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ON TIMELIKE AND SPACELIKE DEVELOPABLE RULED SURFACES

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Abstract. In this study, we have obtained the distribution parameter of a ruled surface generated by a straight line in Frenet trihedron moving along a timelike curve and also along another curve with the same parameter. At this time, the Frenet frames of these timelike curves are not the same. We have moved the director vector of the first curve along the second curve. It is shown that the ruled surface is developable if and only if the base timelike curve is helix. In addition, some smilarities and differences are presented with theorems and results.

Keywords: Timelike curve, Distribution parameter, Minkowski space.

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1. Introduction

In general, it is known that the Lorentz-Minkowski space is the metric space $E_1^3 = (\mathbb{R}^3, \langle , \rangle)$, where the metric $\langle , \rangle$ is given by

$$\langle u, v\rangle = u_1v_1 + u_2v_2 - u_3v_3, \quad u = (u_1, u_2, u_3), \quad v = (v_1, v_2, v_3)$$

The metric $\langle , \rangle$ is called the Lorentzian metric [5]. Also, it can be called E_1^3 as Minkowski space, and $\langle , \rangle$ as the Minkowski metric.

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A surface in 3-dimensional Minkowski space $\mathbb{R}^3_1 = (\mathbb{R}^3, dx^2 + dy^2 - dz^2)$ is called a timelike surface if the induced Lorentzian metric on the surface is a non-degenerate. On the other hand, it can be easily seen that a surface in 3-dimensional Minkowski space $\mathbb{R}^3_1 = (\mathbb{R}^3, dx^2 + dy^2 - dz^2)$ is called a spacelike surface if the induced Lorentzian metric on the surface is a positive [1]. A ruled surface is a surface swept out by a straight line X moving along a curve α. The various positions of the generating line X are called the rulings of the surface. Such a surface, thus, has a parametrization in ruled form as follows,

$$\Phi(s,v) = \alpha(s) + vX(s). \tag{1}$$

We call α to be the base curve, and X to be the director vector. If the tangent plane is constant along a fixed ruling, then the ruled surface is called a developable surface. The remaining ruled surfaces are called skew surfaces[4]. If there exists a common perpendicular to two preceding rulings in the skew surface, then the foot of the common perpendicular on the main ruling is called a central point. The locus of the central points is called the curve of striction.[9]

The ruled surface M is given by the parametrization

$$\begin{aligned} \Phi \quad &: \quad I \times \mathbb{R} \to \mathbb{R}^3_1 \\ (t,v) \quad &\to \quad \Phi(s,v) = \alpha(s) + vX(s) \end{aligned} \tag{2}$$

in $\mathbb{R}^3_1$ where $\alpha : I \to \mathbb{R}^3_1$ is a differentiable timelike curve parametrized by its arc length in $\mathbb{R}^3_1$ that is, $(\langle \alpha'(s), \alpha'(s) \rangle = -1)$ and $X(s)$ is the director vector of the director curve such that X is orthogonal to the tangent vector field T of the base curve α. $\{T, N, B\}$ is an orthonormal frame field along α in $\mathbb{R}^3_1$ where N is the normal vector field of the ruled surface along α [10,11].

At this time, with the assistance of α, we can define curve β. Let $\beta(s)$ be a curve and let its parameter be the same as the parameter of the curve $\alpha(s)$. Nevertheless, we choose the orthonormal frame field $\{T, N, B\}$ along β in IR^3_1 where N is the normal vector field of ruled surface along the curve β. Thus,

$$\langle T, T \rangle = -1, \langle N, N \rangle = 1, \langle X, X \rangle = 1 \tag{3}$$

Smiliarly, as it is said above for the timelike curve α, the ruled surface that produced during the curve $\alpha(s)$is obtained by the parametrization

$$\begin{aligned} \Phi_\alpha &: I \times \mathbb{R} \to \mathbb{R}_1^3 \\ (t,v) &\to \Phi_\alpha(s,v) = \alpha(s) + vX(s) \end{aligned}$$

On the other hand, let P_X be distribution parameter of the ruled surface, then

$$P_X = \frac{\det(\alpha', X, X')}{\langle X', X' \rangle} \tag{4}$$

and also, we can get the ruled surface that produced during the curve $\beta(s)$ with each fixed line X of the moving space H as:

$$\Phi_\beta(s,v) = \beta(s) + vX(s)$$

Just after that, the distribution parameter of the ruled surface for the curve β can be given as:

$$\breve{P}_X = \frac{\det(\beta', X, X')}{\langle X', X' \rangle} \tag{5}$$

Theorem 1.1.1. A ruled surface is a developable surface if and only if the distribution parameter of the ruled surface is zero.[11]

2. Preliminaries

Now we give the basic concepts on differential geometry of cuves and surfaces in Lorentz-Minkowski space about our study.

2.1. The Timelike Case [5]

We suppose that α is a timelike curve. Then $T'(s) \neq 0$ is a spacelike vector independent with $T(s)$. We define the curvature of α at s as $k_1(s) = |T'(s)|$ .The normal vector $N(s)$

is defined by

$$N(s) = \frac{T'(s)}{\kappa(s)} = \frac{\alpha''(s)}{|\alpha''(s)|}. \tag{6}$$

Moreover $k_1(s) = \langle T'(s), N(s) \rangle$. We call the binormal vector $B(s)$ as

$$B(s) = T(s) \times N(s) \tag{7}$$

The vector $B(s)$ is unitary and spacelike. For each s, $\{T, N, B\}$ is an orthonormal base of E_1^3 which is called the Frenet trihedron of α. We define the torsion of α. We define the torsion of α at s as

$$k_2(s) = \langle N'(s), B(s) \rangle . \tag{8}$$

By differentiation each one of the vector functions of the Frenet trihedron and putting in relation with the same Frenet base, we obtain the Frenet equations, namely,

$$\begin{bmatrix} T' \\ N' \\ B' \end{bmatrix} = \begin{bmatrix} 0 & k_1 & 0 \\ k_1 & 0 & k_2 \\ 0 & -k_2 & 0 \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix} \tag{9}$$

3. Timelike Developable Ruled Surfaces

Let

$$\alpha : I \to \mathbb{R}_1^3 \tag{10}$$

be a timelike curve and $\{T, N, B\}$ be Frenet vector, where T, N and B are the tangent, principal normal and binormal vectors of the curve, respectively. $T'(s)$ and $B(s)$ vectors are spacelike, and at the same time $B(s)$ is unitary vector

The two coordinate systems $\{O; T, N, B\}$ and $\{O'; \overrightarrow{e}_1, \overrightarrow{e}_2, \overrightarrow{e}_3\}$ are orthogonal coordinate systems in $\mathbb{R}_1^3$ which represent the moving space H and the fixed space H', respectively. Let us express the displacements (H/H') of H with respect to H'. During the one parameter spatial motion H/H', each line X of the moving space H, generates, in generally, a ruled surface in the fixed space H'.

On the other hand, let X be a unit vector. Thus

$$X \in Sp\{T, N, B\} \text{ and } X = x_1 T + x_2 N + x_3 B \tag{11}$$

such that

$$\langle X, X \rangle = 1 \tag{12}$$

We can obtain the distribution parameter of the ruled surface generated by line X of the moving space H. Different cases can be investigated as following:

Let the vector $T'(s)$ and $B(s)$ be spacelike. Thus, from (11) we have

$$X' = x_1 T' + x_2 N' + x_3 B', \qquad -x_1^2 + x_2^2 + x_3^2 = 1 \tag{13}$$

substituting (10) into (13), then

$$\begin{aligned} X' &= x_1(k_1 N) + x_2(k_1 T + k_2 B) + x_3(-k_2 N) \\ &= x_2 k_1 T + (x_1 k_1 - x_3 k_2) N + x_2 k_2 B \end{aligned} \tag{14}$$

From (4), we obtain

$$P_X = \frac{\det(T, x_1 T + x_2 N + x_3 B, x_2 k_1 T + (x_1 k_1 - x_3 k_2) N + x_2 k_2 B)}{x_2^2(k_1^2 + k_2^2) + (x_1 k_1 - x_3 k_2)^2}$$

And then, it can be easily seen that

$$P_X = \frac{x_2^2 k_2 - x_1 x_3 k_1 + x_3^2 k_2}{x_2^2(k_1^2 + k_2^2) + (x_1 k_1 - x_3 k_2)^2} \tag{15}$$

$$= \frac{k_2(x_2^2 + x_3^2) - x_1 x_3 k_1}{x_2^2(k_1^2 + k_2^2) + (x_1 k_1 + x_3 k_2)^2} \tag{16}$$

The ruled surface is developable if and only if P_X is zero. From (16)

$$P_X = 0 \text{ if and only if } \frac{k_1}{k_2} = \frac{x_2^2 + x_3^2}{x_1 x_3} \tag{17}$$

Hence we state the following theorem:

During the one parameter spatial motion H/H' the ruled surface in the fixed space H' generated by a line X of the moving space H is developable if and only if $\beta'(s)$ is a helix such that harmonic curvature h of the base timelike curve $\alpha(s)$ satisfies the equality

$$h = \frac{k_1}{k_2} = \frac{x_2^2 + x_3^2}{x_1 x_3}$$

Different special situations can occur and some results of these special cases are smiliar in [9,10,11]:

1. The Case $X = T$

In this case $x_1 = 1$, $x_2 = x_3 = 0$, thus from (16)

$$P_X = 0$$

At this time, we can say that the ruled surface is timelike. According to all of these the following theorem can be given:

Theorem 3.0.1. During the one-parameter spatial motion H/H',the timelike ruled surface in the fixed space H' generated by the tangent line T of the timelike curve α in the moving space H is developable.

The Case $X = N$

In this case, $x_2 = 1$, $x_1 = x_3 = 0$, thus from (16)

$$P_N = \frac{k_2}{k_1^2 + k_2^2}$$

If P_N is zero then k_2 is zero. Thus, the timelike curve $\alpha(s)$ is a planar curve. Hence the following theorem is hold:

Theorem 3.0.2. During the one-parameter spatial motion H/H',the timelike ruled surface in the fixed space H' generated by the normal line N of the timelike curve α in the moving space H is developable.

3. The Case $X = B$

In this case, $x_3 = 1$, $x_1 = x_2 = 0$; thus from (16)

$$P_N = \frac{k_2}{k_2^2} = \frac{1}{k_2}$$

At this time, it can be seen that the ruled surface is timelike.

4. The Case X is in the Normal Plane

In this case, x_1 is zero. The ruled surface is timelike.and also it is developable. Now from (16)

$$\begin{aligned} P_X &= \frac{k_2(x_2^2+x_3^2)}{x_2^2(k_1^2+k_2^2)+x_3^2k_2^2} \\ &= \frac{k_2(x_2^2+x_3^2)}{(x_2^2+x_3^2)k_2^2+x_2^2k_1^2} \end{aligned}$$

If P_X is zero hen $k_2=0$ ($or\ x_2^2+x_3^2=0$). Thus, if $k_2=0$, then the timelike curve $\alpha(s)$ is a planar curve. Hence the following theorem can be given as:

Theorem 3.0.3. During the one-parameter spatial motion H/H',the timelike ruled surface in the fixed space H' generated by a line X in the normal plane of H is developable if and only if the timelike curve α is a planar curve in normal plane.

5. The Case X is in the Osculating Plane

In this case, x_3 is zero. Thus the ruled surface is timelike and also it is developable since from (16)

$$\begin{aligned} P_X &= \frac{x_2^2k_2}{x_2^2(k_1^2+k_2^2)+x_1^2k_1^2} \\ &= \frac{x_2^2k_2}{(x_2^2+x_1^2)k_1^2+x_2^2k_2^2} \end{aligned}$$

If $P_X=0$ then $X=T$. This is the **case.1.** If $k_2=0$ then the timelike curve $\alpha(s)$ is a planar curve. Therefore, Theorem(3.0.2) can be restated.

6. The Case X is in the Rectifying Plane

In this case, x_2 is zero. From (16), we can give the distribution parameter of the ruled surface as:

$$P_X = \frac{-x_1x_3k_{1+}x_3^2k_2}{(x_1k_1-x_3k_2)^2}$$

If $P_X=0$ then $\dfrac{k_1}{k_2}=-\dfrac{x_3}{x_1}$. Thus it can be seen that the timelike curve $\alpha(s)$ is a helix if and only if the ruled surface is developable such that $\dfrac{k_1}{k_2}=-\dfrac{x_3}{x_1}$. At this time, the timelike curve is helix if and only if the base curve is striction line such that $\dfrac{k_1}{k_2}=-\dfrac{x_3}{x_1}$.

3.1. Timelike or Spacelike Developable Ruled Surfaces

As it is said in the first part of the study, with the assistance of α, another different cuve β can be defined with the same parameter of the timelike curve $\alpha(s)$, such that

$$\beta' = \lambda_1 T + \lambda_2 N + \lambda_3 B,$$

At this time, we can get the ruled surface that produced during the curve $\beta(s)$ with each line X of the moving space H as

$$\Phi_\beta(s,v) = \beta(s) + vX(s)$$

Smiliarly, the two coordinate systems $\{O; T, N, B\}$ and $\{O'; \overrightarrow{e}_1, \overrightarrow{e}_2, \overrightarrow{e}_3\}$ are orthogonal coordinate systems in IR_1^3 which represent the moving space H and the fixed space H', respectively. Let us express the displacements (H/H') of H with respect to H'. During the one parameter spatial motion H/H', each line X of the moving space H, generates, in generally, a ruled surface in the fixed space H'. Subsequently, it should be seen that the curve can be spacelike or timelike but it can not be null. For this situation, taking $\langle \beta', X \rangle = 0$ is enough.

Let X be a vector and X is fixed. Thus,

$$X \in Sp\{T, N, B\} \text{ and } X = x_1 T + x_2 N + x_3 B \tag{18}$$

such that

$$\langle X, X \rangle = \pm 1 \tag{19}$$

We can obtain the distribution parameter of the ruled surface generated by line X of the moving space H. As it is investigated in the first kind of timelike curve α, let the vector $T'(s)$ and $B(s)$ of the curve β be spacelike. Smiliarly from $X = x_1T + x_2N + x_3B$, we get

$$X' = x_1 T' + x_2 N' + x_3 B', \qquad -x_1^2 + x_2^2 + x_3^2 = \pm 1 \tag{20}$$

substituting (10) into (20), then

$$X' = x_1T' + x_2N' + x_3B' = x_1(k_1N) + x_2(k_1T + k_2B) + x_3(-k_2N) = x_2k_1T + (x_1k_1 - x_3k_2)N + x_2k_2B \tag{21}$$

From (5), we obtain

$$\begin{aligned} P_X &= \frac{\det(\beta', X, X')}{x_2^2k_1^2 + (x_1k_1 + x_3k_2)^2 + x_2^2k_2^2} \\ &= \frac{\det(\lambda_1T + \lambda_2N + \lambda_3B, x_1T + x_2N + x_3B, x_2k_1T + (x_1k_1 - x_3k_2)N + x_2k_2B)}{x_2^2k_1^2 + (x_1k_1 - x_3k_2)^2 + x_2^2k_2^2} \\ &= \frac{\lambda_1(x_2^2k_2 - x_1x_3k_1 + x_3^2k_2) - \lambda_2(x_1x_2k_2 - x_2x_3k_1) + \lambda_3(x_1^2k_1 - x_1x_3k_2 - x_2^2k_1)}{x_2^2k_1^2 + (x_1k_1 - x_3k_2)^2 + x_2^2k_2^2} \\ &= \frac{\lambda_1((x_2^2 + x_3^2)k_2 - x_1x_3k_1) - \lambda_2(x_1x_2k_2 - x_2x_3k_1) + \lambda_3((x_1^2 - x_2^2)k_1 - x_1x_3k_2)}{x_2^2k_1^2 + (x_1k_1 - x_3k_2)^2 + x_2^2k_2^2} \end{aligned} \tag{22}$$

3.2. For $\lambda_1 = 1,\ \lambda_2 = \lambda_3 = 0\ (\beta' = T)$

The same special cases and results have occured. We have found the same results as above investigating the timelike curve $\alpha(s)$.

3.3. For $\lambda_2 = 1,\ \lambda_1 = \lambda_3 = 0\ (\beta' = N)$

$$P_X = \frac{-x_1x_2k_2 - x_2x_3k_1}{x_2^2k_1^2 + (x_1k_1 - x_3k_2)^2 + x_2^2k_2^2} \tag{23}$$

The ruled surface is developable if and only if P_X is zero. From (24) and (25)

$$P_X = 0 \text{ if and only if } \frac{k_1}{k_2} = \frac{-x_1}{x_3} \tag{24}$$

It is the same as the situation "$T'(s)$ is spacelike and $B(s)$ is timelike" in [11]. It can be seen that the curve is helix if and only if the ruled surface is developable such that

$\frac{k_1}{k_2}=\frac{-x_1}{x_3}$. And also the curve is helix if and only if the base curve striction line is constant such that $\frac{k_1}{k_2}=\frac{-x_1}{x_3}$.

SPECIAL CASES

1. The Case $X=T$

In this case, $x_1=1, x_2=x_3=0$.Thus $P_T=0$.

Theorem 3.3.1. During the one parameter spatial motion H/H' the ruled surface in the fixed space H' generated by the tangent line T of the curve $\beta(s)$ in the moving space H is developable.

2. The Case $X=N$

In this case, $x_2=1, x_1=x_3=0$.Thus $P_N=0$.

Theorem 3.3.2. During the one parameter spatial motion H/H' the ruled surface in the fixed space H' generated by the normal line N of the curve $\beta(s)$ in the moving space H is developable.

3. The Case $X=B$

In this case, $x_3=1, x_1=x_2=0$.Thus $P_B=0$.

Theorem 3.3.3. During the one parameter spatial motion H/H' the ruled surface in the fixed space H' generated by the binormal line B of the curve $\beta(s)$ in the moving space H is developable.

4. The Case X is in the Normal Plane

In this case, $x_1=0$.The ruled surface is developable, from (25),

$$P_X=\frac{-x_2x_3k_1}{x_2^2(k_1^2+k_2^2)+x_3^2k_2^2}$$

If $P_X=0$ then $k_1=0$. So, the base curve $\beta(s)$ is a line. The ruled surface is developable if and only if $x_2=0$ or $x_3=0$.

5.The Case X is in the Osculating Plane

In this case, $x_3=0$.The ruled surface is developable, since from (25),

$$P_X=\frac{-x_1x_2k_2}{x_2^2(k_1^2+k_2^2)+x_1^2k_1^2}$$

If P_X is zero then $k_2 = 0$. Thus if $k_2 = 0$ then $\beta(s)$ is a planar curve. Hence following theorem in the first case can be occured:

Theorem 3.3.4. During the one parameter spatial motion he ruled surface in the fixed space H' generated by a line X in the normal plane of H is developable if and only if the curve $\alpha(s)$ is a planar curve in osculating plane.

6.The Case X is in the Rectifying Plane

In this case, $x_2 = 0$, from (25) the distribution parameter of the ruled surface is:

$$P_X = 0$$

If P_X is zero then $X = T$. This is **the case 1.**

3.4. For $\lambda_3 = 1,\ \lambda_1 = \lambda_2 = 0\ (\beta' = B)$

$$P_X = \frac{k_1(x_1^2 - x_2^2) - x_1 x_3 k_2}{x_2^2(k_1^2 + k_2^2) + (x_1 k_1 - x_3 k_2)^2} \tag{25}$$

The ruled surface is developable if and only if P_X is zero. As a result we can easily say that

Result 3.4.1. $P_X = 0$ if and only if $k_1(x_1^2 - x_2^2) = x_1 x_3 k_2$

Result 3.4.2. $P_X = 0$ if and only if $\dfrac{k_1}{k_2} = \dfrac{x_1 x_3}{x_1^2 - x_2^2}$ and also it can be seen that the timelike curve α is helix.

SPECIAL CASES

1. The Case $X = T$,

In this case, $x_1 = 1, x_2 = x_3 = 0$.Thus, from (27),

$$P_T = \frac{k_1}{x_1^2 k_1^2} = .\frac{1}{k_1}$$

It can be seen that the ruled surface is not developable.

2. The Case $X = N$,

In this case, $x_2 = 1, x_1 = x_3 = 0$.So, from (27),

$$P_N = \frac{-k_1}{k_1^2 + k_2^2}$$

If P_N is zero then k_1 is zero. In this case, that can not happen. For this reason, the ruled surface is not developable. And also, it can be seen that the timelike curve α is Mannheim curve if and only if P_N is constant.

3. The Case $X = B$,

In this case, $x_3 = 1, x_1 = x_2 = 0$.Thus, from (27),

$$P_B = 0$$

Result 3.4.3. During the one parameter spatial motion H/H' the ruled surface in the fixed space H' generated by the binormal line B of the curve $\beta(s)$ in the moving space H is developable.

4.The Case X **is in the Normal Plane**

In this case, $x_1 = 0$, so

$$P_X = \frac{-k_1 x_2^2}{x_2^2(k_1^2 + k_2^2) + x_3^2 k_2^2}$$

If $P_X = 0$, then $-k_1 x_2^2 = 0$.From here, x_2 is zero. Hence, this case gives us **The Case** $X = B$.

5.The Case X **is in the Osculating Plane**

In this case, $x_3 = 0$. Then

$$P_X = \frac{k_1(x_1^2 - x_2^2)}{x_2^2(k_1^2 + k_2^2) + x_1^2 k_1^2}$$

$P_X = 0$ if and only if $x_1 = \pm x_2$. From here, it can be easily seen that if $x_1 = |x_2|$, then the ruled surface is developable.

6.The Case X **is in the Rectifying Plane**

In this case, $x_2 = 0$.

$$\begin{aligned} P_X &= \frac{k_1 x_1^2 - x_1 x_3 k_2}{(x_1 k_1 - x_3 k_2)^2} \\ &= \frac{x_1(x_1 k_1 - x_3 k_2)}{(x_1 k_1 - x_3 k_2)^2} \\ &= \frac{x_1}{(x_1 k_1 - x_3 k_2)} \end{aligned}$$

If $P_X = 0$ if and only if $x_1 = 0$. Then $X = x_3 B$ and $x_3^2 = 1$. At this time, the ruled surface is developable.

4. THE CURVE OF THE STRICTION OF A RULED SURFACE

In this section, as it is said in the first part of the study, the various positions of the generating line X are called the rulings of the surface. Such a surface, thus, has a parametrization in ruled form as follows,

$$\Phi_\alpha(s,v) = \alpha(s) + vX(s). \tag{26}$$

the curve of the striction of a ruled surface can be given. This can be investigated for the base timelike curve α and also for another different curve β. If there exists a common perpendicular to two preceding rulings in the ruled surface, then the foot of the common perpendicular on the main ruling is called a central point. The locus of the central points is called the curve of striction.[9]

The ruled surface is obtained by the parametrization

$$\begin{aligned} \Phi_\alpha \quad &: \quad I \times \mathbb{R} \to \mathbb{R}_1^3 \\ (t,v) \quad &\to \quad \Phi_\alpha(s,v) = \alpha(s) + vX(s) \end{aligned} \tag{27}$$

On the other hand, we can get the ruled surface that produced during the curve $\beta(s)$ with each line X of the moving space H as:

$$\Phi_\beta(s,v) = \beta(s) + vX(s) \tag{28}$$

Thus, the curve of the striction of a ruled surface can be given by:

$$\widetilde{\beta}(s) = \beta(s) - \frac{\left\langle \frac{d\beta}{ds}, X' \right\rangle}{\langle X', X' \rangle}.X \tag{29}$$

In this case, some important situations occur, such as it can be easily seen that:

$$\left\langle \frac{d\beta}{ds}, X' \right\rangle = 0 \tag{30}$$

That is,

$$\langle \lambda_1 T + \lambda_2 N + \lambda_3 B, x_2 k_1 T + (x_1 k_1 - x_3 k_2) N + x_2 k_2 B \rangle = 0 \tag{31}$$

and then,

$$-\lambda_1 x_2 k_1 + \lambda_2 (x_1 k_1 - x_3 k_2) + \lambda_3 x_2 k_2 = 0 \tag{32}$$

Accordingly, the following theorem is obtained:

Theorem 4.0.1. $\widetilde{\beta} = \beta$ if and only if $\left\langle \dfrac{d\beta}{ds}, X' \right\rangle = 0$.

Result 4.0.1. If the director vector X and the curve β' are in the rectifying plane of the timelike curve α, then $\widetilde{\beta} = \beta$.

Proof. If X is in the rectifying plane, then $x_2 = 0$. In addition to this, if β' is in the rectifying plane, then $\lambda_2 = 0$.

Result 4.0.2. If the director vector X is in rectifying plane and the timelike curve of α is a helix, such that $\dfrac{k_1}{k_2} = \dfrac{x_3}{x_1}$, then the ruled surface is cylinder. Because,

$$X' = (x_1 k_1 - x_3 k_2) N \tag{33}$$

and

$$X = x_1 T + x_3 B. \tag{34}$$

Thus,

$$X' = 0 \tag{35}$$

then the director vector X is constant.

Result 4.0.3. On the other hand, if the director vector X is in rectifying plane and the timelike curve α is a helix such that the curvature $\dfrac{k_1}{k_2} = \dfrac{x_3}{x_1}$, then the curve $\beta(s)$ is the striction curve of the ruled surface.

5.CONCLUSIONS

In this paper, the distribution parameter of a ruled surface has been investigated by taking a timelike curve and also another curve with the same parameter, with their orthonormal frame fields. By moving the director vector of the first curve along the second curve, some similarities and differences are found. Depending on this, important results and theorems are presented about the cases of the base timelike curve. It is seen that, according to the timelike curve, the values of distribution parameter of ruled surfaces are changed.

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