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SUZUKI-TYPE $\mathscr{Z}$-CONTRACTION PERFORMANCE

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Abstract. We explain the concept of Suzuki type $\alpha\mathscr{T}$-admissible $\mathscr{Z}$-contraction with respect to ζ in the setting of complete metric space in this paper. We investigated the existence and uniqueness of such mappings' common fixed points. We used an example to exemplify the effectiveness. Several existing results in the corresponding literature have been covered by our main conclusions.

Keywords: Suzuki type; $\alpha\mathscr{T}$-admissible; $\mathscr{Z}$-contraction; simulation function.

2010 AMS Subject Classification: 47H09, 47H10.

1. INTRODUCTION

Khojasteh et al. [4] established the notion of $\mathscr{Z}$-contraction in 2015, which generalizes the Banach contraction. The notion of $\mathscr{Z}$-contraction is as follows.

Definition 1.1 ([4]). *Let $\zeta : [0,\infty)\times[0,\infty)\to\mathbb{R}$ be a mapping, then ζ is called a simulation function if it satisfies the following conditions:*

(i) $\zeta(0,0)=0$;

(ii) $\zeta(\sigma,\rho)<\rho-\sigma$ *for all* $\sigma,\rho>0$;

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(iii) if $\{\sigma_n\}, \{\rho_n\}$ *are sequences in* $(0,\infty)$ *such that* $\lim_{n\to\infty}\sigma_n = \lim_{n\to\infty}\rho_n > 0$ *then*

$$\limsup_{n\to\infty} \zeta(\sigma_n, \rho_n) < 0.$$

We denote the set of all simulation functions by $\mathscr{Z}$.

The following functions $\zeta : [0,\infty)\times[0,\infty) \to \mathbb{R}$ belong to $\mathscr{Z}$.

Definition 1.2 ([4]). *Let* $(\mathscr{X}, d)$ *be a metric space,* $\mathscr{S} : \mathscr{X} \to \mathscr{X}$ *a mapping and* $\zeta \in \mathscr{Z}$. *Then* $\mathscr{S}$ *is called a* $\mathscr{Z}$*-contraction with respect to* ζ*, if the following condition is satisfied*

$$\zeta(d(\mathscr{S}x, \mathscr{S}y), d(x,y)) \geq 0 \ \text{for all} \ x, y \in \mathscr{X}.$$

Theorem 1.1 ([8]). *Let* $(\mathscr{X}, d)$ *be a compact metric space and* $\mathscr{S} : \mathscr{X} \to \mathscr{X}$ *be a mapping. Assume that*

$$\frac{1}{2}d(x, \mathscr{S}x) < d(x,y) \Rightarrow d(\mathscr{S}x, \mathscr{S}y) < d(x,y) \ \text{for all distinct} \ x, y \in \mathscr{X}.$$

Then $\mathscr{S}$ *has a unique fixed point in* $\mathscr{X}$.

In 2017, Kumam et al. [6] introduce the motion Suzuki type $\mathscr{Z}$-contraction as follows.

Definition 1.3 ([6]). *Let* $(\mathscr{X}, d)$ *be a metric space,* $\mathscr{S} : \mathscr{X} \to \mathscr{X}$ *a mapping and* $\zeta \in \mathscr{Z}$. *Then* $\mathscr{S}$ *is called a Suzuki type* $\mathscr{Z}$*-contraction with respect to* ζ*, if the following condition is satisfied*

$$\frac{1}{2}d(x, \mathscr{S}x) < d(x,y) \Rightarrow \zeta(d(\mathscr{S}x, \mathscr{S}y), d(x,y)) \geq 0$$

for all distinct $x, y \in \mathscr{X}$.

Theorem 1.2 ([6]). *Let* $(\mathscr{X}, d)$ *be a metric space and* $\mathscr{S} : \mathscr{X} \to \mathscr{X}$ *be a Suzuki type* $\mathscr{Z}$*-contraction with respect to* $\zeta \in \mathscr{Z}$. *Then* $\mathscr{S}$ *has at most one fixed point.*

Definition 1.4 ([12]). *Let* $(\mathscr{X}, d)$ *be a metric space and* $\mathscr{S} : \mathscr{X} \to \mathscr{X}$ *be a mapping and* $\zeta \in \mathscr{Z}$. *Then* $\mathscr{S}$ *is called generalized Suzuki type* $\mathscr{Z}$*-contraction with respect to* ζ *if the following condition is satisfied*

$$\frac{1}{2}d(x, \mathscr{S}x) < d(x,y) \Rightarrow \zeta(d(\mathscr{S}x, \mathscr{S}y), \mathscr{M}(x,y)) \geq 0 \ \text{for all distinct} \ x, y \in \mathscr{X}, \tag{1.1}$$

where

$$\mathscr{M}(x,y) = \max\left\{ d(x,y), d(x,\mathscr{S}x), d(y,\mathscr{S}y), \frac{d(x,\mathscr{S}y)+d(y,\mathscr{S}x)}{2} \right\}.$$

Theorem 1.3 ([12]). *Let $(\mathscr{X},d)$ be a complete metric space, $\mathscr{S}$ is a generalized Suzuki type $\mathscr{Z}$-contraction with respect to ζ. Then $\mathscr{S}$ has fixed point.*

The researcher can see more knowledge in [13, 14, 15, 16].

Theorem 1.4 ([17]). *Let $(\mathscr{X},d)$ be a complete metric space and $\mathscr{S}:\mathscr{X}\to\mathscr{X}$ a continuous mapping satisfying $\alpha(x,y)d(\mathscr{S}x,\mathscr{S}y) \le \psi d(x,y)$ for all $x,y\in\mathscr{X}$ where $\psi:[0,\infty)\to[0,\infty)$ is non-decreasing function such that $\sum_{n=1}^{\infty}\psi^n(\sigma)<\infty$ for all $\sigma>0$. Assume that the following two conditions hold:*

(i) *there exists $x_0\in\mathscr{X}$ such that $\alpha(x_0,\mathscr{S}x_0)\ge 1$;*

(ii) *$\mathscr{S}$ is α-admissible, i.e.,*

$$\alpha(x,y)\ge 1 \Rightarrow \alpha(\mathscr{S}x,\mathscr{S}y)\ge 1 \textit{ for all } x,y\in\mathscr{X}.$$

Then $\mathscr{S}$ has a fixed point.

Karapinar [18] introduce the motion $\alpha\mathscr{Z}$-contraction as follows.

Definition 1.5 ([18]). *A self-mapping $\mathscr{S}$ on a metric space $(\mathscr{X},d)$ is said to be $\alpha\mathscr{Z}$-contraction with respect to ζ if the following condition is satisfied:*

$$\zeta(\alpha(x,y)d(\mathscr{S}x,\mathscr{S}y),d(x,y))\ge 0, \textit{ for all } x,y\in\mathscr{X}. \tag{1.2}$$

Definition 1.6 ([19]). *A self-mapping $\mathscr{S}$ on $\mathscr{X}$ is said to be triangular α-orbital admissible if for all $x,y\in\mathscr{X}$,*

(i) $\alpha(x,\mathscr{S}x)\ge 1 \Rightarrow \alpha(\mathscr{S}x,\mathscr{S}^2x)\ge 1$;

(ii) $\alpha(x,y)\ge 1$ *and* $\alpha(y,\mathscr{S}y)\ge 1 \Rightarrow \alpha(x,\mathscr{S}y)\ge 1$.

Theorem 1.5 ([18]). *Let $(\mathscr{X},d)$ be a complete metric space and $\mathscr{S}:\mathscr{X}\to\mathscr{X}$ an $\alpha\mathscr{Z}$-contraction with respect to ζ. Suppose that*

(i) *$\mathscr{S}$ is triangular α-orbital admissible;*

(ii) *there exists* $x_0 \in \mathscr{X}$ *such that* $\alpha(x_0, \mathscr{S}x_0) \geq 1$;

(iii) *either* $\mathscr{S}$ *is continuous or (if* $\{x_n\}$ *is a sequence in* $\mathscr{X}$ *such that* $\alpha(x_n, x_{n+1}) \geq 1$ *for all* n *and* $x_n \to x \in \mathscr{X}$ *as* $n \to \infty$*, then there exists a subsequence* $\{x_{n_k}\}$ *of* $\{x_n\}$ *such that* $\alpha(x_{n_k}, x) \geq 1$ *for all* k*).*

Then $\mathscr{S}$ *has a fixed point. Moreover, this fixed point is unique if* $\alpha(x,y) \geq 1$ *for all fixed points* x *and* y *of* $\mathscr{S}$.

Motivated by the above results, we establish common fixed point results for Suzuki type $\alpha\mathscr{T}$-admissible $\mathscr{Z}$-contraction.

2. Preliminaries

Definition 2.1 ([20]). *Let* $(\mathscr{S}, \mathscr{T})$ *be a pair of self-mappings on a set* $\mathscr{X}$.

(i) *An element* $x \in \mathscr{X}$ *is said to be a coincidence point of the pair if* $\mathscr{T}x = \mathscr{S}x = \bar{x}$ *for some* $\bar{x} \in \mathscr{X}$*. Here,* $\bar{x}$ *is often termed as the point of coincidence of the pair. Moreover,* $x \in \mathscr{X}$ *is said to be a common fixed point if* $x = \bar{x}$.

(ii) *The pair is said to be weakly compatible if* $\mathscr{S}$ *and* $\mathscr{T}$ *commute at their coincidence points.*

Definition 2.2 ([21]). *A subset* $\mathscr{C}$ *of a metric space* $(\mathscr{X}, d)$ *is said to be precomplete if every Cauchy sequence* $\{x_n\}$ *in* C *converges to a point of* $\mathscr{X}$.

Definition 2.3 ([22]). *A mapping* $\mathscr{S}$ *on a metric space* $(\mathscr{X}, d)$ *is said to be* $\mathscr{T}$*-continuous at* x *if for any sequence* $\{x_n\} \subset \mathscr{X}$,

$$\mathscr{T}x_n \to \mathscr{T}x \Rightarrow \mathscr{S}x_n \to \mathscr{S}x.$$

Moreover, $\mathscr{S}$ *is called* $\mathscr{T}$*-continuous if it is* $\mathscr{T}$*-continuous at every point of* $\mathscr{X}$.

Lemma 2.1 ([23]). *If a pair* $(\mathscr{S}, \mathscr{T})$ *of self-mappings on a set* $\mathscr{X}$ *is a weakly compatible, then every point of coincidence of the pair remains a coincidence point.*

Definition 2.4 ([24]). *A self-mapping* $\mathscr{S}$ *on* $\mathscr{X}$ *is said to be triangular* $\alpha\mathscr{T}$*-admissible if for all* x, y *and* $z \in \mathscr{X}$,

(i) $\alpha(\mathscr{T}x,\mathscr{T}y)\geq 1 \Rightarrow \alpha(\mathscr{S}x,\mathscr{S}y)\geq 1$;

(ii) $\alpha(\mathscr{T}x,\mathscr{T}z)\geq 1$ *and* $\alpha(\mathscr{T}z,\mathscr{T}y)\geq 1 \Rightarrow \alpha(\mathscr{T}x,\mathscr{T}y)\geq 1$.

Definition 2.5 ([24])**.** *A set $\mathscr{X}$ is said to be $\alpha\mathscr{T}$-directed set if for every $x,y\in\mathscr{X}$ there exists $z\in\mathscr{X}$ such that $\alpha(x,\mathscr{T}z)\geq 1$ and $\alpha(y,\mathscr{T}z)\geq 1$.*

Definition 2.6 ([24])**.** *A metric space $(\mathscr{X},d)$ is said to be $\alpha\mathscr{T}$-regular if for every sequence $\{\mathscr{T}x_n\}$ in $\mathscr{X}$ such that $\alpha(\mathscr{T}x_n,\mathscr{T}_{n+1})\geq 1$ for all n and $\{\mathscr{T}x_n\}$ converges to some $\mathscr{T}x\in\mathscr{T}(\mathscr{X})$ then there exists a subsequence $\{\mathscr{T}x_{n_k}\}$ (of $\mathscr{T}x_n$) such that $\alpha(\mathscr{T}x_{n_k},\mathscr{T}x)\geq 1$ for all k.*

3. Main Results

Definition 3.1. *Let $(\mathscr{S},\mathscr{T})$ be a pair of self-mappings on a metric space $(\mathscr{X},d)$. Then, $\mathscr{S}$ is said to be Suzuki type $\alpha\mathscr{T}$-admissible $\mathscr{Z}$-contraction with respect to ζ if for all $x,y\in\mathscr{X}$, we have*

$$\frac{1}{2}d(\mathscr{T}x,\mathscr{S}x)<d(\mathscr{T}x,\mathscr{T}y)\Rightarrow \zeta\left(\alpha(\mathscr{T}x,\mathscr{T}y)d(\mathscr{S}x,\mathscr{S}y),\mathscr{M}(\mathscr{T}x,\mathscr{T}y)\right)\geq 0, \tag{3.1}$$

where

$$M(\mathscr{T}x,\mathscr{T}y) = \max\left\{d(\mathscr{T}x,\mathscr{T}y),d(\mathscr{T}x,\mathscr{S}x),d(\mathscr{T}y,\mathscr{S}y),\frac{d(\mathscr{T}x,\mathscr{S}y)+d(\mathscr{T}y,\mathscr{S}x)}{4}\right\}.$$

Remark 3.1. *Observe that the mapping $\mathscr{S}$ in Definition 3.1 satisfies the following:*

$$\frac{1}{2}d(\mathscr{T}x,\mathscr{S}x)<d(\mathscr{T}x,\mathscr{T}y)\Rightarrow \alpha(\mathscr{T}x,\mathscr{T}y)d(\mathscr{S}x,\mathscr{S}y)<\mathscr{M}(\mathscr{T}x,\mathscr{T}y). \tag{3.2}$$

Theorem 3.1. *Let $(\mathscr{S},\mathscr{T})$ be a pair of self-mappings on a metric space $(\mathscr{X},d)$ such that $\mathscr{S}$ is Suzuki type $\alpha\mathscr{T}$-admissible $\mathscr{Z}$-contraction with respect to ζ, where $\mathscr{S}(\mathscr{X})$ is precomplete in $\mathscr{T}(\mathscr{X})$. Also, suppose that the following conditions hold:*

(i) *there exists $x_0\in\mathscr{X}$ such that $\alpha(x_0,\mathscr{S}x_0)\geq 1$;*

(ii) $\mathscr{S}(\mathscr{X})\subset\mathscr{T}(\mathscr{X})$;

(iii) *$\mathscr{S}$ is triangular $\alpha\mathscr{T}$-admissible;*

(iv) *$\mathscr{S}$ is $\mathscr{T}$-continuous;*

(v) *$\mathscr{X}$ is $\alpha\mathscr{T}$-directed set;*

(vi) *the pair* $(\mathscr{S},\mathscr{T})$ *is weakly compatible.*

Then the pair $(\mathscr{S},\mathscr{T})$ *has a common fixed point.*

Proof. Firstly, we will show that the existence of a Cauchy sequence with the initial point $\mathscr{S}x_0$. Choose x_0 such as in (i). In view of (ii), we can define an increasing sequence $\{\mathscr{T}x_n\}$ in $\mathscr{S}(\mathscr{X})$ such that

$$\mathscr{S}x_{n-1} = \mathscr{T}x_n \text{ for all } n. \tag{3.3}$$

Now, $\alpha(\mathscr{T}x_0,\mathscr{S}x_0) \geq 1$ can be written as $\alpha(\mathscr{T}x_0,\mathscr{T}x_1)$ which observe that that $\alpha(\mathscr{S}x_0,\mathscr{S}x_1) \geq 1 \Rightarrow \alpha(\mathscr{T}x_1,\mathscr{T}x_2) \geq 1$. Continuing this process inductively and using (ii) of Definition 2.4, we find that (for all n,m with $m > n \geq 1$),

$$\alpha(\mathscr{T}x_n,\mathscr{T}x_m) \geq 1. \tag{3.4}$$

If $\mathscr{T}x_m = \mathscr{T}x_{m+1}$ for some $m \in \mathbb{N}$, then x_m is a coincidence point. So, in the rest of the proof, we suppose that

$$0 < d(\mathscr{T}x_n,\mathscr{T}x_{n+1}) \text{ for all } n \in \mathbb{N}.$$

Hence, we have

$$\frac{1}{2}d(\mathscr{T}x_n,\mathscr{S}x_n) < d(\mathscr{T}x_n,\mathscr{T}x_{n+1}).$$

Since $\mathscr{S}$ is a Suzuki type $\alpha\mathscr{T}$-admissible $\mathscr{Z}$-contraction, we have

$$\begin{aligned} 0 &\leq \zeta(\alpha(\mathscr{T}x_n,\mathscr{T}x_{n+1})d(\mathscr{S}x_n,\mathscr{S}x_{n+1}),\mathscr{M}(\mathscr{T}x_n,\mathscr{T}x_{n+1})) \\ &= \zeta(\alpha(\mathscr{T}x_n,\mathscr{T}x_{n+1})d(\mathscr{T}x_{n+1},\mathscr{T}x_{n+2}),\mathscr{M}(\mathscr{T}x_n,\mathscr{T}x_{n+1})). \end{aligned}$$

Then

$$\begin{aligned} &\mathscr{M}(\mathscr{T}x_n,\mathscr{T}x_{n+1}) \\ &= \max\left\{ \begin{array}{c} d(\mathscr{T}x_n,\mathscr{T}x_{n+1}),d(\mathscr{T}x_n,\mathscr{S}x_n),d(\mathscr{T}x_{n+1},\mathscr{S}x_{n+1}), \\ \dfrac{d(\mathscr{T}x_n,\mathscr{S}x_{n+1})+d(\mathscr{T}x_{n+1},\mathscr{S}x_n)}{4} \end{array} \right\} \\ &= \max\left\{ \begin{array}{c} d(\mathscr{T}x_n,\mathscr{T}x_{n+1}),d(\mathscr{T}x_n,\mathscr{T}x_{n+1}),d(\mathscr{T}x_{n+1},\mathscr{T}x_{n+2}), \\ \dfrac{d(\mathscr{T}x_n,\mathscr{T}x_{n+2})+d(\mathscr{T}x_{n+1},\mathscr{T}x_{n+1})}{4} \end{array} \right\} \\ &= \max\left\{ d(\mathscr{T}x_n,\mathscr{T}x_{n+1}),d(\mathscr{T}x_{n+1},\mathscr{T}x_{n+2}),\frac{d(\mathscr{T}x_n,\mathscr{T}x_{n+2})}{4} \right\}. \end{aligned}$$

The triangle inequality yields

$$\frac{d(\mathscr{T}x_n, \mathscr{T}x_{n+2})}{4} \leq \max\{d(\mathscr{T}x_n, \mathscr{T}x_{n+1}), d(\mathscr{T}x_{n+1}, \mathscr{T}x_{n+2})\}.$$

Therefore,

$$\mathscr{M}(\mathscr{T}x_n, \mathscr{T}x_{n+1}) = \max\{d(\mathscr{T}x_n, \mathscr{T}x_{n+1}), d(\mathscr{T}x_{n+1}, \mathscr{T}x_{n+2})\},$$

from (3.1), we get that

$$\begin{aligned}
0 &\leq \zeta(\alpha(\mathscr{T}x_n, \mathscr{T}x_{n+1})d(\mathscr{S}x_n, \mathscr{S}x_{n+1}), \mathscr{M}(\mathscr{T}x_n, \mathscr{T}x_{n+1}))\\
&= \zeta(\alpha(\mathscr{T}x_n, \mathscr{T}x_{n+1})d(\mathscr{T}x_{n+1}, \mathscr{T}x_{n+2}), \mathscr{M}(\mathscr{T}x_n, \mathscr{T}x_{n+1}))\\
&= \zeta(\alpha(\mathscr{T}x_n, \mathscr{T}x_{n+1})d(\mathscr{T}x_{n+1}, \mathscr{T}x_{n+2}),\\
&\quad \max\{d(\mathscr{T}x_n, \mathscr{T}x_{n+1}), d(\mathscr{T}x_{n+1}, \mathscr{T}x_{n+2})\})\\
&< \max\{d(\mathscr{T}x_n, \mathscr{T}x_{n+1}), d(\mathscr{T}x_{n+1}, \mathscr{T}x_{n+2})\}\\
&\quad - \alpha(\mathscr{T}x_n, \mathscr{T}x_{n+1})d(\mathscr{T}x_{n+1}, \mathscr{T}x_{n+2}).
\end{aligned} \tag{3.5}$$

The inequality (3.5) shows that

$$\mathscr{M}(\mathscr{T}x_n, \mathscr{T}x_{n+1}) = d(\mathscr{T}x_n, \mathscr{T}x_{n+1}) \text{ for all } n \in \mathbb{N}, \tag{3.6}$$

which implies that the sequence $\{d(\mathscr{T}x_n, \mathscr{T}x_{n+1})\}$ is a strictly decreasing sequence of positive real numbers. So there is some $r \geq 0$ such that

$$\lim_{n\to\infty} d(\mathscr{T}x_n, \mathscr{T}x_{n+1}) = r.$$

If $r > 0$ then since $\mathscr{S}$ is Suzuki type $\alpha\mathscr{T}$-admissible $\mathscr{Z}$-contraction with respect to $\zeta \in \mathscr{Z}$ therefore by (iii), we have

$$0 \leq \limsup_{n\to\infty} \zeta(\alpha(\mathscr{T}x_n, \mathscr{T}x_{n+1})d(\mathscr{T}x_{n+1}, \mathscr{T}x_{n+2}), d(\mathscr{T}x_n, \mathscr{T}x_{n+1})) < 0.$$

This is a contradiction. Then we conclude that $r = 0$, that is,

$$\lim_{n\to\infty} d(\mathscr{T}x_n, \mathscr{T}x_{n+1}) = 0. \tag{3.7}$$

Next, we will show that $\{\mathscr{T}x_n\}$ is a bounded sequence. Suppose that $\{\mathscr{T}x_n\}$ is not bounded sequence. Then there a subsequence $\{\mathscr{T}x_{n_k}\}$ of $\{\mathscr{T}x_n\}$ such that $n_1 = 1$ and for each $k \in \mathbb{N}$, n_{k+1} is the minimum integer such that

$$d\left(\mathscr{T}x_{n_k}, \mathscr{T}x_{n_{k+1}}\right) > 1 \tag{3.8}$$

and

$$d\left(\mathscr{T}x_{n_k}, \mathscr{T}x_m\right) \le 1 \text{ for } n_k \le m \le n_{k+1}-1. \tag{3.9}$$

Thus, by the triangle inequality, we get

$$\begin{aligned} 1 < d\left(\mathscr{T}x_{n_k}, \mathscr{T}x_{n_{k+1}}\right) &\le d\left(\mathscr{T}x_{n_k}, \mathscr{T}x_{n_{k+1}-1}\right) + d\left(\mathscr{T}x_{n_{k+1}-1}, \mathscr{T}x_{n_{k+1}}\right) \\ &\le 1 + d\left(\mathscr{T}x_{n_{k+1}-1}, \mathscr{T}x_{n_{k+1}}\right). \end{aligned}$$

Letting $k \to \infty$ and by using (3.7), we obtain

$$\lim_{k\to\infty} d\left(\mathscr{T}x_{n_k}, \mathscr{T}x_{n_{k+1}}\right) = 1.$$

By (3.2), we have

$$\begin{aligned} &\frac{1}{2}d\left(\mathscr{T}x_{n_k-1}, \mathscr{T}x_{n_k}\right) < d\left(\mathscr{T}x_{n_k-1}, \mathscr{T}x_{n_{k+1}-1}\right) \\ &\Rightarrow \alpha(\mathscr{T}x_{n_k-1}, \mathscr{T}x_{n_{k+1}-1})d\left(\mathscr{T}x_{n_k}, \mathscr{T}x_{n_{k+1}}\right) < \mathscr{M}\left(\mathscr{T}x_{n_k-1}, \mathscr{T}x_{n_{k+1}-1}\right). \end{aligned}$$

Now,

$$\begin{aligned} 1 &< \alpha(\mathscr{T}x_{n_k-1}, \mathscr{T}x_{n_{k+1}-1})d\left(\mathscr{T}x_{n_k}, \mathscr{T}x_{n_{k+1}}\right) \\ &< \mathscr{M}\left(\mathscr{T}x_{n_k-1}, \mathscr{T}x_{n_{k+1}-1}\right) \\ &= \max\left\{ \begin{array}{c} d\left(\mathscr{T}x_{n_k-1}, \mathscr{T}x_{n_{k+1}-1}\right), d\left(\mathscr{T}x_{n_k-1}, \mathscr{T}x_{n_k}\right), d\left(\mathscr{T}x_{n_{k+1}-1}, \mathscr{T}x_{n_{k+1}}\right), \\ \dfrac{d(\mathscr{T}x_{n_k-1}, \mathscr{T}x_{n_{k+1}}) + d(\mathscr{T}x_{n_{k+1}-1}, \mathscr{T}x_{n_k})}{4} \end{array} \right\} \\ &\le \max\left\{ \begin{array}{c} d\left(\mathscr{T}x_{n_k-1}, \mathscr{T}x_{n_k}\right) + d\left(\mathscr{T}x_{n_k}, \mathscr{T}x_{n_{k+1}-1}\right), d\left(\mathscr{T}x_{n_k-1}, \mathscr{T}x_{n_k}\right), \\ d\left(\mathscr{T}x_{n_{k+1}-1}, \mathscr{T}x_{n_{k+1}}\right), \dfrac{d\left(\mathscr{T}x_{n_k-1}, \mathscr{T}x_{n_{k+1}}\right) + d\left(\mathscr{T}x_{n_{k+1}-1}, \mathscr{T}x_{n_k}\right)}{4} \end{array} \right\} \end{aligned}$$

$$\leq \max\left\{\begin{array}{c} 1+d\left(\mathscr{T}x_{n_k-1},\mathscr{T}x_{n_k}\right),d\left(\mathscr{T}x_{n_k-1},\mathscr{T}x_{n_k}\right),d\left(\mathscr{T}x_{n_{k+1}-1},\mathscr{T}x_{n_{k+1}}\right), \\ \dfrac{1+d\left(\mathscr{T}x_{n_k-1},\mathscr{T}x_{n_{k+1}}\right)}{4} \end{array}\right\}$$

$$\leq \max\left\{\begin{array}{c} 1+d\left(\mathscr{T}x_{n_k-1},\mathscr{T}x_{n_k}\right),d\left(\mathscr{T}x_{n_k-1},\mathscr{T}x_{n_k}\right),d\left(\mathscr{T}x_{n_{k+1}-1},\mathscr{T}x_{n_{k+1}}\right), \\ \dfrac{1+d\left(\mathscr{T}x_{n_k-1},\mathscr{T}x_{n_k}\right)+d\left(\mathscr{T}x_{n_k},\mathscr{T}x_{n_{k+1}}\right)}{4} \end{array}\right\}.$$

Letting $k\to\infty$, we obtain

$$1 \leq \lim_{k\to\infty}\mathscr{M}\left(\mathscr{T}x_{n_k-1},\mathscr{T}x_{n_{k+1}-1}\right) \leq 1,$$

that is,

$$\lim_{k\to\infty}\mathscr{M}\left(\mathscr{T}x_{n_k-1},\mathscr{T}x_{n_{k+1}-1}\right) = \lim_{k\to\infty}\alpha\left(\mathscr{T}x_{n_k-1},\mathscr{T}x_{n_{k+1}-1}\right) = 1.$$

Further, since $\frac{1}{2}d\left(\mathscr{T}x_{n_k-1},\mathscr{T}x_{n_k}\right) < d\left(\mathscr{T}x_{n_k-1},\mathscr{T}x_{n_{k+1}-1}\right)$. Therefore, $\mathscr{S}$ is Suzuki type $\alpha\mathscr{T}$-admissible $\mathscr{Z}$-contraction with respect to $\zeta\in\mathscr{Z}$ therefore by (iii), we have

$$\begin{aligned} 0 &\leq \zeta\left(\alpha(\mathscr{T}x_{n_k-1},\mathscr{T}x_{n_{k+1}-1})d\left(\mathscr{T}x_{n_k},\mathscr{T}x_{n_{k+1}},\right),\mathscr{M}\left(\mathscr{T}x_{n_k-1},\mathscr{T}x_{n_{k+1}-1}\right)\right) \\ &\leq \limsup_{k\to\infty}\zeta\left(\alpha(\mathscr{T}x_{n_k-1},\mathscr{T}x_{n_{k+1}-1})d\left(\mathscr{T}x_{n_k},\mathscr{T}x_{n_{k+1}},\right),\mathscr{M}\left(\mathscr{T}x_{n_k-1},\mathscr{T}x_{n_{k+1}-1}\right)\right) \\ &< 0. \end{aligned}$$

This is a contradiction. Hence, $\{\mathscr{T}x_n\}$ is bounded.

Let $\mathscr{C}_n = \sup\{d\left(x_i,x_j\right) : i,j\geq n\},\ n\in\mathbb{N}$. From above, we know that $\mathscr{C}_n<\infty$ for every $n\in\mathbb{N}$. Since $\mathscr{C}_n$ is a positive monotonically decreasing sequence, there exists $\mathscr{C}\geq 0$ such that $\lim_{n\to\infty}\mathscr{C}_n=\mathscr{C}$. We will show that $\mathscr{C}=0$. If $\mathscr{C}>0$ then by the definition of $\mathscr{C}_n$, for every $k\in\mathbb{N}$, there exists n_k,m_k such that $m_k>n_k\geq k$ and

$$\mathscr{C}_k - \frac{1}{k} < d\left(\mathscr{T}x_{m_k},\mathscr{T}x_{n_k}\right) \leq \mathscr{C}_k.$$

Therefore,

$$\lim_{k\to\infty} d\left(\mathscr{T}x_{m_k},\mathscr{T}x_{n_k}\right) = \mathscr{C}. \tag{3.10}$$

Moreover, by

$$d\left(x_{m_k},x_{n_k}\right) \leq d\left(x_{m_k},x_{m_k-1}\right)+d\left(x_{m_k-1},x_{n_k-1}\right)+d\left(x_{n_k-1},x_{n_k}\right)$$

and

$$d\left(x_{m_k-1},x_{n_k-1}\right) \le d\left(x_{m_k-1},x_{m_k}\right)+d\left(x_{m_k},x_{n_k}\right)+d\left(x_{n_k},x_{n_k-1}\right)$$

Letting $k \to \infty$, using (3.7) and (3.10), we get

$$\lim_{k\to\infty} d\left(x_{m_k-1},x_{n_k-1}\right)=\mathscr{C}. \tag{3.11}$$

By using (3.2) and (3.4), we have

$$\begin{aligned}
&\frac{1}{2}d\left(\mathscr{T}x_{m_k-1},\mathscr{T}x_{m_k}\right) < d\left(\mathscr{T}x_{m_k-1},\mathscr{T}x_{n_k-1}\right)\\
&\Rightarrow \alpha(\mathscr{T}x_{m_k-1},\mathscr{T}x_{n_k-1})d\left(\mathscr{T}x_{m_k},\mathscr{T}x_{n_k}\right) < \mathscr{M}\left(\mathscr{T}x_{m_k-1},\mathscr{T}x_{n_k-1}\right),
\end{aligned}$$

where

$$\begin{aligned}
&\mathscr{M}\left(\mathscr{T}x_{m_k-1},\mathscr{T}x_{n_k-1}\right)\\
&=\max\left\{\begin{array}{c} d\left(\mathscr{T}x_{m_k-1},\mathscr{T}x_{n_k-1}\right), d\left(\mathscr{T}x_{m_k-1},\mathscr{T}x_{m_k}\right), d\left(\mathscr{T}x_{n_k-1},\mathscr{T}x_{n_k}\right),\\ \dfrac{d\left(\mathscr{T}x_{m_k-1},\mathscr{T}x_{n_k}\right)+d\left(\mathscr{T}x_{n_k-1},\mathscr{T}x_{m_k}\right)}{4}\end{array}\right\}\\
&\le\max\left\{\begin{array}{c} d\left(\mathscr{T}x_{m_k-1},\mathscr{T}x_{n_k-1}\right), d\left(\mathscr{T}x_{m_k-1},\mathscr{T}x_{m_k}\right), d\left(\mathscr{T}x_{n_k-1},\mathscr{T}x_{n_k}\right),\\ \dfrac{d\left(\mathscr{T}x_{m_k-1},\mathscr{T}x_{m_k}\right)+d\left(\mathscr{T}x_{n_k-1},\mathscr{T}x_{n_k}\right)}{4}\\ +\dfrac{d\left(\mathscr{T}x_{m_k},\mathscr{T}x_{n_k}\right)+d\left(\mathscr{T}x_{n_k},\mathscr{T}x_{m_k}\right)}{4}\end{array}\right\}.
\end{aligned}$$

Letting $k \to \infty$, using (3.7), (3.10) and (3.11), we get

$$\lim_{k\to\infty}\mathscr{M}\left(\mathscr{T}x_{m_k-1},\mathscr{T}x_{n_k-1}\right)=\mathscr{C} \quad \text{and} \quad \lim_{k\to\infty}\alpha(\mathscr{T}x_{m_k-1},\mathscr{T}x_{n_k-1})=1.$$

As $\mathscr{S}$ is Suzuki type $\alpha\mathscr{T}$-admissible $\mathscr{Z}$-contraction with respect to $\zeta \in \mathscr{Z}$ therefore by (*iii*), we have

$$0 \le \limsup_{k\to\infty}\zeta\left(\alpha(\mathscr{T}x_{m_k-1},\mathscr{T}x_{n_k-1})d\left(\mathscr{T}x_{m_k},\mathscr{T}x_{n_k}\right),\mathscr{M}\left(\mathscr{T}x_{m_k-1},\mathscr{T}x_{n_k-1}\right)\right) < 0.$$

This is a contradiction. Hence, $\mathscr{C}=0$. This is $\{\mathscr{T}x_n\}$ is a Cauchy sequence in $\mathscr{S}(\mathscr{X})$. The precompleteness of $\mathscr{S}(\mathscr{X})$ in $\mathscr{T}(\mathscr{X})$ ensures the existence of some $\bar{x} \in \mathscr{X}$ with

$$\lim_{n\to\infty}\mathscr{T}x_n=\mathscr{T}\bar{x}. \tag{3.12}$$

Secondly, we will show that the existence of a common fixed point of the pair $(\mathscr{S}, \mathscr{T})$. If $\mathscr{S}$ is $\mathscr{T}$-continuous, then $\lim_{n\to\infty} \mathscr{S}x_n = \mathscr{S}\bar{x}$ which (in view of (3.3) and the uniqueness of the limit) implies that $\mathscr{T}\bar{x} = \mathscr{S}\bar{x}$. Let x_* be such that $x_* = \mathscr{S}\bar{x} = \mathscr{T}\bar{x}$. From condition (vi), we have

$$\mathscr{S}x_* = \mathscr{S}(\mathscr{T}\bar{x}) = \mathscr{T}(\mathscr{S}\bar{x}) = \mathscr{T}x_*. \tag{3.13}$$

Suppose that $d(\mathscr{T}x_*, x_*) > 0$. As $\mathscr{S}$ is Suzuki type $\alpha\mathscr{T}$-admissible $\mathscr{Z}$-contraction with respect to $\zeta \in \mathscr{Z}$. Since $0 = \frac{1}{2}d(\mathscr{T}\bar{x}, \mathscr{S}\bar{x}) < d(\mathscr{T}\bar{x}, \mathscr{T}x_*)$, then by applying (3.1), we obtain that

$$0 \le \zeta\left(\alpha(\mathscr{T}\bar{x}, \mathscr{T}x_*)d(\mathscr{S}\bar{x}, \mathscr{S}x_*), \mathscr{M}(\mathscr{T}\bar{x}, \mathscr{T}x_*)\right), \tag{3.14}$$

where

$$\begin{aligned}
\mathscr{M}(\mathscr{T}\bar{x}, \mathscr{T}x_*) &= \max\left\{ \begin{array}{c} d(\mathscr{T}\bar{x}, \mathscr{T}x_*), d(\mathscr{T}\bar{x}, \mathscr{S}\bar{x}), d(\mathscr{T}x_*, \mathscr{S}x_*), \\ \dfrac{d(\mathscr{T}\bar{x}, \mathscr{S}x_*) + d(\mathscr{T}x_*, \mathscr{S}\bar{x})}{4} \end{array} \right\} \\
&= \max\left\{ \begin{array}{c} d(x_*, \mathscr{T}x_*), d(x_*, x_*), d(\mathscr{T}x_*, \mathscr{T}x_*), \\ \dfrac{d(x_*, \mathscr{T}x_*) + d(\mathscr{T}x_*, x_*)}{4} \end{array} \right\} \\
&= d(\mathscr{T}x_*, x_*).
\end{aligned}$$

This together with (3.14) shows that

$$\begin{aligned}
0 &\le \zeta\left(\alpha(\mathscr{T}\bar{x}, \mathscr{T}x_*)d(\mathscr{S}\bar{x}, \mathscr{S}x_*), \mathscr{M}(\mathscr{T}\bar{x}, \mathscr{T}x_*)\right) \\
&= \zeta\left(\alpha(x_*, \mathscr{T}x_*)d(x_*, \mathscr{T}x_*), d(x_*, \mathscr{T}x_*)\right) \\
&< d(x_*, \mathscr{T}x_*) - \alpha(x_*, \mathscr{T}x_*)d(x_*, \mathscr{T}x_*).
\end{aligned}$$

This is a contradiction. Thus, we have $x_* = \mathscr{S}x_* = \mathscr{T}x_*$ and x_* is a common fixed point of the pair $(\mathscr{S}, \mathscr{T})$. If x' is another such point with $d(x_*, x') > 0$, then $0 = \dfrac{1}{2}d(\mathscr{T}x_*, \mathscr{S}x_*) < d(\mathscr{T}x_*, \mathscr{T}x')$ and

$$0 \le \zeta\left(\alpha(\mathscr{T}x_*, \mathscr{T}x')d\left(\mathscr{S}x_*, \mathscr{S}x'\right), \mathscr{M}\left(\mathscr{T}x_*, \mathscr{T}x'\right)\right), \tag{3.15}$$

where

$$\mathscr{M}\left(\mathscr{T}x_*,\mathscr{T}x'\right) = \max\left\{ \begin{array}{c} d\left(\mathscr{T}x_*,\mathscr{T}x'\right), d\left(\mathscr{T}x_*,\mathscr{S}x_*\right), d\left(\mathscr{T}x',\mathscr{S}x'\right), \\ \dfrac{d\left(\mathscr{T}x_*,\mathscr{S}x'\right)+d\left(\mathscr{T}x',\mathscr{S}x_*\right)}{4} \end{array} \right\}$$

$$= \max\left\{ \begin{array}{c} d\left(x_*,x'\right), d\left(x_*,x_*\right), d\left(x',x'\right), \\ \dfrac{d\left(x_*,x'\right)+d\left(x',x_*\right)}{4} \end{array} \right\}$$

$$= d\left(x_*,x'\right).$$

This together with (3.16) shows that

$$\begin{aligned} 0 &\leq \zeta\left(\alpha(\mathscr{T}x_*,\mathscr{T}x')d\left(\mathscr{S}x_*,\mathscr{S}x'\right), \mathscr{M}\left(\mathscr{T}x_*,\mathscr{T}x'\right)\right) \\ &= \zeta\left(\alpha(x_*,x')d\left(x_*,x'\right), d\left(x_*,x'\right)\right) \\ &< d\left(x_*,x'\right) - \alpha(x_*,x')d\left(x_*,x'\right). \end{aligned}$$

This is a contradiction. Hence, the pair $(\mathscr{S},\mathscr{T})$ has a unique common fixed point. $\square$

Theorem 3.2. *Theorem 3.1 remains true if assumption* (iv) *is replaced by the following:*

(iv^*) $\mathscr{T}(\mathscr{X})$ *is* $\alpha\mathscr{T}$*-regular.*

Proof. Firstly, it's following proof of Theorem 3.1, we can deduce a sequence $\{\mathscr{T}x_n\}$ with initial point $\mathscr{S}x_0$ such that (3.13) holds.

Secondly, the $\alpha\mathscr{T}$-regularity of $\mathscr{T}(\mathscr{X})$ implies that there exists a subsequence $\{\mathscr{T}x_{n_k}\}$ of $\{\mathscr{T}x_n\}$ with $\alpha(\mathscr{T}x_{n_k},\mathscr{T}\bar{x}) \geq 1$ for all k. Now, applying (3.1), for all k, we have $\frac{1}{2}d\left(\mathscr{T}x_{n_k},\mathscr{S}x_{n_k}\right) < d\left(\mathscr{T}x_{n_k},\mathscr{T}\bar{x}\right)$ and

$$0 \leq \zeta\left(\alpha(\mathscr{T}x_{n_k},\mathscr{T}\bar{x})d\left(\mathscr{S}x_{n_k},\mathscr{S}\bar{x}\right), \mathscr{M}\left(\mathscr{T}x_{n_k},\mathscr{T}\bar{x}\right)\right), \tag{3.16}$$

where

$$\mathscr{M}\left(\mathscr{T}x_{n_k},\mathscr{T}\bar{x}\right) = \max\left\{ \begin{array}{c} d\left(\mathscr{T}x_{n_k},\mathscr{T}\bar{x}\right), d\left(\mathscr{T}x_{n_k},\mathscr{S}x_{n_k}\right), d\left(\mathscr{T}\bar{x},\mathscr{S}\bar{x}\right), \\ \dfrac{d\left(\mathscr{T}x_{n_k},\mathscr{S}\bar{x}\right)+d\left(\mathscr{T}\bar{x},\mathscr{S}x_{n_k}\right)}{4} \end{array} \right\}$$

$$= \max\left\{ \begin{array}{c} d\left(\mathscr{T}x_{n_k},\mathscr{T}\bar{x}\right), d\left(\mathscr{T}x_{n_k},\mathscr{T}x_{n_{k+1}}\right), d\left(\mathscr{T}\bar{x},\mathscr{S}\bar{x}\right), \\ \dfrac{d\left(\mathscr{T}x_{n_k},\mathscr{S}\bar{x}\right)+d\left(\mathscr{T}\bar{x},\mathscr{T}x_{n_{k+1}}\right)}{4} \end{array} \right\}.$$

This together with (3.16) shows that

$$\begin{aligned}
0 &\leq \zeta\left(\alpha(\mathscr{T}x_{n_k},\mathscr{T}\bar{x})d\left(\mathscr{S}x_{n_k},\mathscr{S}\bar{x}\right),\mathscr{M}\left(\mathscr{T}x_{n_k},\mathscr{T}\bar{x}\right)\right)\\
&= \zeta\left(\alpha(\mathscr{T}x_{n_k},\mathscr{T}\bar{x})d\left(\mathscr{T}x_{n_{k+1}},\mathscr{S}\bar{x}\right),\mathscr{M}\left(\mathscr{T}x_{n_k},\mathscr{T}\bar{x}\right)\right)\\
&< \mathscr{M}\left(\mathscr{T}x_{n_k},\mathscr{T}\bar{x}\right)-\alpha(\mathscr{T}x_{n_k},\mathscr{T}\bar{x})d\left(\mathscr{T}x_{n_{k+1}},\mathscr{S}\bar{x}\right),
\end{aligned}$$

which is equivalent to

$$d\left(\mathscr{T}x_{n_{k+1}},\mathscr{S}\bar{x}\right) \leq \alpha(\mathscr{T}x_{n_k},\mathscr{T}\bar{x})d\left(\mathscr{T}x_{n_{k+1}},\mathscr{S}\bar{x}\right) < \mathscr{M}\left(\mathscr{T}x_{n_k},\mathscr{T}\bar{x}\right).$$

Taking $k\to\infty$, we get $d(\mathscr{T}\bar{x},\mathscr{S}\bar{x})\leq 0$ implying thereby $\mathscr{S}\bar{x}=\mathscr{T}\bar{x}$. The rest of the proof can be completed on the following proof of Theorem 3.1. □

Remark 3.2. *The hypotheses of Theorems 3.1 and 3.2 up to assumption (iv) are enough to ensure the existence of the coincidence point of the underlying pair.*

Corollary 3.1. *Let $(\mathscr{S},\mathscr{T})$ be a pair of self-mappings on a metric space $(\mathscr{X},d)$. Suppose that*

$$\frac{1}{2}d\left(\mathscr{T}x,\mathscr{S}x\right) < d\left(\mathscr{T}x,\mathscr{T}y\right) \Rightarrow \alpha(\mathscr{T}x,\mathscr{T}y)d\left(\mathscr{S}x,\mathscr{S}y\right) \leq \psi(\mathscr{M}\left(\mathscr{T}x,\mathscr{T}y\right)) \tag{3.17}$$

where

$$\begin{aligned}
&\mathscr{M}\left(\mathscr{T}x,\mathscr{T}y\right)\\
&= \max\left\{d\left(\mathscr{T}x,\mathscr{T}y\right),d\left(\mathscr{T}x,\mathscr{S}x\right),d\left(\mathscr{T}y,\mathscr{S}y\right),\frac{d\left(\mathscr{T}x,\mathscr{S}y\right)+d\left(\mathscr{T}y,\mathscr{S}x\right)}{4}\right\}
\end{aligned}$$

for all $x,y\in\mathscr{X}$, and ψ is as in Theorem 1.4. If conditions $(i)-(vi)$ of Theorem 3.1 (resp. Theorem 3.2) are satisfied, then the pair $(\mathscr{S},\mathscr{T})$ has a unique common fixed point.

Proof. It follows proof of Theorem 3.1 (resp. Theorem 3.2). □

Corollary 3.2. *Let $(\mathscr{S},\mathscr{T})$ be a pair of self-mappings on a metric space $(\mathscr{X},d)$. Suppose that*

$$\frac{1}{2}d\left(\mathscr{T}x,\mathscr{S}x\right) < d\left(\mathscr{T}x,\mathscr{T}y\right) \Rightarrow \phi(\alpha(\mathscr{T}x,\mathscr{T}y)d\left(\mathscr{S}x,\mathscr{S}y\right)) \leq \psi(\mathscr{M}\left(\mathscr{T}x,\mathscr{T}y\right)) \tag{3.18}$$

where

$$\begin{aligned}
&\mathscr{M}\left(\mathscr{T}x,\mathscr{T}y\right)\\
&= \max\left\{d\left(\mathscr{T}x,\mathscr{T}y\right),d\left(\mathscr{T}x,\mathscr{S}x\right),d\left(\mathscr{T}y,\mathscr{S}y\right),\frac{d\left(\mathscr{T}x,\mathscr{S}y\right)+d\left(\mathscr{T}y,\mathscr{S}x\right)}{4}\right\}
\end{aligned}$$

for all $x, y \in \mathscr{X}$, *and* $\phi, \psi : [0,\infty) \to [0,\infty)$ *are two continuous mappings such that* $\psi(\sigma) = \phi(\sigma) = 0 \Leftrightarrow \sigma = 0$ *and* $\psi(\sigma) < \sigma \leq \phi(\sigma)$, *for all* $\sigma > 0$. *If conditions* $(i)-(vi)$ *of Theorem 3.1 (resp. Theorem 3.2) are satisfied, then the pair* $(\mathscr{S}, \mathscr{T})$ *has a unique common fixed point.*

Proof. It follows proof of Theorem 3.1 (resp. Theorem 3.2). □

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CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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