Available online at http://scik.org
J. Math. Comput. Sci. 11 (2021), No. 5, 6529-6540
https://doi.org/10.28919/jmcs/6278
ISSN: 1927-5307

SOME GENERALIZATIONS OF L-CLOSED SET

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Abstract. In this paper we study and characterize L-subsets like p-locally L-closed sets, λ-locally L-closed sets, $\Lambda_\lambda L$-closed sets, gL-closed sets, λgL-closed. Further we define and study p-LC-L-continuity, λ-LC-L-continuity and Λ_λ-L-continuity and we obtain decompositions of L-continuity.

Keywords: p-locally L-closed sets; λ-locally L-closed sets; $\Lambda_\lambda L$-closed sets; p-LC-L-continuous; λ-LC-L-continuous; $\Lambda_\lambda L$-continuous.

2010 AMS Subject Classification: 54A40, 03E72.

1. Introduction

The concept of fuzzy set was introduced by Zadeh[13] in his classical paper. Fuzzy topology was introduced by Chang[3] in 1965. Subsequently, many researchers have worked on various basic concepts from general topology using fuzzy sets and developed the theory of fuzzy topological spaces. There are many applications of fuzzy sets in different fields such as information theory[10] and control problems[11].

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Received June 12, 2021

Levine[5] initiated the study of generalized closed sets in topological space in order to extend many of the important properties of closed sets to a larger family. Locally closed set is one of the generalization of closed set. The first step of locally closedness was done by Bourbaki[2]. Ganster and Reily[4] used locally closed set to define LC-continuity and LC-irresoluteness. Several mathematicians generalized this notion by replacing open sets with nearly open sets and/or by replacing closed sets with nearly closed sets. In 1997, G. Balasubramanian and P. Sundaram[1] defined generalized fuzzy closed sets in fuzzy topology. Later many mathematicians extented different generalization of closed sets to fuzzy topology.

In [12] Vinitha and Johnson introduced prime open set in topological spaces. Motivated by those definitions of prime open sets; we introduced a new collection of L-open sets called pL-open sets in L-topological spaces. An L-open G is said to be pL-open if it is not meet of two L-open sets other than G. In this paper we define a new type of L-subsets called p-locally L-closed sets which is a generalization of L-closed set and using this we obtain decomposition of L-closed sets.

The λ-open sets in topological space, which is a new class of generalized open sets was introduced and studied by Shyamapada Modak and Takashi Noirib[8]. Motivated by the definition λ-open sets, in the second part of this paper, we introduce λL-open sets. Based on this new concept, two forms of locally L-closed sets named λ-locally L-closed sets and $\Lambda_\lambda L$-closed sets are being introduced. Properties of these new concepts are studied as well as the interrelations among these types are established with the required counter examples. Also we introduce the concept of p-LC-L-continuous, λ-LC-L-continuous $\Lambda_\lambda L$-continuous and establish some new decompositions of L-continuity.

2. Preliminaries

In this section, we include certain definitions and known results needed for the subsequent development of the study. Throughout this paper, X be a non empty ordinary set and $(L,')$ be a Hutton algebra, that is, a complete, completely distributive lattice L equipped with an order-reversing involution.

Definition 2.1. [6] *Let L be a complete lattice, $C \subseteq L$ then C is a join generating set if $\forall a \in L, \exists C_a \subset C$ such that $\vee C_a = a$.*

Theorem 2.1. [6] *Let L be a completely distributive lattice then $M(L)$ is a join generating set.*

Theorem 2.2. [6] *Let L be a complete lattice, $C \subseteq L$. Then C is a join generating set if and if $\forall a,b \in L$, $a \nleq b$, $\exists c \in C$, $c \leq a$, $c \nleq b$.*

Definition 2.2. [6] *Let X be a nonempty ordinary set, L be a complete lattice. An L-subset on X is a mapping $A : X \to L$. The family of all the L-subsets on X is called L-space denoted by L^X. An L-point on X with support x and value λ is an L-subset $x_\lambda \in L^X$ defined as*

$$x_\lambda(y) = \begin{cases} \lambda & \text{if } y = x \\ 0 & \text{otherwise.} \end{cases}$$

Definition 2.3. [6] *Let X be a non-empty ordinary set and $(L,')$ be a Hutton algebra and $\delta \subseteq L^X$. Then δ is called an L- topology on X and (X,δ) is called an L-toplogical space, if δ satisfies the following three conditions:*
(i) $\underline{0}, \underline{1} \in \delta$
(ii) if $\kappa \subseteq \delta$, then $\vee\kappa \in \delta$
(iii) if $u, v \in \delta$ then $u \wedge v \in \delta$.

Every element in δ is called an L-open subset in L^X, every pseudo-complementary set of an L-open set is called L-closed subset in L^X.

Definition 2.4. *Let (X,δ) be a L-topological space. An L-open set G in (X,δ) is called a prime L-open set if $H \wedge K \leq G$ implies $H \leq G$ or $K \leq G$; where H,K are L-open sets. Prime L-open sets are denoted by pL-open sets.*

Theorem 2.3. *Let (X,δ) be an L-topological space, $x_\lambda \in M(L^X)$. Then $(\overline{x_\lambda})'$ is pL-open.*

Definition 2.5. [9] *An L-subset A of an L-topological space (X,δ) is said to be locally L-closed if $A = U \wedge F$, where U is L-open and F is L-closed.*

Definition 2.6. [1] *Let (X,δ) be an L-topological space and $A \in L^X$. Then A is generalized L-closed set, gL-closed in short if $\overline{A} \leq U$ whenever $A \leq U$ and U is L-open.*

Definition 2.7. [7] *Let (X,δ) be an L-topological space and $A \in L^X$. Then the kernel of A, denoted by $\hat{A}$, is the meet of all L-open set containing A.*

3. p-LOCALLY L-CLOSED SET

Definition 3.1. *An L-subset A of an L-topological space (X,δ) is called p-locally L-closed set if $A = U \wedge F$ where U is pL-open and F is L-closed.*

Remark 3.1. *The join and meet of two p-locally L-closed set is not generally p-locally L-closed.*

Example 3.1. *Let $X = \{a,b,c\}$ and $L = \{0,m,n,1\}$ be the diamond type lattice such that $m' = n$ and $n' = m$. Define $U,V : X \to [0,1]$ as follows.*

$$U(x) = \begin{cases} 1 & x \in \{a,b\} \\ 0 & x = c \end{cases}, V(x) = \begin{cases} 1 & x \in \{a,c\} \\ 0 & x = b \end{cases}$$

Clearly, $\delta = \{\underline{0}, \underline{1}, a_m, a_n, a_1, U, V\}$ is an L-topology on X. It is easy to see that a_m, a_n, U, V are p-locally L-closed but $U \wedge V = a_1$ and $a_m \vee a_n = a_1$ are not p-locally L-closed.

Proposition 3.1. *Let (X,δ) be an L-topological space then the following hold:*
1) Every L-closed set is p-locally L-closed set.
2) Every pL-open set is p-locally L-closed.
3) Every p-locally L-closed is locally L-closed.

Proof. 1) Let A be a L-closed set in (X,δ). Since $\underline{1}$ is pL-open and $A = A \wedge \underline{1}$, A is p-locally L-closed.
2) Follows from the definition.
3) Since every pL-open set is L-open. □

Remark 3.2. *The converse of proposition 3.1.3 need not be true. In example 3.1, $f_1 \wedge f_2$ is locally L-closed but not p-locally L-closed.*

Proposition 3.2. *For an L-subset A of an L-topological space (X,δ), the following are equivalent:*
(1) A is p-locally L-closed,

(2) $A = U \wedge \overline{A}$ for some pL-open set U,

(3) $A' = O \vee E$ where E is pL-closed set and O is an L-open set.

Proof. $(1) \Rightarrow (2)$: Suppose A is p-locally L-closed. Then $A = U \wedge F$ where U is pL-open and F is L-closed. Then $\overline{A} = \overline{(U \wedge F)} \leq \overline{F} = F$. Then $A \leq U \wedge \overline{A} \leq U \wedge F = A$ and hence $A = U \wedge \overline{A}$.
$(2) \Rightarrow (1)$: Follows from the definition 3.1.
$(3) \Leftrightarrow (1)$ Follows from the definition 3.1. □

Theorem 3.1. *Let (X, δ) be an L-topological space and $A \in L^X$. Then kernal of A is the meet of pL-open sets which contains A. i.e, $\hat{A} = \wedge\{G : A \leq G, G$ is pL-open$\}$*

Proof. Let $A \in L^X$. Define $V = \wedge\{G : A \leq G, G$ is pL-open$\}$. We want prove that $\hat{A} = V$. On contradiction suppose that $\hat{A} < V$ then there exists an L-open set U containing A such that $V \nleq U$. Hence we can find $x \in X$ such that $V(x) \nleq U(x)$, which implies $U'(x) \nleq V'(x)$. Since $M(L)$ is a join generating set, we can find $\lambda \in M(L)$ such that $\lambda \nleq V'(x)$ and $\lambda \leq U'(x)$, which implies $\overline{x_\lambda} \nleq V'$ and $\overline{x_\lambda} \leq U'$. Therefore, by theorem 2.3, we have a pL-open set $\overline{x_\lambda}'$ such that $U \leq \overline{x_\lambda}'$ and $V \nleq \overline{x_\lambda}'$. Since $A \leq U$, we have $A \leq \overline{x_\lambda}'$. Hence there exists a pL-open set $\overline{x_\lambda}'$ such that $V \nleq \overline{x_\lambda}'$ which is a contradiction. Thus $\hat{A} = V$. □

lemma 3.1. *Let (X, δ) be an L-topological space and $A \in L^X$. If A is p-locally L-closed then $A = \hat{A} \wedge \overline{A}$.*

Proof. Let A is p-locally L-closed. Then by proposition 3.2, $A = U \wedge \overline{A}$ for some pL-open set U. By theorem 3.1, $\hat{A}$ is the meet of all pL-open set containing A and hence $\hat{A} \leq U$. Now we have $A \leq \hat{A} \wedge \overline{A} \leq U \wedge \overline{A} = A$. Therefore, we obtain $A = \hat{A} \wedge \overline{A}$. □

Theorem 3.2. *Let (X, δ) be an L-topological space and $A \in L^X$. Then A is generalized L-closed set if and only if $\overline{A} \leq U$ whenever $A \leq U$ and U is pL-open.*

Proof. Necessary part trivially follows since every pL-open set L-open. For sufficiency part suppose that A is not generalized L-closed then we can find an L-open set O such that $A \leq O$ but $\overline{A} \nleq O$. Then, $O' \leq A$ and $O' \nleq \overline{A}'$. Since $M(L)$ is a join generating set, we have a $\lambda \in M(L)$ such that $\lambda \leq O', \lambda \nleq \overline{A}'$. Hence $\overline{x_\lambda} \leq O'$ and $\overline{x_\lambda} \nleq \overline{A}'$ which implies, $O \leq \overline{x_\lambda}'$ and $\overline{A} \nleq \overline{x_\lambda}'$.

Since $\lambda \in L^X$, $x_\lambda \in M(L^X)$ and by theorem 2.3, $(\overline{x_\lambda})'$ is pL-open. Thus we have an L-open set $(\overline{x_\lambda})'$ such that $A \leq (\overline{x_\lambda})'$ and $\overline{A} \nleq (\overline{x_\lambda})'$. □

Theorem 3.3. *Let (X,δ) be an L-topological space and $A \in L^X$. Then A is L-closed if and only if A is gL-closed and p-locally L-closed.*

Proof. Every L-closed set is clearly both p-locally L-closed and gL-closed. For converse part suppose that A be gL-closed and p-locally L-closed. Since A is p-locally L-closed, by lemma 3.1, $A = \hat{A} \wedge \overline{A}$. Let V be any pL-open set containing A. Since A is λgL-closed, by theorem 3.2 $\overline{A} \leq V$ and hence $\overline{A} \leq \wedge\{V : A \leq V, V \text{ is } L\text{-open}\} = \hat{A}$. Therefore $\overline{A} \leq \hat{A} \wedge \overline{A} = A$. Thus A is L-closed. □

4. λ-Locally L-Closed Sets

Definition 4.1. *An L-subset A of an L-topological space (X,δ) is said to be λL-open if A contains a nonempty L-open set.*

Proposition 4.1. *Let (X,δ) be an L-topological space. Then*
1) for any index set Δ, if A_α is λL-open, then $\vee\{A_\alpha : \alpha \in \Delta\}$ is λL-open.
2) if A be a λL-open set and B be any L-subset such that $A \leq B$ then B is λL-open set.

Remark 4.1. *The finite meet of λL-open sets need not be λL-open. In example 3.1, a_m and a_n are λL-open but $a_m \wedge a_n = \underline{0}$ is not λL-open.*

Definition 4.2. *An L-subset A of an L-topological space (X,δ) is said to be λ-locally L-closed if $A = U \wedge F$, where U is λL-open and F is L-closed.*

Proposition 4.2. *Let (X,δ) and (Y,μ) be two L-topological spaces and $f : (X,\delta) \to (Y,\mu)$ be an L-continuous function. If A is a λ-locally L-closed subset of (Y,μ), then $f^{-1}(A)$ is λ-locally L-closed in (X,δ).*

Proof. Suppose that A is λ-locally L-closed. Then we can find a λL-open set U and an L-closed set F such that $A = U \wedge F$. Since U is λL-open, there exists an L-open $V \neq \underline{0}$ such that $V \leq U$. Since f is L-continuous, $f^{-1}(V)$ is L-open and $f^{-1}(F)$ is L-closed. Since $f^{-1}(V) \leq$

$f^{-1}(U)$, $f^{-1}(U)$ is λL-open. Now $f^{-1}(A) = f^{-1}(U) \wedge f^{-1}(F)$ therefore $f^{-1}(A)$ is λ-locally L-closed. □

Proposition 4.3. *Let (X,δ) be an L-topological space then the following hold:*

1) Every L-closed set is λ-locally L-closed set.

2) Every λL-open set is λ-locally L-closed.

3) Every p-locally L-closed set is λ-locally L-closed.

Remark 4.2. *The finite join and meet of λ-locally L-closed set need not be λ locally L-closed.*

Example 4.1. *Let $X = \{a,b,c,d\}$ and $L = [0,1]$. Define $\delta = \{\underline{0},\underline{1},A,B\}$ where $A : X \to [0,1]$ is such that $A(a) = 1, A(b) = 1, A(c) = 0, A(d) = 0$ and $B : X \to [0,1]$ is such that $B(a) = 0, B(b) = 0, B(c) = 1, B(d) = 1$. Then (X,δ) be an L-topological space. Consider the L-subsets C,D defined by $C,D : X \to L$ such that $C(a) = 1, C(b) = 1, C(c) = 1, C(d) = 0; D(a) = 1, D(b) = 0, D(c) = 1, D(d) = 1$. Since $A \leq C$ and $B \leq D$, C and D are λL-open. Therefore C,D are λ-locally L-closed. But $C \wedge D = E$ where $E : X \to L$ such that $E(a) = 1, E(b) = 0.E(c) = 1, E(d) = 0$ is not λ locally L-closed. Now $c_1 = A' \wedge C$ and $a_1 = B' \wedge D$, so a_1 and c_1 are λ-locally L-closed. But $a_1 \vee c_1$ is not λ locally L-closed.*

Proposition 4.4. *For an L-subset A of an L-topological space (X,δ), the following are equivalent:*

(1) A is λ-locally L-closed,

(2) $A = U \wedge \overline{A}$ for some pL-open set U,

(3) $A' = O \vee E$ where E is pL-closed set and O is an L-open set.

Proof. $(1) \Rightarrow (2)$: Suppose A is λ-locally L-closed. Then $A = U \wedge F$ where U is λL-open and F is L-closed. Then $\overline{A} = \overline{(U \wedge F)} \leq \overline{F} = F$. Then $A \leq U \wedge \overline{A} \leq U \wedge F = A$ and hence $A = U \wedge \overline{A}$.
$(2) \Rightarrow (1)$: Follows from the definition 4.2.
$(3) \Leftrightarrow (1)$ Follows from the definition 4.2. □

Definition 4.3. *Let (X,δ) be an L-topological space and $A \in L^X$. The L-subset $\Lambda_\lambda(A)$ is defined as follows: $\Lambda_\lambda(A) = \wedge\{U : A \leq U, U$ is λL-open$\}$. An L-subset A is called a Λ_λL-set if $A = \Lambda_\lambda(A)$.*

lemma 4.1. *Let (X,δ) be an L-topological space and $A,B \in L^X$. Then the following hold:*

(1) $A \leq \Lambda_\lambda(A)$,

(2) If $A \leq B$, then $\Lambda_\lambda(A) \leq \Lambda_\lambda(B)$,

(3)$\Lambda_\lambda(\Lambda_\lambda(A)) = \Lambda_\lambda(A)$.

lemma 4.2. *Let (X,δ) be an L-topological space and $A \in L^X$ then the following hold:*

(1) $\Lambda_\lambda(A)$ is a $\Lambda_\lambda L$-set,

(2) If A is λL-open, then A is a $\Lambda_\lambda L$-set,

(3) If A_α is a $\Lambda_\lambda L$-set for each $\alpha \in \Delta$, then $\wedge\{A_\alpha : \alpha \in \Delta\}$ is a $\Lambda_\lambda L$-set.

Definition 4.4. *An L-subset A of an L-topological space (X,δ) is said to be $\Lambda_\lambda L$-closed if $A = K \wedge F$, where K is a $\Lambda_\lambda L$-set and F is a L-closed set.*

Proposition 4.5. *For an L-subset of an L-topological space (X,δ), the following properties hold:*

(1) Every λ-locally L-closed set is $\Lambda_\lambda L$-closed,

(2) Every p-locally L-closed set is $\Lambda_\lambda L$-closed.

Proof. 1) Let A be a λ-locally L-closed set. Then there exist a λL-open set U and L-closed F such that $A = U \wedge F$. By Lemma 4.2, U is a $\Lambda_\lambda L$-set and therefore A is $\Lambda_\lambda L$-closed.

2) By proposition 4.3, every p-locally L-closed set is λ-locally L-closed. Therfore (2) follows from (1) □

Proposition 4.6. *Let (X,δ) be an L-topological space then the following hold:*

(1) A is $\Lambda_\lambda L$-closed,

(2) $A = K \wedge \overline{A}$, where K is a $\Lambda_\lambda L$-set,

(3) $A = \Lambda_\lambda(A) \wedge \overline{A}$.

Proof. $(1) \Rightarrow (2)$: Suppose A is $\Lambda_\lambda L$-closed. Then $A = K \wedge F$ where K is a $\Lambda_\lambda L$-set and F is L-closed set. Then $\overline{A} = \overline{(K \wedge F)} \leq \overline{F} = F$. Thus we have $A \leq K \wedge \overline{A} \leq K \wedge F = A$ and hence $A = U \wedge \overline{A}$.

$(2) \Rightarrow (3)$: Let $A = K \wedge \overline{A}$ for some $\Lambda_\lambda L$-set K. Since $A \leq K$ by lemma 4.1, $\Lambda_\lambda(A) \leq \Lambda_\lambda(K) = K$ and hence $A \leq \Lambda_\lambda(A) \wedge \overline{A} \leq K \wedge \overline{A} = A$. Therefore, we obtain $A = \Lambda_\lambda(A) \wedge \overline{A}$.

$(3) \Rightarrow (1)$: Let $A = \Lambda_\lambda(A) \wedge \overline{A}$. By lemma 4.2, $\Lambda_\lambda(A)$ is a $\Lambda_\lambda L$-set and $\overline{A}$ is L-closed set. Therefore, A is $\Lambda_\lambda L$-closed. □

Proposition 4.7. *Let (X,δ) be an L-topological space. If A_α is a $\Lambda_\lambda L$-closed set for each $\alpha \in \Delta$, then $\wedge\{A_\alpha : \alpha \in \Delta\}$ is $\Lambda_\lambda L$-closed.*

Proof. Let A_α be a $\Lambda_\lambda L$-closed set for each $\alpha \in \Delta$. Then $A_\alpha = U_\alpha \wedge F_\alpha$, where U_α is a $\Lambda_\lambda L$-set and $F\alpha$ is a L-closed set for each $\alpha \in \Delta$. By lemma 4.2, $\wedge\{U_\alpha : \alpha \in \Delta\}$ is a $\Lambda_\lambda L$-set, $\wedge\{F_\alpha : \alpha \in \Delta\}$ is L-closed and $\wedge\{A_\alpha : \alpha \in \Delta\} = (\wedge\{U_\alpha : \alpha \in \Delta\}) \wedge (\wedge\{F_\alpha : \alpha \in \Delta\})$.Therefore, $\wedge\{A_\alpha : \alpha \in \Delta\}$ is $\Lambda_\lambda L$-closed. □

Remark 4.3. *The finite join $\Lambda_\lambda L$-closed need not be $\Lambda_\lambda L$-closed.*

Example 4.2. *Let $X = \{a,b,c\}$ and $\lambda = \{\underline{0}, \underline{1}, a_1, A\}$ where $A : X \to [0,1]$ is such that $A(a) = 1, A(b) = 1, A(c) = 0$. Here b_1 and c_1 are $\Lambda_\lambda L$-closed but their join $B = b_1 \vee c_1$ is not $\Lambda_\lambda L$-closed. Since $\Lambda_\lambda(B) = \underline{1}$, $\overline{B} = \underline{1}$ and $\Lambda_\lambda(B) \wedge \overline{B} = \underline{1}$.*

Definition 4.5. *Let (X,δ) be an L-topological space and $A \in L^X$. Then A is called λ-generalized L-closed, λgL-closed if $\overline{A} \leq U$ whenever $A \leq U$ and U is a λ-L-open set.*

Theorem 4.1. *Let (X,δ) be an L-topological space and $A \in L^X$. Then the following are equivalent:*

(1) A is L-closed,

(2) A is λ-locally L-closed and λgL-closed,

(3) A is $\Lambda_\lambda L$-closed and λgL-closed.

Proof. $(1) \Rightarrow (2)$: Let A be L-closed in (X,δ). By proposition 4.3, A is λ-locally L-closed. Let U be any λL-open set such that $A \leq U$. Then $A = \overline{A} \leq U$ and hence A is λgL-closed.

$(2) \Rightarrow (3)$: By proposition 4.5, every λ-locally L-closed set is $\Lambda_\lambda L$-closed.

$(3) \Rightarrow (1)$: Let A be $\Lambda_\lambda L$-closed and λgL-closed. Since A is $\Lambda_\lambda L$-closed, $A = U \wedge F$, where U is a Λ_λ-set and F is L-closed in (X,δ). Let V be any λL-open set containing A. Since A is λgL-closed, $\overline{A} \leq V$ and hence $\overline{A} \leq \wedge\{V : A \leq V, V \text{ is } \lambda L\text{-open}\} = \Lambda_\lambda(A)$. Therefore, $\overline{A} \leq \Lambda_\lambda(A) \leq \Lambda_\lambda(U) = U$. On the other hand, $A \leq F$and $\overline{A} \leq \overline{F} = F$. Therefore, we obtain $\overline{A} \leq U \wedge F = A$. Thus A is L-closed. □

5. Decompositions of L-Continuity

In this section we obtain some decompositions of L-continuity.

Definition 5.1. *Let (X,δ) and (Y,μ) be two L-topological spaces. A function $f:(X,\delta)\to(Y,\mu)$ is said to be p-LC-L-continuous if $f^{-1}(F)$ is p-locally L-closed in (X,δ) for any L-closed set F in (Y,μ).*

Theorem 5.1. *Let (X,δ) and (Y,μ) be two L-topological spaces. A function $f:(X,\delta)\to(Y,\mu)$ is said to be L-continuous iff f is gLcontinuous and p-LC-L-continuous.*

Proof. This result follows from theorem 3.3. □

Remark 5.1. *p-LC-L-continuity and gL-continuity are independent of each other. This shown in the following examples.*

Example 5.1. *Let $X=Y=\{a,b\}$ and $L=[0,1]$. Define $\delta=\mu=\{\underline{0},\underline{1}\}\cup\{U:X\to[0,1]|U(a)>0\}$. Then (X,δ) and (Y,μ) are L-topological spaces. Also define $f:X\to Y$ as $f(a)=b$; $f(b)=a$. Then f is not gL-continuous and L-continous since $f^{-1}(b_1)=a_1$ and a_1 is not gL-closed. But however f is p-LC-L-continuous.*

Example 5.2. *Let $X=Y=\{a,b,c\}$ and $L=[0,1]$. Define $\delta=\mu=\{\underline{0},\underline{1},a_1,A\}$ where $A:X\to[0,1]$ is such that $A(a)=0, A(b)=1, A(c)=1$. Then (X,δ) and (Y,μ) are L-topological spaces. Also define $f:X\to Y$ as $f(a)=b; f(b)=a; f(c)=c$. Then f is not p-LC-L-continuous and L-continous since $f^{-1}(a_1)=b_1$ and b_1 is not p-locally L-closed. But however f is gL-continuous.*

Definition 5.2. *Let (X,δ) and (Y,μ) be two L-topological spaces and $f:(X,\delta)\to(Y,\mu)$. Then f is said to be*
1) λ-LC-L-continuous if $f^{-1}(F)$ is λ-locally L-closed in (X,δ) for any L-closed set F in (Y,μ).
2) Λ_λL-continuous if $f^{-1}(F)$ is Λ_λL-closed in (X,δ) for any L-closed set F in (Y,μ).
3) λgL-continuous if $f^{-1}(F)$ is λgL-closed in (X,δ) for any L-closed set F in (Y,μ).

Theorem 5.2. *Let (X,δ) and (Y,μ) be L-topological space and $f:(X,\delta)\to(Y,\mu)$. Then the following are equivalent:*

(1) f is L-continuous,

(2) f is λ-LC-L-continuous and λgL-continuous,

(3) f is $\Lambda_\lambda L$-continuous and λgL-continuous.

Proof. This is an immediate consequence of Theorem 4.1. □

Remark 5.2. *(1) λ-LC-L-continuity and λgL-continuity are independent of each other, (2) Λ_λ-L-continuity and λgL-continuity are independent of each other.*

Example 5.3. *Let $X = \{a,b,c\}$, $Y = \{p,q\}$ and $L = [0,1]$. Define $\delta = \{\underline{0},\underline{1}\} \cup \{U \in L^X : U(a) > 0, U(b) = U(c) = 0\}$ and $\mu = \{\underline{0},\underline{1},q_1\}$. Then (X,δ) and (Y,μ) are L-topological spaces. Also define $f : X \to Y$ as $f(a) = f(c) = p$; $f(b) = q$. Then f is not λgL-continuous as well as not L-continous since $f^{-1}(p_1) = A$ where $A : X \to [0,1]$ such that $A(a) = 1, A(b) = 0, A(c) = 1$ and A is not λgL-closed. Clearly f is λ-LC-L-continuous therefore, f is $\Lambda_\lambda L$-continuous.*

Example 5.4. *Let $X = \{a,b,c\}$, $Y = \{p,q\}$ and $L = [0,1]$. Define $\delta = \{\underline{0},\underline{1},A\}$ here $A : X \to [0,1]$ such that $A(a) = 1, A(b) = 1, A(c) = 0$ and $\mu = \{\underline{0},\underline{1},q_1\}$. Then (X,δ) and (Y,μ) are L-topological spaces. Also define $f : X \to Y$ as $f(a) = f(c) = p$; $f(b) = q$. Then f is not λ-LC-L-continuous and not $\Lambda_\lambda L$-continuous since $f^{-1}(p_1) = A$, where $A : X \to [0,1]$ such that $A(a) = 1, A(b) = 0, A(c) = 1$ and A is not λ-locally L-closed. But it is easy to show that f is λgL-continuous.*

Remark 5.3. *Every p-LC-L-continous function is λ-LC-L-continous and every λ-LC-L-continuous function is $\Lambda_\lambda L$-continuous. But none of the reverese implication need not be true. Consider the following examples,*

Example 5.5. *Let $X = Y = R$ and $L = [0,1]$. Fix a point $a \in R$ and define $\delta = \mu = \{\underline{0},\underline{1}\} \cup \{U : X \to [0,1] | U(a) > 0, U(-a) = 0\}$. Then (X,δ) and (Y,μ) are L-topological spaces. Also define $f : X \to Y$ as $f(x) = -x$. Then f is not p-LC-Lcontinuous since $f^{-1}(-a_1) = a_1$ and a_1 is not p-locally L-closed. But f is λ-LC-L-continuous.*

Example 5.6. *Let $X = Y = (-1,0) \cup (0,1)$ and $L = [0,1]$. Define $\delta = \mu = \{\underline{0},\underline{1}\} \cup \{U \in L^X : supp(U) = (-1,0)\} \cup \{V \in L^X : supp(V) = (0,1)\}$. Then (X,δ) and (Y,μ) are L-topological* *spaces. Also define $f : X \to Y$ as,*

$$f(x) = \begin{cases} -x, & \text{if } x \in (-1/2, 0) \cup (0, 1/2), \\ x, & \text{otherwise}. \end{cases}$$

Consider L-subset set F and $f^{-1}(F)$ defined by,

$$F(x) = \begin{cases} 1, & \text{if } x \in (-1, 0) \\ 0, & \text{otherwise.} \end{cases}, f^{-1}(F) = \begin{cases} 1, & \text{if } x \in (-1, -1/2) \cup (0, 1/2) \\ 0, & \text{otherwise} \end{cases}$$

Then F is L-closed but $f^{-1}(F)$ is not λ-locally L-closed. Therefore f is not λ-LC-L-continuous but it is easy to see that f is Λ_λL-continuous.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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