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AN INTERACTING AND NON-INTERACTING TWO-FLUID SCENARIO FOR DARK ENERGY MODELS IN FIVE DIMENSIONAL KALUZA-KLEIN SPACE TIME

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Abstract. In this paper, we studied the evaluation of dark energy parameter in the spatial homogeneous and anisotropic five dimensional Kaluza-Klein space time filled with barotropic fluid and dark energy. To solve the Einstein field equation by considering a variable deceleration parameter here we consider two cases; first, when these fluids are assumed to be not interacting each other and second, when they interact with each other. We have discussed the physical and geometrical importance of the two fluid scenario described in various aspects. We also discuss the jerk parameter and Statefinder parameter in our derived models which found that the model tends to the ΛCDM model.

Keywords: Kaluza-Klein space time; two-fluid; dark energy; variable deceleration parameter.

2010 AMS Subject Classification: 93A30.

1. Introduction

Cosmological observation shows that the universe is undergoing an accelerated expansion[1-4]. It is generally assumed that the dark energy is responsible for the acceleration of the universe [5-10]. So, in a late time accelerating and expanding of the universe has been confirmed from high red shift type Ia Supernovae experiment (SNe.Ia) [11-12], cosmic microwave background

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radiation (CMBR) [13], large scale structure [14], Sloan Digital Sky Survey (SDSS) [15, 16], First Year Wilkinson Microwave Anisotropy Probe(WMAP) [17] and Chandra X-ray observatory [18] combination strongly suggest the existence of the extra constituent in a universe with negative pressure called dark energy. Roughly dark energy occupies about 68.3%, dark matter occupies about 26.8% and baryonic matter occupies about 4.9% of the energy of our Universe [19].

The Kaluza-Klein[20,21] theories have exposed how the gravity and the electromagnetism can be unified from Einstein's field equations generalized to five dimensions. In a certain sense the Kaluza-Klein theory resembles ordinary gravity, except that it is inscribed in five dimensions instead of four. This theory has been regarded as a candidate of fundamental theory due to the possible work of unifying the fundamental principle. Different authors study the five dimensional Kaluza-Klein space-time viz. Chodos et al. [22], Appelquist et al. [23] studied some cosmological models in five dimensional Kaluza-Klein space-time. D. R. K. Reddy et al. [24], R. L. Naidu et al. [25], Reddy et al. [26] , T.Ramprasad et al.[27] investigated five dimensional Kaluza-Klein cosmological model in the framework of Brans-Dicke theory of gravitation. G. S. Khadekar [28] , Namrata I. Jain et al. [29] investigated viscous cosmological model in five dimensional Kaluza-Klein space time. G.S. Khadekar et al. [30] and M. Sharif et al. [31] discussed behavior of gravitational and cosmological term in five dimensional Kaluza-Klein space time. R.Venkateswarlu, K. P. Kumar [32], G. S. Khadekar et al.[33], G. C. Samanta and S. N. Dhal [34] investigated about the higher dimensional FRW cosmological models. H. Amirhashchi et al. [35], B. Saha et al. [36], H. Amirhashchi et al. [37], A. Pradhan et al. [38], V.B.Raut et al. [39] had studied various aspects in dark energy models in FRW universe. Many authors works on FRW universe mentioned above are the motivation behind this work on accelerating and ΛCDM (Lambda cold dark matter) model.

In this paper, we have investigated the evaluation of dark energy parameter in the spatial homogeneous and anisotropic five dimensional Kaluza-Klein space time filled with barotropic fluid and dark energy. To solve the Einstein field equation by considering a variable deceleration parameter $q = -\frac{a\ddot{a}}{\dot{a}^2} = \beta H + \alpha$. In Section 1, we discussed the introduction of Kaluza-Klein cosmology. The Kaluza-Klein models and its field equations are presented in Section 2 and 3.

In Section 4 and 5, we discussed the interacting and non-interacting two fluid model. Section 6, we discussed the jerk parameter and Statefinder parameters in our derived models. In Section 7, we discussed the physical interpretation of our model. Finally, conclusions and summarized are given in the last section 8.

2. METRIC AND FIELD EQUATIONS

We consider the special homogeneous and anisotropic five-dimensional Kaluza-Klein space time given by

(1) $$ds^2 = dt^2 - A^2(dx^2 + dy^2 + dz^2) - B^2 d\varphi^2$$

where A and B are function of cosmic time t only.

Einstein's field equation (with $8\pi G = 1$ and $c = 1$) is given by

(2) $$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij}$$

where R_{ij} is the Ricci tensor R is the Ricci scalar g_{ij} is the metric tensor and T_{ij} is the energy momentum tensor of a perfect fluid and dark energy respectively.

The energy momentum tensor for two fluid is given by

(3) $$T_{ij} = T_{ij}^{m} + T_{ij}^{D}$$

where

(4) $$T_{ij}^{m} = (\rho_m + p_m)]\mu_i\mu_j - p_m g_{ij}$$

(5) $$T_{ij}^{D} = (\rho_D + p_D)\mu_i\mu_j - p_D g_{ij}$$

here $\mu^i\mu_i = 1, \mu^i\mu_j = 0$ and ρ_m and p_m are the energy density and pressure of the perfect fluid and ρ_D and p_D are the energy density and pressure of the dark energy component.

The equation of state parameter (ω) which is consider as an important quantity in describing the dynamic of the universe in the ratio of the pressure (p) and the energy density (ρ) are given by

$$p_m = \omega_m \rho_m \tag{6}$$

and

$$p_D = \omega_D \rho_D \tag{7}$$

The important physical parameter like spatial volume(V), Hubble parameter(H), expansion scalar (θ), shear scalar (σ^2), anisotropic parameter (Δ) and deceleration parameter (q) for the metric (1) are defined as

$$V = a^4 = A^3B \tag{8}$$

$$H = \frac{1}{4}(\frac{3\dot{A}}{A} + \frac{\dot{B}}{B}) \tag{9}$$

$$\theta = 4H = \frac{3\dot{A}}{A} + \frac{\dot{B}}{B} \tag{10}$$

$$\sigma^2 = \frac{1}{2}[\sum_{i=1}^{4} H_i^2 - 4H^2] = \frac{4}{2}\Delta H^2 \tag{11}$$

$$\Delta = \frac{1}{4}\sum_{i=1}^{4}(\frac{\Delta H_i}{H})^2 \tag{12}$$

where $\Delta H_i = H_i - H$

and

(13) $$q = -\frac{\ddot{a}a}{\dot{a}^2} = \frac{d}{dt}\frac{1}{H} - 1$$

where an overhead dot denote derivative with respect to cosmic time t.

The Einstein field equation (2) and (3) for the metric (1) it follows that

(14) $$2\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + 2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}^2}{A^2} = -(p_m + p_D)$$

(15) $$3\frac{\ddot{A}}{A} + 3\frac{\dot{A}^2}{A^2} = -(p_m + p_D)$$

(16) $$3\frac{\dot{A}^2}{A^2} + 3\frac{\dot{A}\dot{B}}{AB} = \rho_m + \rho_D$$

An over dot indicate a derivatives with respect to cosmic time t.

The energy conservation equation $(T^{ij}_{;j} = 0)$ which yields

(17) $$\dot{\rho} + 4H(\rho + p) = 0$$

where $p = p_m + p_D$ and $\rho = \rho_m + \rho_D$.

3. Solution of the Field Equations

We consider deceleration parameter (q) as a linear function of hubble parameter[40-42]:

(18) $$q = -\frac{a\ddot{a}}{\dot{a}^2} = \beta H + \alpha$$

Here α and β arbitrary constants.

we take $\alpha = -1$ in equation (18)

(19) $$q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 + \beta H$$

which yields the following differential equation

$$\frac{a\ddot{a}}{\dot{a}^2}+\beta\frac{\dot{a}}{a}-1=0 \tag{20}$$

which on integration gives

$$a(t)=exp\left[\frac{1}{\beta}\sqrt{2\beta t+c}\right]$$

$$=e^{\frac{1}{\beta}\sqrt{2\beta t+c}} \tag{21}$$

where c is an integrating constant.

Subtracting equation (14) from equation (15) we get,

$$\frac{d}{dt}(\frac{\dot{A}}{A}-\frac{\dot{B}}{B})+(\frac{\dot{A}}{A}-\frac{\dot{B}}{B})\frac{\dot{V}}{V}=0$$

which on integrating gives

$$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=\frac{\lambda}{V} \tag{22}$$

where λ is an integration constant.

Using equation (21) in equation (22) and then integrating we get the scale factors are

$$A(t)=e^{\frac{1}{\beta}\sqrt{2\beta t+c}}exp[-\frac{\lambda\beta}{64}e^{\frac{-4}{\beta}\sqrt{2\beta t+c}}(\frac{4}{\beta}\sqrt{2\beta t+c}+1)] \tag{23}$$

and

$$B(t)=e^{\frac{1}{\beta}\sqrt{2\beta t+c}}exp[\frac{3\lambda\beta}{64}e^{\frac{-4}{\beta}\sqrt{2\beta t+c}}(\frac{4}{\beta}\sqrt{2\beta t+c}+1)] \tag{24}$$

Therefore the metric (1) reduce to

$$ds^2 = dt^2 - e^{\frac{1}{\beta}\sqrt{2\beta t+c}} exp[-\frac{\lambda\beta}{64} e^{\frac{-4}{\beta}\sqrt{2\beta t+c}} (\frac{4}{\beta}\sqrt{2\beta t+c}+1)](dx^2+dy^2+dz^2) - e^{\frac{1}{\beta}\sqrt{2\beta t+c}} exp[\frac{3\lambda\beta}{64} e^{\frac{-4}{\beta}\sqrt{2\beta t+c}} (\frac{4}{\beta}\sqrt{2\beta t+c}+1)]d\phi^2 \tag{25}$$

The physical parameters of the model are given by

$$V = e^{\frac{4}{\beta}\sqrt{2\beta t+c}} \tag{26}$$

$$H = \frac{1}{\sqrt{2\beta t+c}} \tag{27}$$

$$\theta = 4H = \frac{4}{\sqrt{2\beta t+c}} \tag{28}$$

$$\therefore \Delta = \frac{3\lambda^2}{16}(2\beta t+c) e^{\frac{-8}{\beta}\sqrt{2\beta t+c}} \tag{29}$$

$$\sigma^2 = \frac{3\lambda^2}{32} e^{\frac{-8}{\beta}\sqrt{2\beta t+c}} \tag{30}$$

$$q = -1 + \frac{\beta}{\sqrt{2\beta t+c}} \tag{31}$$

In the following sections we deal with two cases (i) Non-interacting two fluid model and (ii) Interacting two fluid model.

4. NON-INTERECTING TWO-FLUID MODEL

The conservation equation for the dark and barotropic fluid separately

(32) $$\dot{\rho}_m + 4H(p_m + \rho_m) = 0$$

(33) $$\dot{\rho}_D + 4H(p_D + \rho_D) = 0$$

Integrating (32) we get,

(34) $$\rho_m = \rho_0 e^{\frac{-4(1+\omega_m)}{\beta}\sqrt{2\beta t+c}}$$

where ρ_0 is an integrating constant.

Using (23), (24) and (34) in equation (15) and (16) we obtain

(35) $$P_D = -3\left[\frac{2+\frac{\lambda^2}{8}(2\beta t+c)e^{\frac{-8}{\beta}\sqrt{2\beta t+c}} - \beta(2\beta t+c)^{\frac{-1}{2}}}{(2\beta t+c)}\right] - \omega_m \rho_0 e^{\frac{-4(1+\omega_m)}{\beta}\sqrt{2\beta t+c}}$$

(36) $$\rho_D = 3\left[\frac{2-\frac{\lambda^2}{8}(2\beta t+c)e^{\frac{-8}{\beta}\sqrt{2\beta t+c}}}{(2\beta t+c)}\right] - \rho_0 e^{\frac{-4(1+\omega_m)}{\beta}\sqrt{2\beta t+c}}$$

Using equations (35) and (36) we can find the expression for EOS of dark filed in terms of time as

$$\omega_D = \frac{P_D}{\rho_D}$$

(37)
$$\omega_D = -\left[\frac{6+\frac{3\lambda^2}{8}(2\beta t+c)e^{\frac{-8}{\beta}\sqrt{2\beta t+c}} - 3\beta(2\beta t+c)^{\frac{-1}{2}} + \omega_m\rho_0(2\beta t+c)e^{\frac{-4(1+\omega_m)}{\beta}\sqrt{2\beta t+c}}}{6-\frac{3\lambda^2}{8}(2\beta t+c)e^{\frac{-8}{\beta}\sqrt{2\beta t+c}} - \rho_0(2\beta t+c)e^{\frac{-4(1+\omega_m)}{\beta}\sqrt{2\beta t+c}}}\right]$$

The expressions of matter-energy-density parameter (Ω_m) and dark energy density parameter (Ω_D) are given by

(38) $$\Omega_m = \frac{\rho_m}{6H^2} = \frac{\rho_0 e^{\frac{-4(1+\omega_m)}{\beta}\sqrt{2\beta t+c}}}{6(2\beta t+c)^{-1}}$$

and

(39) $$\Omega_D = \frac{\rho_D}{6H^2} = 1 - \frac{\lambda^2}{16}(2\beta t+c)e^{\frac{-8}{\beta}\sqrt{2\beta t+c}} - \frac{1}{6}\rho_0(2\beta t+c)e^{\frac{-4(1+\omega_m)}{\beta}\sqrt{2\beta t+c}}$$

The total energy density (Ω) is given by

(40) $$\Omega = \Omega_m + \Omega_D = 1 - \frac{\lambda^2}{16}(2\beta t+c)e^{\frac{-8}{\beta}\sqrt{2\beta t+c}}$$

5. Interecting Two-Fluid Model

In this section, we can write the energy conservation equation for the dark and barotropic fluid as

(41) $$\dot{\rho}_m + 4(1+\omega_m)\rho_m H = Q$$

(42) $$\dot{\rho}_D + 4(1+\omega_D)\rho_D H = -Q$$

where the quantity Q represents the interecting between the componants of matter and dark energy.We consider $Q > 0$, this ensures that the energy is being transferred from dark energy to the matter component.

We consider

(43) $$Q = 4Hk\rho_m$$

where k is coupling constant.

Using (43) in equation (41) we get

$$\rho_m = \rho_0 e^{\frac{-4(1+\omega_m - k)}{\beta}\sqrt{2\beta t + c}} \tag{44}$$

Now we again obtain P_D and ρ_D.

$$P_D = -3\left[\frac{2 + \frac{\lambda^2}{8}(2\beta t + c)e^{\frac{-8}{\beta}\sqrt{2\beta t + c}} - \beta(2\beta t + c)^{\frac{-1}{2}}}{(2\beta t + c)}\right] - \omega_m \rho_0 e^{\frac{-4(1+\omega_m - k)}{\beta}\sqrt{2\beta t + c}} \tag{45}$$

$$\rho_D = 3\left[\frac{2 - \frac{\lambda^2}{8}(2\beta t + c)e^{\frac{-8}{\beta}\sqrt{2\beta t + c}}}{(2\beta t + c)}\right] - \rho_0 e^{\frac{-4(1+\omega_m - k)}{\beta}\sqrt{2\beta t + c}} \tag{46}$$

Also the equation of state parameter (ω_D) is obtained as

(47)

$$\omega_D = -\left[\frac{6 + \frac{3\lambda^2}{8}(2\beta t + c)e^{\frac{-8}{\beta}\sqrt{2\beta t + c}} - 3\beta(2\beta t + c)^{\frac{-1}{2}} + \omega_m \rho_0 (2\beta t + c) e^{\frac{-4(1+\omega_m - k)}{\beta}\sqrt{2\beta t + c}}}{6 - \frac{3\lambda^2}{8}(2\beta t + c)e^{\frac{-8}{\beta}\sqrt{2\beta t + c}} - \rho_0 (2\beta t + c) e^{\frac{-4(1+\omega_m - k)}{\beta}\sqrt{2\beta t + c}}}\right]$$

The expression of matter energy density-parameter (Ω_m) and dark energy density parameter (Ω_D) are given as

$$\Omega_m = \frac{\rho_m}{6H^2} = \frac{\rho_0 e^{\frac{-4(1+\omega_m - k)}{\beta}\sqrt{2\beta t + c_1}}}{6(2\beta t + c_1)^{-1}} \tag{48}$$

and

$$\Omega_D = \frac{\rho_D}{6H^2} = 1 - \frac{\lambda^2}{16}(2\beta t + c)e^{\frac{-8}{\beta}\sqrt{2\beta t + c}} - \frac{1}{6}\rho_0 (2\beta t + c) e^{\frac{-4(1+\omega_m - k)}{\beta}\sqrt{2\beta t + c}} \tag{49}$$

The total energy density (Ω) is given by

$$\Omega = \Omega_m + \Omega_D = 1 - \frac{\lambda^2}{16}(2\beta t + c)e^{\frac{-8}{\beta}\sqrt{2\beta t + c}} \tag{50}$$

6. The Jerk Parameter (j) and Statefinder parameter (r,s)

The Jerk Parameter in cosmology is defined as the dimensionless third derivative of the scale factor with respect to cosmic time t [43,44]. It is defined as

$$j(t)=\frac{\dddot{a}}{aH^3}=\frac{(a^2H^2)''}{2H^2} \tag{51}$$

where over dots and primes denote derivatives with respect to cosmic time and the scale factor respectively.

The Jerk Parameter (j) appears in the fourth term of a Taylor expansion of the scale factor around a_0.

$$\frac{a(t)}{a_0}=1+H_0(t-t_0)-\frac{1}{2}q_0H_0^2(t-t_0)^2+\frac{1}{6}j_0H_0^3(t-t_0)^3+0\left[(t-t_0)^4\right] \tag{52}$$

in equation (37) we can be written as

$$j(t)=q+2q^2-\frac{\dot{q}}{H} \tag{53}$$

From equation (27) and (31) we reduce to

$$j(t)=1-\frac{3\beta}{\sqrt{2\beta t+c}}+\frac{3\beta^2}{2\beta t+c} \tag{54}$$

For flat ΛCDM model, the jerk parameter (j) has the value $j=1$.

The Statefinder parameter (r,s) [45] defined as follows

$$r=\frac{\dddot{a}}{aH^3}=1+3\frac{\dot{H}}{H^2}+\frac{\ddot{H}}{H^3} \tag{55}$$

and

$$s=\frac{r-1}{3(q-\frac{1}{2})} \tag{56}$$

From equations $(27),(31),(55)$ and (56) we obtained

$$r = 1 - \frac{3\beta}{\sqrt{2\beta t + c}} + \frac{3\beta^2}{2\beta t + c} \tag{57}$$

and

$$s = \frac{-2\beta\sqrt{2\beta t + c} + 2\beta^2}{2\beta\sqrt{2\beta t + c} - 3(2\beta t + c)} \tag{58}$$

From the above result it is observed that as $t \to \infty$, $(r,s) \to (1,0)$ which gives that the model tends to the ΛCDM model as in recent observation[45].

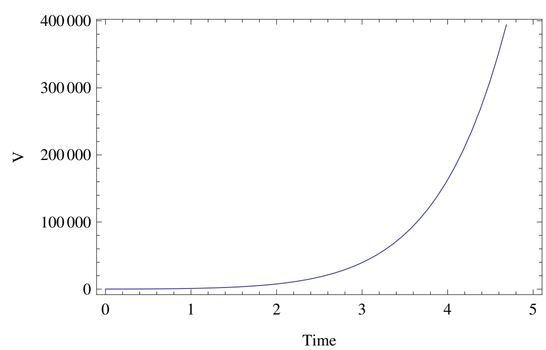

FIGURE 1. The plot of spatial volume V versus cosmic time t, for $\beta = 1$ and $c = 1$.

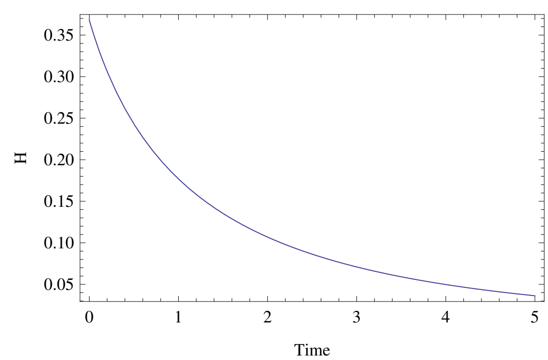

FIGURE 2. The plot of hubble parameter H versus cosmic time t, for $\beta = 1$ and $c = 1$.

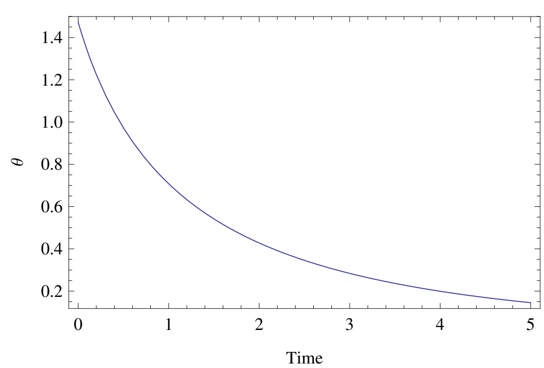

FIGURE 3. The plot of expansion scalar θ versus cosmic time t,for $\beta = 1$ and $c = 1$.

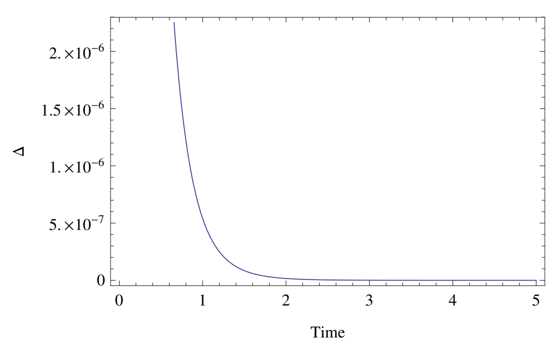

FIGURE 4. The plot of anisotropic parameter Δ versus cosmic time t,for $\beta = 1$, $\lambda = 1$ and $c = 1$.

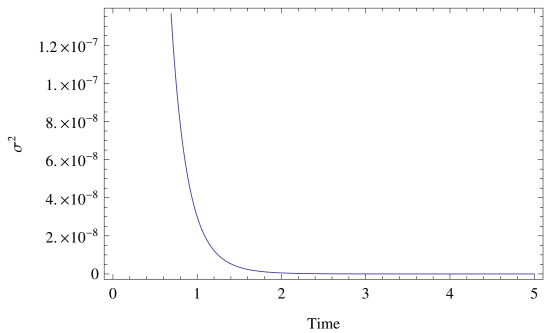

FIGURE 5. The plot of shear scalar σ^2 versus cosmic time t,for $\beta = 1$, $\lambda = 1$ and $c = 1$.

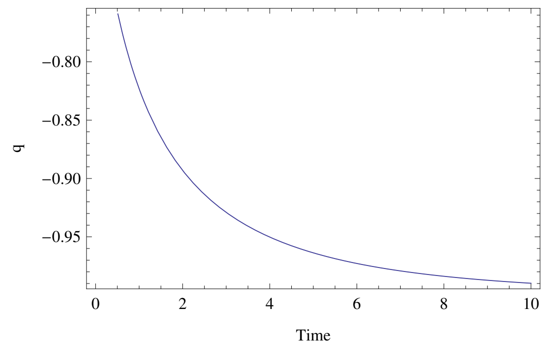

FIGURE 6. The plot of deceleration parameter q versus cosmic time t,for $\beta = 1$ and $c = 1$.

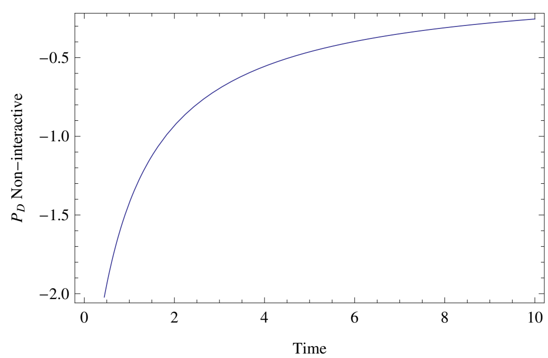

FIGURE 7. The plot of pressure p_D versus cosmic time t for $\beta = 1, c = 1, \lambda = 1$, $\rho_0 = 1$ and $\omega_m = 0.5$ for non-interactive case.

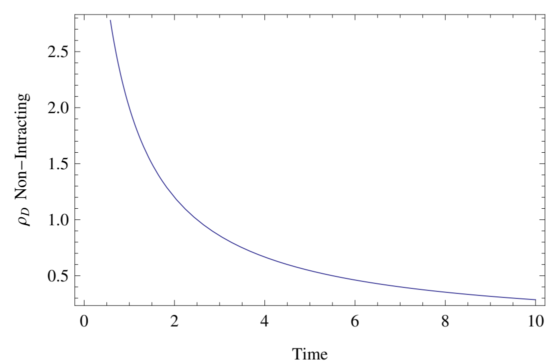

FIGURE 8. The plot of energy density ρ_D versus cosmic time t for $\beta = 1, c = 1, \lambda = 1$, $\rho_0 = 1$ and $\omega_m = 0.5$ for non-interactive case.

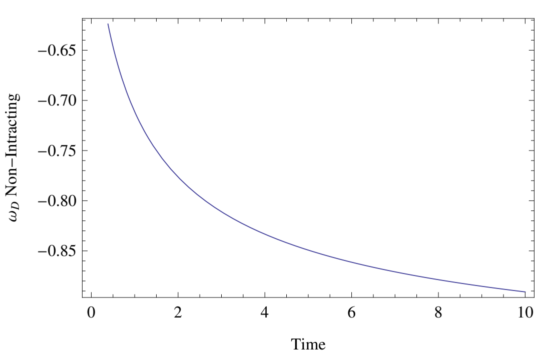

FIGURE 9. The plot of EoS parameter ω_D versus cosmic time t for $\beta = 1, c = 1, \lambda = 1$, $\rho_0 = 1$ and $\omega_m = 0.5$ for non-interactive case.

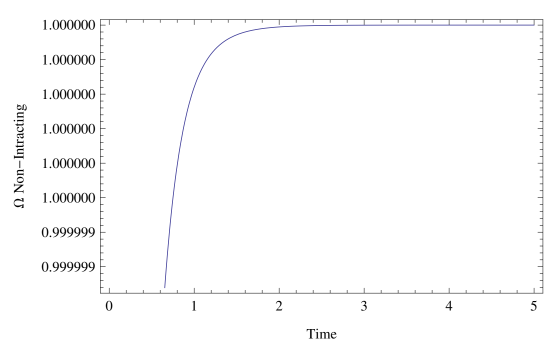

FIGURE 10. The plot of total energy density Ω versus cosmic time t for $\beta = 1, c = 1, \lambda = 1$, $\rho_0 = 1$ and $\omega_m = 0.5$ for non-interactive case.

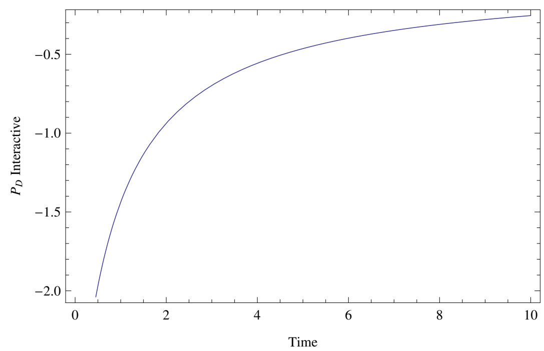

FIGURE 11. The plot of pressure p_D versus cosmic time t for $\beta = 1, c = 1, \lambda = 1$, $\rho_0 = 1$ and $\omega_m = 0.5$ for interactive case.

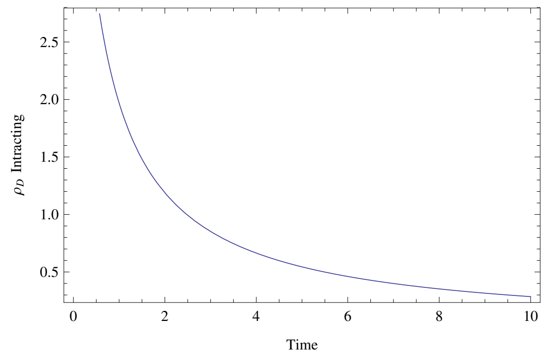

FIGURE 12. The plot of energy density ρ_D versus cosmic time t for $\beta = 1, c = 1, \lambda = 1$, $\rho_0 = 1$ and $\omega_m = 0.5$ for interactive case.

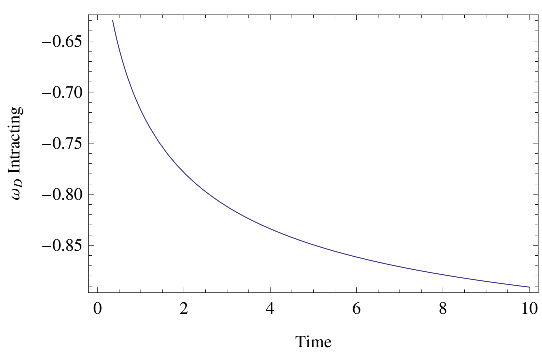

FIGURE 13. The plot of EoS parameter ω_D versus cosmic time t for $\beta = 1, c = 1, \lambda = 1$, $\rho_0 = 1$ and $\omega_m = 0.5$ for interactive case.

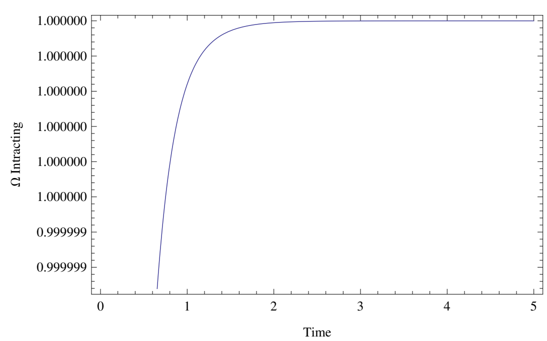

FIGURE 14. The plot of total energy density Ω versus cosmic time t for $\beta = 1, c = 1, \lambda = 1$, $\rho_0 = 1$ and $\omega_m = 0.5$ for interactive case.

7. Physical Interpretation

Figure 1 depicts the spatial volume V versus cosmic time t. It is observed that the behavior of spatial volume V is zero as cosmic time $t = 0$ and increase as t tends to infinity. In figures 2 and 3 shows that both Hubble parameter H and expansion scalar θ positive, decrease with cosmic time t and tends to zero as $t \to \infty$. Figures 4 and 5 shown that the anisotropic parameter Δ and shear scalar σ^2 tends to zero as cosmic time $t \to \infty$. From figure 6 corresponding to the equation (31) it is shows that the deceleration parameter q is decreasing function of cosmic time t. It is observed that $q > 0$ for $t < \frac{\beta^2 - c}{2\beta}$ which indicates that the universe is a decelerating phase and $q < 0$ for $t > \frac{\beta^2 - c}{2\beta}$ which indicates that the universe is a accelerating phase, which is in the good agreement with the recent observations.

Figures 7 and 11 shows that the pressure p_D for both non-interacting and interacting cases it is observed that pressure p_D is negative, which indicates that the universe is a accelerating of expansion. The energy density ρ_D shown in figures 8 and 12 for both cases is divers as $t \to 0$ and becomes zero as $t \to \infty$. From figures 9 and 13 We observed that for both the cases that during the evolution of the universe the behavior of EoS parameter ω_D is a decreasing function of cosmic time t. From equation 40 and 50 we observed that in non-interacting case the total energy density parameter has the same properties as in interacting case.

8. Conclusions

In the present paper, we have studied the system of two fluid scenario for dark energy parameter in the spatial homogenous and anisotropic five dimensional Kaluza-Klein space time field with a barotropic fluid and dark energy by considering a variable deceleration parameter. For the model the spatial volume increase as $t \to \infty$. The anisotropic parameter Δ and shear scalar σ^2 tends to zero as cosmic time $t \to \infty$. The pressure p_D is negative, which indicates that the universe is a accelerating of expansion. The energy density ρ_D is divers as $t \to 0$ and becomes zero as $t \to \infty$. The EoS parameter ω_D is a decreasing function of cosmic time t in both the cases and is always varying in the quintessence region. The total energy density parameter Ω_D tends to one at late time. The value of jerk parameter $j = 1$, deceleration parameter $q = -1$ predict that the universe decelerating expansion to accelerating expansion passes through a transition phase. This solution gives the model tends to the ΛCDM model at late time.

Conflict of Interests

The author(s) declare that there is no conflict of interests.

References

[1] S. Perlmutter, et al., Measurements of the cosmological parameters omega and lambda from the first 7 supernovae at $z \geq 0.35$, Astrophys. J. 483 (1997) 565-581.

[2] S. Perlmutter et al., Discovery of a supernova explosion at half the age of the Universe, Nature 391 (1998) 51-54.

[3] S. Perlmutter et al., Measurements of omega and lambda from 42 high-redshift supernovae, Astrophys. J. 517 (1999) 565-586.

[4] A. G. Riess et al., Observational evidence from supernovae for an accelerating Universe and a cosmological constant, Astrophys. J. 116 (1998) 1009-1038

[5] P. Garnavich, et al.,Constraints on cosmological models from hubble space telescope observations of high-z supernovae, Astrophys. J. 493(1998), L53-L57.

[6] B. P. Schmidt, et al.,The High-Z Supernova Search: Measuring Cosmic Deceleration and Global Cur vature of the Universe Using Type Ia Supernovae, Astrophys. J. 507(1998), 46-63.

[7] N. A. Bachall, et al., The Cosmic Triangle: Revealing the State of the Universe, Science, 284(1999), 1481-1488.

[8] V. Sahni, et al., The case for a positive cosmological Λ term, Int. J. Mod. Phys. D9(2000), 373-444.

[9] P. J. E. Peebles and B. Ratra,The cosmological constant and dark energy, Rev. Mod. Phys. 75(2003), 559.

[10] T. Padmanabhan,Cosmological Constant - the Weight of the Vacuum, Phys. Rept.380(2003), 235-320.

[11] Adam G. Riess. et al., Observational Evidence from Supernova for an Accelerating Universe and a cosmological constant, Astron. J. 116 (1998), 1009-1038.

[12] Adam G. Riess, The Case for an Accelerating Universe from Supernovae, Astron. Soc. Pac. 112 (2000), 1284-1299.

[13] Charles H. Lineweaver, The Cosmic Microwave Background and Observational Convergence in the $\Omega_m - \Omega_\Lambda$ Plane, Astrophys. J. 505 (1998), 69-73.

[14] Max Tegmark et al., Cosmological parameters from SDSS and WMAP, Phys. Rev. D69 (2004), 103501.

[15] Uros. Seljak. et al., Cosmological parameter analysis including SDSS Lyα forest and galaxy bias: Constraints on the primordial spectrum of fluctuations, neutrino mass, and dark energy, Phys. Rev. D 71 (2005), 103515 .

[16] Jennifer K. Adelman-McCarthy, et al., The Fourth Data Release of the Sloan Digital Sky Survey, Astrophys. J. Sup. S. 162 (2006), 38-48.

[17] C. L. Bennett. et al., First Year Wilkinson Microwave Anisotropy Probe (WMAP) observation Preliminary Maps and Results, Astrophys. J. Sup. S. 148 (2003), 1-27.

[18] S. W. Allen et al., Constraints on dark energy from Chandra observations of the largest relaxed galaxy clusters, Mon. Not. Roy. Astron. Soc. 353 (2004), 457-467.

[19] H.R Ghate, Kaluza-Klein Anisotropic Dark Energy Cosmological Model with hybrid Deceleration Parameter, Afr. Rev. Phys. 11 (2016), 0026.

[20] T. Kaluza, On the Unification Problem in Physics, Preuss. Akad. Wiss. Berlin. Math. Phys. K1 (1921), 966-972.

[21] Oskar. Klein, Quantentheorie and Fundamental Relativistic Theory. Zeitschrift Physik. 37 (1926), 895-906.

[22] A. Chodos and S. Detweiler, Where has the Fifth Dimension Gone, Phys. Rev. D 21 (1980), 2167-2170.

[23] Appelquist, T. et al., Modern Klein Theories. Addison Wesley, Reading (1987).

[24] D. R. K. Reddy and R. S. Kumar :Two-fluid atmosphere from decelerating to accelerating FRW dark energy models, Int. J. Theor. Phys. 52 (2013),1362-1369.

[25] R. L. Naidu et al., Five dimensional Kaluz-Klein a bulk viscous string cosmological model in Brans-Dicke scalar-tensor theory of gravitation. Astrophys. Space Sci. 347 (2013),197-201.

[26] D.R.K. Reddy, and G.V.V. Lakshmi, Five Dimensional Radiating Model in Brans-Dicke Theory of Gravitation, Astrophys. Space Sci. 354 (2014), 633-636.

[27] T. Ramprasad et al., Five dimensional FRW bulk viscous cosmological models in Brans Dicke theory of gravitation, Astrophys. Space Sci. 357 (2015), 132-135.

[28] G.S. Khadekar, FRW Viscous Fluid Cosmological Model with Time-dependent Inhomogeneous equation of state, Int. J. of Mod. Phys. 15 (2018), 1830001-1830015.

[29] Namrata I. Jain • Shyamsunder S., Kaluza-Klein Bulk Viscous Cosmological Model with Time Dependent Gravitational Constant and Cosmological Constant. Int. J. Theor. Phys. 54 (2015), 2991-3003.

[30] G.S. Khadekar, Vrushali Patki, :Kaluza-Klein Type Robertson Walker Cosmological Model with Dynamical Cosmological Term Λ, Int. J. Theor. Phys. 47 (2008), 1751-1763 .

[31] M. Sharif , Farida Khanum, Kaluza-Klein cosmology with varying G and Λ, Astrophys. Space Sci. 334 (2011),209-214.

[32] R. Venkateswarlu and K.P Kumar, Higher Dimensional FRW Cosmological Models in Self-Creation Theory, Astrophys. Space Sci. 301 (2006), 73-77.

[33] G. S. Khadekar, Vaishali Kamdi, Higher Dimensional Cosmological Model Of the Universe With Variable Equation of State Parameter in the Presence of G and Λ, Rom. J. Phys. 55 (2010), 871-880 .

[34] G.C. Samanta, and S.N. Dhal., Higher Dimensional Cosmological Models Filled with Perfect Fluid in $f(R,T)$ Theory of Gravity, Int. J. Theor. Phys. 52 (2013), 1334-1344.

[35] H. Amirhashchi, A. Pradhan and B. Saha, An Interacting Two-Fluid Scenario for Dark Energy in an FRW Universe, Chin. Phys. Lett. 28 (2011),039801 .

[36] B. Saha, H. Amirhashchi and A. Pradhan, Two-Fluid Scenario for Dark Energy Models in an FRW Universe-Revisited, Astrophys. Space Sci. 342 (2012), 257-267.

[37] H. Amirhashchi, A. Pradhan and H. Zainuddin, An interacting and non-interacting two-fluid dark energy models in FRW universe with time dependent deceleration parameter, Int. J. Theor. Phys. 50 (2011), 3529-3543.

[38] A. Pradhan, H. Amirhashchi and B. Saha, An interacting and non-interacting two-fluid scenario for dark energy in FRW universe with constant deceleration parameter, Astrophys. Space Sci. 333 (2011), 343-350.

[39] V.B.Raut, et al., Kaluza-Klein cosmological models with anisotropic dark energy and special form of deceleration parameter, Int. J. Sci. Eng. R. 6 (2015), 2229.

[40] R.K. Tiwari, et al., A cosmological model with variavle deceleration parameter,Afr. Rev. Phys. 10(2015) 0048.

[41] U.K. Sharma, et al., Stability of LRS Bianchi type-I cosmological modelsin $f(R,T)$ gravity, Rev. Astron. Astrophys. 19 (2019), 055-068.

[42] R.K. Tiwari, et al., Scenario of two fluid dark energy models in Bianchi type-III universe, Int. J. of Mod. Phys. 15, (2018) 1850189.

[43] M. Visser, Jerk, snap, and the cosmological equation of state, Class. Quantum Grav. 21 (2004) 2603-2605.

[44] V. Sahni, et al., Statefinder-A new geometrical diagnostic of dark energy, J. Exp. Theor. Phys. Lett. 77 (2003) 201-206.

[45] Nasr Ahmed, et al., Bianchi Type-V Cosmology in $f(R,T)$ Gravity with $\Lambda(T)$, Int. J. Theor. Phys. 53 (2014) 289-306.