A NOVEL k – MEAN CLUSTERING BASED GRAPH CUT FOR BRAIN MR IMAGE SEGMENTATION

NARESH GHORPADE[†,*], H. R. BHAPKAR

Department of Mathematics, MIT School of Engineering, MIT ADT University, Loni Kalbhor, Pune - 412201, India

Abstract: Image Segmentation is an influential method to detect and distinguish the diseased and normal sections of an image. Image segmentation extracts the regions of interest for precise diagnosis of tumours and scheduling appropriate line of treatment. The distinct tumours in brain have varied outlies, position and intensity values. Consequently, it is difficult to develop a common technique for brain MRI segmentation. Moreover, the extraction of anomalies from the brain MRI turns out to be a challenging task. In this research, we have developed a self-regulating method for selection of seed points for partitioning the graph of MR images with tumor to attain graph cut segmentation. The proposes method overcomes the key problem of primary seed point selection by utilizing the balanced brain layout and combines k - mean clustering into graph cut for segmenting an image. The outcomes attained by the proposed technique facilitates improved segmentation of the diseased region.

Keywords: graph partitioning; clustering; brain MRI segmentation.

2010 Subject Classification: 65D18.

1. INTRODUCTION

Over last few decades, Magnetic Resonance Imaging (MRI) is multimodality technique progressed amidst other extrusive imaging techniques. Considering the needs of the today's

world, MRI provides large amount of data with the high-quality level. To study the brain anatomy and function, For the doctors and clinicians MRI technique is quite convenient. Due to the larger and composite dataset manual analysis is challenging and difficult. In Magnetic Resonance Imaging, brain MRI is the best application. The brain is basically a combination of different soft tissues such as neurons, glial cells, neural stem cells, and blood vessels. MRI offers larger divergence and hence it is highly preferred imagining technique.

Although, the brain tumor is a life intimidating ailment, if it is diagnosed at an early stage, the chances of survival are high. Uncharacteristic development in tissue frame leads to tumor is formation inside the brain and subsequently it damages the brain cells. A tumor is classified into two categories such as malignant and benign [1]. It may differ in shape, position and might contain intersecting intensities along with the normal tissue. In depth study reveals that the developed countries are facing problem of high mortality percentage because of brain tumor [2]. There are some of the common tumors like Gliomas, Astrocytoma, Meningiomas and nerve sheath tumors which are mainly observed inside the human brain [3]. Accuracy in segmenting and identifying the affected regions plays vital role in the diagnosis.

In order to ease improved diagnosis and treatments, computer aided methods are developed [4]. Image segmentation method is more efficient among all developed methods for tumor identification. Brain MRI analysis is done with the help of image segmentation which is precisely explain in this paper. With the help of image segmentation, the image is partitioned into distinct subparts such that every subpart is an expressive fragment of that image. Which plays crucial role in exact finding and helps to decide the further line of treatment with analysis and visualization of different brain structures. It is also useful for delimiting injuries, image guided interferences and clinical arrangement. The brain MRI segmentation approaches are categorized as region based, splitting and Merging, Clusterization, Graph theoretical approaches etc. [6]. An image is converted into binary image in thresholding segmentation which is again divided into two regions viz; foreground region and background region. It is very easy technique to apply and it require a minimum time. but it is little complicated to reach proper threshold value. Also, it is mandatory to have substantial variance among the intensity values of forefront

and background [7]. There are two techniques involved in the Region based segmentation, region growing and splitting- Merging [8]. Both the methods are modified with seed points [9]) and process of segmentation is continued till the uniformity criterion get satisfied. Through the preset factors like same intensity, color and texture, grouping is formed in region growing. Based on certain criteria in splitting- Merging process, the image is divided into reduced sections. Thereafter, those smaller regions get merged or get divided. Above two methods are totally relying on intensity standards and there is no smooth boundary in the images obtained by them. Gambatto *et.al.* [10], have combined region growing and contour recognition to generate an image having smooth end. Clustering is an alternative technique that recognizes alike sections. Data organized with lesser inter cluster resemblance and larger inter cluster likeness arises in Clustering method. The intensity variance among image pixels is determined to obtain the likeness and/or unlikeness between two information sets. It imitates the degree of closeness or separation within the data points. The well-known clusterization techniques are: k-mean clusterization and Fuzzy C-mean clusterization. The groups of identical pixels are created in FCM clustering [11], by assigning relationship that provides the grade of likeness. As compared to FCM C – mean clustering, *k*-mean Clusterization [12] is a difficult method which splits an image into distinct groups step by step. The pixels are clubbed in such a way that pixels having the same characteristics will occur in the same region and those having distinct characteristics will fall in another region. The resemblance and divergence is restrained on the basis of Euclidean distance, and then the clustering is executed. These techniques rely on preliminary centroid and maximum efforts are required for the proper choice of preliminary centroid [13]. Ghorpade et al. [14] have developed an approach to overcome this problem via energy centered method projected by labeled as graph cut. An objective function obtained in this technique touches a minimal value after segmenting the image. In a broader aspect, Techniques proposed in [15 - 16] used graph cut scheme consisting the cost function in which image segmentation is initiated. Subsequently, optimization techniques have been strongly used for the numerous research objectives. Here, the binary variables are used by objective function to specify the pixel grouping. The method of Graph cut is very efficient, iterative and robust in nature, although

distinct connective minimum cut and maximum flow criteria for obtaining least *s-t* cut are used to get the same optimization function and identical segments [17 - 18]. By studying the practical efficiency, Boykov et al. [17] has explained that, in image segmentation for two dimensional and three-dimensional problems are solved by maximum flow criteria. Graph cut adopts the provincial and border span ranging for the multidimensional problem in the segmentation process. Contrasting live wire [18] as well as intelligent scissor. Graph theoretical approach appraises the small weighted edge which partitions the image. In addition to this the provincial removal graph theoretical approach shows improvement in segmentation process by removing an edge that configures items which are converted into binary digital images [19].

In this paper graph theoretical approach is proposed for brain MRI segmentation. In Section II, mathematical explanation of the different techniques is well explained. Whereas, in Section III the experimental set up, the proposed approaches like CBSS and KMSS used for finding the seed points are precisely explained. Results and performance analysis is presented in Section – IV. Finally, the paper is concluded in Section – V.

2. Graph Theoretical Technique (Graph Cut Method)

In a Graph theoretical approach, an image is partitioned in two subparts where the surrounding area is entirely black and the mined object area clasps unique pixel value. The image represented graphically shows that the pixel positions are determined by the vertices and thickness among the neighbouring pixels is determined by the edges. The minor thickness in the edges leads to lesser similarity among the neighbouring pixels next cut is performed. Optimal cut value in the graph comprising the provincial and border value occurs due to the energy function [20]. In order to tag a pixel as a surrounding or entity, provincial value is evaluated to determine the drawbacks. Partition of the graph can be done using various parameters. For the distribution approximation, Gaussian Mixture model (GMM) introduced in [21] reorganizes collaboratively and gives a constant illustration of the entity. In [22] the provincial value from local images is restructured for every iteration. In order to signify pixel as background, the classic way of intensity

distribution from the histogram is chosen. The penalty of discontinuity is represented by border value in the objective function and the border value consists of the terms that are assessed among the pixels having same intensity and distinct labelling. Favaro et.al [23] has described that, this value provides the evenness constraint. Further, Boykov et al. [24] represented two dimensional images graphically for visualizations and illustrations. Introductory terminologies of graph are presented in Eq. (1) and Eq. (2).

$$G = (L, M) \tag{1}$$

$$L = (u, v) \cup Q \tag{2}$$

where, L represents set of nodes and M represents set of edges. L encircles the extreme nodes: (u, v) and internal nodes are listed in set $Q = \{q_1, q_2, \ldots, q_n\}$. Extreme nodes n (u, v) are two exceptional extreme nodes; u represents the group of nodes that are lying in the segmented objects area and v represents the group of nodes that are lying in the background of segmented object. The graph consists of nonterminal nodes called as pixels or vertices and they are denoted by P. The edge is labelled as, v – link if extreme node and nonterminal nodes are connected inside it; whereas an - link, if two nonterminal nodes are connected inside it. The weight w is allocated to every edge of the graph; it is also named as cost of the edge. The cut imposed depend on these weights divides the desired region. Distinct sections belong to segmented image share equal degree of similarity. The criteria for good segmentation is defined as the specific segmented region having properties like uniformity and homogeneity reliant on the following factors: intensity, brightness, colour, etc. and internally segmented parts should not be similar. Boykov et al. [24] have suggested that the following conditions should be satisfied by the cut of graph:

- all the object nodes must be linked with the extreme node in object and all the background nodes must be linked with the extreme node in the background.
- If the maximum source is same as the dimensions of the smallest cut in the graph then the maximal flow from source to sink is attainable.

Initially, minimum cut and maximum flow technique was introduced by Greig *et. al.* [25]. Enhanced energy function is defined in Eq. (3)

$$M(A) = \psi F_p(A) + G_P(A) \tag{3}$$

$$A = \sum_{k=1}^{n} a_k \tag{4}$$

$$a_k = \begin{cases} a_k \epsilon (0,1) \\ a_1 \cup a_2 \cup ... \cup a_n = G \\ a_i \cap a_j = \emptyset \end{cases} \tag{5}$$

where $F_p(A)$ is the regional term that generates the measure of allocating A to P, $G_P(A)$ determines the border term and ψ is the relative importance factor. Regional term designates t – link and boundary term designate n – link in the graph.

a) Regional term: It is an ordinary mode to denote pixel as an object with the intensity distribution. It computes the cost of every pixel with respect to object and background as defined in Eq. (6).

$$F(A) = \sum_{p \in P} F_p(a_p) \tag{6}$$

where $a_p \in (0,1)$, reliant to the associateship with a specific seed.

Primarily, to estimate source and sink, seed points inside object and background are determined. Once the entire set of nodes in a graph is partitioned into two sets which lie exclusively into object and background terminal, then the segmentation occurs. R_o and R_b are the primary grey values of object seed and background seed which are calculated for the evaluation of object mean and background mean which is defined in Eq. 7 and Eq. 8. Consider $\{z_{o1}, z_{o2}, z_{o3}, ...\}$ and $\{z_{b1}, z_{b2}, z_{b3}, ...\}$ as occurring frequency of selected grey values representing the existence of the grey values on the other hand $\{o_1, o_2, o_3, ...\}$ and $\{b_1, b_2, b_3, ...\}$ are pixel intensities belong to object and background region.

$$m_o = \frac{z_{o1}o_1 + z_{o2}o_2 + z_{o3}o_3 + \ ...}{z_{o1} + z_{o2} + z_{o3} + \cdots} \tag{7}$$

$$m_B = \frac{z_{b1}b_1 + z_{b2}b_2 + z_{b3}b_3 + \cdots}{z_{b1} + z_{b2} + z_{b3} + \cdots} \tag{8}$$

The cost computation gives the variation in the intensity value of a pixel from m_o or m_B. Value of cost supports in labelling them as $'1'$ for object and $'0'$ for background determined by using Eq. (9) and Eq. (10)

$$w_0^p = 1 - F_p(B) \tag{9}$$

$$w_B^p = 1 - F_p(O) \tag{10}$$

where, w_0^p, w_B^p are weights of object and background pixel. $F_p(B)$, $F_p(O)$ are the assuring limits of object and background pixel. Which signifies the presence projected by the pixel for a specific part and are calculated by using Eq. (11) and Eq. (12).

$$F_p(B) = \frac{\lambda_B^p}{QL} = \frac{\sqrt{(I_p - m_B)^2}}{QL} \tag{11}$$

$$F_p(O) = \frac{\lambda_O^p}{QL} = \frac{\sqrt{(I_p - m_o)^2}}{QL} \tag{12}$$

λ_B^p and λ_o^p represents the average distances of object and background from the pixel respectively, I_p is the intensity value of the pixel, QL is the pixel intensity value. The edges of a graph have been assigned the weight. Weights are considered as the width of the edge. Bigger the width, maximum is weight and exactly the opposite is applicable for smaller edge widths. For assignment of labels, the weights are inspected from Table 1.

Table1. Edge Weight Description

Edge	Constraint	Label Assigned
(q, u)	q ∈ object	1
	q ∈ background	0
	q ∈ object U background	$m_o\left(1 - \dfrac{\lambda_o^p}{QL}\right)$
(r, v)	r ∈ object	$m_B\left(1 - \dfrac{\lambda_B^p}{QL}\right)$
	r ∈ background	
	r ∈ object U background	

If $w_0^p < w_B^p$ the pixel is labelled as 0, else it is 1. From the table 1 it is shown that, two weight values are evaluated for $w_0^p = w_B^p$. Identical weighted pixels lie on the boundary therefore they belong to both the regions and are additionally supportive in computing the boundary term. The proclivity of a pixel between the object or background terminal is represented by the weight comparison. Pixels within the graph are distributed amongst two equivalent groups comprising of t – link those are (q, u) and (r, v). They represent the terminal edges of the graph, by creating segmented u – v graph.

The graph cut is carried out on the pixels where intensity of the neighbouring node is same but the labelling Hence the minimum cut is executed along the object boundaries. Min cut value is defined in Eq. (13).

$$\text{Cut}\,(O, B) = \sum_{q \in O, r \in B} w(q, r) \tag{13}$$

where B is back ground label, O is object label, and $w(q, r)$ represents the node weight of the graph.

b) Boundary Term: The edges which are elements of cut set for the optimum segmentation contains the pixels of the borderline. To curtail the energy function, the boundary term subsidizes cost of discontinuity. Any two pixel with distinct labels and identical intensities reflect in the boundary term determined by using Eq. (14)

$$B(A) = B(q, r) = e^{-\left(\frac{|I_q - I_r|}{\lambda}\right)} \tag{14}$$

where I_q and I_r determines the intensity values of neighbouring pixel having same intensity and distinct label.

c) k-mean Clusterization:

This method is also known as hard clusterization. It is simple and time saving but mainly if it is malignant, it is not capable to segment the tumor effectively. It follows the iterative process in which image is divided into different cluster. On the basis of characteristics, the data pixels are grouped such a way that if a data point occurring in a certain cluster then it wills not lie in any other cluster. The selection of initial centroid point decides the position of clusters, while

selecting conventional *k*-mean clustering. On the basis of K-mean by assuming Euclidean distance defined in Eq. (15), the equality and inequality is measured and the clusterization takes place.

$$Distance = \sqrt{\sum (q_k - r_k)^2} \tag{15}$$

 where q_k and r_k are the intensities of the pixels and k is quantity of cluster. For the *k*-mean clusterization, initial seed points are chosen arbitrarily for the complete image and then distance between all the seed points and pixels is obtained. Pixels with less distance to the respective seed point are gathered together. The iteration process carried out and for every step, a new average value is calculated which is a centroid. Unless there is any change or variation in the mean value the process will be continued.

3. EXPERIMENTAL SETUP

Simulations are carried out on Intel(R) Core i5-1135G7/8GB RAM /Windows 10/MSO using MATLAB 2015a. For simulation 100 images are chosen from the standard dataset which is available online [26]. Results for three sample images are presented here in Fig. (1). For the simulation of different resolution, axial view of the images is considered in the following manner: 180*218, 800*450 and 674*594 respectively. Tumour, astrocytoma, glioma diseases are properly visible in the dataset of abnormal brain MRI images
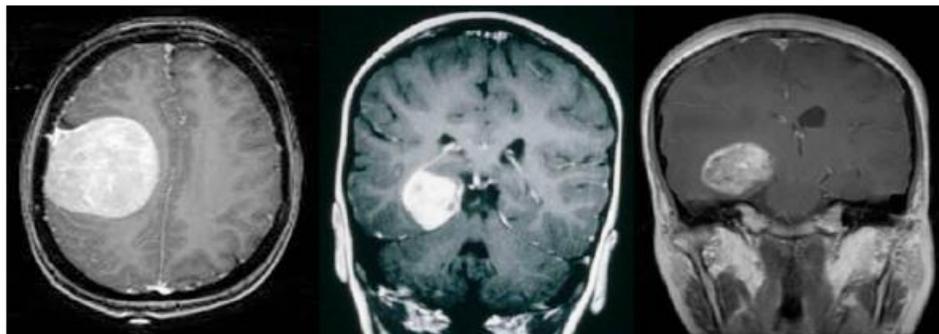
Fig. 1: MR Images with Distinct Tumours

A) CENTROID BASED SEED SELECTION (CBSS)

By using centroid based seed assortment technique, we observe that, preliminary seed points are automatically generated for segmentation process. The difference in the pixel intensity is so high in the diseased area of the brain MRI. The probable pixel position lying in the infected part is encompassed by the pixel values neighbouring to this maximum variance. This range is restricted to the seed points lying in the object region, and all the remaining pixels are situated in the background region. R_B and R_O defined in Eq. (16) and Eq. (17) includes background and object seed points which contribute the evaluation of the centroids by initializing the process of segmentation.

$$R_O = [d - \beta, d + \beta]; \{o_1, o_2, o_3, \dots.\} \tag{16}$$

$$R_B = [d_1 - \beta, d_1 + \beta]; \{b_1, b_2, b_3, \dots.\} \tag{17}$$

$$d_1 = 255 - d \tag{18}$$

d represents the large variance intensity value achieved in the diseased part or the object region which is to be segmented, β is maximum range and d_1 is intensity in the background region with the maximum value.

For graph cut segmentation using centroid the flow chart is presented in Diagram 2. Primarily, the RGB image is transformed into grey scale image and then it is separated in to vertical parts. Balanced nature of the brain is exploited for selecting the initial seed points automatically. Every vertical half of a MRI image of a brain with healthy condition is almost similar to its other half. However, the contrary is valid in the diseased area in case of unhealthy or tumour affected brain. Points of centroid are obtained in the following manner.

Step 1: Among the two vertical halves of the brain, the maximum intensity difference is calculated.

Step 2: For the centroid points above and below the range, the maximum variance cost for object and background area are chosen.

Hence the segmentation is performed where the graph cut technique is implemented for extracting the region of interest. Process flow of graph cut is shown in Fig. 2
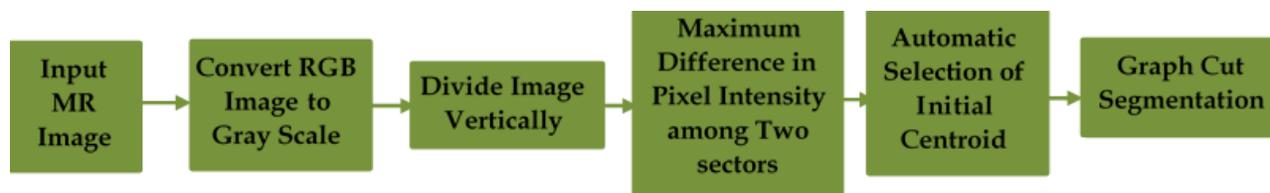
Fig. 2: Process flow of Graph Cut

B) ᴋ-MEAN SEED SELECTION (KMSS):

By using *k*-mean clustering, different method in this research work is used for selecting seed points. In order to develop the accuracy in seed selection in a short span of time, KMSS technique is implemented. This technique is applied to obtain effective centroid points for segmentation. The flow diagram shown in figure 3 describes the proposed technique. Where input image is primarily transformed to grey scale image. then select the cluster number $k>2$. As soon as k increases, consequently there is increase in clusters which in turn includes the micro centroid points. In all there are ten clusters selected in this paper. Out of which two are manually chosen for source and sink terminal and further graph cut segmentation process is followed. Process flow of KMSS is shown in Fig. 3.
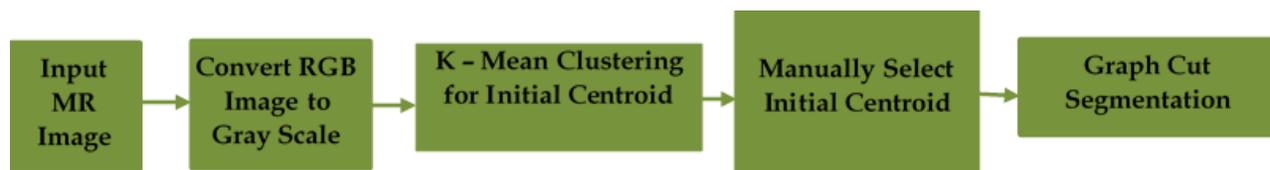
Fig. 3: Process flow of KMSS

4. RESULTS AND DISCUSSIONS

Fig.4(a), Fig.5(a) and Fig.6(a) represents the original brain MR images affected due to tumour, astrocytoma and glioma respectively. Whereas Fig.4(d), Fig.5(d) and Fig.6(d) are corresponding histograms. Output images by CBSS graph cut segmentation are shown in Fig.4(b), Fig.5(b) and Fig.6(b). Their corresponding histogram are showed in diagram Fig.4(e), Fig.5(e) and Fig.6(e).

Fig. 4 (a)   **Fig. 4 (b)**   **Fig. 4 (c)**
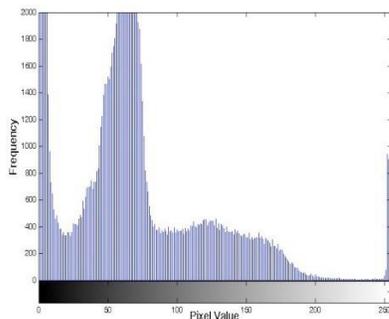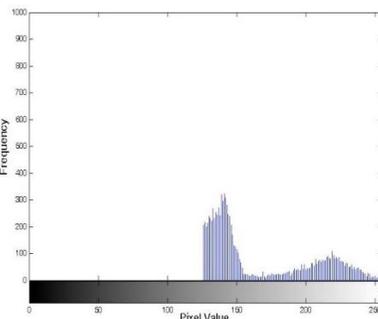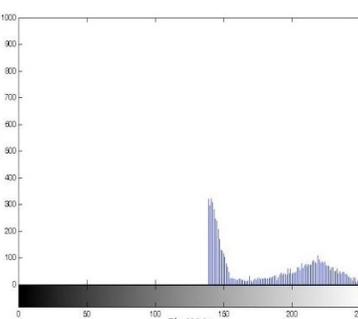
Fig. 4 (d)   **Fig. 4 (e)**   **Fig. 4 (f)**

Fig. 4: (a) and (d): Original MR Image affected by tumor and its histogram; (b) and (e): Image segmented by CBSS graph cut and its histogram; (c) and (f): Image segmented by KMSS Graph cut and its histogram.
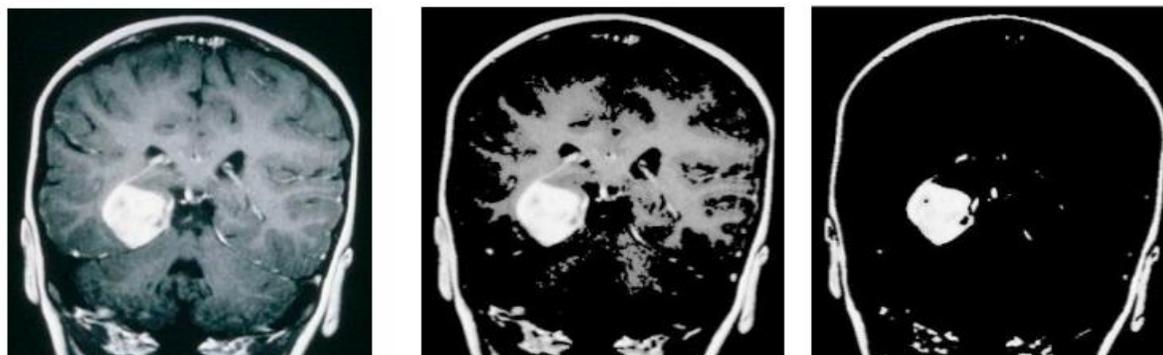
Fig. 5 (a)   **Fig. 5 (b)**   **Fig. 5 (c)**
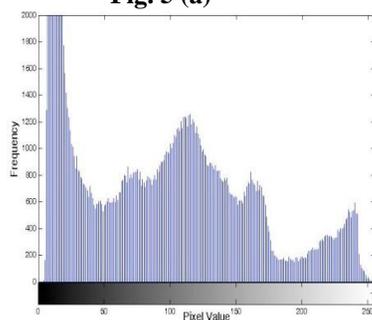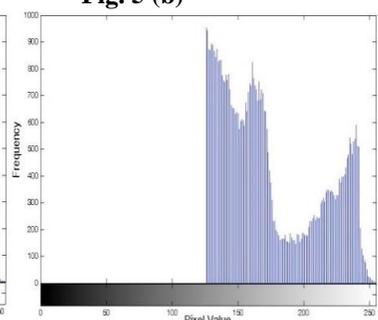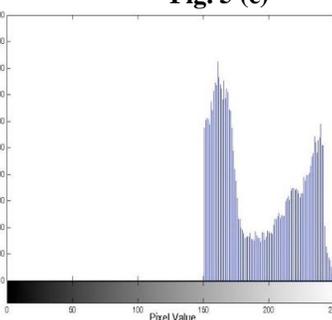
Fig. 5 (d)   **Fig. 5 (e)**   **Fig. 5 (f)**

Fig. 5: (a) and (d): Original MR Image affected by Astrocytoma and its histogram; (b) and (e): Image segmented by CBSS graph cut and its histogram; (c) and (f): Image segmented by KMSS Graph cut and its histogram.
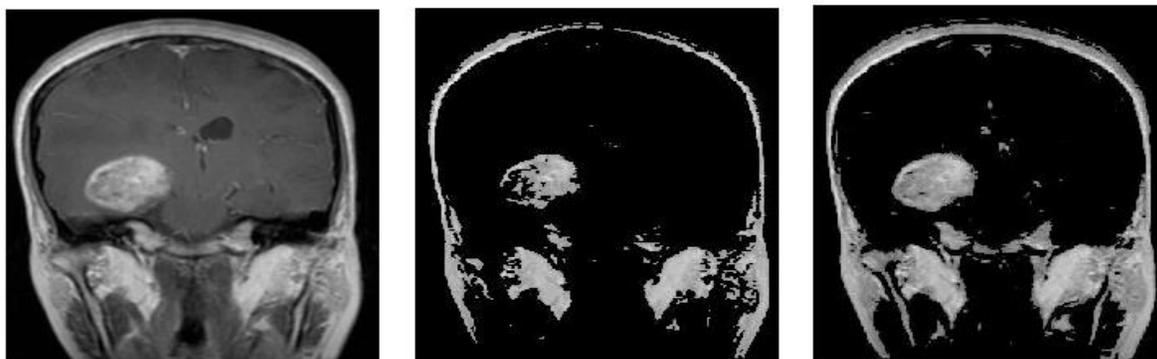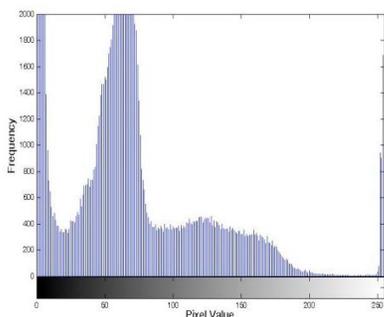
| Fig. 6 (a) | Fig. 6 (b) | Fig. 6 (c) |

| Fig. 6 (d) | Fig. 6 (e) | Fig. 6 (f) |
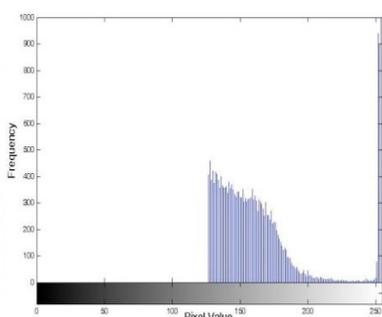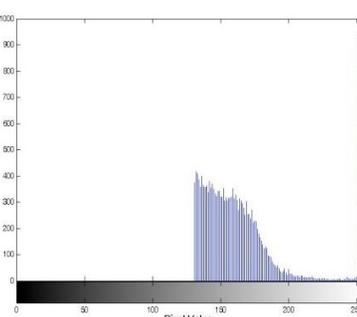
Fig. 6: (a) and (d): Original MR Image affected by Glioma and its histogram; (b) and (e): Image segmented by CBSS graph cut and its histogram; (c) and (f): Image segmented by KMSS Graph cut and its histogram.

By analysing Fig.4(b), Fig.5(b) and Fig.6(b) we can conclude that the region of interests are excavated along with undesirable region. The key reason behind the occurrence of undesirable region is the inaccurate pixels are getting involved in the region of interest.

Output images by KMSS are shown in Fig.4(c), Fig.5(c) and Fig.6(c). Their corresponding histogram are showed in diagram Fig.4(f), Fig.5(f) and Fig.6(f). The region of interests in the figure Fig.4(c), Fig.5(c) and Fig.6(c) are completely extracted which are purely perceptible. A substantial transformation is perceived by comparison of these images with the images attained by CBSS method, it avoids the unwanted region.

The KMSS graph cut segmentation method efficiently extracts the region of interest. This separation provides clarity of the tumour regardless of the shape, location or type. In the KMSS method, histogram clearly displays lesser number of pixels occurring in area of segmentation, indicating improved extraction compared to the CBSS method.

5. CONCLUSION

In this paper, we have proposed CBSS and KMSS techniques to locate the centroid points, and used graph cut segmentation for the extraction of tumour region existing in the brain MR image. In CBSS method, by exploiting brain symmetry two centroid points are randomly chosen. On the other hand, in KMSS method a hybrid process is implemented for evaluating the effective centroid points. With the deep analysis, it has been observed that the recommended k-mean clusterization with graph cut segmentation approach is more preferable for detecting the tumour with asymmetrical shape carrying the characteristics of segmented part. This work will be continued in the future for quantitative analysis by assessing the precision level for different parameters to assure the perimeter, shape and area of the extracted tumour.

CONFLICT OF INTEREST

The author(s) declare that there is no conflict of interests.

REFERENCES

[1] W. L. Nowinski, Human brain atlasing: past, present and future, Neuroradiol. J. 30(6) (2017), 504-519.

[2] C.-T. Lin, C.-M. Yeh, S.-F. Liang, J.-F. Chung, N. Kumar, Support-vector-based fuzzy neural network for pattern classification. IEEE Trans. Fuzzy Syst. 14(1) (2006), 31-41.

[3] N. Ghorpade, H.R. Bhapkar, Brain MRI Segmentation and Tumor Detection: Challenges, Techniques and Applications, in: 2021 5th International Conference on Intelligent Computing and Control Systems (ICICCS), IEEE, Madurai, India, 2021: pp. 1657–1664.

[4] S.D. Kapade, Swarm intelligence-based graph partitioning for image segmentation, in Chapter 3: Review of image segmentation methods. Thesis, Suresh Gyan Vihar University, Shodhganga. https://shodhganga.infibnet.ac.in//handle/10603/46071, (2015).

[5] E. Abdel-Maksoud, M. Elmogy, R. Al-Awadi, Brain tumor segmentation based on a hybrid clustering technique, Egypt. Inform. J. 16(1) (2015), 71-81.

[6] D. Jyotsna, P. Navdeep, J. Shruti, S. Meenakshi, Improved methods for analyzing MRI brain images, Network Biol. 8(1) (2018), 1- 11.

[7] I. Manousakas, P. Undrill, G. Cameron, T. Redpath, Split-and-merge segmentation of magnetic resonance medical images: performance evaluation and extension to three dimensions, Computers and Biomedical

Research, 31(6) (1998), 393-412.

[8] J. Fan, D.K. Yau, A.K. Elmagarmid, W.G. Aref, Automatic image segmentation by integrating color-edge extraction and seeded region growing, IEEE Trans. Image Proc. 10(10) (2001), 1454-1466.

[9] E. Hancer, D. Karaboga, A comprehensive survey of traditional, merge-split and evolutionary approaches proposed for determination of cluster number, Swarm Evol. Comput. 32 (2017), 49-67.

[10] S.D. Kapade, S.M. Khairnar, B.S. Chaudhari, Enhanced graph based normalized cut methods for image segmentation. ICTACT J. Image Video Process. 5(2) (2014), 907–912.

[11] N. Ghorpade, H.R. Bhapkar, Evaluation and Performance Analysis of Brain MRI Segmentation Methods, Int. J. Computer Sci. Eng. 6(8) (2018), 687-696.

[12] A.K. Jain, Data clustering: 50 years beyond K-means, Pattern Recogn. Lett. 31(8) (2010), 651-666.

[13] N. Dhanachandra, Y.J. Chanu, Image Segmentation Method Using K-Means Clustering Algorithm for Color Image, Adv. Res. Electric. Electron. Eng. 2(11) (2015), 68-72.

[14] S.D. Kapade, S.M. Khairnar, B.S. Chaudhari, A new particle swarm intelligence-based graph partitioning technique for image segmentation, J. Electric. Syst. Inf. Technol. 7 (2020), 4.

[15] Z. Wu, R. Leahy, An optimal graph theoretic approach to data clustering: Theory and its application to image segmentation, IEEE Trans. Pattern Anal. Mach. Intell. 15(11) (1993), 1101-1113.

[16] L.R. Ford Jr, D.R. Fulkerson, Flows in networks, Princeton University Press, (2015).

[17] Boykov, Kolmogorov, Computing geodesics and minimal surfaces via graph cuts, in: Proceedings Ninth IEEE International Conference on Computer Vision, IEEE, Nice, France, 2003: pp. 26–33 vol.1.

[18] S.D. Kapade, S.M. Khairnar, B.S. Chaudhari, Image Segmentation using min – cut and swarm intelligence, Int. J. Math. Sci. Eng. Appl. 9 (2015), 165-177.

[19] A.X. Falcão, J.K. Udupa, F.K. Miyazawa, An ultra-fast user-steered image segmentation paradigm: live wire on the fly, IEEE Trans. Med. Imaging, 19(1) (2000), 55-62.

[20] A.D. Parakkat, J. Peethambaran, P. Joseph, R. Muthuganapathy, A Graph-based Geometric Approach to Contour Extraction from Noisy Binary Images, Computer-Aided Design Appl. 12(4) (2015), 403-413.

[21] D. Mahapatra, Semi-supervised learning and graph cuts for consensus based medical image segmentation, Pattern Recogn. 63 (2017), 700-709.

[22] N. Ghorpade, H.R. Bhapkar, Enhanced N-Cut and Watershed based Model for Brain MRI Segmentation, Int. J. Eng. Adv. Technol. 9(5) (2020), 346-351.

[23] B. Peng, L. Zhang, D. Zhang, A survey of graph theoretical approaches to image segmentation, Pattern Recogn. 46(3) (2013), 1020- 1038.

[24] Y.Y. Boykov, M.-P. Jolly, Interactive graph cuts for optimal boundary & region segmentation of objects in N-D images, in: Proceedings Eighth IEEE International Conference on Computer Vision. ICCV 2001, IEEE

Comput. Soc, Vancouver, BC, Canada, 2001: pp. 105–112.

[25] D.M. Greig, B.T. Porteous, A.H. Seheult, Exact Maximum A Posteriori Estimation for Binary Images, J. R. Stat. Soc.: Ser. B (Methodol.) 51 (1989), 271–279.

[26] Stubberfield, L. Big Picture. https://bigpictureeducation.com