



Available online at <http://scik.org>

J. Math. Comput. Sci. 11 (2021), No. 5, 6599-6613

<https://doi.org/10.28919/jmcs/6267>

ISSN: 1927-5307

## **FIVE DIMENSIONAL BIANCHI TYPE-I STRING COSMOLOGICAL MODEL WITH ELECTROMAGNETIC FIELD**

JAGAT DAIMARY, RAJSHEKHAR ROY BARUAH\*

Department of Mathematical Sciences, Bodoland University, Kokrajhar, BTC, Assam, PIN-783370, India

Copyright © 2021 the author(s). This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

**Abstract:** In this paper we defined the five dimensional Bianchi type-I string cosmological models with electromagnetic field. Here we considered that the energy momentum tensor is equal to the sum of the energy density, tension density and electromagnetic field. In order to get determinate solutions of the Einstein's field equations, we assume the average scale factor as a connecting function of time. Some physical and kinematical parameters are also discussed.

**Keywords:** five dimension; scalar-tensor theory; Bianchi type-I; electromagnetic field.

**2010 AMS Subject Classification:** 83C50.

### **1. INTRODUCTION**

In various conditions of beginning moment of cosmological problems higher dimensional cosmological models play a significant role. The investigation of higher dimensional space-time gives a knowledge that our universe is much smaller at beginning moment of expansion as defined presently. A. Chodos and S. Detweiler [1], K.D. Krori and M. Barua [2] have investigated about

---

\*Corresponding author

E-mail address: [rsroybaruah007@gmail.com](mailto:rsroybaruah007@gmail.com)

Received June 11, 2021

the higher dimension. Several authors [3, 4, 5, 6] have studied higher-dimensional cosmological model using different type of scalar-tensor theory of gravitation. G. Samanta and S. Debata [7], have investigated five dimensional Bianchi Type-I string cosmological models in lyra manifold. Several types of research suggest that our universe is expanding with an accelerated rate. Einstein's theory of gravitation is assumed to be a perfect fluid to derive our expanding universe in FRW space-time. Bull et al. [8] have investigated the alternative cosmology and encapsulate the current position of  $\Lambda$ CDM like a physical theory besides the standard model  $\Lambda$ CDM in expansion cosmology. FRW space-time explains only isotropic and uniform universe on a large scale. On the other hand, present conclusions and arguments propose that there survive an anisotropic stage before approaching to an isotropic one throughout the cosmic development of the universe. R. Bali, and N. K. Chandnani [9], G. Mohanty et al. [10] studied five dimensional LRS Bianchi type-I string cosmological model in Saez and Ballester theory. The anisotropic universe has acquired a greater majority than the isotropic model universes in the theoretical view. Several researchers [11, 12, 13, 14, 15, 16, 17, 18] studying anisotropic Bianchi Type-I cosmological models and higher dimensional bianchi type-I cosmological model in several aspects. Presently A. Banerjee et al. [19] have defined Bianchi type I cosmological model with a viscous fluid.

Since string play a very important role in explaining the expansion of the beginning period of our universe, hence lots of authors in current period comprehensively investigated the string cosmological models of the universe. The string has ability to explain together the character and theoretical structure of the beginning universe. String theory is the largest sincerely investigated access to quantum gravity, and by applying this theory we can debate the physics of beginning universe. String theory gives us with a single theoretical configuration, in which the entire matters and forces are homogeneous and explain the beginning period of our universe in terms of vibrating strings in place of particles. Kibble [20] had suggested that, cosmic string are steady line like such as topological objects that are developed at some period in the stage transition in immature beginning stage of our universe. Also, many authors [21, 22, 23, 24, and 25] have investigated that there is a similarity collapsing in the time of the stage transition in the beginning period of universe

later the big-bang and these strings derive during the cosmic temperature decline under some critical temperature. Hence before the investigation about the beginning period of universe, string can take part important role. Moreover, large scale configurations such as galaxies, and cluster of galaxies are developed by massive closed loops of string. All of these thinks attract many authors to investigate about the string cosmological models. Several authors [26, 27, 28, 29, 30, and 31] have studied about the string cosmological models using different scalar-tensor.

In the presence of electromagnetic fields, the cosmological model takes a crucial part in the expansion of universe and the development of huge scale configuration such as galaxies and other stellar bodies. The current period of accelerated extension of the universe is for the existence of a cosmological electromagnetic field developed throughout expansion. J.B. Jimenez and A.L. Maroto [32] have investigated that the existence of a temporal electromagnetic field on cosmological scales develops a sufficient cosmological constant that is explained for accelerated extension of the universe. Also, many researchers [33, 34, 35, 36, 37, 38 and 39] have studied about the electromagnetic field by taking different aspects.

In this paper we investigated five dimensional Bianchi type-I string cosmological models with electromagnetic field. In section 1, we discussed about the higher dimension, Bianchi Type-I cosmological model, string and electromagnetic field. In section 2, we assumed the Bianchi Type-I metric and the field equation in general relativity are defined. We have defined the solution of the survival field equations in section 3. In section 4, physical and kinematic parameters are discussed. Section 5, we discussed physical and geometrical character of our model with the help of the graph. Finally, we give conclusion in section 6.

## 2. METRIC AND FIELD EQUATIONS

Let us assume the five dimensional Bianchi type-I metric as follows

$$(1) \quad ds^2 = -dt^2 + A^2(dx^2 + dy^2) + B^2dz^2 + C^2d\psi^2$$

Here A, B, C are functions of cosmic time t.

Also we assume the Einstein field equation  $\left(\frac{8\pi G}{c^4} = 1\right)$  is as given below

$$(2) \quad R_i^j - \frac{1}{2} R g_i^j = -T_i^j$$

where  $R = g^{ij} R_{ij}$  represents the Ricci scalar and  $R_i^j$  denotes the Ricci tensor.

Here, for cloud string the energy momentum tensor with having the electromagnetic field can be written as follows

$$(3) \quad T_{ij} = \rho u_i u_j - \lambda x_i x_j + E_{ij}$$

where,  $\rho$  denotes the rest energy density of the cloud strings with particles attached to them,  $\lambda$  denotes the tension density and  $p = \rho_p + \lambda$ ,  $\rho_p$  denotes the particle density of the string. Here  $u^i$  represents the five velocity vector and  $x^i$  represents the unit space-like vector with the following condition

$$(4) \quad u^i = (0,0,0,0,1) \quad \text{and} \quad x^i = \left(\frac{1}{A}, 0,0,0,1\right)$$

such that,

$$(5) \quad u^i u_i = -x^i x_i = -1 \quad \text{and} \quad u^i x_i = 0$$

We consider the Electromagnetic field  $E_{ij}$  as follows,

$$(6) \quad E_{ij} = \frac{1}{4\pi} \left( g^{\alpha\beta} F_{i\alpha} F_{j\beta} - \frac{1}{4} g_{ij} F_{\alpha\beta} F^{\alpha\beta} \right)$$

Here  $F_{\alpha\beta}$  denotes the electromagnetic field tensor.

When we compute the magnetic field along the x axis then we get that  $F_{15}$  is the only non-zero component of the electromagnetic field tensor  $F_{ij}$ . Hence, when we consider infinite electromagnetic conductivity, we get that  $F_{12} = F_{13} = F_{14} = F_{23} = F_{24} = F_{25} = F_{34} = F_{35} = 0$ .

Therefore from equation (6), we get

$$(7) \quad E_1^1 = -E_2^2 = -E_3^3 = -E_4^4 = E_1^1 = -\frac{1}{2} g^{11} g^{55} F_{15}^2 = \frac{1}{2A^2} F_{15}^2$$

For co-moving coordinates when we compute the magnetic field along x- axis then we get  $F_{34}$  is the only non-zero component of the electromagnetic tensor field  $F_{ij}$ . Hence when we consider

infinite electromagnetic conductivity, then we get as follows  $F_{15} = F_{25} = F_{35} = F_{12} = F_{13} = F_{14} = 0$ .

Again from equation (6), we get

$$(8) \quad E_1^1 = -E_2^2 = -E^3 = -E^4 = E_1^1 = \frac{1}{2A^2} F_{34}^2$$

Here we define the spatial volume  $V$ , Scalar expansion  $\theta$ , Hubble parameter  $H$ , the deceleration parameter  $q$ , the shear scalar  $\sigma^2$  and the mean isotropy parameter  $\Delta$  for the metric (1) are given by as follows,

$$(9) \quad V = R^4 = A^2 BC$$

$$(10) \quad \theta = u^i_{;i} = 2 \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}$$

$$(11) \quad H = \frac{\dot{R}}{R} = \frac{1}{4} \left( 2 \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right)$$

$$(12) \quad q = -\frac{R\ddot{R}}{\dot{R}^2}$$

$$(13) \quad \sigma^2 = \frac{1}{2} \sigma_{ij} \sigma^{ij} = \frac{1}{2} \left( 2 \frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} \right) - \frac{\theta^2}{8}$$

$$(14) \quad \Delta = \frac{1}{4} \sum_{i=1}^4 \left( \frac{H_i - H}{H} \right)^2$$

Here an overhead dot denoted differentiation with respect to cosmic time  $t$ .

Using Einstein's field equation (2) with the energy momentum tensor (3) for the metric equation (1), we get the following equations

$$(15) \quad \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} = \lambda - \frac{1}{2A^2} F_{15}^2$$

$$(16) \quad 2 \frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} + \frac{\ddot{C}}{C} + 2 \frac{\dot{A}\dot{C}}{AC} = \frac{1}{2A^2} F_{15}^2$$

$$(17) \quad 2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} + \frac{\ddot{B}}{B} + 2\frac{\dot{A}\dot{B}}{AB} = \frac{1}{2A^2}F_{15}^2$$

$$(18) \quad \frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{B}}{AB} + 2\frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} = \rho - \frac{1}{2A^2}F_{15}^2$$

### 3. SOLUTIONS OF THE FIELD EQUATIONS

Here we have to assume two relations. Since the field equations (15)-(18) are four equations with six unknown parameters  $A, B, C, \rho, \lambda,$  and  $F_{15}$ . Hence we have assume the following two relations.

(i) Since the shear scalar  $\sigma$  is proportional to the scalar expansion  $\theta$  (Collins et. al. [40] VUM Rao et.al [41]) so we may assume

$$(19) \quad B = C^n$$

Here  $n$  is a constant.

(ii) We also assume the special law of variation for Hubble parameter defined by Berman [42] as given below,

$$(20) \quad H = \frac{a_1}{R^m}$$

Where  $a_1, m (\neq 0)$  are constant and  $H$  is the Hubble parameter given by

$$(21) \quad H = \frac{\dot{R}}{R}$$

If  $m (\neq 0)$  from equation (20) and (21) we get,

$$(22) \quad R = (a_1mt + a_2)^{\frac{1}{m}}$$

and from equation (12)

$$(23) \quad q = (m - 1)$$

If  $m=0$  from equation (20) and (21) we get,

$$(24) \quad R = a_3e^{a_1t}$$

again from (12) we get,

$$(25) \quad q = -1$$

where  $a_2$  and  $a_3$  are constants.

For  $m \neq 0$  we get the following values

From equations (9), (16), (17), (19) and (22) we get,

$$(26) \quad C = b_2 e^{\frac{b_1(a_1mt+a_2)^{\frac{m-4}{m}}}{a_1(m-4)}}$$

Here  $b_1$  and  $b_2$  are constants so without loss of generality we can assume that  $b_1 = b_2 = 1$ , hence we have,

$$(27) \quad C = e^{\frac{(a_1mt+a_2)^{\frac{m-4}{m}}}{a_1(m-4)}}$$

And from equation (9), (19) (22) and (27) we have,

$$(28) \quad A = (a_1mt + a_2)^{\frac{2}{m}} e^{-\frac{(a_1mt+a_2)^{\frac{m-4}{m}}(n+1)}{2a_1(m-4)}}$$

Also from equation (19)

$$(29) \quad B = e^{\frac{n(a_1mt+a_2)^{\frac{m-4}{m}}}{a_1(m-4)}}$$

Using equations (27), (28) and (29) in metric equation (1) we get as follows

$$(30) \quad ds^2 = -dt^2 + (a_1mt + a_2)^{\frac{4}{m}} e^{-\frac{(a_1mt+a_2)^{\frac{m-4}{m}}(n+1)}{a_1(m-4)}} (dx^2 + dy^2) \\ + e^{\frac{2n(a_1mt+a_2)^{\frac{m-4}{m}}}{a_1(m-4)}} dz^2 + e^{\frac{2(a_1mt+a_2)^{\frac{m-4}{m}}}{a_1(m-4)}} d\psi^2$$

#### 4. PHYSICAL AND KINEMATICAL PARAMETERS

We define the energy density  $\rho$ , string tensor density  $\lambda$ , and electromagnetic field tensor  $F_{15}$  for the models given by equation (30) from equations (15)-(18) as follows,

$$(31) \quad \rho = -4a_1^2(m-4)(a_1mt + a_2)^{-2}$$

$$(32) \quad \lambda = \frac{1}{2}(3n^2 + 2n + 3)(a_1mt + a_2)^{-\frac{8}{m}} - 2a_1^2(3m-8)(a_1mt + a_2)^{-2} \\ - 4a_1(n+1)(a_1mt + a_2)^{-\frac{(m+4)}{m}}$$

$$(33) \quad F_{15} = \sqrt{2} \left[ \frac{1}{4}(3n^2 + 2n + 3)(a_1mt + a_2)^{-\frac{4}{m}} - 4a_1^2(m-3)(a_1mt + a_2)^{-\frac{2(m-2)}{m}} \right. \\ \left. - 2a_1(n+1)(a_1mt + a_2)^{-1} \right]^{\frac{1}{2}} e^{-\frac{(a_1mt+a_2)^{\frac{m-4}{m}}(n+1)}{2a_1(m-4)}}$$

Since  $\rho = \rho_p + \lambda$ , so have

Particle density  $\rho_p$  is

$$(34) \quad \rho_p = 2a_1^2m(a_1mt + a_2)^{-2} - \frac{1}{2}(3n^2 + 2n + 3)(a_1mt + a_2)^{-\frac{8}{m}} \\ + 4a_1(n+1)(a_1mt + a_2)^{-\frac{(m+4)}{m}}$$

The important physical quantities such as spatial volume  $V$ , Hubble parameter  $H$ , scalar expansion  $\theta$ , Shear stress  $\sigma^2$ , mean anisotropy parameter  $\Delta$  are defined by in the following way

Spatial volume  $V$  is

$$(35) \quad V = (a_1mt + a_2)^{\frac{4}{m}}$$

The Hubble parameter  $H$  is

$$(36) \quad H = \frac{a_1}{(a_1mt + a_2)}$$

The scalar expansion  $\theta$  is given by

$$(37) \quad \theta = \frac{4a_1}{(a_1mt + a_2)}$$



The shear scalar  $\sigma^2$  is

$$(38) \quad \sigma^2 = \frac{(3n^2 + 2n + 3)}{4} (a_1 mt + a_2)^{-\frac{8}{m}} - 2a_1(n + 1)(a_1 mt + a_2)^{-\frac{(m+4)}{m}} + 2a_1^2(a_1 mt + a_2)^{-2}$$

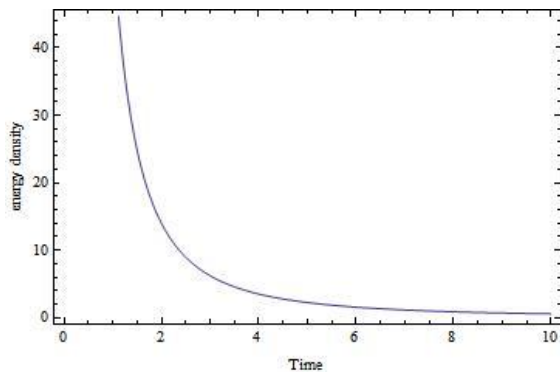
And the anisotropy parameter  $\Delta$  is given by

$$(39) \quad \Delta = 1 - \frac{(n + 1)}{a_1} (a_1 mt + a_2)^{\frac{m-4}{m}} + \frac{(3n^2 + 2n + 3)}{2a_1^2} (a_1 mt + a_2)^{\frac{2(m-4)}{m}}$$

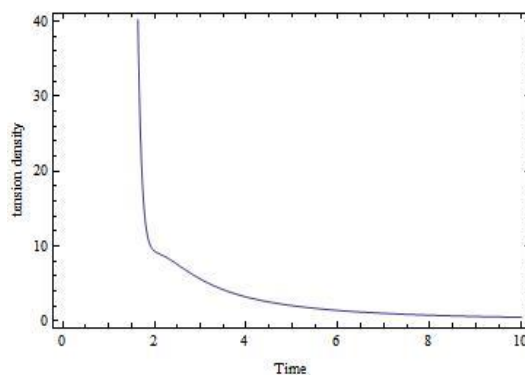
Also, from Equation (37) and (38) we get,

$$(40) \quad \lim_{t \rightarrow \infty} \frac{\sigma^2}{\Theta^2} \neq 0 \quad \text{for } m \neq 1$$

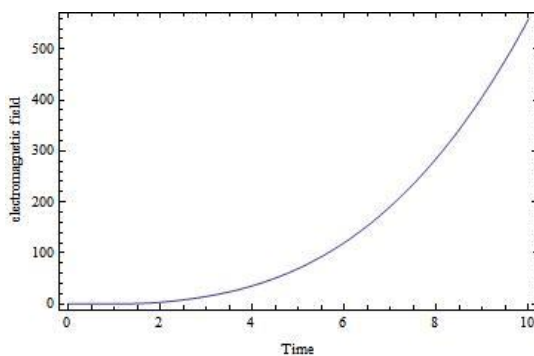
Here we plot all the graphs by taking  $a_1 = 1$ ,  $a_2 = 0$ , and  $m = n = .5$



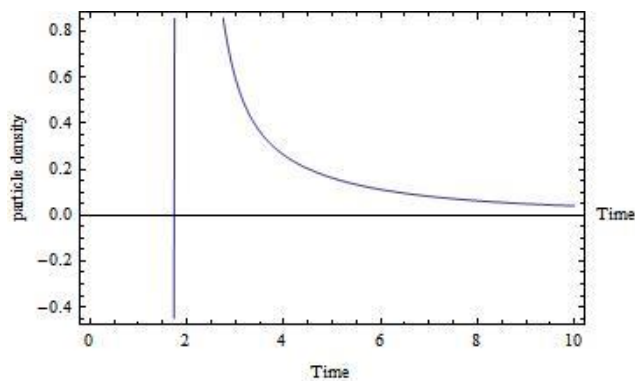
**Fig-1. Energy density  $\rho$  vs. time**



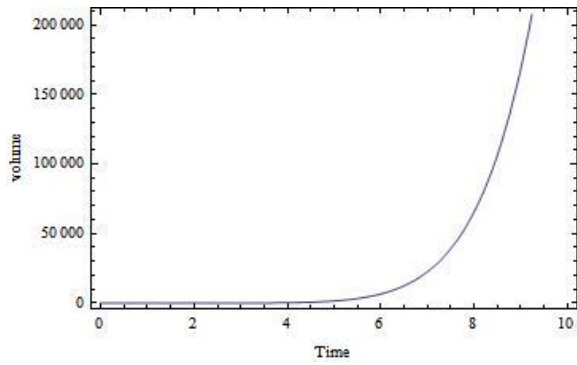
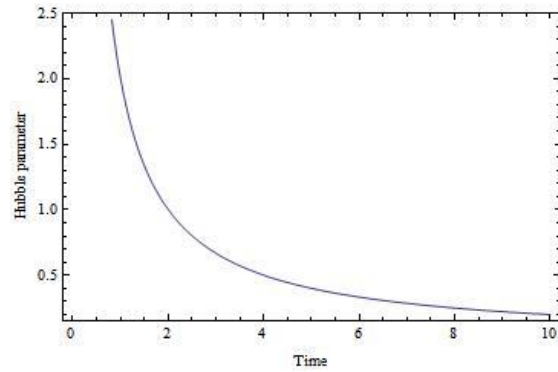
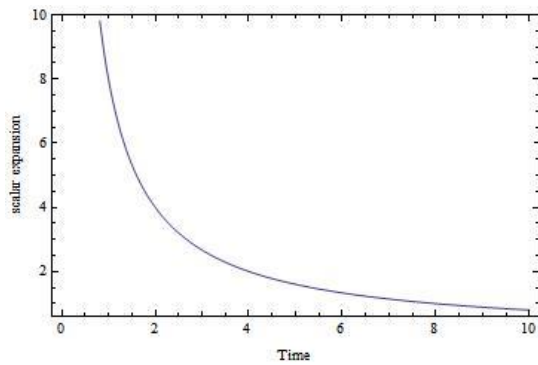
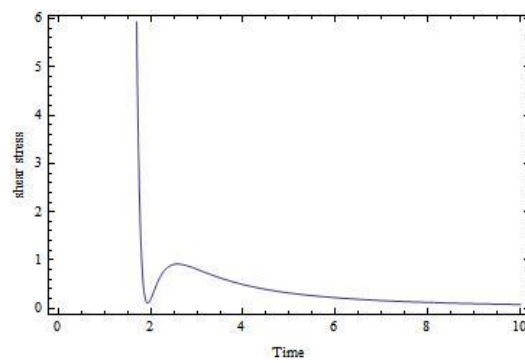
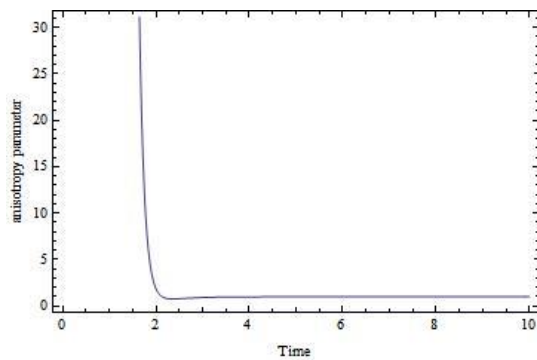
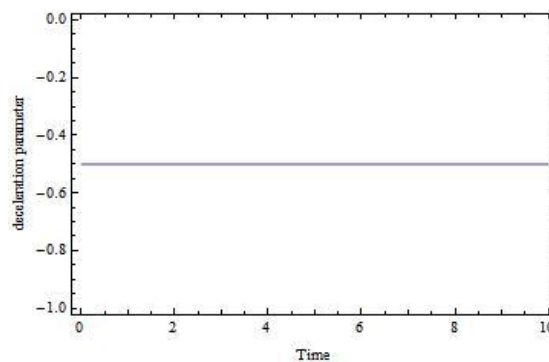
**Fig-2. Tension Density  $\lambda$  vs. time**



**Fig-3. Electromagnetic field  $F_{15}$  vs. time**



**Fig-4. Particle Density  $\rho_p$  vs. time**

Fig-5. Volume  $V$  vs. timeFig-6. Hubble Parameter  $H$  vs. timeFig-7. Scalar expansion  $P$  vs. timeFig-8. Shear stress  $\sigma^2$  vs. timeFig-9. Anisotropy parameter  $\Delta$  vs. timeFig-10. Deceleration parameter  $q$  vs. time

#### 4. PHYSICAL AND GEOMETRICAL INTERPRETATION

(i) From Equation (31), (32) and (34) we obtained that the energy density  $\rho$ , tension density  $\lambda$  and particle density  $\rho_p$  are decreases when time is increases and they will approach a constant finite value at  $t \rightarrow \infty$ . All of them will increase when time is decrease (i.e. when time is passes) and

diverge at the initial epoch (i.e.,  $t = 0$ ) (as shown in fig-1, fig-2 and fig-4). From there we obtain that the particle controls the universe as time decreases and communicates to total constant finite number of particles in the universe. Also, since the string tension density vanishes more quickly than the particle density so that at late time period our model defines a matter controls universe, this helps the present day's information data.

(ii) When time increases the electromagnetic field  $F_{15}$  and the volume  $V$  given by equations (33), (35) increases and they become infinity at infinitely large time. Here for  $a_1 \neq 0$  the electromagnetic field does not vanish. At beginning period of expansion of the universe it has appreciable effect in construction up strings. Also, we obtain that the volume is  $a_2^{\frac{4}{m}}$  at  $t=0$ . This shows that at  $t = 0$  the universe first begins with finite volume and later it is expand with increasing time. (as shown in fig-3, fig-5).

(iii) The Hubble parameter  $H$ , scalar expansion  $\theta$  given by equations (36), (37) are decreases when time is increases and vanish for  $t \rightarrow \infty$ . They will become infinity at initial epoch (see fig-6, fig-7).

(iv) From equation (23) we obtain that the deceleration parameter  $q$  is constant and negative. From there it is define that our assume model universe inflates and expanding with constant acceleration (since cosmic time  $t=0$ ) [see fig-10]. At  $t \rightarrow \infty$  the shear scalar  $\sigma^2$  and anisotropy parameter  $\Delta$  which are given by equation (38), (39) are vanish and they will decrease when time is increase. At initial epoch they will diverge (see fig-8, fig-9).

(v) Here we have  $\lim_{t \rightarrow \infty} \frac{\sigma^2}{\theta^2} \neq 0$  for  $m \neq 1$ , hence our model given by equation (30) is an anisotropic. Though we get our models as anisotropic but it cannot bring any contradiction to the current day's conclusion which shows that our universe is isotropy. Because in the time of development of expansion, the initial isotropy do not appear after some process and reaches to isotropy and ultimately it derives into a FRW universe as advised by [43].

## 5. CONCLUSION

Here we have investigated five dimensional Bianchi type-I string cosmological models with electromagnetic field. In this paper we have obtain the model given by equation (30) where  $H > 0$  and  $q < 0$ . This represents that the universe displays power law-expansion after the big-bang influence which begin with some finite volume at initial epoch and increases with an acceleration. Here the tension density vanishes more quickly than particle density. Hence the particle density controls the universe when time increases. For the model given by equation (30), the universe may have some expectation to be anisotropy during the expansion from beginning to late time period. Current conclusion gives us that there is a distinction in measuring strength of microwaves approaching from various directions of the sky. This motivated us to investigate the universe with anisotropic Bianchi type-I metric as follows to explain our universe in many sensible positions. Again, many CMB anomalies such as incompatible of temperature anisotropy in the CMB with perfect homogeneous and isotropic FRW model defined by WMAP satellites, systematics and import topologies are an indication that we stay in a globally anisotropic universe. At the stage of inflation, shear decrease and ultimately it becomes into an isotropic step with unacceptable shear. Hence currently, we need to promote anisotropy in space-time to develop any considerable quantity of shear.

## CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

## REFERENCES

- [1] A. Chodos and S. Detweiler, Where has the fifth dimension gone? *Phys. Rev. D* 21(1980), 2167-2170.
- [2] K.D. Krori and M. Barua, Higher-dimensional bianchi type-I cosmologies, *Phys. Lett. A*, 123(1987), 379-381.
- [3] G.P. Singh, R.V. Deshpande, and T Singh, Higher-dimensional cosmological model with variable gravitational constant and bulk viscosity in Lyra geometry, *J. Phys.* 63(2004), 937-945.

- [4] I. Yilmaz, Higher-dimensional cosmological models with strange quark matter, *Int. J. Mod. Phys. D*, 15(2006), 477–483.
- [5] D.R.K. Reddy, A Higher Dimensional Cosmological Model in a Scale-Covariant Theory of Gravitation, *Int. J. Theor. Phys.* 48(2009), 3044–3048.
- [6] V.U.M. Rao and K.V.S. Sireesha, A higher-dimensional string cosmological model in a scalar-tensor theory of gravitation, *Eur. Phys. J. Plus.* 127(2012), 1-6.
- [7] G. Samanta and S. Debata, Five Dimensional Bianchi Type-I String Cosmological Models in Lyra Manifold, *J. Mod. Phys.* 3(2012), 180-183.
- [8] P. Bull1 et al., Beyond  $\Lambda$ CDM Problems, solutions, and the road ahead, *Phys. Dark Univ.* 12(2016), 56-59.
- [9] R. Bali, and N. K. Chandnani, Bianchi type-I cosmological model for perfect fluid distribution in Lyra geometry, *J. Math. Phys.* 49(2008), 1-8.
- [10] G. Mohanty, R. R. Sahoo, K. L. Mahanta, Five dimensional LRS Bianchi type-I string cosmological model in Saez and Ballester theory. *Astrophys. Space Sci.* 312(2007), 321–324.
- [11] A. Pradhan and O. P. Pandey, Bianchi type-I anisotropic magnetized cosmological models with varying  $\Lambda$ , *Int. J. Mod. Phys. D*, 12(2003), 1299–1314.
- [12] O. Akarsu and C. B. Kilinc, LRS Bianchi Type I Models with Anisotropic Dark Energy and Constant Deceleration Parameter, *Gen. Rel. Grav.* 42(2009), 1-20.
- [13] A. P. and D.S. Chouhan, Anisotropic Bianchi type-I models in string cosmology, *Astrophys. Space. Sci.* 331(2011), 697-704.
- [14] A. K. Yadav and B. Saha, LRS Bianchi-I anisotropic cosmological model with dominance of dark energy, *Astrophys. Space. Sci.* 337(2012), 1-9.
- [15] S. Kotambkar et al., Anisotropic Bianchi Type-I Cosmological Models with Generalized Chaplygin Gas and Dynamical Gravitational and Cosmological Constants, *Theor. Phys.* 67(2017), 222–230.
- [16] A. Banerjee et al., Bianchi type-I cosmological models in higher dimensions, *Astrophys. J.* 358(1990), 23-27.
- [17] K.S. Adhav et al., Higher Dimensional Bianchi Type-I Universe in Creation-field Cosmology, *Int. J. Theor. Phys.* 49(2010), 957-966.

- [18] S. D. Katore et al., A higher-dimensional Bianchi type-I inflationary Universe in general relativity, *J. Phys.* 78(2012), 101-107.
- [19] A. Banerjee, S. B. Duttachoudhury, A.K. Sanyal, Bianchi type I cosmological model with a viscous fluid, *J. Math. Phys.* 26 (1985), 3010–3015.
- [20] T.W. B. Kibble, Topology of cosmic domains and strings, *J. Phys. A*, 9(1976), 1387-1398.
- [21] Y.B. Zel'dovich, et al., Cosmological consequences of a spontaneous breakdown of a discrete symmetry, *Zh. Eksp. Teor.* 67(1974), 1-5.
- [22] T.W.B. Kibble, Some Implications of a Cosmological Phase Transition, *Phys. Rep.* 67(1980), 183-199.
- [23] A.E. Everett, Cosmic strings in unified gauge theories, *Phys. Rev. D*, 24(1981), 858-868.
- [24] A. Vilenkin, Cosmic strings, *Phys. Rev. D*, 24(1981), 2082-2089.
- [25] A. Vilenkin, Gravitational field of vacuum domain walls and strings, *Phys. Rev. D*, 23(1981), 852-857.
- [26] G. Mohanty and G. C. Samanta, Some Bulk Viscous String Cosmological Models, *Turk. J. Phys.* 32(2008), 251 – 260.
- [27] K.S. Adhav, et al., Bianchi Type-I String Cosmological Model in Creation-Field, *Int. J. Theor. Phys.* 50(2011), 2720–2736.
- [28] P.K. Sahoo, et al., Bianchi type string cosmological models in  $f(R, T)$  gravity, *Eur. Phys. J. Plus*, 131(2016), 1-12.
- [29] A. K. Sethi, et al., String Cosmological Models with Bulk Viscosity in Lyra Geometry, *J. Phys.* 1344(2019), 1-8.
- [30] R.K. Mishra and H. Dua, Bulk viscous string cosmological models in Saez-Ballester theory of gravity, *Astrophys. Space. Sci.* 195(2019), 1-12.
- [31] K. D. Raju, et al., Bianchi type-V string cosmological model with a massive scalar field, *Astrophys. Space. Sci.* 28(2020), 1-6.
- [32] J. B. Jiménez, A. L. Maroto, Cosmological electromagnetic fields and dark energy, *J. Cosmol. Astropart. Phys.* 2009 (2009), 016.
- [33] R. Bali and G. S. Singh, Inhomogeneous cosmological models with electromagnetic field in general relativity, *Astrophys. Space Sci.* 134 (1987), 47-54.

- [34] A. Pradhan and V. K. Yadav, Plane-Symmetric Inhomogeneous viscous fluid cosmological models with electromagnetic field, *Int. J. Mod. Phys. D*, 6 (2002), 857-868.
- [35] S. K. Tripathy, et al., Bulk viscous string cosmological models with electromagnetic field, *Astrophys. Space. Sci.* 321 (2009), 247–252.
- [36] A. Pradhan, et al., Massive String Cosmology in Bianchi Type-III Space-Time with Electromagnetic Field, *Commun. Theor. Phys.* 54(2010), 950–956.
- [37] M. A. Megied, et al., Inhomogeneous Bianchi type-I cosmological model with electromagnetic field in lyra geometry, *Assiut Univ. J. Math. Computer Sci.* 28(2014), 1-10.
- [38] E. A. Hegazy and F. Rahaman, Bianchi type VI<sub>0</sub> cosmological model with electromagnetic field and variable deceleration parameter in general theory of relativity, *Indian J. Phys.* 94 (2020), 1847-1852.
- [39] K. P. Singh, et al., Interaction of Bianchi Type-I Anisotropic Cloud String Cosmological Model Universe with Electromagnetic Field, *Int. J. Geom. Meth. Mod. Phys.* 21 (2020), 1-15.
- [40] C. B. Collins et al., Exact Spatially Homogeneous Cosmologies, *Gen. Relat. Gravit.* 12(1980), 805-823.
- [41] V.U.M. Rao, M. Kiran, D.R.K Reddy, Minimally interacting holographic Dark energy model in Brans-Dicke theory, *Astrophys. Space. Sci.* 356(2015), 407-411.
- [42] M.S. Berman, A Special Law of Variation for Hubble's Parameter, *Nuovo Cimento B.* 74B (1983), 182-186.
- [43] K. C. Jacobs, Spatially homogeneous and Euclidean cosmological models with shear, *Astrophys. J.* 153(2) (1968), 661–678.