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INTUITIONISTIC (α, β)-FUZZY H_v-SUBMODULES

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Abstract. The notion of intuitionistic fuzzy sets was introduced by Atanassov as a generalization of the notion of fuzzy sets. Using the notion of "belongingness ($\in$)" and "quasi-coincidence (q)" of fuzzy points with fuzzy sets, we introduce the concept of an intuitionistic (α, β)-fuzzy H_v-submodules of an H_v-modules, where $\alpha \in \{\in, q\}$, $\beta \in \{\in, q, \in \vee q, \in \wedge q\}$ and, then we investigate the basic properties of these notions.

Keywords: Hyperstructure, H_v-Module, Fuzzy set, Intuitionistic fuzzy set, Intuitionistic (α, β)-fuzzy H_v-submodule.

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1. Introduction

The notion of a hypergroup introduced by Marty in 1934 [16]. Algebraic hyperstructures are a suitable generalization of classical algebraic structures. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a set. Since then, hundreds of papers and

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several books have been written on this topic, see [11, 12, 19]. Vougiouklis [19] introduced a new class of hyperstructures, the so-called H_v-structures. The H_v-structures are hyperstructures where equality is replaced by non-empty intersection.

The notion of a fuzzy subset introduced by Zadeh in 1965 [21] as a function from a nonempty set H to unit real interval $I = [0, 1]$.
After the introduction of fuzzy sets by Zadeh, there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Atanassov [2, 3] is one among them. An intuitionistic fuzzy set gives both a membership degree and a non-membership degree. The membership and non-membership values induce an indeterminacy index, which models the hesitancy of deciding the degree to which an object satisfies a particular property. Many concepts in fuzzy set theory were also extended to intuitionistic fuzzy set theory, such as intuitionistic fuzzy relations, intuitionistic L-fuzzy sets, intuitionistic fuzzy implications, intuitionistic fuzzy grade of hypergroups, intuitionistic fuzzy logics, and the degree of similarity between intuitionistic fuzzy sets, etc., [1, 9, 10]. In [4] Biswas applied the concept of intuitionistic fuzzy sets to the theory of groups and studied intuitionistic fuzzy subgroups of a group. Davvaz et al. [14] considered the intuitionistic fuzzy sets for H_v-modules.

The idea of quasi-coincidence of a fuzzy point with a fuzzy set, which is mentioned in [17], played a vital role to generate some different types of fuzzy subgroups. Bhakat and Das [6, 7] gave the concepts of (α, β)-fuzzy subgroups by using the notion of "belongingness $(\in)$" and "quasi-coincidence (q)" between a fuzzy point and a fuzzy subgroup, where α, β are any two of $\{\in, q, \in \vee q, \in \wedge q\}$ with $\alpha \neq \in \wedge q$, and introduced the concept of an $(\in, \in \vee q)$-fuzzy subgroup. In [8] $(\in, \in \vee q)$- fuzzy subrings and ideals defined. In [15] Jun and Song initiated the study of (α, β)-fuzzy interior ideals of a semigroup. In [5] Bhakat defined $(\in \vee q)$-level subsets of a fuzzy set. In [18] Shabir, Jun et al. studied characterizations of regular semigroups by (α, β)-fuzzy ideals. In [20] Yuan, Li et al. redefined (α, β)-intuitionistic fuzzy subgroups. Davvaz and Corsini initiated the study of (α, β)-Fuzzy H_v-Ideals of H_v-Rings in [13]. This paper continues this line of research.

The paper is organized as follows: in Section 2 some fundamental definitions on H_v-structures and fuzzy sets are explored, in Section 3 we define intuitionistic (α,β)-fuzzy with H_v-submodules and then establish some useful theorems.

2. Preliminaries

Let H be a nonempty set and let $\wp^*(H)$ be the set of all nonempty subsets of H. A *hyperoperation* on H is a map $\circ : H \times H \longrightarrow \wp^*(H)$ and the couple $(H,\circ)$ is called a *hypergroupoid* (or hyperstructure).

If A and B are nonempty subsets of H, then we denote

$$A \circ B = \bigcup_{a\in A,\, b\in B} a\circ b, \quad x\circ A = \{x\}\circ A \ \text{ and } \ A\circ x = A\circ\{x\}.$$

A hypergroupoid $(H,\circ)$ is called a *semihypergroup* if for all x, y, z of H, we have $(x\circ y)\circ z = x\circ(y\circ z)$, which means that

$$\bigcup_{u\in x\circ y} u\circ z = \bigcup_{v\in y\circ z} x\circ v.$$

We say that a semihypergroup $(H,\circ)$ is a *hypergroup* if for all $x\in H$, we have $x\circ H = H\circ x = H$.

A hyperstructure $(H,\circ)$ is called an H_v*-semigroup* if

$$((x\circ y)\circ z)\cap(x\circ(y\circ z)) \neq \emptyset,$$

for all $x, y, z \in H$.

Definition 2.1. [19] *An* H_v-ring *is a system* $(R,+,.)$ *with two hyperoperations satisfying the following axioms:*

(i) $(R,+)$ *is an* H_v*-group, i.e.,*

$$((x+y)+z)\cap(x+(y+z)) \neq \emptyset, \quad \textit{for all} \ \ x,y,z\in R,$$

$$x+R = R+x = R, \quad \textit{for all} \ \ x\in R;$$

(ii) $(R,.)$ *is an* H_v*-semigroup;*

(iii) "." *is weak distributive with respect to* "+", *i.e., for all* $x,y,z\in R$,

$(x.(y+z)) \cap (x.y + x.z) \neq \emptyset,$

$(x+y).z)) \cap (x.z + y.z)) \neq \emptyset.$

An H_v-group $(R,+)$ is called a *weak commutative H_v-group* if $(x+y)\cap(y+x) \neq \emptyset$ for all $x, y \in R$.

Definition 2.2. [19] *A nonempty set M is called an H_v-module over an H_v-ring R if $(M,+)$ is a weak commutative H_v-group and there exists a map*

$$.: R \times M \longrightarrow \wp^*(M), \quad (r,x) \longmapsto r.x$$

such that for all $a, b \in R$ and $x, y \in M$, we have

$(a.(x+y)) \cap (a.x + a.y) \neq \emptyset,$

$(a.(x+y)) \cap (a.x + a.y) \neq \emptyset,$

$(a.(b.x)) \cap ((ab).x) \neq \emptyset.$

We note that an H_v-module is a generalization of a module. For more definitions, results and applications on H_v-ring, we refer the reader to [19]. Note that by using fuzzy sets, we can consider the structure of H_v-module on any ordinary module.

Definition 2.3. [14] *An intuitionistic fuzzy set $A = (\mu_A, \ \lambda_A)$ in M is called an* intuitionistic fuzzy H_v-submodule *of M if*

(1) $\mu_A(x) \wedge \mu_A(y) \leq \bigwedge_{z \in x+y} \mu_A(z)$ *for all $x, y \in M$,*

(2) *for all $x, a \in M$, there exist $y, z \in M$ such that $x \in (a+y) \cap (z+a)$ and $\mu_A(x) \wedge \mu_A(a) \leq \mu_A(y) \wedge \mu_A(z)$,*

(3) $\mu_A(y) \leq \bigwedge_{z \in x.y} \mu_A(z)$ *for all $y \in M$ and $x \in R$,*

(4) $\bigvee_{z \in x+y} \lambda_A(z) \leq \lambda_A(x) \vee \lambda_A(y)$ *for all $x, y \in M$,*

(5) *for all* $x, a \in M$, *there exist* $y, z \in M$ *such that* $x \in (a+y) \cap (z+a)$ *and* $\lambda_A(y) \vee \lambda_A(z) < \lambda_A(x) \vee \lambda_A(a)$,

(6) $\bigvee_{z \in x.y} \lambda_A(z) \leq \lambda_A(y)$ *for all* $y \in M$ *and* $x \in R$.

The concept of a fuzzy set in a non-empty set was introduced by Zadeh [21] in 1965. Let H be a non-empty set. A mapping $\mu : H \longrightarrow [0;1]$ is called a *fuzzy set* in H. The *complement* of μ, denoted by μ^c, is the fuzzy set in H given by $\mu^c(x) = 1 - \mu(x)$ for all $x \in H$.

Definition 2.4. *An* intuitionistic fuzzy set A *in a non-empty set* X *is an object having the form* $A = \{(x, \mu_A(x), \lambda_A(x)) | x \in X\}$, *where the functions* $\mu_A : X \longrightarrow [0;1]$ *and* $\lambda_A : X \longrightarrow [0;1]$ *denote the degree of membership (namely* $\mu_A(x)$*) and the degree of non-membership (namely* $\lambda_A(x)$*) of each element* $x \in X$ *with respect to the set* A, *respectively, and* $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$ *for all* $x \in X$. *For the sake of simplicity, we shall use the symbol* $A = (\mu_A, \lambda_A)$ *for the intuitionistic fuzzy set* $A = \{(x, \mu_A(x), \lambda_A(x)) | x \in X\}$.

Definition 2.5. [2] *Let* $A = (\mu_A, \lambda_A)$ *and* $B = (\mu_B, \lambda_B)$ *be intuitionistic fuzzy sets in* X. *Then*

(1) $A \subseteq B$ *iff* $\mu_A(x) \leq \mu_B(x)$ *and* $\lambda_A(x) \geq \lambda_B(x)$ *for all* $x \in X$,

(2) $A^c = \{(x, \lambda_A(x), \mu_A(x)) | x \in X\}$,

(3) $A \cap B = \{(x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\lambda_A(x), \lambda_B(x)\}) | x \in X\}$,

(4) $A \cup B = \{(x, \max\{\mu_A(x), \mu_B(x)\}, \min\{\lambda_A(x), \lambda_B(x)\}) | x \in X\}$,

(5) $\Diamond A = \{(x, \lambda_A^c(x), \lambda_A(x)) | x \in X\}$.

3. Intuitionistic (α, β)-Fuzzy H_v-Submodules

Definition 3.1. [6] *Let* μ *be a fuzzy subset of* R. *If there exist a* $t \in (0,1]$ *and an* $x \in R$ *such that*

$$\mu(y) = \begin{cases} t & \text{if } y = x \\ 0 & \text{otherwise.} \end{cases}$$

Then μ *is called a fuzzy point with support* x *and value* t *and is denoted by* x_t.

Definition 3.2. [6] *Let μ be a fuzzy subset of R and x_t be a fuzzy point.*

(1) *If $\mu(x) \geq t$, then we say x_t belongs to μ, and write $x_t \in \mu$.*

(2) *If $\mu(x) + t > 1$, then we say x_t is quasi-coincident with μ, and write $x_t q\mu$.*

(3) $x_t \in \vee q\mu \Longleftrightarrow x_t \in \mu$ *or* $x_t q\mu$.

(4) $x_t \in \wedge q\mu \Longleftrightarrow x_t \in \mu$ *and* $x_t q\mu$.

In what follows, unless otherwise specified, α and β will denote any one of $\in, q, \in \vee q$ or $\in \wedge q$ with $\alpha \neq \in \wedge q$, which was introduced by Bhakat and Das [7].

Definition 3.3. [13] *Let R be an H_v-ring. A fuzzy subset A of R is said to be an (α, β)-fuzzy left (right) H_v-ideals of R if for all $t, r \in (0, 1]$,*

(1) *$x_t\alpha A$, $y_r\alpha A$ implies $z_{t\wedge r}\beta A$ for all $z \in x + y$,*

(2) *$x_t\alpha A$, $a_r\alpha A$ implies $y_{t\wedge r}\beta A$ for some $y \in R$ with $x \in a + y$,*

(3) *$x_t\alpha A$, $a_r\alpha A$ implies $z_{t\wedge r}\beta A$ for some $z \in R$ with $x \in z + a$,*

(4) *$y_t\alpha A$ and $x \in R$ imply $z_t\beta A$ for all $z \in x.y$*

($x_t\alpha A$ and $y \in R$ imply $z_t\beta A$ for all $z \in x.y$).

In what follows, let M denote an H_v-module over an H_v-Ring R unless other wise specified. We start by defining the notion of intuitionistic (α, β)-fuzzy H_v-submodules.

Definition 3.4. *An intuitionistic fuzzy set $A = (\mu_A,\ \lambda_A)$ in M is said to be an intuitionistic (α, β)-fuzzy left (right) H_v-submodule of M if for all $t, r \in (0, 1]$,*

(1) *For all $x, y \in M$, $x_t, y_r\alpha\mu_A$ implies $z_{t\wedge r}\beta\mu_A$ for all $z \in x + y$,*

(2) *For all $x, a \in M$, $x_t, a_r\alpha\mu_A$ implies $(y \wedge z)_{t\wedge r}\beta\mu_A$ for some $y, z \in M$ with $x \in (a + y) \cap (z + a)$,*

(3) *For all $y \in M$, $x \in R$, $y_t\alpha\mu_A$ implies $z_t\beta\mu_A$ for all $z \in x.y$*

(For all $y \in M$, $x \in R$, $y_t\alpha\mu_A$ implies $z_t\beta\mu_A$ for all $z \in y.x$),

(4) *For all $x, y \in M$, $x_t, y_r\overline{\alpha}\lambda_A$ implies $z_{t\wedge r}\overline{\beta}\lambda_A$ for all $z \in x + y$,*

(5) *For all $x, a \in M$, $x_t, a_r\overline{\alpha}\lambda_A$ implies $(y \wedge z)_{t\wedge r}\overline{\beta}\lambda_A$ for some $y, z \in M$ with $x \in (a + y) \cap (z + a)$,*

(6) *For all $y \in M$, $x \in R$, $y_t\overline{\alpha}\lambda_A$ implies $z_t\overline{\beta}\lambda_A$ for all $z \in x.y$*

(For all $y \in M$, $x \in R$, $y_t\overline{\alpha}\lambda_A$ implies $z_t\overline{\beta}\lambda_A$ for all $z \in y.x$),

where $(y \wedge z)_{t\wedge r}\alpha\mu_{_A}$ $((y \wedge z)_{t\wedge r}\overline{\beta}\lambda_{_A})$*, i.e.,* $y_{t\wedge r}\alpha\mu_{_A}$ *and* $z_{t\wedge r}\alpha\mu_{_A}$ $(y_{t\wedge r}\overline{\beta}\lambda_{_A}$ *and* $z_{t\wedge r}\overline{\beta}\lambda_{_A})$*. And, the symbol* $\overline{\beta}$ *means* β *does not hold for all* $\beta \in \{\in, q, \in\vee q, \in\wedge q\}$*.*

Let R be an H_v-ring. Then a fuzzy subset λ_A of M is said to be an *anti* (α,β)*-fuzzy left (right)* H_v*-submodule* of M if it satisfies the conditions (4)-(6) of Definition 3.4 for all $t, r \in (0, 1]$.

In this paper we present all the proofs for left H_v-submodules. Similar results hold for right H_v-submodules.

Example 3.5. *Let* $M = \{a, b, c, d\}$ *and* $R = \{a, b, c\}$*. Let operation "." and hyperoperation "+" and defied by the following tables*

.	a	b	c	d
a	a	a	a	a
b	a	b	b	b
c	a	c	c	c
d	a	d	d	d

and

+	a	b	c	d
a	a	b	c	d
b	b	{a,b}	d	c
c	c	d	{a,c}	b
d	d	c	b	{a,d}

Let μ *and* λ *be two fuzzy subset of* M *such that* $\mu(a) = 0.6$*,* $\mu(b) = \mu(c) = \mu(d) = 0.8$ *and* $\lambda(a) = \lambda(b) = \lambda(c) = \lambda(d) = 0.3$*. Then* $(\mu,\ \lambda)$ *is an intuitionistic* $(\in, \in\vee q)$*-fuzzy* H_v*-submodule of* M*.*

Proof. μ is an $(\in, \in\vee q)$-fuzzy H_v-ideal of M (see [13]). So, it is easy to see that λ satisfies the conditions (4)-(6) of Definition 3.4.

Lemma 3.6. *Let* $A = (\mu_{_A},\ \lambda_{_A})$ *be an intuitionistic fuzzy set in* M*. Then for all* $x \in M$ *and* $r \in (0, 1]$*, we have*

(1) $x_t q\mu_{_A} \Longleftrightarrow x_t \overline{\in} \mu^c_{_A}$*;*

(2) $x_t \in\vee q\mu_{_A} \Longleftrightarrow x_t \overline{\in\wedge q}\mu^c_{_A}$*.*

Proof. (1) Let $x \in M$ and $r \in (0, 1]$. Then, we have

$$\begin{aligned} x_t q\mu_{_A} &\Longleftrightarrow \mu_{_A}(x) + t > 1 \\ &\Longleftrightarrow 1 - \mu_{_A}(x) < t \\ &\Longleftrightarrow \mu^c_{_A}(x) < t \\ &\Longleftrightarrow x_t \overline{\in} \mu^c_{_A}. \end{aligned}$$

(2) Let $x \in M$ and $r \in (0,1]$. Then, we have

$$\begin{aligned} x_t \in \vee q \mu_A &\Longleftrightarrow x_t \in \mu_A \ \text{ or } \ x_t q \mu_A \\ &\Longleftrightarrow \mu_A(x) \geq t \ \text{ or } \ \mu_A(x) + t > 1 \\ &\Longleftrightarrow 1 - \mu_A^c(x) \geq t \ \text{ or } \ 1 - \mu_A^c(x) + t > 1 \\ &\Longleftrightarrow x_t \overline{q} \mu_A^c \ \text{ or } \ x_t \overline{\in} \mu_A^c \\ &\Longleftrightarrow x_t \overline{\in \wedge q} \mu_A^c. \end{aligned}$$

If $A = (\mu_A,\ \lambda_A)$ is an intuitionistic (α, β)-fuzzy H_v-submodule of M. Since $\alpha \neq \in \wedge q$, by Lemma 3.6(2) and the Definition 3.4, we have $\alpha \neq \in \vee q$.

Let $\beta = \in,\ q,\ \in \wedge q,\ \in \vee q$. We write $\beta' = q,\ \in,\ \in \vee q,\ \in \wedge q$, respectively. It is obvious that $\beta'' = \beta$.

Theorem 3.7. *If $A = (\mu_A,\ \lambda_A)$ is an intuitionistic $(\in, \in)$-fuzzy H_v-submodule of M, then $A = (\mu_A,\ \lambda_A)$ is an intuitionistic fuzzy H_v-submodule of M.*

Proof. *Condition(1).* Let $x, y \in M$ and $\mu_A(x) \wedge \mu_A(y) = t$. Then $x_t, y_t \in \mu_A$. By condition (1) of Definition 3.4, we have

$$z_t \in \mu_A \text{ for all } z \in x + y,$$

and so $\mu_A(z) \geq t$ for all $z \in x + y$. Consequently

$$\mu_A(x) \wedge \mu_A(y) = t \leq \bigwedge_{z \in x+y} \mu_A(z)$$

for all $x, y \in M$.

Condition(2). Now, let $x, a \in M$ and $\mu_A(x) \wedge \mu_A(a) = t$. Then $x_t, a_t \in \mu_A$. It follows from condition (2) of Definition 3.4 that

$$(y \wedge z)_t \in \mu_A, \text{ for some } y, z \in M \text{ with } x \in (a + y) \cap (z + a).$$

Thus

$$y_t, z_t \in \mu_A \text{ for some } y, z \in M \text{ with } x \in (a + y) \cap (z + a).$$

So, for all $x, a \in M$, there exist $y, z \in M$ such that $x \in (a + y) \cap (z + a)$ and

$$\mu_A(x) \wedge \mu_A(a) = t \leq \mu_A(y) \wedge \mu_A(z).$$

Condition(3). Let $y \in M,\ x \in R$ and $\mu_A(y) = t$. Thus $y_t \in \mu_A$. From condition (3) of Definition 3.4, we have

$$z_t \in \mu_A \text{ for all } z \in x.y,$$

and so

$$\mu_A(z) \geq t \text{ for all } z \in x.y.$$

This proves that

$$\mu_A(y) = t \leq \bigwedge_{z \in x.y} \mu_A(z)$$

for all $y \in M$ and $x \in R$.

Condition(4). Let $x, y \in M$ and $\lambda_A(x) \vee \lambda_A(y) = s$. If $s = 1$, then $\lambda_A(z) \leq 1 = s$ for all $z \in x + y$. It is easy to see that

$$\bigvee_{z \in x+y} \lambda_A(z) \leq \lambda_A(x) \vee \lambda_A(y) \text{ for all } x, y \in M.$$

If $s < 1$, there exists a $t \in (0, 1]$ such that

$$\lambda_A(x) \vee \lambda_A(y) = s < t.$$

Then $x_t, y_t \overline{\in} \lambda_A$. By condition (4) of Definition 3.4, we have

$$z_t \overline{\in} \lambda_A, \text{ for all } z \in x + y,$$

and so $\lambda_A(z) < t$. Consequently

$$\bigvee_{z \in x+y} \lambda_A(z) \leq \lambda_A(x) \vee \lambda_A(y)$$

for all $x, y \in M$.

Condition(5). Let $x, a \in M$ and $\lambda_A(x) \vee \lambda_A(a) = s$. If $s < 1$, there exists a $t \in (0, 1]$ such that $\lambda_A(x) \vee \lambda_A(a) = s < t$. Then $x_t, a_t \overline{\in} \lambda_A$. By condition (5) of Definition 3.4, we have

$$(y \wedge z)_t \overline{\in} \lambda_A \text{ for some } y, z \in M \text{ with } x \in (a + y) \cap (z + a).$$

Hence,

$$\lambda_A(y) < t \text{ and } \lambda_A(z) < t.$$

Thus

$$\lambda_A(y) \vee \lambda_A(z) < t.$$

This implies that, for all $x, a \in M$, there exist $y, z \in M$ such that $x \in (a + y) \cap (z + a)$ and

$$\lambda_A(y) \vee \lambda_A(z) \leq \lambda_A(x) \vee \lambda_A(a).$$

If $s = 1$, the proof is obvious.

Condition(6). Let $y \in M,\ x \in R$ and $\lambda_A(y) = s$. If $s < 1$, there exists a $t \in (0, 1]$ such that $\lambda_A(y) = s < t$. Thus $y_t \overline{\in} \lambda_A$. From condition (6) of Definition 3.4, we have

$$z_t \overline{\in} \lambda_A \text{ for all } z \in x.y,$$

and so

$$\lambda_A(z) < t \text{ for all } z \in x.y.$$

Then $\lambda_A(z) \leq \lambda_A(y)$. This proves that

$$\bigvee_{z \in x.y} \lambda_A(z) \leq \lambda_A(y),$$

for all $y \in M$ and $x \in R$. If $s = 1$, the proof is obvious.

Theorem 3.8. *If $A = (\mu_A,\ \lambda_A)$ is an intuitionistic $(\in, \in \vee q)$ and $(\in, \in \wedge q)$-fuzzy H_v-submodule of M, then $A = (\mu_A,\ \lambda_A)$ is an intuitionistic fuzzy H_v-submodule of M.*

Proof. The proof is similar to the proof of Theorem 3.7.

Theorem 3.9. *$\Box A = (\mu_A,\ \mu_A^c)$ is an intuitionistic (α, β)-fuzzy H_v-submodule of M if and only if $\Box A = (\mu_A,\ \mu_A^c)$ is an intuitionistic (α', β')-fuzzy H_v-submodule of M, where $\alpha \in \{\in, q\}$, $\beta \in \{\in, q, \in \vee q, \in \wedge q\}$.*

Proof. $(\Longrightarrow)$ We only prove the case of $(\alpha, \beta) = (\in, \in \vee q)$. The others are analogous. Let $\Box A = (\mu_A,\ \mu_A^c)$ is an intuitionistic $(\in, \in \vee q)$-fuzzy H_v-submodule of M.

Condition(1). Let $x, y \in M$, $t, r \in (0, 1]$ be such that $x_t, y_r q \mu_A$. It follows from Lemma 3.6 that $x_t, y_r \overline{\in} \mu_A^c$. Since μ_A^c is an anti $(\in, \in \vee q)$-fuzzy H_v-submodule of M. Thus, by condition (4) of Definition 3.4, we have

$$z_{t \wedge r} \overline{\in \vee q} \mu_A^c \text{ for all } z \in x + y.$$

By Lemma 3.6, this is equivalence with

$$z_{t\wedge r} \in \wedge q\mu_A \text{ for all } z \in x + y.$$

Thus condition (1) of Definition 3.4 is valid.

Condition(2). Suppose that $x, a \in M$ and $t, r \in (0, 1]$ be such that $x_t, a_r q\mu_A$. By Lemma 3.6, we have $x_t, a_r q\mu_A$ if and only if $x_t, a_r \overline{\in} \mu_A^c$. By hypotheses, μ_A^c is an anti $(\in, \in \vee q)$-fuzzy H_v-submodule of M. Thus, from condition (5) of Definition 3.4, we have

$$(y \wedge z)_{t\wedge r} \overline{\in \vee q} \mu_A^c,$$

for some $y, z \in M$ with $x \in (a + y) \cap (z + a)$. This is equivalence with

$$y_{t\wedge r} \overline{\in \vee q} \mu_A^c \text{ and } z_{t\wedge r} \overline{\in \vee q} \mu_A^c,$$

for some $y, z \in M$ with $x \in (a + y) \cap (z + a)$. By Lemma 3.6, it is easy to see that

$$y_{t\wedge r} \in \wedge q\mu_A \text{ and } z_{t\wedge r} \in \wedge q\mu_A,$$

for some $y, z \in M$ with $x \in (a + y) \cap (z + a)$ if and only if

$$(y \wedge z)_{t\wedge r} \in \wedge q\mu_A,$$

for some $y, z \in M$ with $x \in (a+y) \cap (z+a)$. Thus condition (2) of Definition 3.4 is valid.

Condition(3). Let $y \in M, x \in R$ and $t \in (0, 1]$ be such that $y_t q\mu_A$. It follows from Lemma 3.6 that $y_t \overline{\in} \mu_A^c$. Since $\Box A = (\mu_A,\ \mu_A^c)$ is an intuitionistic $(\in, \in \vee q)$-fuzzy H_v-submodule of M. From condition (6) of Definition 3.4, we have

$$z_t \overline{\in \vee q} \mu_A^c \text{ for all } z \in x.y.$$

It is equivalence with

$$z_t \in \wedge q\mu_A \text{ for all } z \in x.y.$$

Which verify conditions (3) of Definition 3.4.

Condition(4). Suppose that $x, y \in M$ and $t, r \in (0, 1]$ be such that $x_t, y_r \overline{q} \mu_A^c$. It follows from Lemma 3.6 that $x_t, y_r \overline{q} \mu_A^c$ if and only if $x_t, y_r \in \mu_A$. Since $\Box A = (\mu_A,\ \mu_A^c)$ is an intuitionistic $(\in, \in \vee q)$-fuzzy H_v-submodule of M. By condition (1) of Definition 3.4, we have

$$z_{t\wedge r} \in \vee q\mu_{_A} \text{ for all } z \in x+y.$$

This is equivalence with

$$z_{t\wedge r}\overline{\in \wedge q}\mu^c_{_A} \text{ for all } z \in x+y.$$

Thus condition (4) of Definition 3.4 is valid.

Condition(5). Suppose that $x, a \in M$ and $t, r \in (0,1]$ be such that $x_t, a_r\overline{q}\mu^c_{_A}$. This is equivalence with $x_t, a_r\in\mu_{_A}$. By hypotheses, $\mu_{_A}$ is an $(\in, \in \vee q)$-fuzzy H_v-submodule of M. From condition (2) of Definition 3.4, we have

$$(y \wedge z)_{t\wedge r}\in \vee q\mu_{_A},$$

for some $y, z \in M$ with $x \in (a+y) \cap (z+a)$, and so

$$y_{t\wedge r}\in \vee q\mu_{_A} \text{ and } z_{t\wedge r}\in \vee q\mu_{_A},$$

for some $y, z \in M$ with $x \in (a+y) \cap (z+a)$. It follows from Lemma 3.6 that

$$y_{t\wedge r}\overline{\in \wedge q}\mu^c_{_A} \text{ and } z_{t\wedge r}\overline{\in \wedge q}\mu^c_{_A},$$

for some $y, z \in M$ with $x \in (a+y) \cap (z+a)$ if and only if

$$(y \wedge z)_{t\wedge r}\overline{\in \wedge q}\mu^c_{_A},$$

for some $y, z \in M$ with $x \in (a+y) \cap (z+a)$. Thus condition (5) of Definition 3.4 is valid.

Condition(6). Let $y \in M, x \in R$ and $t \in (0,1]$ be such that $y_t\overline{q}\mu^c_{_A}$. Then, we have $y_t\in\mu_{_A}$. Since $\square A = (\mu_{_A},\ \mu^c_{_A})$ is an intuitionistic $(\in, \in \vee q)$-fuzzy H_v-submodule of M, by condition (3) of Definition 3.4, we have

$$z_t\in \vee q\mu_{_A} \text{ for all } z \in x.y.$$

It is equivalence with

$$z_t\overline{\in \wedge q}\mu^c_{_A} \text{ for all } z \in x.y.$$

Which verify conditions (6) of Definition 3.4.

$(\Longleftarrow)$ The proof is similar to the proof of above.

Theorem 3.10. *$\Diamond A = (\lambda^c_{_A},\ \lambda_{_A})$ is an intuitionistic (α,β)-fuzzy H_v-submodule of M if and only if $\Diamond A = (\lambda^c_{_A},\ \lambda_{_A})$ is an intuitionistic (α',β')-fuzzy H_v-submodule of M, where $\alpha \in \{\in, q\}$, $\beta \in \{\in, q, \in\vee q, \in\wedge q\}$.*

Proof. The proof is similar to the proof of Theorem 3.9.

Theorem 3.11. *$A = (\mu_{_A},\ \lambda_{_A})$ is an intuitionistic (α,β)-fuzzy H_v-submodule of M if and only if $\mu_{_A}$ is an (α,β)-fuzzy H_v-submodule of M and $\lambda^c_{_A}$ is an (α',β')-fuzzy H_v-submodule of M, where $\alpha \in \{\in, q\}$, $\beta \in \{\in, q, \in\vee q, \in\wedge q\}$.*

Proof. We only prove the case of $(\alpha,\beta) = (\in, \in\vee q)$. The others are analogous. It is sufficient to show that, $\lambda^c_{_A}$ is an $(q, \in\wedge q)$-fuzzy H_v-submodule of M if and only if $\lambda_{_A}$ is an anti $(\in, \in\vee q)$-fuzzy H_v-submodule of M. This is true, because

$$x_t q \lambda_{_A} \Longleftrightarrow x_t \overline{\in} \lambda^c_{_A},$$

and

$$x_t \in\wedge q \lambda_{_A} \Longleftrightarrow x_t \overline{\in\vee q} \lambda^c_{_A},$$

for all $x \in M$ and $t \in (0,1]$.

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