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ON AN $M^x/G/1$ QUEUE WITH A RANDOM SET UP TIME, RANDOM BREAKDOWNS AND DELAYED DETERMINISTIC REPAIRS

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Abstract: We study a single server queue with Poisson arrivals in batches of variable size. The server provides one by one general service to customers with a set-up time of random length before starting the first service at the start of the system as well as after every idle period of the system. The set-up time has been assumed to be general. Further, the server is subject to random breakdowns. The repair time has been assumed to be deterministic with a further delay time before starting repairs. The delay time in starting repairs has been assumed to be general. We find steady state queue length of various states of the system in terms of probability generating functions. Steady state results of a few interesting special cases have been derived.

Keywords: Poisson arrivals; set-up time; random breakdowns; delay time; repair time; steady state.

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1. INTRODUCTION

In real life situations, most of the queuing systems are subject to interruptions and delays in service

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due to many factors such as random breakdowns due to power failure or failure of the vital parts of the system, server vacations, delays in starting the service, etc. Such interruptions and delays have a definite effect on the system efficiency, the quality and cost of service and waiting time of customers. When all customers waiting for service in a system are served, the system becomes idle and waits for a new batch of customers to arrive. At the end of such idle periods, the system needs extra time termed as 'set-up time' or 'warming up time' before serving the first customer of the next busy period of the system. We refer the reader to Choudhury and Madan [2] who dealt with such a set-up time in their work. In this paper, we study a $M^x/G/1$ queueing system with a random set up time preceding the first service after each idle period.

This random set up time has been assumed to be general. In addition, we assume that the system is subject to random breakdowns from time to time and when a breakdown occurs the repairs on the system do not start immediately. We further assume that there is a delay time in starting the repairs. This delay time too has been assumed to be general. Such a delay time was assumed by Madan [7]. On completion of delay time, the system undergoes repairs immediately. The repair time has been assumed to be deterministic with constant duration of the repair time. In order to refer the reader to additional literature involving breakdowns, delays, general and deterministic repairs, we mention Aissani and Artalejo [1], Takine and Sengupta [8], Krishnamurthy et al [5], Khalaf et al [4], Ke [3], Shi and Chao [9] and Madan [7, 8]. Symbolically, we denote our system as $\boldsymbol{M^x/G}$**/S/D/D/1** queueing system, where $\boldsymbol{M^x}$ stands for Poisson arrivals in batches of variable size, **G** stands for one by one general service, **S** stands for general set-up time, **D** stands for general delay time in starting repairs, the second **D** stands for deterministic repair time and 1 stands for a single server. We have obtained explicit and tractable results under the steady state in terms of probability generating functions of various states of the system. In real life situations, most of the queuing systems are subject to interruptions and delays in service due to many factors such as random breakdowns due to power failure or failure of the vital parts of the system, server vacations, delays in starting the service, etc. Such interruptions and delays have a definite effect on the system efficiency, the quality and cost of service and waiting time of customers. When all customers waiting for service in a system are served, the system becomes idle and waits for a new batch of customers to arrive. At the end of such idle periods, the system needs extra time termed as 'set-up time' or 'warming up time' before serving the first customer of the next busy period of the system. We refer the reader to Choudhury and Madan [2] who dealt with such a set-up time in their work. In this paper, we study a M(X)/G/1 queueing system with a random set up time, random breakdowns and delayed deterministic repairs.

2. THE MATHEMATICAL MODEL

- Customers (units) arrive at the system in batches of variable size in a compound Poisson process. Let $\lambda\, c_i dt\, (i = 1,2,3,...)$ be the first order probability that a batch of i customers arrives at the system during a short interval of time $(t, t+dt]$, where $0 \le c_i \le 1$, $\sum_{i=1}^{\infty} c_i = 1$ and $\lambda > 0$ is the mean arrival rate of batches.
- Customers are served one by one on a first come, first served basis. The service time 'S' follows a general distribution. Let $\boldsymbol{B(x)}$ and $\boldsymbol{b(x)}$ respectively be the distribution function and the density function of the service time S and let $\mu(x)dx$ be the conditional probability of completion of service during the short interval of time $(\boldsymbol{t}, \boldsymbol{t} + \boldsymbol{dt}]$, given that the elapsed time is x , so that

  $$\mu(x) = \frac{b(x)}{1-B(x)} \text{ ; and therefore, } \quad b(x) = \mu(x)e^{-\int_0^x \mu(t)dt} \quad . \tag{2.1}$$

- Whenever a new customer arrives at the system at the end of an idle period, the system needs warming up time called 'set-up time'. We assume that the set-up time 'W' follows a general distribution. Let $\boldsymbol{G(x)}$ and $\boldsymbol{g(x)}$ respectively be the distribution function and the density function of the set-up time 'W' and let $\delta(x)$ be the conditional probability of completion of delay during the short interval of time $(\boldsymbol{t}, \boldsymbol{t} + \boldsymbol{dt}]$, given that the elapsed time is x , so that

  $$\delta(x) = \frac{g(x)}{1-G(x)} \text{ ; and therefore, } \quad g(x) = \delta(x)e^{-\int_0^x \delta t)dt} \tag{2.2}$$

- It is assumed that the server is subject to random breakdowns. Let $\propto dt$ be the first order probability that the server may breakdown while providing service to a customer during the short interval of time$(t, t+dt]$.
- As soon as the server fails, the customer whose service is interrupted comes back to the head of the queue and would be taken up first for service immediately alter the server becomes operative.
- We assume that as soon as the server breaks down, its repairs do not start immediately. There is a delay in the start of its repairs.
- The delay time 'D' in starting repairs follows a general distribution. Let $\boldsymbol{H(x)}$ and $\boldsymbol{h(x)}$ respectively be the distribution function and the density function of the delay time 'D' and let $\varphi(x)$ be the conditional probability of completion of delay time during the short interval of time $(\boldsymbol{t}, \boldsymbol{t}+\boldsymbol{dt}]$, given that the elapsed time is x , so that

  $$\varphi(x) = \frac{h(x)}{1-H(x)} \quad \text{and, therefore,} \quad h(x) = \varphi(x)e^{-\int_0^x \varphi t)dt} \tag{2.3}$$
- We assume that the repair time of the server is deterministic with constant repair time 'd'.
- All stochastic processes involved in the system are independent of each other.

3. DEFINITIONS AND NOTATIONS

- Let $W_n(x,t)$ be the probability that at time *t* there are n (≥0) customers in the queue excluding one customer in service with elapsed service time *x*. Accordingly, $W_n(t) = \int_{x=0}^{\infty} W_n(x,t)\,dx$ denotes the probability that at time t there are n ≥0 customers in the queue excluding one customer in service irrespective of the value of x.
- Let $S_n(x,t)$ be the probability that at time *t* there are $n \geq 1$ customers in the queue and the server is under set-up time with elapsed time *x*. Accordingly, $S_n(t) = \int_{x=0}^{\infty} S_n(x,t)\,dx$ denotes the probability that at time t there are n ≥1customers in the queue and the server is under set-up time irrespective of the value of x.

- Let $D_n(x,t)$ be the probability that at time *t* there are $n \geq 1$ customers in the queue and the server is in the failed state and the system is under delay time with elapsed time *x* before starting repairs. Accordingly, $D_n(t) = \int_{x=0}^{\infty} D_n(x,t)\,dx$ denotes the probability that at time t there are $n \geq 1$ customers in the queue and the server is under failed state and the system is under delay time before starting repairs irrespective of the value of x.
- Let $R_n(t)$ be the probability that at time t, there are $n \geq 1$ customers in the queue and the server is under deterministic repairs
- Let $P_n(t) = W_n(t) + S_n(t) + D_n(t) + R_n(t)$ denote the probability that at time *t* there are n (≥0) customers in the queue irrespective of whether the server is providing service or is under set-up time, or delay time or is under repairs.
- Let Q (t) be the steady state probability that there is no customer in the system and the server is idle.
- We assume that all stochastic processes involved in the system are independent of each other.

Now, if the steady state exists, we define the following limiting probabilities as the steady state probabilities corresponding to the probabilities defined above for the various states of the system:

$$\underset{t\to\infty}{Lim}\, W_n(x,t) = W_n(x),\;\; \underset{t\to\infty}{Lim}\, W_n(\,t) = W_n,\; n \geq 0$$

$$\underset{t\to\infty}{Lim}\, S_n(x,t) = S_n(x),\;\; \underset{t\to\infty}{Lim}\, S_n(t) = S_n,\; n \geq 1$$

$$\underset{t\to\infty}{Lim}\, D_n(x,t) = D_n(x),\;\; \underset{t\to\infty}{Lim}\, D_n(t) = D_n,\; n \geq 1$$

$$\underset{t\to\infty}{Lim}\, R_n(t) = R_n \tag{3.1}$$

We further assume that K_r is the probability of r arrivals during the deterministic period of repairs and therefore,

$$K_r = \frac{exp(\lambda d)(\lambda d)^r}{r!},\quad r = 0,1,2,.\;.\;. \tag{3.2}$$

Next, we define the following Probability Generating Functions (PGFs):

$$W(x,z) = \sum_{n=0}^{\infty} W_n(x) z^n,\quad W(z) = \sum_{n=0}^{\infty} W_n z^n, \tag{3.3}$$

$$S(x,z) = \sum_{n=1}^{\infty} S_n(x) z^n, \quad S(z) = \sum_{n=1}^{\infty} S_n z^n, \tag{3.4}$$

$$D(x,z) = \sum_{n=1}^{\infty} D_n(x)\, z^n, \quad D(z) = \sum_{n=1}^{\infty} D_n z^n, \tag{3.5}$$

$$R(z) = \sum_{n=1}^{\infty} R_n\, z^n, \tag{3.6}$$

$$K(z) = \sum_{i=0}^{\infty} K_n z^n = \sum_{n=0}^{\infty} \frac{exp(-\lambda d)(\lambda d)^n}{n!} z^n$$

$$= \exp\left[-\lambda d(1-z)\right], \quad |z| \le 1 \tag{3.7}$$

$$C(z) = \sum_{i=1}^{\infty} c_i z^i, \ |z| \le 1 \tag{3.8}$$

4. STEADY STATE EQUATIONS

Based on the underlying assumptions of the model, we obtain the following steady state equations for our system:

$$\frac{d}{dx} W_n(x) + (\lambda + \propto + \mu(x)) W_n(x) = \lambda \sum_{i=1}^{n} c_i W_{n-i}(x), \ n \ge 1 \tag{4.1}$$

$$\frac{d}{dx} W_0(x) + (\lambda + \propto + \mu(x)) W_0(x) = 0 \tag{4.2}$$

$$\frac{d}{dx} S_n(x) + \big(\lambda + \delta(x)\big) S_n(x) = \lambda \sum_{i=1}^{n} c_i S_{n-i}(x), \ n \ge 1 \tag{4.3}$$

$$\frac{d}{dx} D_n(x) + (\lambda + \varphi(x)) D_n(x) = \lambda \sum_{i=1}^{n} c_i D_{n-i}(x)\, n \ge 1 \tag{4.4}$$

$$R_{n=} \int_0^{\infty} D_n(x) \varphi(x) dx, \ n \ge 1 \tag{4.5}$$

$$\lambda\, Q = \int_0^{\infty} W_0(x)\, \mu(x) dx \tag{4.6}$$

Above equations must be solved subject to the following boundary conditions:

$$W_n(0) = \int_0^{\infty} W_{n+1}(x) \mu(x) dx \ + \int_0^{\infty} S_{n+1}(x)\, \delta(x) dx$$

$$+ (R_1 K_n + R_2 K_{n-1} + \ldots + R_{n+1} K_0), \ \ n \ge 1 \tag{4.7}$$

$$W_0(0) = \int_0^{\infty} W_1(x) \mu(x) dx \ + \ \int_0^{\infty} S_1(x)\, \delta(x) dx + \ R_1 K_0 \tag{4.8}$$

$$S_n(0) = \ \lambda C_n\, Q \ , \quad n \ge 1 \tag{4.9}$$

$$D_{n+1}(0) = \propto \ W_n, \ n \ge 0 \tag{4.10}$$

5. STEADY STATE SOLUTION

Multiplying both sides of equation (4.1) by suitable powers of z, adding equation (4.2) in the result and using (3.3) and (3.8) and on simplifying we obtain,

$$\frac{d}{dz}W(x,z) + (\lambda - \lambda C(z) + \propto + \mu(x))W(x,z) = 0. \tag{5.1}$$

Similar operations on (4.3), (4.4) and (4.5) separately, using (3.4), (3.5), (3.6) and (3.8) lead to

$$\frac{d}{dz}S(x,z) + \big(\lambda - \lambda C(z) + \delta(x)\big)S(x,z) = 0. \tag{5.2}$$

$$\frac{d}{dz}D(x,z) + \big(\lambda - \lambda C(z) + \varphi(x)\big)D(x,z) = 0. \tag{5.3}$$

$$R(z) = \int_0^\infty D(x,z)\varphi(x)dx \tag{5.4}$$

Next, similar operations on (4.7), (4.8), (4.9) and |(4.10) lead to

$$z\,W(0,z) = \int_0^\infty W(x,z)\mu(x)dx + \int_0^\infty S(x,z)\,\delta(x)dx$$
$$+R(z)K(z) - \lambda Q \tag{5.5}$$

Which on using (5.4) becomes

$$z\,W(0,z) = \int_0^\infty W(x,z)\mu(x)dx + \int_0^\infty S(x,z)\,\delta(x)dx$$
$$+\int_0^\infty D(x,z)\varphi(x)dx\,K(z) - \lambda Q \tag{5.6}$$

$$S(0,z) = \lambda C(z)Q \tag{5.7}$$

$$D(0,z) = \propto z\,W(z) \tag{5.8}$$

Now, we integrate equations (5.1), (5.2) and (5.3) between the limits 0 and x and obtain

$$W(x,z) = W(0,z)\,exp\big[-\big((\lambda - \lambda C(z) + \propto)x\big) - \int_0^x \mu(t)dt\big] \tag{5.9}$$

$$S(x,z) = S(0,z)\,exp\big[-\big((\lambda - \lambda C(z))x\big) - \int_0^x \delta(t)dt\big] \tag{5.10}$$

$$D(x,z) = D(0,z)\,exp\big[-\big((\lambda - \lambda C(z))x\big) - \int_0^x \varphi(t)dt\big] \tag{5.11}$$

Where $W(0,z)$, $S(0,z)$ and $D(0,z)$ have been obtained above in (5.6), (5.7) and (5.8) respectively.

Next, we again integrate (5.8), (5.9) and (5.10) and with respect to x and obtain

$$W(z) = W(0,z)\left(\frac{1-\bar{B}[\lambda - \lambda C(z) + \propto]}{\lambda - \lambda C(z) + \propto}\right) \tag{5.12}$$

$$S(z) = S(0,z)\left(\frac{1-\bar{G}[\lambda-\lambda C(z)]}{\lambda-\lambda C(z)}\right) \tag{5.13}$$

$$D(z) = D(0,z)\left(\frac{1-\bar{H}[\lambda-\lambda C(z)]}{\lambda-\lambda C(z)}\right) \tag{5.14}$$

Where $\bar{B}(\lambda-\lambda z+\propto) = \int_0^\infty exp[-(\lambda-\lambda C(z)+\propto]x\, dB(x)$ is the Laplace-Stieltjes transform of the service time S , and $\bar{G}(\lambda-\lambda z) = \int_0^\infty exp[-(\lambda-\lambda C(z)]x\, dG(x)$ is Laplace- Stieltjes transform of the set up time W and $\bar{H}(\lambda-\lambda z) = \int_0^\infty exp[-(\lambda-\lambda z)]x\, dH(x)$ is the Laplace-Stieltjes transform of the delay time *D* in starting repairs.

Next, we multiply equations (5.9), (5.10) and (5.11) respectively by $\mu(x)$, $\delta(x)$ and $\varphi(x)$ and integrate each with respect to x. Thus, we obtain

$$\int_0^\infty W(x,z)\mu(x)dx = W(0,z)\,\bar{B}(\lambda-\lambda z+\propto) \tag{5.15}$$

$$\int_0^\infty S(x,z)\delta(x)dx = S(0,z)\,\bar{G}(\lambda-\lambda z) \tag{5.16}$$

$$\int_0^\infty D(x,z)\varphi(x)dx = D(0,z)\,\bar{H}(\lambda-\lambda z) \tag{5.17}$$

Now, we use (5.15), (5.16) and (5.17) into the result (5.6) and simplify. Thus, we obtain

$$\begin{aligned} z\,W(0,z) = {} & W(0,z)\,\bar{B}(\lambda-\lambda z+\propto) + S(0,z)\,\bar{G}(\lambda-\lambda z) \\ & + D(0,z)\,\bar{H}(\lambda-\lambda z)K(z) - \lambda Q \end{aligned} \tag{5.18}$$

Next, using (5.7) and (5.8) into (5.18) and simplifying we get

$$\begin{aligned} [z-\bar{B}(\lambda-\lambda z+\propto)]W(0,z) = {} & \propto z\,W(z)\bar{H}(\lambda-\lambda z)K(z) \\ & + [\lambda C(z)\,\bar{G}(\lambda-\lambda z)-\lambda]Q \end{aligned} \tag{5.19}$$

We now use (5.19) into (5.12) and simplify to obtain

$$\mathrm{W(z)} = \frac{[\lambda C(z)\bar{G}(\lambda-\lambda z)-\lambda]\left(\frac{1-\bar{B}[\lambda-\lambda C(z)+\propto]}{\lambda-\lambda C(z)+\propto}\right)Q}{z-\bar{B}(\lambda-\lambda z+\propto)-\propto z\,\bar{H}(\lambda-\lambda z)K(z)\left(\frac{1-\bar{B}[\lambda-\lambda C(z)+\propto]}{\lambda-\lambda C(z)+\propto}\right)} \tag{5.20}$$

Next, using (5.7) and (5.8) in (5.13) and (5.14), we get

$$S(z) = \lambda C(z)Q\left(\frac{1-\bar{G}[\lambda-\lambda C(z)]}{\lambda-\lambda C(z)}\right) \tag{5.21}$$

$$\boldsymbol{D(z)} = \frac{\propto z\,[\lambda C(z)\bar{G}(\lambda-\lambda z)-\lambda]\left(\frac{1-\bar{B}[\lambda-\lambda C(z)+\propto]}{\lambda-\lambda C(z)+\propto}\right)\left(\frac{1-\bar{H}[\lambda-\lambda C(z)]}{\lambda-\lambda C(z)}\right)Q}{\boldsymbol{z}-\bar{B}(\lambda-\lambda z+\propto)-\propto z\,\bar{H}(\lambda-\lambda z)K(z)\left(\frac{1-\bar{B}[\lambda-\lambda C(z)+\propto]}{\lambda-\lambda C(z)+\propto}\right)} \tag{5.22}$$

From (5.4)

$R(z) = \propto z\, W(z)\, \bar{H}(\lambda-\lambda z)$

Which on using (5.20) gives

$$\mathrm{R(z)} = \frac{\propto z\,\bar{H}(\lambda-\lambda z)[\lambda C(z)\bar{G}(\lambda-\lambda z)-\lambda]\left(\frac{1-\bar{B}[\lambda-\lambda C(z)+\propto]}{\lambda-\lambda C(z)+\propto}\right)Q}{\boldsymbol{z}-\bar{B}(\lambda-\lambda z+\propto)-\propto z\,\bar{H}(\lambda-\lambda z)K(z)\left(\frac{1-\bar{B}[\lambda-\lambda C(z)+\propto]}{\lambda-\lambda C(z)+\propto}\right)} \tag{5.23}$$

Now, from (5.20) to (5.23) we find

$$W(1) = \lim_{z\to 1} W(z) = \frac{(\lambda E(S)+\lambda E(I))\left(\frac{1-\bar{B}[\lambda-\lambda C(z)+\propto]}{\lambda-\lambda C(z)+\propto}\right)Q}{1-(\propto+\propto\lambda E(D)+\propto\lambda d+\propto\lambda E(I)\left(\frac{1-\bar{B}[\lambda-\lambda C(z)+\propto]}{\lambda-\lambda C(z)+\propto}\right)} \tag{5.24}$$

Which is the steady state probability that the server is proving service to customers

$$S(1) = \lim_{z\to 1} S(z) = \lambda E(I)\lambda E(W)Q \tag{5.25}$$

Which is the steady sate probability that the system is under set-up time

$$D(1) = \lim_{z\to 1} D(z) = \frac{\propto(\lambda E(S)+\lambda E(I))E(W)\left(\frac{1-\bar{B}[\lambda-\lambda C(z)+\propto]}{\lambda-\lambda C(z)+\propto}\right)Q}{1-(\propto+\propto\lambda E(D)+\propto\lambda d+\propto\lambda E(I)\left(\frac{1-\bar{B}[\lambda-\lambda C(z)+\propto]}{\lambda-\lambda C(z)+\propto}\right)} \tag{5.26}$$

Which is the steady state probability that the system is in the failed satte and waiting for repairs to start

$$R(1) = \lim_{z\to 1} R(z) = \frac{\propto(\lambda E(S)+\lambda E(I))E(D)\left(\frac{1-\bar{B}[\lambda-\lambda C(z)+\propto]}{\lambda-\lambda C(z)+\propto}\right)Q}{1-(\propto+\propto\lambda E(D)+\propto\lambda d+\propto\lambda E(I)\left(\frac{1-\bar{B}[\lambda-\lambda C(z)+\propto]}{\lambda-\lambda C(z)+\propto}\right)} \tag{5.27}$$

Which is the steady state probability that the system is under repairs.

Now, in order to determine the only unknown probability Q, we use the normalizing condition

Q+W 1) +S (1) +D (1) +R (1) =1 (5.28)

Utilizing the results found in (5.25), (5.26) and (5.27) into (5.28) and simplifying we obtain

$$Q = \frac{1-(\lambda E(I)+\propto+\propto\lambda E(I)E(D)+\propto\lambda d)\left(\frac{1-\bar{B}[\propto]}{\propto}\right)}{\left\{\begin{array}{c}(1+\lambda E(W))\ (1-(\lambda E(I)+\propto+\propto\lambda E(I)E(D)+\propto\lambda d))\left(\frac{1-\bar{B}[\propto]}{\propto}\right)\\ +(\lambda E(W)+\lambda E(I))(1+\propto+\propto E(D))\left(\frac{1-\bar{B}[\propto]}{\propto}\right)\end{array}\right\}} \tag{5.29}$$

Where

E (I) is the average batch size of arriving customers, E (D) is the average delay time in starting repairs and E (W) is the average warming up (set-up) time.

Finally, on substituting the value of Q from (5.29) in the results (5.25), (5.26) and (5.27) we can completely and explicitly determine all the steady state probability generating functions of the queue length.

6. PARTICULAR CASES

CASE 1: NO WARMING-UP TIME

In this case we substitute E (W) =0 and $\bar{G}(\lambda-\lambda z)=1$ in the main results (5.20), (5.21), (5.22), (5.23) and (5.29). Thus, we obtain

$$\mathrm{W(z)}=\frac{[\lambda C(z)-\lambda]\left(\frac{1-\bar{B}[\lambda-\lambda C(z)+\propto]}{\lambda-\lambda C(z)+\propto}\right)Q}{\boldsymbol{z}-\bar{B}(\lambda-\lambda z+\propto)-\propto z\,\bar{H}(\lambda-\lambda z)K(z)\left(\frac{1-\bar{B}[\lambda-\lambda C(z)+\propto]}{\lambda-\lambda C(z)+\propto}\right)} \tag{6.1}$$

$$S(z)=0 \tag{6.2}$$

$$\boldsymbol{D}(\boldsymbol{z})=\frac{\propto z\,[\lambda C(z)-\lambda]\left(\frac{1-\bar{B}[\lambda-\lambda C(z)+\propto]}{\lambda-\lambda C(z)+\propto}\right)\left(\frac{1-\bar{H}[\lambda-\lambda C(z)]}{\lambda-\lambda C(z)}\right)Q}{\boldsymbol{z}-\bar{B}(\lambda-\lambda z+\propto)-\propto z\,\bar{H}(\lambda-\lambda z)K(z)\left(\frac{1-\bar{B}[\lambda-\lambda C(z)+\propto]}{\lambda-\lambda C(z)+\propto}\right)} \tag{6.3}$$

$$\mathrm{R(z)}=\frac{\propto z\,\bar{H}(\lambda-\lambda z)[\lambda C(z)\bar{G}(\lambda-\lambda z)-\lambda]\left(\frac{1-\bar{B}[\lambda-\lambda C(z)+\propto]}{\lambda-\lambda C(z)+\propto}\right)Q}{\boldsymbol{z}-\bar{B}(\lambda-\lambda z+\propto)-\propto z\,\bar{H}(\lambda-\lambda z)K(z)\left(\frac{1-\bar{B}[\lambda-\lambda C(z)+\propto]}{\lambda-\lambda C(z)+\propto}\right)} \tag{6.4}$$

$$Q=\frac{1-(\lambda E(I)+\propto+\propto\lambda E(I)E(D)+\propto\lambda d)\left(\frac{1-\bar{B}[\propto]}{\propto}\right)}{\left\{\begin{array}{c}(1-(\lambda E(I)+\propto+\propto\lambda E(I)E(D)+\propto\lambda d))\left(\frac{1-\bar{B}[\propto]}{\propto}\right)\\ +(\lambda E(I))(1+\propto+\propto E(D))\left(\frac{1-\bar{B}[\propto]}{\propto}\right)\end{array}\right\}} \tag{6.5}$$

CASE 2: NO DELAY IN STARTING REPAIRS

In this case we substitute E (D) =0 and $\bar{H}(\lambda-\lambda z)=1$ in the main results (5.20), (5.21), (5.22), (5.23) and (5.29). Thus, we obtain

$$\mathrm{W(z)}=\frac{[\lambda C(z)\bar{G}(\lambda-\lambda z)-\lambda]\left(\frac{1-\bar{B}[\lambda-\lambda C(z)+\propto]}{\lambda-\lambda C(z)+\propto}\right)Q}{\boldsymbol{z}-\bar{B}(\lambda-\lambda z+\propto)-\propto z\,K(z)\left(\frac{1-\bar{B}[\lambda-\lambda C(z)+\propto]}{\lambda-\lambda C(z)+\propto}\right)} \tag{6.6}$$

$$S(z) = \lambda C(z) Q \left(\frac{1-\bar{G}[\lambda-\lambda C(z)]}{\lambda-\lambda C(z)}\right) \tag{6.7}$$

$$\boldsymbol{D}(\mathrm{z}) = 0 \tag{6.8}$$

$$\mathrm{R(z)} = \frac{\propto z\,[\lambda C(z)\bar{G}(\lambda-\lambda z)-\lambda]\left(\frac{1-\bar{B}[\lambda-\lambda C(z)+\propto]}{\lambda-\lambda C(z)+\propto}\right)Q}{\boldsymbol{z}-\bar{B}(\lambda-\lambda z+\propto)-\propto z\,K(z)\left(\frac{1-\bar{B}[\lambda-\lambda C(z)+\propto]}{\lambda-\lambda C(z)+\propto}\right)} \tag{6.9}$$

$$Q = \frac{1-(\lambda E(I)+\propto+\propto\lambda d)\left(\frac{1-\bar{B}[\propto]}{\propto}\right)}{\left\{\begin{matrix}(1+\lambda E(W))\,(1-(\lambda E(I)+\propto+\propto\lambda d))\left(\frac{1-\bar{B}[\propto]}{\propto}\right) \\ +(\lambda E(W)+\lambda E(I))(1+\propto)\left(\frac{1-\bar{B}[\propto]}{\propto}\right)\end{matrix}\right\}} \tag{6.10}$$

CASE 3: WARMING-UP TIME WITH NO BREAKDOWNS

In this case we substitute $\propto = 0$, $\bar{H}(\lambda-\lambda z) = 1$, E(D)=0, E(W)=0 in the main results (5.20), (5.21), (5.22), (5.23) and (5.29). Thus, we obtain

$$\mathrm{W(z)} = \frac{[\lambda C(z)\bar{G}(\lambda-\lambda z)-\lambda]E(S)Q}{\boldsymbol{z}-\bar{B}(\lambda-\lambda z)} \tag{6.11}$$

$$S(z) = \lambda C(z) Q \left(\frac{1-\bar{G}[\lambda-\lambda C(z)]}{\lambda-\lambda C(z)}\right) \tag{6.12}$$

$$\mathrm{D(z)} = 0 \tag{6.13}$$

$$R(z) = 0 \tag{6.14}$$

$$Q = \lambda E(I)E(S) \tag{6.15}$$

CASE 4: NO WARMING-UP TIME AND NO BREAKDOWNS

In this case we substitute $\propto = 0$, $\bar{H}(\lambda-\lambda z) = 1$, $\bar{G}(\lambda-\lambda z) = 1$, E(D)= 0, E(W) = 0 in the main results (5.20), (5.21), (5.22), (5.23) and (5.29). Thus, we obtain

$$\mathrm{W(z)} = \frac{[\lambda C(z)-\lambda]E(S)Q}{\boldsymbol{z}-\bar{B}(\lambda-\lambda z)} \tag{6.16}$$

$$S(z) = 0 \tag{6.17}$$

$$\mathrm{D(z)} = 0 \tag{6.18}$$

$$R(z) = 0 \tag{6.19}$$

$$Q = \lambda E(I)E(S) \tag{6.20}$$

Results of this case are known results of the M^X/G/1 queue.

CONCLUSION

The paper studies a new queueing model in which a single server providers general service to customers arriving at the system in a compound Poisson process. The system needs warming up time at the start of service of the first customer whenever the system starts first time or before serving the first customer after every idle period. Further, the system is subject to random breakdowns and there is a delay time before starting the repairs. We assume that the service time, the set-up time and the delay time all follow a general distribution and the repair time is deterministic. We obtain new, clear and explicit tractable steady state results of the system. The results of many interesting particular cases have been derived from the main results.

CONFLICT OF INTERESTS

The author declares that there is no conflict of interests.

REFERENCES

[1] A. Assini, J. R. Artalejo, On the single server retrial queue subject to breakdowns, Queue. Syst. 30 (1988), 309-321.

[2] G. Choudhury, K. C. Madan, A batch arrival Bernoulli vacation queue with a random set-up time under restricted admissibility policy, Int. J. Oper. Res. 2(1)1 (2007), 81-97.

[3] J. C. Ke, Batch arrival queues under vacation policies with server breakdowns and startup/closedown times, Appl. Math. Model. 31 (7) (2007), 1282-1292.

[4] R. F. Khalaf, K. C. Madan, C. A. Lucas, A $M^X/G/1$ queue with Bernoulli schedule general vacation times, general extended vacations, random breakdowns, general delay time for repairs to start and general repair times, J. Math. Res. 3 (4) (2011), 8-20.

[5] A. Krishnamurthy, P. K. Pramod, S. R. Chakravarty, Queues with interruptions-a survey, TOP 22 (2014), 290–320.

[6] K. C. Madan, On an M/G/1 type single server queue subject to two types of random failures with heterogeneous repair time distributions, Far East J. Math. Sci. 106 (2018), 183-194.

[7] K. C. Madan, On a queueing system with random failures and delayed repairs, J. Indian Stat. Assoc. 32 (1994), 39-48.

[8] T. Takine, B. A. Sengupta, A single server queue with service interruptions, Queueing Systems, 26 (1997), 285-300.

[9] W. Li, D. Shi, X. Chao, Reliability analysis of M/G/1 queueing system with server breakdowns and vacations, J. Appl. Probab. 34 (1997), 546-555.