(1) = -1$.

(2) $\{a, (\delta^+_\mu(a), \delta^-_\mu(a), \delta^+_\lambda(a), \delta^-_\lambda(a)) = (\delta^+_\mu(a^{-1}), \delta^-_\mu(a^{-1}), \delta^+_\lambda(a^{-1}), \delta^-_\lambda(a^{-1})) = (1,0,0,-1)\}$.

Theorem 3.16. G is a Hasse diagram iff for any collection $a_1, a_2, \ldots, a_r$ of vertices in V such that $r \ge 2$ and $\delta^+_\mu(a_i) > 0, \delta^+_\lambda(a_i) > 0, \delta^-_\mu(a_i) < 0, \delta^-_\lambda(a_i) < 0$ for $i = 1,2,\ldots,r$ we have $\delta^+_\mu(a_1a_2\ldots a_r) = 0, \delta^-_\mu(a_1a_2\ldots a_r) = 0, \delta^+_\lambda(a_1a_2\ldots a_r) = 0$ and $\delta^-_\lambda(a_1a_2\ldots a_r) = 0$.

Proof. $\Rightarrow$ It is abvious that $R(a_1a_2\ldots a_{i-1}, a_1a_2\ldots a_i, a_1a_2\ldots a_{i-1}, a_1a_2\ldots a_i) = (\delta^+_\mu(a_i), \delta^-_\mu(a_i), \delta^+_\lambda(a_i), \delta^-_\lambda(a_i))$ for $i = 1,2,\ldots,r$ such that $a_0 = 1$. Therefore $(1, a_1, a_1a_2, \ldots, a_1a_2, \ldots, a_r)$ is a path from 1 to $a_1a_2\ldots a_r$ but G is a Hasse diagram, thus $R(1,a_1a_2,\dots,a_r) = 0$. This implies that $\delta_\mu^+(a_1a_2\dots a_r) = 0, \delta_\mu^-(a_1a_2\dots a_r) = 0, \delta_\lambda^+(a_1a_2\dots a_r) = 0$ and $\delta_\lambda^-(a_1a_2\dots a_r) = 0$.

$\Leftarrow$ Let $(a_0,a_1,a_2,\dots,a_r)$ be a path in G from a_0 to a_n with $n \geq 2$. Then $R(a_{i-1},a_i) > 0$, for $i = 1,2,\dots,n$. Thus $\delta_\mu^+(a_{i-1}^{-1},a_i) > 0, \delta_\mu^-(a_{i-1}^{-1},a_i) = 0, \delta_\lambda^+(a_{i-1}^{-1},a_i) = 0$ and $\delta_\lambda^-(a_{i-1}^{-1},a_i) < 0$ for $i = 1,2,\dots,r$. Now consider the elements $a_0^{-1}a_1, a_1^{-1}a_2,\dots,a_{r-1}^{-1}a_r$ in H. Then by assamption $\delta_\mu^+(a_0^{-1}a_1,a_1^{-1}a_2,\dots,a_{r-1}^{-1}a_r) = 0, \delta_\mu^-(a_0^{-1}a_1,a_1^{-1}a_2,\dots,a_{r-1}^{-1}a_r) = 0, \delta_\lambda^+(a_0^{-1}a_1, a_1^{-1}a_2,\dots,a_{r-1}^{-1}a_r) = 0$ and $\delta_\lambda^-(a_0^{-1}a_1,a_1^{-1}a_2,\dots,a_{r-1}^{-1}a_r) = 0$, that is, $\delta_\mu^+(a_0^{-1}a_r) = 0$, $\delta_\mu^-(a_0^{-1}a_r) = 0, \delta_\lambda^+(a_0^{-1}a_r) = 0$ and $\delta_\lambda^-(a_0^{-1}a_r) = 0$. Hence $R(a_0,a_n) = 0$. Therefore G is a Hasse diagram. □

Remark 3.17. Let $G = (H,R)$ is any bipolar intuitionistic fuzzy graph, then G is connected (weakly connected, semi connected, locally connected or quasi connected) iff the induce fuzzy graph (H,R_0^+) is connected (weakly connected, semi connected, locally connected or quasi connected).

Definition 3.18. If $(H,\star)$ is a semigroup and if δ is a bipolar intuitionistic fuzzy set of H. Then the subsemigroup generated by δ is the meeting of all bipolar intuitionistic fuzzy subsemigroup of H which contains δ and it is denoted by $< \delta >$.

Proposition 3.19. If $(H,\star)$ is a semigroup and δ is a bipolar intuitionistic fuzzy set of H. Then the intuitionistic fuzzy subset $< \delta >$ is precisely given by

$< \delta_\mu^+ > (a) = \max\{\min\{\delta_{\mu^+}(a_1),\delta_{\mu^+}(a_2),\dots,\delta_{\mu^+}(a_r); a = a_1a_2\dots a_r$ with $\delta_{\mu^+}(a_i) > 0$ for $i = 1,2,\dots,r\}\}$,

$< \delta_\mu^- > (a) = \min\{\max\{\delta_{\mu^-}(a_1),\delta_{\mu^-}(a_2),\dots,\delta_{\mu^-}(a_r); a = a_1a_2\dots a_r$ with $\delta_{\mu^-}(a_i) \leq 0$ for $i = 1,2,\dots,r\}\}$,

$< \delta_\lambda^+ > (a) = \min\{\max\{\delta_{\lambda^+}(a_1),\delta_{\lambda^+}(a_2),\dots,\delta_{\lambda^+}(a_r); a = a_1a_2\dots a_r$ with $\delta_{\lambda^+}(a_i) \geq 0$ for $i = 1,2,\dots,r\}\}$,

$< \delta_\lambda^- > (a) = \max\{\min\{\delta_{\lambda^-}(a_1),\delta_{\lambda^-}(a_2),\dots,\delta_{\lambda^-}(a_r); a = a_1a_2\dots a_r$ with $\delta_{\lambda^-}(a_i) < 0$ for $i = 1,2,\dots,r\}\}$, for any $a \in H$.

Proof. If $\delta' = (\delta'_{\mu^+},\delta'_{\mu^-},\delta'_{\lambda^+},\delta'_{\lambda^-})$ is a bipolar intuitionistic fuzzy set of H given by:

$\delta'_{\mu^+}(a) = \max\{\min\{\delta_{\mu^+}(a_1),\delta_{\mu^+}(a_2),\dots,\delta_{\mu^+}(a_r), a = a_1a_2\dots a_r$ such that $\delta_{\mu^+}(a_i) >$

0 for $i = 1,2,\ldots,r\}\}, \delta'_{\mu^-}(a) = \min\{\max\{\delta_{\mu^-}(a_1), \delta_{\mu^-}(a_2), \ldots, \delta_{\mu^-}(a_r), a = a_1a_2\ldots a_r$ such that $\delta_{\mu^-}(a_i)$
≤ 0 for $i = 1,2,\ldots,r\}\}, \delta'_{\lambda^+}(a) = \min\{\max\{\delta_{\lambda^+}(a_1), \delta_{\lambda^+}(a_2), \ldots, \delta_{\lambda^+}(a_r), a = a_1a_2\ldots a_r$ such that $\delta_{\lambda^+}(a_i) \geq 0$ for $i = 1,2,\ldots,r\}\}, \delta'_{\lambda^-}(a) = \max\{\min\{\delta_{\lambda^-}(a_1), \delta_{\lambda^-}(a_2), \ldots, \delta_{\lambda^-}(a_r),$ $a = a_1a_2\ldots a_r$ such that $\delta_{\lambda^-}(a_i) < 0$ for $i = 1,2,\ldots,r\}\}$.

Let $a,b \in H$, if $\delta_{\mu^+}(a) = 0$ or $\delta_{\mu^+}(b) = 0$ then $\min\{\delta_{\mu^+}(a), \delta_{\mu^+}(b)\} = 0, \delta_{\mu^-}(a) = 0$ or $\delta_{\mu^-}(b) = 0$ then $\max\{\delta_{\mu^-}(a), \delta_{\mu^-}(b)\} = 0, \delta_{\lambda^+}(a) = 0$ or $\delta_{\lambda^+}(b) = 0$ then $\max\{\delta_{\lambda^+}(a), \delta_{\lambda^+}(b)\} = 0, \delta_{\lambda^-}(a) = 0$ or $\delta_{\lambda^-}(b) = 0$ then $\min\{\delta_{\lambda^-}(a), \delta_{\lambda^-}(b)\} = 0$. Thus $\delta_{\mu^+}(ab) \geq \min\{\delta_{\mu^+}(a), \delta_{\mu^+}(b)\}$,
$\delta_{\mu^-}(ab) \leq \max\{\delta_{\mu^-}(a), \delta_{\mu^-}(b)\}, \delta_{\lambda^+}(ab) \leq \max\{\delta_{\lambda^+}(a), \delta_{\lambda^+}(b)\}, \delta_{\lambda^-}(ab) \geq \min\{\delta_{\lambda^-}(a), \delta_{\lambda^-}(b)\}$.

Again, if $\delta_{\mu^+}(a) \neq 0, \delta_{\mu^-}(a) \neq 0, \delta_{\lambda^+}(a) \neq 0$ and $\delta_{\lambda^-}(a) \neq 0$ then by definition of $\delta_{\mu^+}(a), \delta_{\mu^-}(a), \delta_{\lambda^+}(a)$ and $\delta_{\lambda^-}(a)$ we have $\gamma_{\mu^+}(ab) \geq \min\{\delta_{\mu^+}(a), \delta_{\mu^+}(b)\}, \gamma_{\mu^-}(ab) \leq \max\{\delta_{\mu^-}(a),$
$\delta_{\mu^-}(b)\}, \gamma_{\lambda^+}(ab) \leq \max\{\delta_{\lambda^+}(a), \delta_{\lambda^+}(b)\}$ and $\gamma_{\lambda^-}(ab) \geq \min\{\delta_{\lambda^-}(a), \delta_{\lambda^-}(b)\}$. Therfore δ is a bipolar intuitionistic fuzzy subsemigroup of H containing δ. Now if η be any bipolar intuitionistic fuzzy subsemigroup of H containig δ then for any $a \in H$ with $a = a_1a_2\ldots a_r$ with $\delta_{\mu^+}(a_i) > 0, \delta_{\mu^-}(a_i) \leq 0, \delta_{\lambda^+}(a_i) \geq 0$ and $\delta_{\lambda^-}(a_i) < 0$ for $i = 1,2,\ldots,r$, we have

$\eta_{\mu^+}(a_i) \geq \min\{\eta_{\mu^+}(a_1), \eta_{\mu^+}(a_2), \ldots, \eta_{\mu^+}(a_r)\} \geq \min\{\delta_{\mu^+}(a_1), \delta_{\mu^+}(a_2), \ldots, \delta_{\mu^+}(a_r)\}$,

$\eta_{\mu^-}(a_i) \leq \max\{\eta_{\mu^-}(a_1), \eta_{\mu^-}(a_2), \ldots, \eta_{\mu^-}(a_r)\} \leq \max\{\delta_{\mu^-}(a_1), \delta_{\mu^-}(a_2), \ldots, \delta_{\mu^-}(a_r)\}$,

$\eta_{\lambda^+}(a_i) \leq \max\{\eta_{\lambda^+}(a_1), \eta_{\lambda^+}(a_2), \ldots, \eta_{\lambda^+}(a_r)\} \leq \max\{\delta_{\lambda^+}(a_1), \delta_{\lambda^+}(a_2), \ldots, \delta_{\lambda^+}(a_r)\}$,

$\eta_{\lambda^-}(a_i) \geq \min\{\eta_{\lambda^-}(a_1), \eta_{\lambda^-}(a_2), \ldots, \eta_{\lambda^-}(a_r)\} \geq \min\{\delta_{\lambda^-}(a_1), \delta_{\lambda^-}(a_2), \ldots, \delta_{\lambda^-}(a_r)\}$.

Thus $\eta_{\mu^+}(a) \geq \max\{\min\{\delta_{\mu^+}(a_1), \delta_{\mu^+}(a_2), \ldots, \delta_{\mu^+}(a_r); a = a_1a_2\ldots a_r$ such that $\delta_{\mu^+}(a_i) > 0$ for $i = 1,2,\ldots,r\}\}$,

$\eta_{\mu^-}(a) \leq \min\{\max\{\delta_{\mu^-}(a_1), \delta_{\mu^-}(a_2), \ldots, \delta_{\mu^-}(a_r); a = a_1a_2\ldots a_r$ such that $\delta_{\mu^-}(a_i) \leq 0$ for $i = 1,2,\ldots,r\}\}$,

$\eta_{\lambda^+}(a) \leq \min\{\max\{\delta_{\lambda^+}(a_1), \delta_{\lambda^+}(a_2), \ldots, \delta_{\lambda^+}(a_r); a = a_1a_2\ldots a_r$ such that $\delta_{\lambda^+}(a_i) \geq 0$ for $i = 1,2,\ldots,r\}\}$,

and $\eta_{\lambda^-}(a) \geq \max\{\min\{\delta_{\lambda^-}(a_1), \delta_{\lambda^-}(a_2), \ldots, \delta_{\lambda^-}(a_r); a = a_1a_2\ldots a_r$ such that $\delta_{\lambda^-}(a_i) <$

0 for $i = 1,2,\dots,r\}\}$, forall $a \in H$.
Hence $\eta_{\mu^+}(a) \geq \delta_{\mu^+}(a), \eta_{\mu^-}(a) \leq \delta_{\mu^-}(a), \eta_{\lambda^+}(a) \leq \delta_{\lambda^+}(a)$ and $\eta_{\lambda^-}(a) \geq \delta_{\lambda^-}(a)$. Thus δ' is the maeting of all bipolar intuitionistic fuzzy subsemigroup containing δ. □

Theorem 3.20. If $(H,\star)$ is a semigroup & δ is a bipolar intuitionistic fuzzy set of H, then for any $\beta \in [0,1]$, $(<\delta_{\mu^+_\beta}>, <\delta_{\mu^-_\beta}>, <\delta_{\lambda^+_\beta}>, <\delta_{\lambda^-_\beta}>) = (<\delta_{\mu^+}>_\beta, <\delta_{\mu^-}>_\beta,$
$<\delta_{\lambda^+}>_\beta, <\delta_{\lambda^-}>_\beta)$.

Proof. if $a \in (<\delta_{\mu^+}>_\beta, <\delta_{\mu^-}>_\beta, <\delta_{\lambda^+}>_\beta, <\delta_{\lambda^-}>_\beta)$ if and only if there exist $a_1,a_2,\dots,a_r \in (\delta_{\mu^+_\beta}, \delta_{\mu^-_\beta}, \delta_{\lambda^+_\beta}, \delta_{\lambda^-_\beta})$ such that $a = a_1a_2\dots a_r$.
If and only if there exist $a_1,a_2,\dots,a_r \in H$ where $\delta_{\mu^+}(a_i) \geq \beta, \delta_{\mu^-}(a_i) \leq \beta, \delta_{\lambda^+}(a_1) \geq \beta, \delta_{\lambda^-}(a_1) \leq \beta$ for all $i = 1,2,\dots,r$ and $a = a_1a_2\dots a_r$.
Iff $<\delta_{\mu^+}>(a) \geq \beta, <\delta_{\mu^-}>(a) \leq \beta, <\delta_{\lambda^+}>(a) \geq \beta, <\delta_{\lambda^-}>(a) \leq \beta$.
Iff $a \in (<\delta_{\mu^+}>_\beta, <\delta_{\mu^-}>_\beta, <\delta_{\lambda^+}>_\beta, <\delta_{\lambda^-}>_\beta)$, therefore $(<\delta_{\mu^+_\beta}>, <\delta_{\mu^-_\beta}>, <\delta_{\lambda^+_\beta}>$ $, <\delta_{\lambda^-_\beta}>) = (<\delta_{\mu^+}>_\beta, <\delta_{\mu^-}>_\beta, <\delta_{\lambda^+}>_\beta, <\delta_{\lambda^-}>_\beta)$. □

Note

If $(H,\star)$ is a semigroup and δ is a bipolar intuitionistic fuzzy set of H. Hence by above theorem, we have $Supp(\delta) = \delta^+) = Supp <\delta>$.
If G represent the Cayley bipolar intuitionistic fuzzy graphs $G = (H,R)$ induced by $(H,\star,\delta_{\mu^+},\delta_{\mu^-},$
$\delta_{\lambda^+},\delta_{\lambda^-})$, thus we have the following results.

Corollary 3.21. (1)- If X is any subset of H' & $G' = (H',R')$ is the Cayley graph induced by $(H',\star,X)$. Thus:
(i) - G' is connected iff $H - a_i \subseteq <X>$.
(ii)- G' is weakly connected iff $H - a_i \subseteq <X \cup X^{-1}>$ such that $X^{-1} = \{a^{-1}, a \in X\}$.
(2)- G is connected iff $H - a_i \subseteq Supp <X>$.

Definition 3.22. Let $(G,\star)$ be a group and δ is a bipolar intuitionistic fuzzy subset of H. Then δ^{-1} is defined as a bipolar intuitionistic fuzzy set of H given by $\delta^{-1}(a) = \delta(a^{-1}), \forall a \in H$.

Theorem 3.23. $H - a_i \subseteq Supp(<\delta \cup \delta^{-1}>)$ iff G is weakly connected.

Proof. $H - a_i \subseteq Supp(< \delta \cup \delta^{-1} >)$
if and only if $H - a_i \subseteq Supp < \delta \cup (\delta)^{-1} >$
if and only if $H - a_i \subseteq < Supp(\delta) \cup Supp(\delta)^{-1} >$
if and only if $H - a_i \subseteq < (\delta_0^+) \cup (\delta_0^+)^{-1} >$.
If and only if (H, R_0^+) is weakly connected. Thus G is weakly connected. □

Corollary 3.24. $H - a_i \subseteq Supp(< \delta > \cup < \delta^{-1} >)$ iff G is semi-connected.

Proof. $H - a_i \subseteq Supp(< \delta > \cup < \delta^{-1} >)$
if and only if $H - a_i \subseteq Supp(< \delta > \cup < \delta >^{-1})$
if and only if $H - a_i \subseteq < Supp(\delta) > \cup < Supp(\delta)^{-1} >$
if and only if $H - a_i \subseteq < \delta_0^+ > \cup < (\delta_0^+)^{-1} >$.
If and only if (H, R_0^+) is semi-connected. Therefore G is semi-connected. □

Corollary 3.25. $Supp(< \delta >) = Supp(< \delta^{-1} >)$ iff G is locally connected.

Proof. $Supp(< \delta >) = Supp(< \delta^{-1} >)$
if and only if $< Supp(\delta) > = < Supp(< \delta^{-1}) >$
if and only if $< \delta_0^+ > = < (\delta_0^+)^{-1}) >$
If and only if (H, R_0^+) is locally connected. Therefore G is locally connected. □

Corollary 3.26. A finite Cayley bipolar intuitionistic fuzzy graph G is connected if and only if it is quai connected.

Proof. G is connected iff (H, R_0^+) is connected if and only if (H, R_0^+) is quai connected if and only if G is quai connected. □

CONCLUSION

A bipolar intuitionistic fuzzy graph is generalized structure of fuzzy graph notion, which provides more precision, consistency and pliability in comarison with bipolar intuitionistic fuzzy also fuzzy graph model's. In this paper, we have studied various properties of bipolar intuitionistic fuzzy graph with respect to Cayley group. The extension of this article work is application of a bipolar intuitionistic fuzzy digraph in the soft computing area including geographical information systems, neural network's and decision making.

Conflict of Interests

The author(s) declare that there is no conflict of interests.

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