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TRIGONOMETRIC FUZZY ENTROPIC MODELS AND THEIR APPLICATIONS TOWARDS MAXIMUM ENTROPY PRINCIPLE

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Abstract: There exist both types of uncertainties, viz. probabilistic as well as fuzzy but both types of uncertainties are poles apart but participate with a crucial responsibility towards reduction in uncertainties and consequently making the system under study more proficient. It has also been realized that the principle of maximum entropy plays an imperative responsibility for the study of optimization problems associated with the theoretical information measures. We have generated two new trigonometric entropic models for discrete fuzzy distributions and applied them for the knowledge of maximum entropy principle under a set of fuzzy constraints.

Keywords: probabilistic entropy; fuzzy event; fuzzy set; fuzzy entropy; maximum entropy principle; concavity.

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1. INTRODUCTION

The quantitative entropic model introduced by Shannon [22] has incredibly pleasant properties and provides wonderful applications in a series of disciplines. The fundamental discovery of the

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probabilistic entropy led the researchers to investigate new mathematical models measuring uncertainty contained in probabilistic experiments. The following quantitative expression of this information entropy made a revolution for the researchers:

$$H(P) = -\sum_{i=1}^{n} p_i \log p_i \tag{1.1}$$

Observing the wonderful properties of Shannon's [22] entropy, many authors introduced a diversity models and applied them towards a variety of disciplines. These researchers include Huang and Zhang [8], Bulinski and Kozhevin [3], Cincotta and Giordano [4], Majumdar and Jayachandran [15], Markechová, Mosapour and Ebrahimzadeh [16] etc.

On the other hand, Zadeh's [24] fuzzy set theory established confession from diverse quarters and became trendy because of the following specifics:

"A fuzzy set A is a subset of universe of discourse U, characterized by a membership function ${}_A\mu(x)$ which associates to each $x \in U$, a membership value from [0, 1], that is, ${}_A\mu(x)$ represents the grade of membership of x in A. When ${}_A\mu(x)$ takes a value only in [0, 1], A reduces to a crisp or non fuzzy set and ${}_A\mu(x)$ represents the characteristic function of set A. Zadeh [24] defined the entropy of a fuzzy event as weighted Shannon [22] entropy, where the membership values are taken as weights".

Observing the idea of fuzzy sets, De Luca and Termini [4] suggested that corresponding to Shannon's [22] entropy, the measure of fuzzy entropy should be:

$$H(A) = -\frac{1}{n\log 2}\sum_{i=1}^{n}\left\{ {}_A\mu(x_i)\log {}_A\mu(x_i) + (1 - {}_A\mu(x_i))\log(1 - {}_A\mu(x_i)) \right\} \tag{1.2}$$

A quantity of efforts related with the continuity of fuzzy entropic models has been made by Bassanezi and Roman-Flores [1] while lots of other fuzzy models have been originated by Kapur [11], Parkash [18], Mishra et. al. [17], Lam et. al. [13], Lin, Duan and Tian [14], Singh, Lalotra and Sharma [21] etc.

It was Jaynes [9] who anticipated a regulation to distribute numerical probabilities with the availability of guaranteed partial information. Kapur and Kesavan [12] emphasized this rule by commenting:

"Today this rule, known as Maximum Entropy Principle, is used in many fields, ranging from Physical, Biological and Social Sciences to Stock Market analysis. The authors have studied the use of extended MaxEnt in all these applications. They have also studied the use of measures of entropy of a proportions distribution as a measure of uncertainty, equality, spread out, diversity, interactivity, flexibility, system complexity, disorder, dependence and similarity. Generalized measures of entropy and cross entropy can be used for the study of optimization principles. Reliability Theory, Marketing, Measurement of Risk in Portiofolio Analysis and Quality Control are some of the areas where these generalized optimization principles can be successfully applied. Herniter [7] used Shannon's [22] measure in studying the market behavior and found an anomalous result which was later overcome on using parametric measure of entropy. Similarly various other methods can be explained if we use generalized measures of entropy and directed divergence".

Observing the incredible applications of Jayne's [9] principle, many other authors provided the applications of maximum entropy principle to a diversity of disciplines including goods selection, image restoration, population heterogeneity in continuous cell cultures, statistical inference, network systems etc. These pioneers include Zehua et. al. [25], Fernandez-de-Cossio-Diaz and Mulet [6], Jizba and Korbel [10], Xu, Crodelle, Zhou and Cai [23], Parkash, Sharma and Mahajan [19, 20], Du et.al. [5] etc.

The above mentioned principles relate with probability distributions only but for unavoidable circumstances where these probabilistic models cannot be employed, we discover the opportunity of fuzzy models to broaden the extent of applications. In this paper, we have extended Jayne's [9] principle of maximum entropy for discrete fuzzy distributions for studying uppermost fuzziness under fuzzy constraints.

2. GROWTH OF TRIGONOMETRIC FUZZY ENTROPIC MODELS

I. We, recommend the following trigonometric fuzzy entropic model:

$$H_1(A)=\sum_{i=1}^{n}\left[Sin\frac{\beta\,{}_A\mu(x_i)}{n}+Sin\frac{\beta(1-{}_A\mu(x_i))}{n}-Sin\frac{\beta}{n}\right];0<\beta\leq\pi,n>1 \tag{2.1}$$

Upon differentiating equation (2.1), we have

$$\frac{\partial^2 H_1(A)}{\partial\,{}_A\mu^2(x_i)}=-\frac{\beta^2}{n^2}\left[Sin\frac{\beta\,{}_A\mu(x_i)}{n}+Sin\frac{\beta\{1-{}_A\mu(x_i)\}}{n}\right]<0\ \forall i$$

which proves the concavity of $H_1(A)$. Also $\left[H_1(A)\right]_{Max}$ which arises at ${}_A\mu(x_i)=\frac{1}{2}\forall i$ is given by

$$\left[H_1(A)\right]_{Max}=\sum_{i=1}^{n}\left[2Sin\frac{\beta}{2n}-Sin\frac{\beta}{n}\right]=4nSin\frac{\beta}{2n}Sin^2\frac{\beta}{4n}>0$$

Thus, we monitor:

(i) $H_1(A)$ is concave w.r.t. ${}_A\mu(x_i)$.

(ii) $H_1(A)$ is an increasing function of ${}_A\mu(x_i)$ when $0\leq{}_A\mu(x_i)\leq\frac{1}{2}$

because $\left\{H_1(A)=0\,when\ {}_A\mu(x_i)=0\right\}$ and $\left\{H_1(A)=4nSin\frac{\beta}{2n}Sin^2\frac{\beta}{4n}\text{ when }{}_A\mu(x_i)=\frac{1}{2}\right\}$

(iii) $H_1(A)$ is a decreasing function of ${}_A\mu(x_i)$ when $\frac{1}{2}\leq{}_A\mu(x_i)\leq 1$

because $\left\{H_1(A)=4nSin\frac{\beta}{2n}Sin^2\frac{\beta}{4n}\text{ when }{}_A\mu(x_i)=\frac{1}{2}\right\}$ and $\left\{H_1(A)=0\,when\ {}_A\mu(x_i)=0\right\}$

(iv) $H_1(A)$ does not change when ${}_A\mu(x_i)$ is changed to $1-{}_A\mu(x_i)$.

(v) $H_1(A)$ = 0 when ${}_A\mu(x_i)$ = 0 or 1.

(vi) $H_1(A)\geq 0$

The study of above six properties proves the validity of the trigonometric fuzzy entropic model projected in (2.1). For graphical purposes, we have displayed the computations of the trigonometric measure in table-2.1.

Table-2.1

${}_A\mu(x_i)$	$H_1(A)$
0.0	0.0000
0.1	0.6180
0.2	1.1754
0.3	1.6180
0.4	1.9021
0.5	2.0000
0.6	1.9021
0.7	1.6180
0.8	1.1754
0.9	0.6180
1.0	0.0000

The subsequent Fig.-2.1 verifies the concavity of $H_1(A)$ w.r.t. ${}_A\mu(x_i)$.

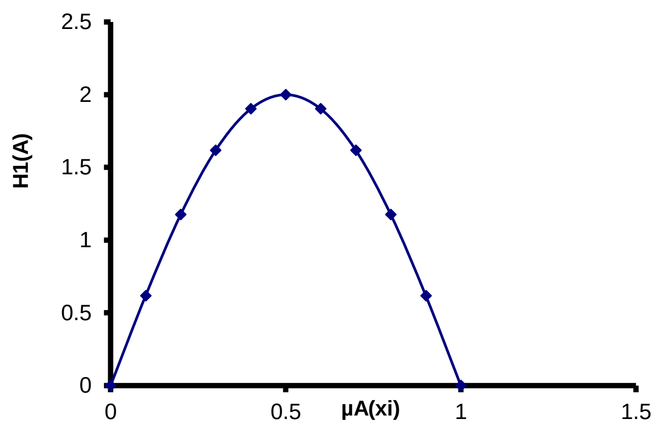

Fig.-2.1. Concavity of $H_1(A)$

II. We suggest another trigonometric fuzzy entropic model given by

$$H_2(A) = \sum_{i=1}^{n}\left[Cos\beta\,{}_A\mu(x_i) + Cos\beta(1-{}_A\mu(x_i)) - 2Cos^2\beta\right]; 0 < \beta < \pi \qquad (2.2)$$

We have

$$\frac{\partial H_2(A)}{\partial\, {}_A\mu(x_i)} = -\beta\left[Sin\beta\, {}_A\mu(x_i) - Sin\beta\left\{1 - {}_A\mu(x_i)\right\}\right]$$

Also

$$\frac{\partial^2 H_2(A)}{\partial\, {}_A\mu^2(x_i)} = -\beta^2\left[Cos\beta\, {}_A\mu(x_i) + Cos\beta\left\{1 - {}_A\mu(x_i)\right\}\right] < 0\ \forall i$$

which proves the concavity of $H_2(A)$. Also $\left[H_2(A)\right]_{Max}$ which arises at ${}_A\mu(x_i) = \frac{1}{2}\forall i$ is given by

$\left[H_2(A)\right]_{Max} = \sum_{i=1}^{n}\left[2Cos\frac{\beta}{2} - 2Cos^2\frac{\beta}{2}\right] = 2n\left[Cos\frac{\beta}{2} - Cos^2\frac{\beta}{2}\right] > 0$ as $Cos\frac{\beta}{2}$ lies in the first quadrant only.

Thus, we see that $H_2(A)$ satisfies the following properties:

(i) $H_2(A)$ is a concave function of ${}_A\mu(x_i)$.

(ii) $H_2(A)$ is an increasing function of ${}_A\mu(x_i)$ when $0 \le {}_A\mu(x_i) \le \frac{1}{2}$

as $\left\{H_2(A) = 0\ when\ {}_A\mu(x_i) = 0\right\}$ and $\left\{H_2(A) = 2n\left[Cos\frac{\beta}{2} - Cos^2\frac{\beta}{2}\right]\text{when}\ {}_A\mu(x_i) = \frac{1}{2}\right\}$

(iii) $H_2(A)$ is a decreasing function of ${}_A\mu(x_i)$ when $\frac{1}{2} \le {}_A\mu(x_i) \le 1$

as $\left\{H_2(A) = 2n\left[Cos\frac{\beta}{2} - Cos^2\frac{\beta}{2}\right]\text{when}\ {}_A\mu(x_i) = \frac{1}{2}\right\}$ and $\left\{H_2(A) = 0\ when\ {}_A\mu(x_i) = 0\right\}$

(iv) $H_2(A)$ does not change when ${}_A\mu(x_i)$ is changed to $1 - {}_A\mu(x_i)$.

(v) $H_2(A) = 0$ when ${}_A\mu(x_i) = 0$ or 1.

(vi) $H_2(A) \ge 0$

The learning of above six properties proves the validity of the trigonometric fuzzy entropic model predicted in (2.2).

In the sequel, we elaborate the maximum entropy principles with the assistance of above trigonometric fuzzy entropic models.

3. PRINCIPLE OF MAXIMUM FUZZY ENTROPY UNDER A SET OF FUZZY CONSTRAITS

To provide these applications, we have considered the following mathematical problems related with the fuzzy entropic models and shown that the maximum fuzzy entropy in each case is concave.

Problem-I: Here, we consider the problem of maximizing fuzzy entropy (2.1) subject to following fuzzy constraints:

$$\sum_{i=1}^{n} {}_A\mu(x_i) = \alpha_0 \tag{3.1}$$

and

$$\sum_{i=1}^{n} {}_A\mu(x_i)\, g_r(x_i) = K;\ r = 1, 2, \ldots, m \ \ and\ m+1 < n \tag{3.2}$$

where K is some positive constant, $g_r(x_i)$ are assumed to be known values and ${}_A\mu(x_i)$ are the values of fuzzy distribution. Since $g_r(x_i)$ are assumed to be known, the expected fuzzy values given in equation (3.2) are assumed to be known exactly.

Consider the following Lagrangian:

$$\begin{aligned} L = \sum_{i=1}^{n}\left[Sin\frac{\beta\, {}_A\mu(x_i)}{n} + Sin\frac{\beta(1-{}_A\mu(x_i))}{n} - Sin\frac{\beta}{n}\right] + \lambda_1\left\{\sum_{i=1}^{n} {}_A\mu(x_i) - \alpha_0\right\} \\ + \lambda_2\left\{\sum_{i=1}^{n} {}_A\mu(x_i) g_r(x_i) - K\right\} \end{aligned} \tag{3.3}$$

Thus $\frac{\partial L}{\partial\, {}_A\mu(x_i)} = 0$ gives

$$\frac{2\beta}{n}Sin\frac{\beta}{2n}Sin\frac{\beta}{2n}\left\{2\,{}_{A}\mu\left(x_i\right)-1\right\}=\left\{\lambda_1+\lambda_2 g_r(x_i)\right\}$$

or $$Sin\frac{\beta}{2n}\left\{2\,{}_{A}\mu\left(x_i\right)-1\right\}=\frac{n}{2\beta}\frac{\left\{\lambda_1+\lambda_2 g_r(x_i)\right\}}{Sin\frac{\beta}{2n}}$$

or $$\left\{\frac{\beta}{2n}\left\{2\,{}_{A}\mu\left(x_i\right)-1\right\}\right\}=Sin^{-1}\left\{\frac{n}{2\beta}\frac{\left\{\lambda_1+\lambda_2 g_r(x_i)\right\}}{Sin\frac{\beta}{2n}}\right\}$$

or $${}_{A}\mu\left(x_i\right)=\frac{1}{2}\left[\frac{2n}{\beta}\left\{Sin^{-1}\left\{\frac{n}{2\beta}\frac{\left\{\lambda_1+\lambda_2 g_r(x_i)\right\}}{Sin\frac{\beta}{2n}}\right\}\right\}+1\right]$$

Applying (3.1) and (3.2), we get

$$\frac{1}{2}\sum_{i=1}^{n}\left[\frac{2n}{\beta}\left\{Sin^{-1}\left\{\frac{n}{2\beta}\frac{\left\{\lambda_1+\lambda_2 g_r(x_i)\right\}}{Sin\frac{\beta}{2n}}\right\}\right\}+1\right]=\alpha_0$$

and $$\frac{1}{2}\sum_{i=1}^{n}\left[\frac{2n}{\beta}\left\{Sin^{-1}\left\{\frac{n}{2\beta}\frac{\left\{\lambda_1+\lambda_2 g_r(x_i)\right\}}{Sin\frac{\beta}{2n}}\right\}\right\}+1\right]g_r\left(x_i\right)=K$$

When $\lambda_2\to 0$, we have

$$\alpha_0=\frac{1}{2}\sum_{i=1}^{n}\left[\frac{2n}{\beta}\left\{Sin^{-1}\left\{\frac{n}{2\beta}\frac{\left\{\lambda_1+\lambda_2 g_r(x_i)\right\}}{Sin\frac{\beta}{2n}}\right\}\right\}+1\right]$$

and $$K=\frac{1}{2}\sum_{i=1}^{n}\left[\frac{2n}{\beta}\left\{Sin^{-1}\left\{\frac{n}{2\beta}\frac{\left\{\lambda_1+\lambda_2 g_r(x_i)\right\}}{Sin\frac{\beta}{2n}}\right\}\right\}+1\right]g_r\left(x_i\right)$$

Thus when $\lambda_2>0$, we have

$$[H_1(A)]_{\max}=\sum_{i=1}^{n}\left[\begin{array}{l}\frac{1}{2}Sin\frac{\beta}{n}\left[\frac{2n}{\beta}\left\{Sin^{-1}\left\{\frac{n}{2\beta}\frac{\{\lambda_1+\lambda_2 g_r(x_i)\}}{Sin\frac{\beta}{2n}}\right\}\right\}+1\right]\\ +Sin\frac{\beta}{n}\left[1-\frac{1}{2}\left[\frac{2n}{\beta}\left\{Sin^{-1}\left\{\frac{n}{2\beta}\frac{\{\lambda_1+\lambda_2 g_r(x_i)\}}{Sin\frac{\beta}{2n}}\right\}\right\}+1\right]\right]-Sin\frac{\beta}{n}\end{array}\right]$$

Let $\frac{\beta}{n}\left[\frac{2n}{\beta}\left\{Sin^{-1}\left\{\frac{n}{2\beta}\frac{\{\lambda_1+\lambda_2 g_r(x_i)\}}{Sin\frac{\beta}{2n}}\right\}\right\}+1\right]=\theta$

Thus $[H_1(A)]_{\max}=\sum_{i=1}^{n}f(\theta)$

where $f(\theta)=\frac{1}{2}Sin\theta+Sin\frac{\beta}{n}\left\{1-\frac{n\theta}{2\beta}\right\}-Sin\frac{\beta}{n}$

$f'(\theta)=\frac{1}{2}Cos\theta-Cos\frac{\beta}{n}\left\{1-\frac{n\theta}{2\beta}\right\}$

and $f''(\theta)=-\left\{\frac{1}{2}Sin\theta+Sin\frac{\beta}{n}\left\{1-\frac{n\theta}{2\beta}\right\}\right\}<0$

which shows that $f(\theta)$ are concave functions and since the sum of concave functions is concave, we see that $[H_1(A)]_{\max}$ is concave.

Problem-II: Here, we consider the problem of maximizing trigonometric fuzzy entropic model (2.2) under the constraints (3.1) and (3.2).

Let

$$L=\sum_{i=1}^{n}\left[Cos\beta\,{}_A\mu(x_i)+Cos\beta(1-{}_A\mu(x_i))-2Cos^2\beta\right]+\lambda_1\left\{\sum_{i=1}^{n}{}_A\mu(x_i)-\alpha_0\right\}+\lambda_2\left\{\sum_{i=1}^{n}{}_A\mu(x_i)g_r(x_i)-K\right\} \quad (3.4)$$

Thus $\frac{\partial L}{\partial_A \mu(x_i)} = 0$ gives

$$_A\mu(x_i) = \frac{1}{2}\left[\frac{2}{\beta}\left\{Sin^{-1}\left\{\frac{\{\lambda_1 + \lambda_2 g_r(x_i)\}}{2\beta Cos\frac{\beta}{2}}\right\}\right\} + 1\right]$$

Applying equations (3.1) and (3.2), we have

$$\frac{1}{2}\sum_{i=1}^{n}\left[\frac{2}{\beta}\left\{Sin^{-1}\left\{\frac{\{\lambda_1 + \lambda_2 g_r(x_i)\}}{2\beta Cos\frac{\beta}{2}}\right\}\right\} + 1\right] = \alpha_0$$

and

$$\frac{1}{2}\sum_{i=1}^{n}\left[\frac{2}{\beta}\left\{Sin^{-1}\left\{\frac{\{\lambda_1 + \lambda_2 g_r(x_i)\}}{2\beta Cos\frac{\beta}{2}}\right\}\right\} + 1\right] g_r(x_i) = K$$

Now when $\lambda_2 \to 0$, we have

$$\alpha_0 = \frac{1}{2}\sum_{i=1}^{n}\left[\frac{2}{\beta}\left\{Sin^{-1}\left\{\frac{\lambda_1}{2\beta Cos\frac{\beta}{2}}\right\}\right\} + 1\right]$$

and $K = \frac{1}{2}\sum_{i=1}^{n}\left[\frac{2}{\beta}\left\{Sin^{-1}\left\{\frac{\lambda_1}{2\beta Cos\frac{\beta}{2}}\right\}\right\} + 1\right] g_r(x_i)$

Thus when $\lambda_2 > 0$, we have

$$\left[H_2(A)\right]_{\max} = \sum_{i=1}^{n}\left[\begin{array}{l} Cos\beta\left[\frac{1}{2}\left[\frac{2}{\beta}\left\{Sin^{-1}\left\{\frac{\{\lambda_1+\lambda_2 g_r(x_i)\}}{2\beta Cos\frac{\beta}{2}}\right\}\right\}\right]+1\right] \\ +Cos\beta\left[\left[1-\frac{1}{2}\left[\frac{2}{\beta}\left\{Sin^{-1}\left\{\frac{\{\lambda_1+\lambda_2 g_r(x_i)\}}{2\beta Cos\frac{\beta}{2}}\right\}\right\}\right]\right]+1\right]-2Cos^2\frac{\beta}{2}\end{array}\right] \quad (3.6)$$

Let $\beta\left[\frac{1}{2}\left[\frac{2}{\beta}\left\{Sin^{-1}\left\{\frac{\{\lambda_1+\lambda_2 g_r(x_i)\}}{2\beta Cos\frac{\beta}{2}}\right\}\right\}\right]+1\right]=\theta$

Thus $\left[H_2(A)\right]_{\max} = \sum_{i=1}^{n} f(\theta)$

where $f(\theta) = Cos\theta + Cos\{\beta-\theta\} - 2Cos^2\frac{\beta}{2}$

$f'(\theta) = -Sin\theta + Sin\{\beta-\theta\}$ and $f''(\theta) = -\{Cos\theta + Cos\{\beta-\theta\}\} < 0$

Thus equation (3.6) shows that $\left[H_2(A)\right]_{\max}$ is concave.

CONCLUDING REMARKS

The principles of maximum entropy and minimum discrimination have found tremendous applications associated with probability distributions but under unavoidable situations where probabilistic models cannot be engaged, we ascertain the opportunity of fuzzy measures. Such trigonometric fuzzy measures have been created for fuzzy distributions. Our findings make available the study of maximum fuzziness under a set of equality fuzzy constraints. It is supplementary that such a study can be extended to additional well recognized fuzzy entropic models.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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