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ON FEEBLY PAIRWISE EXPANDABLE SPACE

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Abstract. In this paper we introduce the concept of pairwise feebly-expandable (f-expandable) spaces as a significant variation of pairwise expandable spaces. This paper appears some properties of pairwise f-expandable spaces and given the equivalent conditions on space to be pairwise f-expandable.

Keywords: pairwise expandable; pairwise paracompact; pairwise subparacompact; pairwise metacompact; bitopological space.

2010 AMS Subject Classification: 54A05.

1. INTRODUCTION

A bitopological space (X,τ_1,τ_2) is a non-empty set X with two arbitrary topologies τ_1,τ_2. The idea of the bitopological space was induced by topologies generalized by the following two sets:

$$B_{\rho_\varepsilon} = \{y \in X \mid \rho(x,y) \leq \varepsilon\}$$

and

$$B_{\delta_\varepsilon} = \{y \in X \mid \delta(x,y) \leq \varepsilon\},$$

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where ρ and δ are quasi-metric spaces of X with $\rho(x,y) = \delta(y,x)$. From 1963, when Kellay introduced the concept of bitopological space, several topological properties, which are already included in a single topology, are generalized into bitopological spaces[1]. Some of these properties are compactness, paracompactness, separation axioms, connectedness, some special types of functions and many others [1]. Many authors studied and investigated these bitopological spaces after Kelly, like Fletcher et al. (1969)[2], Birsan (1969) [3], Reilly (1970) [4], Datta (1972) [5], Hdeib and Fora (1982, 1983) [6], Bose et al. (2008)[7], Killiman and salleh (2011) [8], Abushaheen et al. (2016) [9], and Qoqazeh et al. (2018) [10]. For further exploration in this field, this work aims to present some properties of the Feebly pairwise expandable spaces and study their properties and their relations with other a bitopological spaces. several examples are discussed and many will known theorems are generalized concerning Feebly pairwise expandable spaces. and we shall investigate subspace of Feebly pairwise expandable spaced also bitopological spaces which are related to Feebly pairwise expandability.

To go forward in this paper and for more simplification, we state some notations and notions which will be used later on. In particular, the closure and interior of the set A will be denoted respectively by $CL(A)$ and $Int(A)$ with noting that (X,τ) is a topological space.

2. PRELIMINARIES

In the following content, we state certain definitions and preliminaries associated with the bitopological space for completeness.

Definition 1. *[11] A collection subset $\tilde{F} = \{F_\alpha : \alpha \in \Delta\}$ of a bitopological space (X,τ_1,τ_2) is said to be pairwise locally finite if for each $x \in X$ there exist an τ_1–open set U containing x such that U intersects only finitely many members of $\tilde{F}$, or there exist τ_2– open set V containing x such that V intersects only finitely many members of $\tilde{F}$.*

Definition 2. *[11] A P–open cover $\tilde{V}$ of a bitopological space (X,τ_1,τ_2) is called parallel refinement of a P–open cover $\tilde{U}$ of X if each τ_i–open set of $\tilde{V}$ is contained in some τ_i–open set of $\tilde{U}$, where $i = 1,2$.*

Definition 3. *[5] A bitopological space X is called P–m–paracompact, if every P–open cover $\tilde{U}$ of X, so that $\left|\tilde{U}\right| \leq m$, has a pairwise locally finite open parallel refinement. If $m = \omega_0$, then* *the space X is called P–countably paracompact. If the space X is P–m–paracompact for every m, then X is called P–paracompact.*

Definition 4. *[11] Let m be an infinite cardinal, then the bitopological space (X,τ_1,τ_2) is called τ_ι–m–expandable space with respect to τ_j if for every τ_ι–locally finite $\tilde{F}=\{F_\alpha:\alpha\in\Delta\}$ with $|\triangle|\leq m$, there exist τ_j–locally finite collection $\tilde{G}=\{G_\alpha:\alpha\in\Delta\}$ of open subsets of X such that $F_\alpha\subset G_\alpha$ for all $\alpha\in\Delta$ and for $i\neq j$, where $i,j=1,2$.*

Definition 5. *[11] A bitopological space (X,τ_1,τ_2) is called τ_ι–expandable with respect to τ_j, if it is an τ_ι–m–expandable for every cardinal m and $i\neq j$ where $i,j=1,2$.*

Definition 6. *A bitopological space (X,τ_1,τ_2) is called a pairwise expandable (or simply P–expandable), if it is P–T_2–space and it is τ_1–expandable with respect to τ_2 and τ_2– expandable with respect to τ_1.*

Definition 7. *[11] Let $X=(X,\tau_1,\tau_2)$ be a bitopological space and $A\subset X$. A is called τ_1 -feebly open ($\tau_1-f-open$)) set if and only if $A\subset\overline{A^o}^o$ with respect to τ_1. It is clear that every τ_1-f-open set is τ_1- open set in X, and A is pairwise feebly open (P-f -open) set if it is τ_1 $-f-$open and τ_2- f -open in X. The complement of f-open set is f-open set.*

Definition 8. *Let $X=(X,\tau_1,\tau_2)$ be a bitopological space. Then a collection $F=\{F_\alpha:\alpha\in\triangle\}$ of subsets of X is called pairwise locally finite collection (P$-$ locally finite) if for each $x\in X$ there exist an τ_1- open set U containing X such that U intersects finitely many members of $\underline{F}$, or there exist τ_2- open V containing X such that V intersects finitely many members of $\underline{F}$.*

Definition 9. *A topological space X is called m-expandable if for every locally finite $F=\{F_\alpha:\alpha\in\triangle\}$ of subset of X with $|\triangle|\leq m$, there exist a locally. finite collection of open subsets $G=\{G_\alpha:\alpha\in\Lambda\}$ such that $F_\alpha\subseteq G_\alpha$, for all $\alpha\in\triangle$, where m be an infinite cardinal. If X is m-expandable for each m, then X will be called expandable Since if $F=\{F_\alpha:\alpha\in\triangle\}$ is locally finite collection, then $CL(F)=\{CL(F_\alpha):\alpha\in\triangle\}$. is also locally finite then a sufficient condition for X to be expandable is to show that for every closed collection of locally finite subsets of X there exists an open locally finite expansion.*

Definition 10. *Let m be an infinite cardinal, then a bitopological space (X,τ_1,τ_2) is called $\tau_\imath - m -$ expandable space if for every $\tau_{\imath -}$locally finite $F=\{F_\alpha : \alpha \in \triangle\}$ of subsets of X, there exist τ_{j-}locally finite collection $G=\{G_\alpha : \alpha \in \triangle\}$ of open subsets of X such that $F_\alpha \subseteq G_\alpha$, for all $\alpha \in \triangle$ and for $i \neq j, i, j = 1,2$, in this case X is said to be $\tau_\imath$ $-m-$ expandable space with respect to τ_j for every cardinal m and $i \neq j, i, j = 1,2$.*

Definition 11. *A bitopological space (X,τ_1,τ_2) is called a pairwise expandable space, proved that it is pairwise T_2- space and it is τ_1- expandable with respect to τ_2 and τ_2- expandable with respect to τ_1.*

Definition 12. *Let m be an infinite cardinal, then a bitopological space (X,τ_1,τ_2) is called $\tau_\imath - m - f-$expandable space if for every $\tau_{\imath -}$locally finite $F=\{F_\alpha : \alpha \in \triangle\}$ of subsets of X, there exist τ_{j-}locally finite collection $G=\{G_\alpha : \alpha \in \triangle\}$ of $f-$open subsets of X such that $F_\alpha \subseteq G_\alpha$, for all $\alpha \in \triangle$ and for $i \neq j, i, j = 1,2$, in this case X is said to be $\tau_\imath$ $-m-$ $f-$ expandable space with respect to τ_j for every cardinal m and $i \neq j, i, j = 1,2$.*

Definition 13. *A bitopological space (X,τ_1,τ_2) is called a pairwise feebly expandable ($P-f-$expandable) space, proved that it is pairwise T_2- space and it is $\tau_1- f-$ expandable with respect to τ_2 and $\tau_2- f-$expandable with respect to τ_1.*

3. Main Theoretical Results

In this part, we intend to state and derive some novel significant theoretical results in light of the aforesaid definitions. At the beginning, one can observe that it is easy to prove the following theorem:

Theorem 1. *Let $F=\{F_\alpha : \alpha \in \triangle\}$be $P-$ locally finite family of $P-f-$ closed subsets of a bitopological space $X=(X,\tau_1,\tau_2)$ then $\bigcup\limits_{\lambda\in\Delta} \overline{F_\lambda}^f = \overline{\bigcup\limits_{\lambda\in\Delta} F_\lambda^f}$*

Theorem 2. *Let $X=(X,\tau_1,\tau_2)$ be a bitopological space. If Y is an open subset of X and A is $P-f-$open set in Y, then there exists an $P-f-$open set B in X such that $A=B\bigcap Y$.*

Next, based on the two lemmas provided above, we state and prove the following important theorem that concerns with the *P–m*-expandable space.

Theorem 3. *Let $X=(X,\tau_1,\tau_2)$ be a bitopological space.Then the following condition are equivalent:*

1. *X is P-f-expandable.*
2. *Every $P-$locally finite collection $\{F_\alpha : \alpha \in \triangle\}$ of P-f-closed subsets of X there exists a $P-$locally finite collection of P-f-open subsets $\{G_\alpha : \alpha \in \triangle\}$ of X such that $F_\alpha \subseteq G_\alpha$ for each $\alpha \in \triangle$.*
3. *Every $P-$locally finite collection of closed subsets $\{F_\alpha : \alpha \in \triangle\}$ of X there exists a $P-$locally finite collection of P-f-open subsets $\{G_\alpha : \alpha \in \triangle\}$ such that $F_\alpha \subseteq G_\alpha$ for each $\alpha \in \triangle$.*

Proof. $1 \to 2 \to 3$ is clear.

$3 \to 1$

Let $\{F_\alpha : \alpha \in \triangle\}$ be a $P-$locally finite collection of subsets of X. Then $\{\overline{F}_\alpha : \alpha \in \triangle\}$ is a $P-$locally finite of closed subsets of X.Therefore $\{\overline{F}_\alpha : \alpha \in \triangle\}$ has a $P-$locally finite collection $\{G_\alpha : \alpha \in \triangle\}$ of P-f-open subsets of X such that $\overline{F}_\alpha \subseteq G_\alpha$ for each $\alpha \in \triangle$. Hence $F_\alpha \subseteq \overline{F}_\alpha \subseteq G_\alpha$ for each $\alpha \in \triangle$. □

Theorem 4. *A space $X=(X,\tau_1,\tau_2)$ is P-f-expandable if and only if for each $P-$locally finite collection $\{\overline{F}_\alpha : \alpha \in \triangle\}$ of closed sets in X, there exists a $P-$locally finite P-f-open cover G of X such that for each $G \in G$.$\{F_\alpha \in \triangle$: $G \bigcap F_\alpha \neq \phi\}$ is finite .*

Proof. Assume that $\{F_\alpha : \alpha \in \triangle\}$ be a $P-$locally finite collection of closed sets. Let D be the union of all finite subsets of $\triangle$. for each $d \in D$, let $U(d) = X \setminus \bigcup\{F_\alpha : \alpha \in \triangle \setminus d\}$. Then $\{U(d) : d \in D\}$ is an $P-$open cover of X. Let $B(\phi) = U(\phi)$ and $B(d) = U(d) \bigcap (\bigcap_{\alpha \in d} F_\alpha)$, $d \in D \setminus \{\phi\}$.Then $\{B(d) : d \in D\}$ is a $P-$locally finite cover of X with $B(d) \subset U(d)$ for each $d \in D$. There is a $P-$locally finite P-f-open cover $\{G(d) : d \in D\}$ of X such that $B(d) \subset G(d)$ for each $d \in D$. We may assume $G(d) \subset U(d)$ for each $d \in D$. Since for each $\alpha \in \triangle \setminus d$, $G(d) \bigcap F_\alpha \subset U(d) \bigcap (\bigcup_{\alpha \in \triangle \setminus d} F_\alpha)$. Then for each $d \in D$, $\{\alpha \in \triangle : G \bigcap F_\alpha \neq \phi\} \subset d$ finite. conversely, let $\{F_\alpha : \alpha \in \triangle\}$ be a $P-$locally finite collection of closed sets in X and G is a $P-$locally finite P-f-open cover of X such that for each $G \in G$, $A(G) = \{\alpha \in \triangle : G \bigcap F_\alpha \neq \phi\}$ is finite. For each $\alpha \in \triangle$,let $U_\alpha = st(F_\alpha, G)$.Then $F_\alpha \subset U_\alpha$ for each $\alpha \in \triangle$. Let $x \in X$, there is a neighborhood

H of x such that intersects finitely many member of G. Let $C = \alpha \in \triangle : H \bigcap U_\alpha \neq \phi$, then $C \subset \bigcup\{A(G) : G \in (G)_H\}$. Therefore C is finite set. It follows that $\{U_\alpha : \alpha \in \triangle\}$ is a $P-$locally finite. □

Definition 14. *A bitopological space $X = (X, \tau_1, \tau_2)$ is said to be paracompact if each $P-$open covering of X has a $P-$locally finite open refinement.*

Definition 15. *A bitopological space $X = (X, \tau_1, \tau_2)$ is said to be P-f-paracompact if each $P-$open covering of X has a $P-$locally finite P-f-open refinement.*

Theorem 5. *Every P-f-paracompact space is P-f-expandable.*

Proof. Suppose $\{F_\alpha : \alpha \in \triangle\}$ is a $P-$locally finite collection of closed sets in P-f-par compact space X, let D be the union of all finite subsets of $\triangle$. For each $d \in D$ define $G(d) = X \backslash \bigcup F_\alpha : \alpha \in \triangle \backslash d$. For each $x \in X$, let $d(x) = \alpha \in \triangle : x \in F_\alpha$, then $x \in G(d(x))$, and so $G = G(d) : d \in D$ is an $P-$open cover of X. Let W be a $P-$locally finite P-f-open refinement of G .For each $\alpha \in \triangle$, let $U_\alpha = st(F_\alpha, W)$. Then $F_\alpha \subset U_\alpha$ for each $\alpha \in \triangle$. To see $U = \{U_\alpha : \alpha \in \triangle\}$ is a $P-$locally finite, let $x \in X$. There is a neighborhood V of x with $(W)_V = \{W_1, \ldots\ldots.., W_n\}$ is finite. For each $i \leq n$ there exists $d_i \in D$, $W_i \subset G(d_i)$. If $C = \{\alpha \in \triangle : V \bigcap U_\alpha \neq \phi\}$, then $C \subset \bigcup_{i=1}^{n}$, so C is finite. Hence U is a $P-$locally finite. □

Definition 16. *A space $X = (X, \tau_1, \tau_2)$ is called countable P-f-expandable if and only if every $P-$locally finite collection $\{F_i\}_{i=1}^{\infty}$ of countable subsets of X has a $P-$locally finite collection $\{G_i\}_{i=1}^{\infty}$ of P-f-open subsets of X such that $F_i \subseteq G_i$ for each i .*

Theorem 6. *A space $X = (X, \tau_1, \tau_2)$ is countable P-f-expandable if and only if every countable $P-$open cover of X has a $P-$locally finite P-f-open refinement.*

Proof. Assume that $U = \{U_i\}_{i=1}^{\infty}$ be a countable $P-$open cover of X. Let $F_1 = U_1$ and $F_i = U_i \backslash \bigcup_{k=1}^{i-1} U_k$ for each $i = 1, 2, \ldots..$ then the collection $\{F_i : i = 1, 2, \ldots\}$ is a $P-$locally finite cover of X. Since X is countable P-f-expandable, there is a $P-$locally finite P-f-open collection $\{G_i : i = 1, 2, \ldots.\}$ such that $F_i \subseteq G_i$ for each i. Let $V_i = U_i \bigcap G_i$ for each i. Then the collection $\{V_i : i = 1, 2, \ldots..\}$ is a $P-$locally finite P-f-open refinement of U. Conversely, let $F = \{F_i\}_{i=1}^{\infty}$ be a $P-$locally finite closed collection of countable subsets of X. Let $U_i = X\setminus \bigcup_{j=i+1} F_j, i = 1,2,....$ Now U_i is open, U_i meet only finitely many elements of F, and $U_{i_{i=1}}^{\infty}$ is countable $P-$open cover of X. Then there is a $P-$locally finite P-f-open refinement $V = V_i i = 1^{\infty}$ of $U_i i = 1^{\infty}$. Let $G_i = st(F_i,V) = \bigcup\{v \in V : v \bigcap F_i \neq \phi,\ i = 1,2,...,\}$ clearly $F_i \subseteq G_i$ and G_i is P-f-open for each i. We claim that $\{G_i\}_{i=1}^{\infty}$ is a $P-$locally finite. Each $x \in X$ belongs to an open set O which meets only finitely many members of V. Thus $O \bigcap F_i \neq \phi$ if and only if $O \bigcap V \neq \phi$ such that $V \bigcap F_i \neq \phi$ for some i. Since V contained in some U_i. So it meet only finitely many F_i. Thus $\{G_i\}_{i=1}^{\infty}$ is a $P-$locally finite. □

Lemma 1. *If a space $X = (X, \tau_1, \tau_2)$ is P-f-expandable then every countable $P-$open cover of X has a $P-$locally finite P-f-open refinement*

Proof. Form Theorem 6. □

Theorem 7. *If every $P-$open cover of a space $X = (X, \tau_1, \tau_2)$ has $P-\delta$-locally finite open refinement, then X is P-f-par compact if and only if X is countable P-f-expandable.*

Proof. $\Rightarrow$) Clear since every P-f-par compact is f -expandable and so is countable P-f-expandable.

$\Leftarrow$) Let $U = \{U_\alpha : \alpha \in I\}$ be an $P-$open cover of X, then there exists an $P-$open refinement $W = \bigcup_{n=1}^{\infty} W_n$ of U such that for each n and $x \in X$ there is a neighborhood G_n of x such that intersection finitely many member of W_n. Since $\{\bigcup_{n=1}^{\infty} W_n : n = 1,2,...\}$ is a countable $P-$open cover of X and since X is countable P-f-expandable, by Theorem (4.1.6), there exists $V_n : n = 1,2,.....$ a P-f-open refinement of $\{\bigcup_{n=1}^{\infty} W_n\}$ such that $V_n \subset \bigcup_{n=1}^{\infty} W_n$ and for each $x \in X$ there is a neighborhood O of x such that intersection finitely many member V_n. Let $G = \{V_n \bigcap W : W \in W_n, n = 1,2,...\}$ hence G is P-f-open refinement of U. To show that G is a $P-$locally finite, let $x \in X$ there exists a neighborhood O of x such that $\{O \bigcap V_n \neq \phi\} = \{n_1, ..., n_k\}$. For each $i = 1,2,...,k$, there is a neighborhood H_i of x such that intersection many member of W_{ni}. Let $H = O \bigcap (\bigcap_{i=1}^{k} H_i)$, then H is a neighborhood of x.

Let $C = \{(n,W) : H \bigcap (V_n \bigcap W) \neq \phi, W \in W_n, n = 1,2,.....\}$.

Then $C \subset \bigcup_{i=1}^{k} [\{n_i\} \times W_{ni}]_{Hi}$ Therefore C is finite. □

Lemma 2. *If every $P-$open cover of a space $X = (X, \tau_1, \tau_2)$ has $P - \delta$ -locally finite open refinement, then the following are equivalent.*

(i) *X is P-f-par compact.*

(ii) *X is P-f-expandable.*

(iii) *X is countable P-f-expandable.*

Proof. Follows from Theorem 6 and Theorem 7 . □

Theorem 8. *The following condition are equivalent for a space $X = (X, \tau_1, \tau_2)$:*

(i) *X is P-f-expandable.*

(ii) *Every A-cover of X has a $P-$locally finite P-f-open refinement.*

(iii) *Every directed A-cover of X a $P-$locally finite P-f-open refinement.*

Proof. $(i) \Rightarrow (ii)$ Let U be an A-covering of X, then U has a $P-$locally finite refinement $V = \{V_\alpha : \alpha \in \triangle\}$. Since the family $\{\overline{V_\alpha} : \alpha \in \triangle\}$ is a $P-$locally finite of closed sets in X, hence by Theorem (4.1.4), there exists a $P-$locally finite P-f-open cover G of X such that every $G \in G$, $G \bigcap \overline{V_\alpha} \neq \phi, \alpha \in \triangle$ is finite. Let $W_\alpha = G \bigcap U_\alpha : \alpha \in \triangle$, then the collection $W_\alpha : \alpha \in \triangle$ is a $P-$locally finite P-f-open refinement of U.
$(ii) \Rightarrow (iii)$ Clear since every directed A-cover is A-cover.
$(iii) \Rightarrow (i)$ Let $F_\alpha : \alpha \in \triangle$ be a $P-$locally finite collection of closed sets of X, let Γ be the set of all finite subsets of $\triangle$. For each $\gamma \in \Gamma$, let $U_\gamma = X \setminus \bigcup\limits_{\alpha \notin \gamma} F_\alpha$, then $U = \{U_\gamma : \gamma \in \Gamma\}$ is directed $P-$open cover of X. Let $H_\gamma = \bigcap\limits_{\alpha \in \gamma} F_\alpha$ for each $\gamma \in \Gamma$, then it is easily show that$\{U_\gamma \bigcup H_\gamma : \gamma \in \Gamma\}$ is a $P-$locally finite refinement of U. Hence U is directed A-cover of X, by assumption, U has a $P-$locally finite P-f-open refinement G such that $G_\gamma \subset U_\gamma$ for each $\gamma \in \Gamma$.
Since $G_\gamma \bigcap F_\gamma \subset U_\gamma \bigcap F_\alpha \subset U_\gamma \bigcap \bigcup\limits_{\alpha \notin \gamma} F_\alpha = \phi$, then for each $\gamma \in \Gamma$, $\{\alpha \in \triangle : G_\gamma \bigcap F_\gamma \neq \phi\} \subset \gamma$, so it is finite. Hence X is P-f-expandable. □

Lemma 3. *A clopen subspace of a P-f-expandable space is P-f-expandable*

Proof. Let Y be a clopen subset in X and let $U = \{U_\lambda : \lambda \in \triangle\}$ be an A-open covering of Y. For each λ there exists V_λ open set in X such that $U_\lambda = Y \bigcap V_\lambda$. The an A-open covering $\{U_\lambda : \lambda \in$

$\triangle\}\bigcup\{X\backslash Y\}$ of X has a $P-$locally finite P-f-open refinement $W=\{W_\gamma:\gamma\in\Gamma\}$, then $\{Y\bigcap W_\gamma:\gamma\in\Gamma\}$ be a $P-$locally finite P-f-open refinement of U, hence Y is P-f-expandable. □

Theorem 9. *Let $\{U_\alpha:\alpha\in I\}$ be a $P-$locally finite clopen covering of $X=(X,\tau_1,\tau_2)$. Then X is P-f-expandable if and only if each U_α is*
P-f-expandable.

Proof. $\Rightarrow$) from Proposition 3
$\Leftarrow$) Let $G=\{G_\lambda:\lambda\in\triangle\}$ be an A-cover of X. Then for each $\alpha\in I$ $G_\lambda\bigcap U_\alpha:\alpha\in\triangle$ is an A-cover of G_α (since G is A-cover, hence G has a $P-$locally finite refinement V, then $V\bigcap U_\alpha$ is a $P-$locally finite refinement of $\{G_\lambda U_\alpha:\lambda\in\triangle\}$). Since U_α is P-f-expandable, there exists a $P-$locally finite P-f-open refinement W of $\{G_\lambda\bigcap U_\alpha:\lambda\in\triangle\}$ such that $W\subset G_\lambda\bigcap U_\alpha\subset G_\lambda, W\in W$. Since U_α is clopen subset in X, then W is a $P-$locally finite P-f-open refinement of $G=\{G_\lambda:\lambda\in\triangle\}$. □

Lemma 4. *Let $X=(X,\tau_1,\tau_2)$ is extremely disconnected and let $\{U_\alpha:\alpha\in\triangle\}$ be a $P-$locally finite open cover of X .Then X is P-f-expandable space if and only if $\{\overline{U_\alpha}:\alpha\in\triangle\}$ be a P-f-expandable .*

Theorem 10. *Let $\{G_\alpha:\alpha\in\triangle\}$ be any family of a space $X=(X,\tau_1,\tau_2)$ such that $X=\bigcup\limits_{\alpha\in\triangle}G_\alpha$ where G_α are pair wise disjoint and open, then X is P-f-expandable if each G_α is P-f-expandable.*

Proof. Let $\{U_\lambda:\lambda\in\triangle\}$ be an A-cover of X. Then for each α, $\{G_\alpha\bigcap U_\lambda:\lambda\in\triangle\}$ is an A-cover of G_α. Since G_α is P-f-expandable, there exists a $P-$locally finite (in G_α) family of P-f-open refinement $\{V_\gamma:\gamma\in\Gamma^\alpha\}$ of $\{G_\alpha\bigcap U_\lambda:\lambda\in\triangle\}$ which cover of G_α .By Proposition (4.1.2) ,there exists an P-f-open family $\{W_\gamma:\gamma\in\Gamma^\alpha\}$ in X such that $\{W_\gamma\bigcap G_\alpha:\gamma\in\Gamma^\alpha\}$ is $P-$locally finite refinement of $\{G_\alpha\bigcap U_\lambda:\lambda\in\triangle\}$ in G_α. Let $A^\alpha=\{W_\gamma\bigcap G_\alpha:\alpha\in\Gamma^\alpha\}$, hence A^α is a P-f-open in X. To show that $A=\{A^\alpha:\alpha\in\triangle\}$ is $P-$locally finite refinement of $\{U_\lambda:\lambda\in\triangle\}$. Let x and intersects finitely many sets of the family A^α. Since $A_x\subset G_\alpha$ and G_α are pair wise disjoint, therefore x A intersect only finitely many sets of A and hence A is a $P-$locally finite P-f-open refinement of $\{U_\lambda:\lambda\in\triangle\}$. Therefore X is P-f-expandable. □

4. CONCLUSIONS

This paper has studied some properties of the Feebly pairwise expandable spaces and study their properties and their relations with other a bitopological spaces. several examples are discussed and many will known theorems are generalized concerning Feebly pairwise expandable spaces. and we shall investigate subspace of Feebly pairwise expandable space also bitopological spaces which are related to Feebly pairwise expandability.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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