FIVE DIMENSIONAL COSMOLOGICAL MODEL IN THE FORM OF TSALLIS HDE

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Abstract: Here, in the context of Tsallis holographic dark energy, the Kaluza-Klein five dimensional metric is explored. The time based deceleration parameter is found by solving the field equations using the hybrid scale factor. It depicts the universe from the initial stages of deceleration to the current state of acceleration. The model's physical features are also addressed.

Keywords: Kaluza-Klein metric; hybrid scale factor; THDE.

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1. INTRODUCTION

Recent astronomical experiments such as Type Ia supernovae [1], CMB [2], and LSS [3], highly recommend that the universe is governed by a non-positive pressure component known as dark energy (DE) [4]. As per Ade et al. [5], the universe's present matter energy density is close to its critical value, with DE accounting for 68.3%, cold dark matter for 26.8%, and conventional baryonic matter accounting for only 4.9%. As a reason, one of the most intriguing and difficult

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questions in modern cosmology is clarifying the character of DE and cosmic growth.

The cosmological constant (Λ, Lambda) is the simplest theoretical choice for DE, and it closely reflects the facts. However, it faces fine-tuning and cosmic coincidence issues [6]. As a consequence, a dynamically developing entity is preferable to a Λ. Apart from dynamical models, numerous alternative DE models have been proposed to tackle the problem in the last decade, notably quintessence [7], k-essence [8], tachyon [9], phantom [10], chaplygin gas [11], quintom [12], agegraphic DE [13] and many more.

The holographic DE (HDE) [14] model of the cosmos has also become increasingly popular to comprehend cosmic growth. In the field of black hole physics, the HDE model is founded on the holographic principle, which was first presented by G. 't Hooft [15] who defined the energy density as $\rho_{HE} = 3c^2 M^2 L^{-2}$ (M, c, L are Planck mass, infrared cut off radius, and constant respectively). Tsallis HDE (THDE) is a novel version of HDE model that incorporates generalized entropy $S_\mu = \delta B^\mu$ to explain the universe's growth where δ is an unknown constant and μ signifies the non-additive parameter. Cohen et al. [16] have used the holographic principle to formulate a relationship among the system entropy (S), IR cut-off (L), and UV (Λ) cut-offs as $L^3 \Lambda^3 \leq s^{0.75}$, and if joined with $S_\mu = \delta B^\mu$ leads to $\Lambda^4 \leq (\gamma(4\pi)^\mu)L^{2\mu-4}$. The THDE density can be computed $\rho_{TE} = BL^{2\mu-4}$ using this inequality, where B is just an unknown parameter [17]. The standard HDE is provided by this expression, where $B = 3c^2 M^2$ and $\mu = 1$. By accepting L as the future horizon in HDE, Saridakis et al. [18] produced a coherent formulation of THDE. THDE has been investigated by numerous researchers [19, 20, 21].

It is widely recognized that within 4 dimensional space-time, a merger of gravitational forces with other natural forces is unfeasible. Due to recent advances in super gravity and superstring theory, the research of higher dimensional models has gained significance. Kaluza [22] and Klein [23] intended to combine electromagnetic and gravitational forces, leading to the development of the Kaluza-Klein 5 dimensional theory. This theory appeals to me because it has a beautiful geometric presentation. The 5 dimensional Kaluza-Klein metric [24, 25, 26] is now extensively

performed to explore the character of DE in many scenarios.

As a product of the foregoing explanation, we have constructed THDE in Kaluza-Klein metric, adopting Hubble horizon as the IR cut-off and a hybrid scale factor. The current work is unique from the earlier researches. The below is a description of the paper's arrangement: Section 2 presents the metric and field equations. Section 3 describes the solutions and the model. Section 4 represents the physical features of the model. In Section 5, we sum up our findings.

2. METRIC AND FIELD EQUATIONS

The 5 dimensional Kaluza-Klein metric is expressed in the form

(1) $$ds^2 = dt^2 - a^2(t)[dx^2 + dy^2 + dz^2] - b^2(t)d\varphi^2$$

where the fifth dimension φ is considered to be space-like co-ordinate.

We presume that the universe is made up of DM and THDE components and the Einstein's field equations are

(2) $$R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R = -\left(T_{\alpha\beta} + \bar{T}_{\alpha\beta}\right)$$

where $R_{\alpha\beta}$ and R denote the Ricci tensor and scalar respectively.

For the physical interpretation, the matter energy momentum tensor is

(3) $$T_{\alpha\beta} = diag[\rho_m, 0,0,0,0]$$

where the matter energy density is ρ_m.

The THDE momentum tensor is

$$\bar{T}_{\alpha\beta} = diag[\rho_{TE}, -p_{TE}{}^x, -p_{TE}{}^y, -p_{TE}{}^z, -p_{TE}{}^\varphi]$$

$$= diag[1, -\omega_{TE}{}^x, -\omega_{TE}{}^y, -\omega_{TE}{}^z, -\omega_{TE}{}^\varphi]\rho_{TE}$$

(4) $$= diag[1, -\omega_{TE}, -\omega_{TE}, -\omega_{TE}, -\omega_{TE}]\rho_{TE}$$

where ρ_{TE} is the THDE density and $p_{TE}{}^x$, $p_{TE}{}^y$, $p_{TE}{}^z$, $p_{TE}{}^\varphi$ are the pressures in the x, y, z, φ directions respectively and $\omega_{TE}{}^x = \omega_{TE}{}^y = \omega_{TE}{}^z = \omega_{TE}{}^\varphi = \omega_{TE}$ is the EOS parameter of THDE.

The field equations (2) for the metric (1) with the help of (3) and (4) can be found as

(5) $$3\frac{\dot{a}^2}{a^2} + 3\frac{\dot{a}}{a}\frac{\dot{b}}{b} = \rho_m + \rho_{TE}$$

(6) $$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + 2\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{\ddot{b}}{b} = \omega_{TE}\rho_{TE}$$

(7) $$3\frac{\dot{a}^2}{a^2} + 3\frac{\ddot{a}}{a} = \omega_{TE}\rho_{TE}$$

For the metric (1), we now define some cosmological parameters that are vital for solving the field equations. Spatial volume (V) and the mean scalar factor (R) are defined as

(8) $$V = R^4 = a^3 b$$

The mean Hubble parameter (H) is defined as

(9) $$H = \frac{\dot{R}}{R} = \frac{1}{4}\frac{\dot{V}}{V} == \frac{1}{4}\left(3\frac{\dot{a}}{a} + \frac{\dot{b}}{b}\right)$$

The expansion (θ) and shear (σ^2) scalars are defined as

(10) $$\theta = 4\frac{\dot{R}}{R}$$

and

(11) $$\sigma^2 = \frac{1}{2}\left(\sum_{\alpha=1}^{4} H_\alpha{}^2 - 4H^2\right)$$

The deceleration (q) and anisotropic (Δ) parameters are given by

(12) $$q = -1 + \frac{d}{dt}(H^{-1})$$

and

(13) $$\Delta = \frac{1}{4}\sum_{\alpha=1}^{4}\left(\frac{H_\alpha - H}{H}\right)^2$$

3. SOLUTIONS AND THE MODEL

Subtracting (6) from (7), we get

(14) $$2\frac{\dot{a}}{a}\left(\frac{\dot{a}}{a} - \frac{\dot{b}}{b}\right) + \frac{\ddot{a}}{a} - \frac{\ddot{b}}{b} = 0$$

With the help of (9), the above equation can be written as

(15) $$\frac{d}{dt}\left(\frac{\dot{a}}{a} - \frac{\dot{b}}{b}\right) + \left(\frac{\dot{a}}{a} - \frac{\dot{b}}{b}\right)\frac{\dot{V}}{V} = 0$$

Integrating (15), we get

(16) $$ab^{-1} = d_2 exp[d_1 \int V^{-1} dt]$$

where d_1 and d_2 are constants.

This gives

(17) $$a = d_2^{\frac{1}{4}} V^{\frac{1}{4}} exp\left[\frac{d_1}{4} \int V^{-1} dt\right]$$

(18) $$b = d_2^{-\frac{3}{4}} V^{\frac{1}{4}} exp\left[-\frac{3d_1}{4} \int V^{-1} dt\right]$$

The conservation of energy $T_{;\beta}^{\alpha\beta} = 0$ gives the continuity equation as

(19) $$\dot{\rho}_m + 4\dot{R}R^{-1}\rho_m + \dot{\rho}_{TE} + 4\dot{R}R^{-1}(\rho_{TE} + p_{TE}) = 0$$

The continuity equation (19) will be used independently because the two fluids examined here non interacting. Therefore

(20) $$\dot{\rho}_m + 4\dot{R}R^{-1}\rho_m = 0$$

and

(21) $$\dot{\rho}_{TE} + 4\dot{R}R^{-1}(\rho_{TE} + p_{TE}) = 0$$

We have added two more criteria to solve the field equations (5)-(7) adequately.

Firstly, we take the THDE density (ρ_{TE}) as

(22) $$\rho_{TE} = BH^{4-2\mu}$$

and secondly we take the hybrid scale factor (R) as

(23) $$R = a_0 \left(\frac{t}{t_0}\right)^k e^{l\left(\frac{t}{t_0}-1\right)}$$

where k, l are non negative constants.

Using the scale factor (Equation (23)) in (8), we get

(24) $$V = R^4 = a_0^4 \left(\frac{t}{t_0}\right)^{4k} e^{4l\left(\frac{t}{t_0}-1\right)}$$

Equations (17), (18) and (24) together give

(25) $$a = d_2^{\frac{1}{4}} a_0 \left(\frac{t}{t_0}\right)^k e^{l\left(\frac{t}{t_0}-1\right)} exp\left[\frac{d_1 a_0^{-4}}{4} \int \left(\frac{t}{t_0}\right)^{-4k} e^{-4l\left(\frac{t}{t_0}-1\right)} dt\right]$$

(26) $$b = d_2^{-\frac{3}{4}} a_0 \left(\frac{t}{t_0}\right)^k e^{l\left(\frac{t}{t_0}-1\right)} exp\left[-\frac{3d_1 a_0^{-4}}{4} \int \left(\frac{t}{t_0}\right)^{-4k} e^{-4l\left(\frac{t}{t_0}-1\right)} dt\right]$$

With the help of (25) and (26), the metric (1) can be expressed as

(27) $$ds^2 = dt^2 - \left[d_2^{\frac{1}{2}} a_0^2 \left(\frac{t}{t_0}\right)^{2k} e^{2l\left(\frac{t}{t_0}-1\right)} exp\left[\frac{d_1 a_0^{-4}}{2} \int \left(\frac{t}{t_0}\right)^{-4k} e^{-4l\left(\frac{t}{t_0}-1\right)} dt\right]\right](dx^2 + dy^2 +$$

$$dz^2) - \left[d_2{}^{-\frac{3}{2}} a_0{}^2 \left(\frac{t}{t_0}\right)^{2k} e^{2l\left(\frac{t}{t_0}-1\right)} exp\left[-\frac{3 d_1 a_0{}^{-4}}{2} \int \left(\frac{t}{t_0}\right)^{-4k} e^{-4l\left(\frac{t}{t_0}-1\right)} dt \right] \right] d\varphi^2$$

4. PHYSICAL FEATURES OF THE MODEL

The physical properties of the model (Equation (27)) are obtained as follows

(28)
$$H_x = H_y = H_z = \frac{\dot{a}}{a} = \frac{k}{t} + \frac{l}{t_0} + \frac{d_1 a_0{}^{-4}}{4} \left(\frac{t}{t_0}\right)^{-4k} e^{-4l\left(\frac{t}{t_0}-1\right)}$$

(29)
$$H_\varphi = \frac{\dot{b}}{b} = \frac{k}{t} + \frac{l}{t_0} - \frac{3 d_1 a_0{}^{-4}}{4} \left(\frac{t}{t_0}\right)^{-4k} e^{-4l\left(\frac{t}{t_0}-1\right)}$$

where H_x, H_y, H_z, and H_φ are Hubble's parameters in x, y, z and φ directions respectively.

(30)
$$H = \frac{k}{t} + \frac{l}{t_0}$$

(31)
$$\theta = 4\left(\frac{k}{t} + \frac{l}{t_0}\right)$$

(32)
$$\sigma^2 = \frac{3 d_1{}^2 a_0{}^{-8}}{8} \left(\frac{t}{t_0}\right)^{-8k} e^{-8l\left(\frac{t}{t_0}-1\right)}$$

(33)
$$q = -1 + k t_0{}^2 (k t_0 + l t)^{-2}$$

(34)
$$\Delta = \frac{3}{16} d_1{}^2 a_0{}^{-8} \left(\frac{t}{t_0}\right)^{-8k} e^{-8l\left(\frac{t}{t_0}-1\right)} \left(\frac{k}{t} + \frac{l}{t_0}\right)^{-2}$$

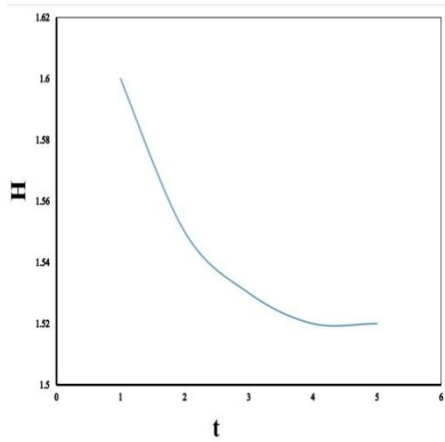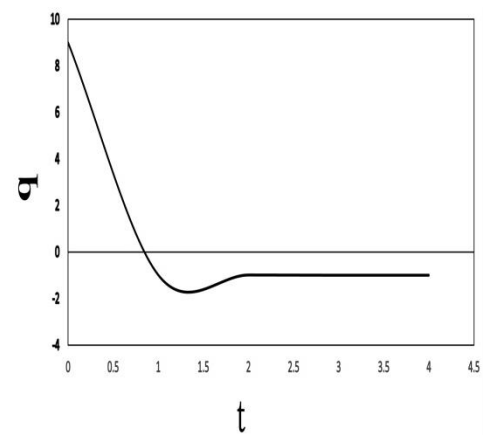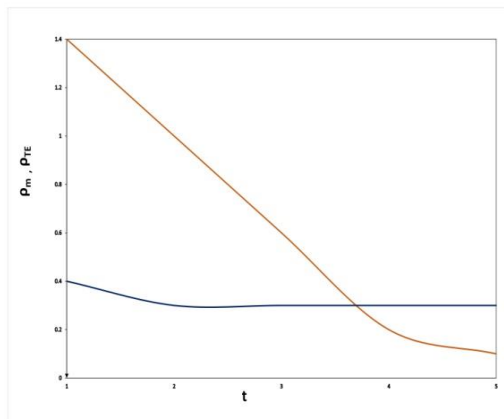Now, from equations (20), (21), (25) and (26), the energy densities of matter, THDE and EOS parameter are obtained as
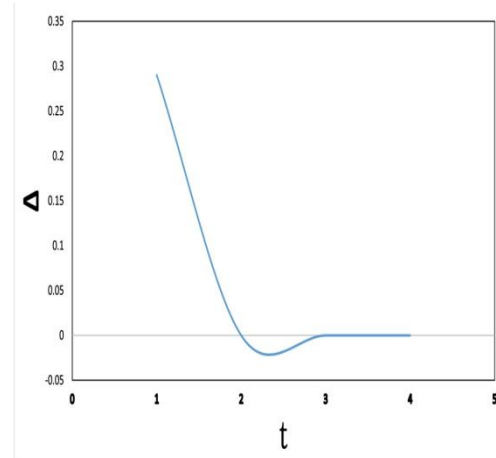
(35)
$$\rho_m = d_3 a_0{}^{-4} \left(\frac{t}{t_0}\right)^{-4k} e^{-4l\left(\frac{t}{t_0}-1\right)}$$

(36)
$$\rho_{TE} = B\left(\frac{k}{t} + \frac{l}{t_0}\right)^{4-2\mu}$$

(37)
$$\omega_{TE} = -1 + \frac{k(2-\mu)t_0{}^2}{2(k t_0 + l t)^2}$$

The graphical illustrations of these parameters are given below.

Figure1. H vs. t

Figure2. q vs. t

Figure 3. ρ_m, ρ_{TE} vs. t

Figure 4. Δ vs. t

We can see from Figure 1 that H (Hubble parameter) is a decreasing function of t (time).

We can see that from Figure 2, how q (deceleration parameter) is positive at first and then becomes negative. It signifies that the universe is transitioning from a stage of deceleration to another of acceleration.

Both ρ_m (matter energy density, red line) and ρ_{TE} (THDE density, blue line) gradually decline with time (t), as shown in Figure 3. Late in the process, the THDE density becomes a constant, while the matter energy density reaches 0.

In Figure 4,   Δ (anisotropic parameter) reaches to zero when $t \to \infty$. As a consequence, our model approaches isotropy a later point in time.
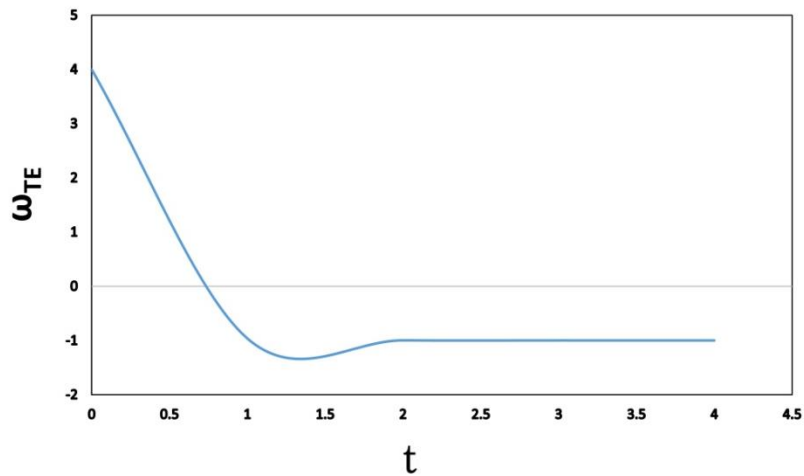
Figure 5. ω_{TE} **vs. t**

Figure 5 illustrates that ω_{TE} (THDE's EOS parameter) goes to -1 with time. As an outcome, it performs as a cosmological constant in later periods of times.

5. CONCLUSION

The focus of this research was to give the new solutions to the field equations acquired using the hybrid scale factor for the Kaluza-Klein metric filled with dark matter and THDE. We noticed that the model starts out with 0 volumes and afterwards expands at an unlimited rate. The deceleration parameter (q) is positive at first and then turns negative as time progress. When $t \to \infty$, the anisotropic parameter (Δ) approaches to 0, it is a reducing function of time.

The THDE is seen to tend to a constant value, but the matter energy density is seen to become zero at a later point in time. The value of the EOS parameter of THDE is also determined to be -1, demonstrating that THDE performs like such a cosmological constant. In the accelerated model, the THDE was used as the DE for Kaluza-Klein metric at least mathematical abstractions. The above results indicate that our model accurately reflect contemporary observations.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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