SOME RESULTS AND APPLICATIONS ON CONFORMABLE FRACTIONAL KAMAL TRANSFORM

R. S. TEPPAWAR[1,*], R. N. INGLE[2] AND S. N. THORAT[1]

[1]P.G Department of Mathematics, N.E.S. Science College, Nanded - 431602, (MH), India

[2]Department of Mathematics, Bahirji Smarak Mahavidyalaya, Basmathnagar, Hingoli - 431512, (MH), India

Abstract. In this paper, we derive conformable fractional Kamal transform from the classical Kamal transform and we present their properties. We discuss the conformable fractional kamal transform of some functions and relationship with conformable fractional Laplace transform. Moreover we solve the general analytical solution of a general conformable Bernoulli's fractional differential equation by this new transform and Adomain polynomial method. Also we use this method to find the solution of linear and nonlinear conformable fractional differential equations.

Keywords: conformable fractional derivative; Kamal transform; Adomain decomposition method; fractional linear and non-linear differential equation.

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1. INTRODUCTION

The theory of fractional derivation has known great importance in mathematical research these last decades. The definition of fractional derivative don't have a standard form. But the most used definitions are those from the integration. Caputo definition, Riemann– Liouville,

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*Corresponding author

E-mail address: rajeshteppawar@gmail.com

Hadamard, Grunwald [22, 23, 24]. Here, these fractional derivatives do not provide some properties of algebra of derivative. To overcome these difficulties, Khalil et al.[18], came up with an idea that extends the limit definition of the derivative. He derived some results of fractional derivative by using his new definition of fractional derivative. Almeida et al[7]. introduced different definition of the fractional derivative. He also discussed some important results of fractional derivative by using his definition if fractional derivative.

Definition 1.1. [18] If $\phi \colon [0, \infty) \to \mathbb{R}$ be a function and $\forall \ \alpha \in (0, 1)$, then the conformable fractional derivative of ϕ of order α is defined as

$$D^\alpha(\phi)(t) = \lim_{\mu \to 0} \frac{\phi\left(t + \mu t^{1-\alpha}\right) - \phi(t)}{\mu}, \quad t > 0 \tag{1.1}$$

If $D_\alpha(\phi(t))$ and $\lim_{\mu \to 0^+} \phi^{(\alpha)}(t)$ is exist in $(0, c)$, where $c > 0$, then α-derivative is defined as $\phi^{(\alpha)}(0) = \lim_{t \to 0^+} \phi^{(\alpha)}(t)$.

Definition 1.2. [18] Let $\phi \colon [0, \infty) \to \mathbb{R}$ and $\alpha \in (n, n+1]$ be an n-differentiable at t, where $t > 0$. Then conformable fractional derivative of ϕ is defined as

$$D^{n\alpha}(\phi)(t) = \lim_{\mu \to 0} \frac{\phi^{\lceil \alpha \rceil - 1}\left(t + \mu t^{\lceil \alpha \rceil - \alpha}\right) - \phi^{\lceil \alpha \rceil - \alpha}(t)}{\mu}, \quad n - 1 < \alpha \le n, t > 0 \tag{1.2}$$

where $n \in \mathbb{N}$ and $\lceil \alpha \rceil$ is the smallest integer number greater than or equal to α. Provided $D^{n\alpha}(\phi)(0) = \lim_{\mu \to 0} D^{n\alpha}(\phi)(t), \phi(t)$ is n-differentiable and $D^{n\alpha}(\phi)(0) = \lim_{\mu \to 0} D^\alpha(\phi)(t), \phi(t)$ exists.

Definition 1.3. [18] Let , $0 \le \gamma \le t$ and ϕ be a function defined on $(\gamma, t]$, then New α-fractional integral is defined by

$$I^\alpha \phi(t) = \int_0^t \phi(t) t^{\alpha - 1} dt, \quad 0 < \alpha \le 1$$

provided integral exists.

Remark 1.4. [18] The most useful result is that

$$D^{n\alpha}(\phi)(t) = t^{\lceil \alpha \rceil - \alpha} \phi^{\lceil \alpha \rceil}(t). \tag{1.3}$$

where $\alpha \in (n, n+1]$ and ϕ is an $(n+1)$-differentiable function at $t > 0$.

Recently introduced Kamal transform by Abdelilah Kamal and H. Sedeeg [2] . It is defined for

functions of exponential order in the set A by:

$$A = \{\phi(t) : \exists \, M, \lambda_1, \lambda_2 > 0, |\phi(t)| < M e^{\frac{|t|}{\lambda_i}}, if \, t \in (-1)^i \times [0, \infty), i = 1, 2\} \qquad (1.4)$$

where M is a constant but finite number, λ_1, λ_2 are finite or infinite. The Kamal transform is defined by the integral equation and it is denoted by $K(.)$

$$K\{\phi(t)\} = G(v) = \int_0^\infty e^{\frac{-t}{v}} \phi(t) dt, \quad v > 0. \qquad (1.5)$$

The most useful rules of classical Kamal transform are [2]

   (1) **Shifting property:** If the Kamal transform of functions $\phi(t)$ is $G(v)$, then Kamal transform of functions $e^{at}\phi(t)$ is

$$K(e^{at}\phi(t)) = G\left(\frac{v}{1 - av}\right). \qquad (1.6)$$

   (2) **Kamal transform of $t\phi(t)$:** If $K\{\phi(t)\} = G(v)$, then

$$K(t\phi(t)) = v^2 \frac{d}{dv} G(v). \qquad (1.7)$$

   (3) **Kamal transform of the derivatives of the function $\phi(t)$:** If $K\{\phi(t)\} = G(v)$, then

$$K\{(\phi(t)'\} = \frac{1}{v} G(v) - \phi(0). \qquad (1.8)$$

   (4) **Convolution property:** If the Kamal transform of functions $\phi(t)$ and $\psi(t)$ are $\Phi(v)$ and $\Psi(v)$ respectively, then the convolution of their

$$K\{(\phi * \psi)(t)\} = \Phi(v)\Psi(v), \qquad (1.9)$$

where $\phi, \psi \colon [0, \infty) \to \mathbb{R}$ are given functions, $\phi * \psi$ is the convolution product of ϕ and ψ, $K\{\phi(t)\} = \Phi(v)$ and $K\{\psi(t)\} = \Psi(v)$.

The conformable fractional Laplace transform (CFLT) for a given function $\phi \colon [0, \infty) \to \mathbb{R}$ at $t > 0$ is defined by [1, 11]

$$L_\alpha\{\phi(t)\} = \Phi_\alpha(s) = \int_0^\infty e^{-s\frac{t^\alpha}{\alpha}} \phi(t) t^{\alpha - 1} dt, \quad 0 < \alpha \leq 1. \qquad (1.10)$$

In particular, if $\alpha = 1$, then Eq. (1.10) is convert to the definition of the fractional Laplace transform:

$$L\{\phi(t)\} = \Phi(s) = \int_0^\infty e^{-st}\phi(t)dt. \tag{1.11}$$

Note: The relationship between Kamal transform $G(v) = K\{\phi(t)\}$ and Laplace transform $\Phi(s) = L\{\phi(t)\}$ [6, 29]

$$G\left(\frac{1}{v}\right) = \Phi(s), \tag{1.12}$$

and also between conformable fractional laplace transform (CFLT) $L_\alpha\{\phi(t)\} = \Phi_\alpha(s)$ and Laplace transform $\Phi(s) = L\{\phi(t)\}$ is given by [1]

$$\Phi_\alpha(s) = L\left\{\phi\left((\alpha t)^{\frac{1}{\alpha}}\right)\right\}. \tag{1.13}$$

The most important results for the conformable fractional laplace transform (CFLT) are [1, 11].

If $\phi\colon [0,\infty) \to \mathbb{R}$ is a given function and $0 < \alpha \leq 1$, then following are

(1) If $\phi\colon [0,\infty) \to \mathbb{R}$ be real valued differentiable function and $0 < \alpha \leq 1$, then

$$L_\alpha\{D^\alpha\phi(t)\} = s\Phi_\alpha(s) - \phi(0), \quad s > 0, \tag{1.14}$$

(2) If $\phi\colon [0,\infty) \to \mathbb{R}$ be the function and $0 < \alpha \leq 1$, then

$$L_\alpha\{I^\alpha\phi(t)\} = \frac{\Phi_\alpha(s)}{s}, \quad s > 0, \tag{1.15}$$

2. PROPERTIES OF CONFORMABLE FRACTIONAL KAMAL TRANSFORM (CFKT)

In this section, we introduce the definition of conformable fractional kamal transform (CFKT) and derive some properties and rules of this fractional transform for some functions and a relationship between conformable fractional kamal transform (CFKT) and conformable fractional laplace transform (CFLT) are determinate which are important role for solving conformable fractional linear and nonlinear differential equations (CFDEs).

Definition 2.1. Consider the conformable fractional kamal transform for functions of exponential order in the set A by

$$A = \left\{\phi(t) : \exists\, M, \lambda_1, \lambda_2 > 0, |\Phi(t)| < Me^{\frac{|t|}{\lambda_i}}, \; if\; t \in (-1)^i \times [0,\infty), i = 1, 2\right\} \tag{2.1}$$

where M is a constant but finite number, λ_1, λ_2 are finite or infinite, then the conformable fractional kamal transform (CFKT) of ϕ can be defined as

$$K_\alpha\{\phi(t)\} = G_\alpha(v) = \int_0^\infty e^{\frac{-t^\alpha}{v\alpha}} \phi(t) d_\alpha t, \quad v > 0 \tag{2.2}$$

where $d_\alpha t = t^{\alpha-1} dt, \quad 0 < \alpha \le 1$ and provided the integral exists.

Theorem 2.2. If $\phi: [0,\infty) \to \mathbb{R}$ such that $K_\alpha\{\phi(t)\} = G_\alpha(v)$ and $0 < \alpha \le 1$, then

$$G_\alpha(v) = K\left\{\phi\left((\alpha t)^{\frac{1}{\alpha}}\right)\right\} \tag{2.3}$$

Proof By using the Definition 2.1, we have

$$G_\alpha(v) = \int_0^\infty e^{\frac{-t^\alpha}{v\alpha}} \phi(t) d_\alpha t$$

Putting $u = \frac{t^\alpha}{\alpha}$, then $du = t^{\alpha-1} dt$, we have

$$G_\alpha(v) = \int_0^\infty e^{\frac{-u}{v}} \phi\left((\alpha u)^{\frac{1}{\alpha}}\right) du = \int_0^\infty e^{\frac{-t}{v}} \phi\left((\alpha t)^{\frac{1}{\alpha}}\right) dt$$

$$= K\left\{\phi\left((\alpha t)^{\frac{1}{\alpha}}\right)\right\}.$$

Theorem 2.3. If $\phi: [0,\infty) \to \mathbb{R}$ be a given function and $0 < \alpha \le 1$, then

$$K_\alpha\{\phi(t)\} = G_\alpha\left(\frac{1}{v}\right), \tag{2.4}$$

where $G_\alpha(v)$ and $\Phi_\alpha(s)$ are the conformable fractional kamal transform (CFKT) and CFLTs, respectively.

Proof By Using Definition 2.1, we have

$$K_\alpha\{\phi(t)\} = G_\alpha(v) = \int_0^\infty e^{\frac{-t^\alpha}{v\alpha}} \phi(t) d_\alpha t.$$

Putting $u = \frac{t^\alpha}{\alpha}$, then $du = t^{\alpha-1} dt$, and substitute in above equation, then by using equations (1.5) and (1.6), we have

$$G_\alpha(v) = \int_0^\infty e^{\frac{-u}{v}} \phi\left((\alpha u)^{\frac{1}{\alpha}}\right) du$$

$$= \int_0^\infty e^{-\frac{t}{v}} \phi\left((\alpha t)^{\frac{1}{\alpha}}\right) dt = L\{\phi\left((\alpha t)^{\frac{1}{\alpha}}\right)\}_{s \to \frac{1}{v}}$$

$$= \Phi_\alpha\left(\frac{1}{v}\right).$$

Hence proved.

Theorem 2.4. If $\phi: [0,\infty) \to \mathbb{R}$ be differentiable function and $0 < \alpha \leq 1$, then

$$K_\alpha\{D^\alpha\phi(t)\} = \frac{1}{v}G_\alpha(v) - \phi(0). \tag{2.5}$$

Proof By Applying Theorem 2.3 and equation (1.14), we have

$$K_\alpha\{D^\alpha\phi(t)\} = L_\alpha\{D^\alpha\phi(t)\}_{s\to\frac{1}{v}}$$

$$= \frac{1}{v}G_\alpha(v) - \phi(0).$$

Hence the proof is complete the Theorem 2.4.

If $\alpha = \frac{1}{n+1}(n \in \mathbb{N})$, where α satisfies $0 < \alpha = \frac{1}{n+1} \leq 1$, then we have $0 < n\alpha = \frac{n}{n+1} \leq 1$ in the above Theorem 2.4 and we have generalised follow Theorem.

Theorem 2.5. Let $\phi: [0,\infty) \to \mathbb{R}$ be n-differentiable function and $0 < \alpha \leq 1$, then

$$K_\alpha\{D^{n\alpha}\phi(t)\} = \frac{1}{v^n}G_\alpha(v) - \frac{1}{v^{n-1}}\phi(0). \tag{2.6}$$

Proof Proof is follows by using induction and Theorem 2.4.

Note: In the following example we solve the fractional kamal transform for certain functions.

Example 2.6. Consider the $a, c \in \mathbb{R}$, then by using Theorem 2.4, we have

(1) $K_\alpha\{c\} = v$.

(2) $K_\alpha\left\{e^{a\frac{t^\alpha}{\alpha}}\right\} = K\left\{e^{\frac{a}{\alpha}(\alpha t)^{\frac{1}{\alpha}\alpha}}\right\} = K\{e^{at}\} = \frac{v}{1-av}, \quad v > \frac{1}{a}$.

(3) $K_\alpha\left\{\sin\left(a\frac{t^\alpha}{\alpha}\right)\right\} = K\left\{\sin\left(\frac{a}{\alpha}(\alpha t)^{\frac{1}{\alpha}\alpha}\right)\right\} = K\{\sin(at)\} = \frac{av^2}{1+a^2v^2}, \quad v > 0$.

(4) $K_\alpha\left\{\cos\left(a\frac{t^\alpha}{\alpha}\right)\right\} = K\left\{\cos\left(\frac{a}{\alpha}(\alpha t)^{\frac{1}{\alpha}\alpha}\right)\right\} = K\{\cos(at)\} = \frac{v}{1+a^2v^2}, \quad v > 0$.

(5) $K_\alpha\left\{\sinh\left(a\frac{t^\alpha}{\alpha}\right)\right\} = K\left\{\sinh\left(\frac{a}{\alpha}(\alpha t)^{\frac{1}{\alpha}\alpha}\right)\right\} = \frac{av^2}{1-a^2v^2}, \quad v > \frac{1}{|a|}$.

(6) $K_\alpha\left\{\cosh\left(a\frac{t^\alpha}{\alpha}\right)\right\} = K\left\{\cosh\left(\frac{a}{\alpha}(\alpha t)^{\frac{1}{\alpha}\alpha}\right)\right\} = \frac{v}{1-a^2v^2}, \quad v > \frac{1}{|a|}$.

(7) $K_\alpha\left\{\left(\frac{t^{n\alpha}}{\alpha^n}\right)\right\} = K\left\{\frac{(\alpha t)^{\frac{1}{\alpha}n\alpha}}{\alpha^n}\right\} = K\{t^n\} = \Gamma(n+1)v^{n+1}, \quad v > 0$.

Where $\Gamma()$ denotes to the gamma function.

Theorem 2.7. Let $\phi, \psi: [0,\infty) \to \mathbb{R}$ be the functions such that $K_\alpha\{\phi(t)\} = \Phi_\alpha(v), K_\alpha\{\psi(t)\} = \Psi_\alpha(v)$ and $L_\alpha\{\phi(t)\} = \Phi_\alpha(s)$, for $\sigma, \zeta \in \mathbb{R}$ and $0 < \alpha \leq 1$, we have

(1) Linearly property

$$K_\alpha\{\sigma\phi(t) + \zeta\psi(t)\} = \sigma\Phi_\alpha(v) + \zeta\Psi_\alpha(v). \tag{2.7}$$

(2) Shifting property

$$K_\alpha\left\{e^{a\frac{t^\alpha}{\alpha}}\phi(t)\right\} = \frac{1-av}{v}. \tag{2.8}$$

(3) Integration property

$$K_\alpha\{I_\alpha\phi(t)\} = v\Phi_\alpha(\frac{1}{v}). \tag{2.9}$$

(4) Convolution property

$$K_\alpha\{(\phi * \psi)(t)\} = \Phi_\alpha(v)\Psi_\alpha(v). \tag{2.10}$$

(5) Power product property

$$K_\alpha\left\{\left(\frac{t^\alpha}{\alpha}\phi(t)\right)\right\} = v^2\frac{d}{dv}\Phi_\alpha(v). \tag{2.11}$$

Proof

(1) Proof of (a) follows from Definition 2.1.

(2) By Using Theorem 2.3, we have

$$K_\alpha\left\{e^{a\frac{t^\alpha}{\alpha}}\phi(t)\right\} = L\left\{e^{at}\phi\left((\alpha t)^{\frac{1}{\alpha}}\right)\right\}_{s\to\frac{1}{v}}$$

$$= \Phi_\alpha(s-a)_{s\to\frac{1}{v}}$$

$$= \Phi_\alpha\left(\frac{1-av}{v}.\right)$$

(3) By using Theorem 2.3 and equation (1.12)

$$K_\alpha\{I_\alpha\phi(t)\} = L_\alpha\{I_\alpha\phi(t)\}_{s\to\frac{1}{v}}$$

$$= \left\{\frac{\Phi_\alpha(s)}{s}\right\}_{s\to\frac{1}{v}} = \frac{\Phi_\alpha(1/v)}{1/v}$$

$$= v\Phi_\alpha\left(\frac{1}{v}\right).$$

(4) By using Theorem 2.3 and equation (1.7)

$$
\begin{aligned}
K_\alpha\{(\phi * \psi)(t)\} &= K\left\{(\phi * \psi)\left((\alpha t)^{\frac{1}{\alpha}}\right)\right\} \\
&= K\left\{(\phi((\alpha t)^{\frac{1}{\alpha}})\right\} K\left\{(\psi((\alpha t)^{\frac{1}{\alpha}})\right\} \\
&= \Phi_\alpha(v)\Psi_\alpha(v).
\end{aligned}
$$

(5) By using Theorem 2.3 and equation (1.9)

$$
\begin{aligned}
K_\alpha\left\{\left(\frac{t^\alpha}{\alpha}\phi(t)\right)\right\} &= K\left\{t\phi((\alpha t)^{\frac{1}{\alpha}})\right\} \\
&= v^2 \frac{d}{dv}\Phi_\alpha(v).
\end{aligned}
$$

Hence completes the proof of Theorem 2.7.

3. APPLICATIONS OF CONFORMABLE FRACTIONAL KAMAL TRANSFORM (CFKT)

In this section, we solve the general analytical solution of the generalized conformable Bernoulli's fractional differential equations by using conformable fractional kamal transform (CFKT) and ADM. The ADM in [3, 4, 25, 26, 27] that used a very spontaneous method and has been successfully applied to solve nonlinear ordinary and fractional differential equations of various kinds. The ADM calculates the solutions of nonlinear equations as infinite series solution determined. Each term of these series is a generalized polynomial called Adomian's polynomial. The convergence, the order of convergence and the principle and convergence of ADM has been studied by Cherruault, Babolian and Biazar, Jiao et al. respectively [10, 9, 30]. We solve the general solutions of some linear and nonlinear conformable fractional differential equations (CFDEs) to new approach and to verify that this method can be applied successfully for finding the general solutions of many other linear and nonlinear conformable fractional differential equations (CFDEs).

Example 3.1. Consider the conformable Bernoulli's fractional differential equation of form

$$
D^\alpha y + P(t)y = Q(t)y^n + R(t), \quad 0 < \alpha \leq 1. \tag{3.1}
$$

where $P(t), Q(t)$ and $R(x)$ are α-differentiable functions and $n \in \mathbb{N}$. Clearly this equation is a nonlinear if $n \neq 0$ or 1.

Applying K_α on both sides of equation (3.1), we get

$$Y_\alpha(v) = K_\alpha[Q(t)y^n] + K_\alpha[R(t)] - K_\alpha[P(t)y] \tag{3.2}$$

On taking the inverse K_α^{-1} of both sides of equation (3.2), we get

$$y(t) = K_\alpha^{-1}\{K_\alpha[Q(t)y^n]\} + K_\alpha^{-1}\{K_\alpha[R(t)]\} - K_\alpha^{-1}\{K_\alpha[P(t)y]\} \tag{3.3}$$

An infinite general solution $y(t)$ of equation (3.1) as

$$y(t) = \sum_{k=0}^{\infty} y_k(t) = y_0 + y_1 + y_2 + \cdots \tag{3.4}$$

Now equation (3.3) can be rewritten as

$$\sum_{k=0}^{\infty} y_k(t) = K_\alpha^{-1}\{K_\alpha[Q(t)A_k]\} + K_\alpha^{-1}\{K_\alpha[R(t)]\} - K_\alpha^{-1}\{K_\alpha[P(t)B_k]\}$$

where A_k and B_k are the Adomian polynomials of nonlinear functions $\phi(y) = y^n$ and $\psi(y) = y$, respectively.

Now define

$$A_0 = \phi(y_0),$$

$$A_1 = y_1\phi'(y_0),$$

$$A_2 = y_1\phi'(y_0) + \frac{1}{2!}y_1^2\phi''(y_0),$$

$$A_3 = y_3\phi'(y_0) + y_1y_2\phi''(y_0) + \frac{1}{3!}y_1^3\phi'''(y_0),\dots,$$

and

$$B_0 = \psi(y_0),$$

$$B_1 = y_1\psi'(y_0),$$

$$B_2 = y_1\psi'(y_0) + \frac{1}{2!}y_1^2\psi''(y_0),$$

$$B_3 = y_3\psi'(y_0) + y_1y_2\psi''(y_0) + \frac{1}{3!}y_1^3\psi'''(y_0),\dots,$$

Now, the recursive relation of $\{y_k\}_{n=0}^{\infty}$ as fallows

$$y_0(t) = K_\alpha^{-1}\{K_\alpha[R(t)]\}$$

$$y_1(t) = K_\alpha^{-1}\{K_\alpha[Q(t)A_0]\} - K_\alpha^{-1}\{K_\alpha[P(t)B_0]\}$$

$$y_2(t) = K_\alpha^{-1}\{K_\alpha[Q(t)A_1]\} - K_\alpha^{-1}\{K_\alpha[P(t)B_1]\}$$

$$y_3(t) = K_\alpha^{-1}\{K_\alpha[Q(t)A_2]\} - K_\alpha^{-1}\{K_\alpha[P(t)B_2]\}$$

$$\vdots$$

Now the general recursive relation can be found by:

$$y_0(t) = K_\alpha^{-1}\{K_\alpha[R(t)]\}$$

$$y_{k+1}(t) = K_\alpha^{-1}\{K_\alpha[Q(t)A_k]\} - K_\alpha^{-1}\{K_\alpha[P(t)B_k]\}.$$

$$where\ n = 0,1,2,3,...$$

Example 3.2. Consider the initial value problem of conformable fractional differential equation(CFDE)

$$D^\alpha y = e^{-y(t)}, \quad y(0) = 0, \quad 0 < \alpha \leq 1. \tag{3.5}$$

By applying K_α of both sides of above equation (3.5), we have

$$\frac{1}{v}Y_\alpha(v) - y(0) = K_\alpha\left[e^{-y(t)}\right]$$

$$Y_\alpha(v) = vK_\alpha[e^{-y(t)}] \tag{3.6}$$

Taking inverse transform K_α^{-1} of both sides of equation (3.6), we get the solution of this problem as

$$y(t) = K_\alpha^{-1}\{vK_\alpha[e^{-y(t)}]\} \tag{3.7}$$

By using ADM, then the equation (3.7) can be rewritten as

$$\sum_{k=0}^{\infty} y_k(t) = K_\alpha^{-1}\left\{K_\alpha\left\{\sum_{k=0}^{\infty} A_k(t)\right\}\right\},$$

where $\{A_k\}_{k=0}^{\infty}$ are the Adomian polynomials representing to $\phi(y) = e^{-y(t)}$. Then the recursive relation as follows

$$y_0(t) = 0$$

$$y_1(t) = K_\alpha^{-1}\{vK_\alpha[A_0]\}$$

$$y_2(t) = K_\alpha^{-1}\{vK_\alpha[A_1]\}$$

$$y_3(t) = K_\alpha^{-1}\{vK_\alpha[A_2]\}$$

$$\vdots$$

Thus, the general recursive relation is given by

$$y_0(t) = 0$$

$$y_{k+1}(t) = K_\alpha^{-1}\{K_\alpha[A_k]\}, \quad k = 0, 1, 2, 3, \ldots$$

By using this recursive relation, we have

$$y_1(t) = K_\alpha^{-1}\left\{vK_\alpha\left[e^{-y_0}\right]\right\} = K_\alpha^{-1}\{v^2\} = \frac{t^\alpha}{\alpha},$$

$$y_2(t) = K_\alpha^{-1}\left\{vK_\alpha\left[-e^{-y_0}y_1\right]\right\} = K_\alpha^{-1}\{v^3\} = -\frac{1}{2}\frac{t^{2\alpha}}{\alpha^2},$$

$$y_3(t) = K_\alpha^{-1}\left\{vK_\alpha\left[\frac{e^{-y_0}}{2}(-2y_2 + y_1^2)\right]\right\} = \frac{1}{3}\frac{t^{3\alpha}}{\alpha^3},$$

$$\vdots$$

Then the general solution of equation (3.5) is series form and with the help of we obtain (see Figure 1)

$$y(t) = \sum_{n=0}^{\infty} y_n(t) = \frac{t^\alpha}{\alpha} - \frac{1}{2}\frac{t^{2\alpha}}{\alpha^2} + \frac{1}{3}\frac{t^{3\alpha}}{\alpha^3} + \cdots$$

$$= \ln\left(\frac{t^\alpha}{\alpha} + 1\right).$$

Example 3.3. Consider the following non-linear CFDE

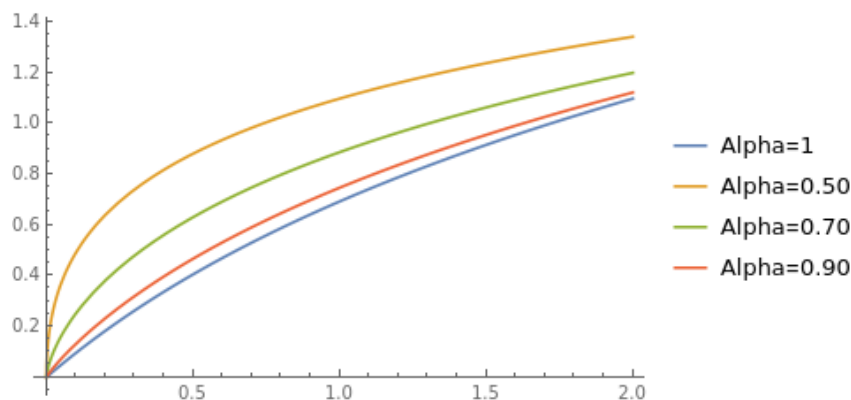$$D^\alpha y - 1 = y^2, \quad y(0) = 0, \quad 0 < \alpha \le 1. \tag{3.8}$$

FIGURE 1. The solution of equation (3.5) considering several values of α.

This equation is directly solved by using equation (1.1) to convert into the ordinary differential equation as

$$t^{1-\alpha}\frac{dy}{dt} = y^2 + 1, \quad y(0) = 0, \quad 0 < \alpha \leq 1.$$

Now it is solve by separable, we have

$$\int \frac{dy}{y^2+1} = \int t^{1-\alpha}dt, y(0) = 0 \Rightarrow y(t) = \tan\left(\frac{t^\alpha}{\alpha}\right).$$

This problem can also solved by using our technique as in above. Comparing equation (3.8) with equation (3.1), we have $P(t) = 0, Q(t) = R(t) = 1$ and $n = 2$. Then by applying K_α on both sides of equation (3.8), we get

$$Y_\alpha(v) = v^2 + K_\alpha[y^2] \tag{3.9}$$

Taking inverse K_α^{-1} of both sides of equation (3.9), we get the solution of this problem as

$$y(t) = \frac{t^\alpha}{\alpha} + K_\alpha^{-1}\left\{K_\alpha[y^2]\right\} \tag{3.10}$$

By using ADM, then the equation (3.10) can be rewritten as

$$\sum_{k=0}^{\infty} y_k(t) = \frac{t^\alpha}{\alpha} + K_\alpha^{-1}\left\{K_\alpha\left\{\sum_{k=0}^{\infty} A_k(t)\right\}\right\} \tag{3.11}$$

where $\{A_k\}_{k=0}^{\infty}$ are the Adomian polynomials of function $\phi(y) = y^2$. Then the recursive relation as follows:

$$y_0(t) = \frac{x^\alpha}{\alpha}$$

$$y_1(t) = K_\alpha^{-1}\{K_\alpha[A_0]\}$$

$$y_2(t) = K_\alpha^{-1}\{K_\alpha[A_1]\}$$

$$y_3(t) = K_\alpha^{-1}\{K_\alpha[A_2]\}$$

$$\vdots$$

Thus, the general recursive relation is given by

$$y_0(t) = \frac{t^\alpha}{\alpha}$$

$$y_{k+1}(t) = K_\alpha^{-1}\{K_\alpha[A_k]\}, \quad k = 0, 1, 2, 3, \ldots$$

By using this recursive relation, we have

$$y_1(t) = K_\alpha^{-1}\left\{K_\alpha[y_0^2]\right\} = K_\alpha^{-1}\{2!\} = \frac{1}{3}\frac{t^\alpha}{\alpha},$$

$$y_2(t) = K_\alpha^{-1}\{K_\alpha[2y_0y_1]\} = \frac{2(4!)}{3}K_\alpha^{-1}\{v^6\} = \frac{2}{15}\frac{t^{5\alpha}}{\alpha^5},$$

$$y_3(t) = K_\alpha^{-1}\left\{K_\alpha[2y_0y_2 + y_1^2]\right\} = \frac{17}{315}\frac{t^{7\alpha}}{\alpha^7},$$

$$\vdots$$

Then the general solution of equation (3.8) is series form and with the help of we obtain (see Figure 2)

$$y(x) = \sum_{n=0}^{\infty} y_n(t) = \frac{t^\alpha}{\alpha} + \frac{1}{3}\frac{t^{3\alpha}}{\alpha^3} + \frac{2}{15}\frac{t^{5\alpha}}{\alpha^5} + \cdots$$

$$= \tan\left(\frac{t^\alpha}{\alpha}\right).$$

Ntoe: If $\alpha = 1$ in the equation (3.8), then the equation becomes ordinary differetial equation it's exact solution is $\tan(t)$.
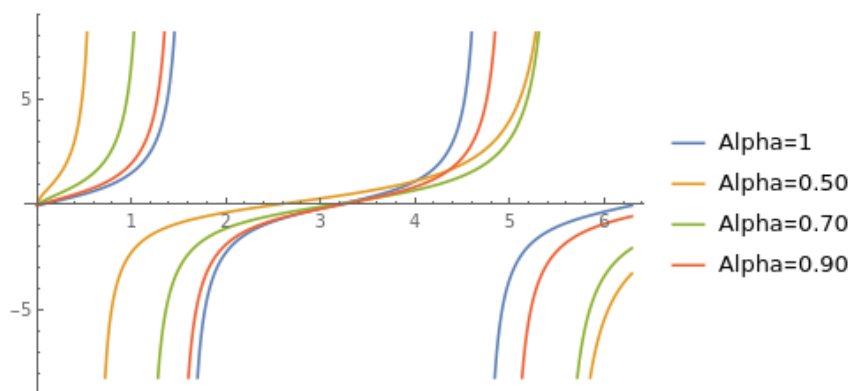
FIGURE 2. The solution of equation (3.8) considering several values of α.

Example 3.4. Consider a Logistic or Verhulst CFDE

$$D^\alpha y = y^2 - y, \quad y(0) = -1, \quad 0 < \alpha \leq 1. \tag{3.12}$$

By applying K_α of both sides of above equation (3.12), we have

$$\frac{1}{v}Y_\alpha(v) + 1 + Y_\alpha(v) = K_\alpha\left[y^2\right]$$

$$Y_\alpha(v) = \frac{v}{1+v}K_\alpha[y^2] - \frac{v}{1+v} \tag{3.13}$$

Taking inverse transform K_α^{-1} of both sides of equation (3.13), we get the solution of this problem as

$$y(t) = K_\alpha^{-1}\left\{\frac{v}{1+v}K_\alpha[y^2]\right\} - e^{\frac{-t^\alpha}{\alpha}} \tag{3.14}$$

By using ADM, then the equation (3.14) can be rewritten as

$$\sum_{k=0}^{\infty} y_k(t) = -e^{\frac{-t^\alpha}{\alpha}} + K_\alpha^{-1}\left\{K_\alpha\left\{\sum_{k=0}^{\infty} A_k(t)\right\}\right\},$$

where $\{A_k\}_{k=0}^{\infty}$ are the Adomian polynomials representing to $\phi(y) = y^2$. Then the recursive relation as follows

$$y_0(t) = -e^{\frac{-t^\alpha}{\alpha}}$$

$$y_1(t) = K_\alpha^{-1}\{vK_\alpha[A_0]\}$$

$$y_2(t) = K_\alpha^{-1}\{vK_\alpha[A_1]\}$$

$$y_3(t) = K_\alpha^{-1}\{vK_\alpha[A_2]\}$$

$$\vdots$$

Thus, the general recursive relation is given by

$$y_0(t) = -e^{\frac{-t^\alpha}{\alpha}}$$

$$y_{k+1}(t) = K_\alpha^{-1}\{K_\alpha[A_k]\}, \quad k = 0, 1, 2, 3, \dots$$

By using this recursive relation, we have

$$y_1(t) = K_\alpha^{-1}\left\{\frac{v}{1+v}K_\alpha[y_0^2]\right\} = K_\alpha^{-1}\left\{\frac{v^2}{(1+v)(1+2v)}\right\} = e^{\frac{-t^\alpha}{\alpha}} - e^{\frac{-2t^\alpha}{\alpha}},$$

$$y_2(t) = K_\alpha^{-1}\left\{\frac{v}{1+v}K_\alpha[2y_0y_1]\right\} = -e^{\frac{-3t^\alpha}{\alpha}} + 2e^{\frac{-2t^\alpha}{\alpha}} - e^{\frac{-t^\alpha}{\alpha}},$$

$$y_3(t) = K_\alpha^{-1}\left\{\frac{v}{1+v}K_\alpha[2y_0y_2 + y_1^2]\right\} = 4e^{\frac{-4t^\alpha}{\alpha}} + 3e^{\frac{-3t^\alpha}{\alpha}} - 3e^{\frac{-2t^\alpha}{\alpha}} + e^{\frac{-t^\alpha}{\alpha}},$$

$$\vdots$$

Then the approximate solution of equation (3.12) is series form and with the help of we obtain (see Figure 3)

$$y(t) = y_0(t) + y_1(t) + y_2(t) + y_3(t) + \cdots = \sum_{n=0}^{\infty} y_n(t),$$

advandage to the exact solution is of the form

$$y(t) = \left(\frac{1}{1 - 2e^{\frac{t^\alpha}{\alpha}}}\right).$$

Example 3.5. Consider the following linear CFDE

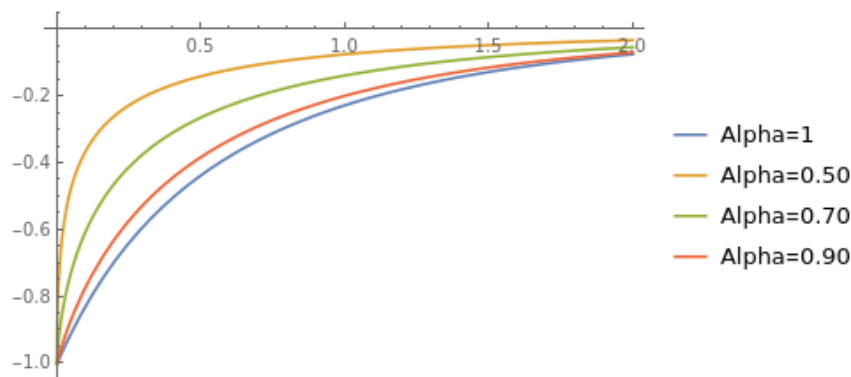$$D^{2\alpha}y - 4y = 0, \quad y(0) = 1, D^\alpha y(0) = 0 \quad 1 \le \alpha < 2. \tag{3.15}$$

FIGURE 3. The solution of equation (3.12) considering several values of α.

By applying K_α on both sides of equation (3.15), we obtain

$$v^2 Y_\alpha(v) - y(0) - 4 Y_\alpha(v) = 0$$

$$Y_\alpha(v) = \frac{1}{1 - 4v^2}. \tag{3.16}$$

Taking inverse transform K_α^{-1} of equation (3.16), then we get

$$y(x) = K_\alpha^{-1} \left\{ \frac{1}{1 - 4v^2} \right\} = \cosh\left(\frac{t^\alpha}{\alpha} \right).$$

Hence the solution of equation (3.15).

4. CONCLUSION

In this paper we derived some important results of the conformable fractional kamal transform (CFKT) which are main roles for solving conformable linear and nonlinear fractional differential equations. Also we have discussed the general analytical solution of the generalized conformable Bernoulli fractional differential equation based on the conformable fractional kamal transform (CFKT) and adomain decomposition method (ADM). Moreover we solved conformable linear and nonlinear fractional differential equations with help of this conformable fractional kamal transform (CFKT) and ADM we gives most appropriate solution.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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