Available online at http://scik.org
J. Math. Comput. Sci. 11 (2021), No. 5, 6629-6637
https://doi.org/10.28919/jmcs/6442
ISSN: 1927-5307

# IMPACT OF BERNOULLI AND NON-BERNOULLI WORKING VACATION SCHEDULE ON SINGLE SERVER MARKOVIAN QUEUE WITH DISASTER

M. LAKSHMI PRIYA*, N. KALAIVANI

Department of mathematics, Vel Tech Rangarajan Dr Sagunthala R & D Institute of Science and Technology, Chennai, India



**Abstract.** We discussed SSMQM (Single Server Markovian Queueing Model) with disaster. In Bernoulli trial, vacation period for working, after work the server either wait for the next service with probability $p$ (or) go for a vacation with probability $q$. By letting the disaster to happen in busy state, time dependent probabilities of the model under Bernoulli and Non-Bernoulli are investigated under Laplace transform techniques and recurrence relation for generating function at power series.

**Keywords:** Bernoulli working vacation; generating function; Laplace transform; transient probabilities.

**2010 AMS Subject Classification:** 90B15.

## 1. Introduction

It has been discussed for Queueing system under vacation model by lots of authors as it plays a vital role in networking, communication systems queueing models under disasters and vacations were first studied by Mytalas and Zazanis. In multiple adapted vacation policy, they considered $M/G/1$ queueing model with disaster and repair under in the year 2015.

*Corresponding author
E-mail address: vishnuvcstrust@gmail.com
Received June 30, 2021

Kumar and Arivudainambi considered $M/M/1$ queueing model with catastrop and derived transient solution in the year 2000. Kumar and Madheswari dealt with queueing model at single server subject to catastrophes and server failures in the year 2005. Oliver etal dealt with single server multiple vacation queueing system with differentiated vacations in the year 2014. Ye, Liu and Jiang in the year 2016 derived transient solution for under Bernoulli vacation schedule $M/M/1$ with disaster and repairs. Extending in Janani and Lakshmi Priya considered $M/M/1$ with disaster and repair under Bernoulli working vacation schedule. They also derived explicit analytical expressions for methods such as Laplace transform and generating function techniques using time dependent probabilities .

This models aims at analysing $M/M/1$ system for disaster under working vacation schedule as Bernoulli and Non-Bernoulli. We are motivate to analyse the impact of system under above mentioned schedules.

The paper is organized as below for overall discussion:

- In section 2, representation of probability with $M/M/1$ queue
- In section 3,transient probabilities in Bernoulli working vacation
- In section 4, transient probabilities in non-Bernoulli working vacation
- Conclusion and Future scope of the model.

## 2. Model Description

The Bernoulli and Non-Bernoulli working vacation schedule as $M/M/1$ is considered. $M/M/1$ system is analyzed at two cases Primarily in Bernoulli working vacation schedule and secondarily in Non-Bernoulli working vacation schedule. Arrivals follows Poisson distribution with parameter $\lambda$ whereas service takes place exponentially with parameter $\mu$. Disaster occurs at busy time and vacation time and disaster time are exponentially distributed with parameter $\eta$ and $\sigma$. System is assumed to be idle initially.

Mathematically

- $\rho_{2,0}(t)$ represents probability of idle state.
- $\rho_{0,n}(t)$ represents probability of working vacation state.
- $\rho_{1,n}(t)$ represents probability of busy state with initial condition $\rho_{2,0}(0) = 1$.

## 3. Transient Probabilities in Bernoulli Working Vacation Schedule

Defining generating function for $\rho_{0,n}(t)$ as

$$Q(z,t)=\sum_{n=1}^{\infty}\rho_{0,n}(t)z^n$$

Kolmogorov differential equations are

$$\rho'_{0,0}=-(\lambda+\eta)\rho_{0,0}+\mu q\rho_{0,0}+\mu_\rho\rho_{0,1} \tag{1}$$

$$\rho'_{0,n}=-(\lambda+\eta+\mu_\rho)\rho_{0,n}+\mu q\rho_{1,n+1}+\lambda\rho_{0,n-1}\mu_\rho\rho_{0,n+1}, n>1 \tag{2}$$

$$\rho'_{1,1}=-(\lambda+\eta+\sigma)\rho_{1,1}+\lambda\rho_{2,0}+\mu\rho\rho_{1,2}+\eta\rho_{0,1} \tag{3}$$

$$\rho'_{1,n}=-(\lambda+\eta+\sigma)\rho_{1,n}+\lambda\rho_{1,n-1}+\mu\rho\rho_{1,n+1}+\eta\rho_{0,n} \tag{4}$$

$$\rho'_{2,0}=-\lambda\rho_{2,0}+\mu\rho\rho_{1,1}+\eta\rho_{0,0} \tag{5}$$

Evaluation of $\rho_{0,n}(t)$:
Define $Q(z,t)=\sum\limits_{n=1}^{\infty}\rho_{0,n}(t)z^n$
then

$$Q'(z,t)=\sum_{n=1}^{\infty}\rho'_{0,n}(t)z^n \tag{6}$$

Substituting equations (1) and (2) and after some simplification equations (6) becomes

$$Q'(z,t)-((-\lambda+\eta+\mu_\rho)+\lambda z+\frac{\mu_\rho}{z})Q(z,t)=\lambda z\rho_{0,0}(t)-\mu_\rho\rho_{0,1}(t)+\frac{\mu q}{z}\sum_{n=1}^{\infty}Q(z,t) \tag{7}$$

Inverting equation (7) and applying modified Bessel functions we get

$$\begin{aligned}Q(z,t)=\lambda\int_0^t z\rho_{0,0}(t)e^{-(\lambda+\eta+\mu_\rho)(t-x)}\sum_{n=-\infty}^{\infty}(\tau,z)^nI_n(\sigma_1(t-x))dx\\ -\mu_v\int_0^t \rho_{0,1}(x)e^{-(\lambda+\eta+\mu_\rho)(t-x)}\sum_{n=-\infty}^{\infty}(\tau,z)^nI_n(\sigma_1(t-x))dx\\ +\mu_q\int_0^t\frac{1}{z}\left(\sum_{n=1}^{\infty}\rho_{1,n+1}(x)z^{n+1}\right)e^{-(\lambda+\eta+\mu_\rho)(t-x)}\sum_{n=-\infty}^{\infty}(\tau,z)^nI_n(\sigma_1(t-x))dx\end{aligned} \tag{8}$$

Coefficients subtraction of $z^n$ and $z^{-n}$ equation (8) we get

$$\rho_{0,n}(t) = \lambda \int_0^t \rho_{0,0}(x) e^{-(\lambda+\eta+\mu_\rho)(t-x)} (I_{n-1}(\sigma_1(t-x)) - I_{n+1}(\sigma_1(t-x)))dx$$

$$\mu_q \int_0^t \sum_{n=2}^{\infty} \rho_{1,m}(x) e^{-(\lambda+\eta+\mu_\rho)(t-x)} \tau_1^{n-m+1} (I_{n-m+1}(\sigma_1(t-x)) - I_{n-m-1}(\sigma_1(t-x))dx \tag{9}$$

By taking Laplace transform of equation (9) and substituting $n=1$ we get $\widehat{\rho}_{0,1}(s)$ in terms of $\widehat{\rho}_{0,0}(s)$ and $\widehat{\rho}_{1,n}(s)$.

Hence $\rho_{0,n}(s), n \geq 1$ is the form of $\rho_{0,0}(t)$ and $\rho_{1,n}(s)$ for $n = 2,3,\cdots$

To find $\rho_{0,0}(t)$:

Taking LT (Laplace transform) of (1) on both sides, we have

$$(s+\lambda+\eta)\widehat{\rho}_{0,0}(s) = \mu_q \rho_{1,1}(s) + \mu_\rho \rho_{0,1}(s) \tag{10}$$

Substituting $\rho_{0,1}(s)$ and after simplifying, then we have

$$\rho_{0,0}(s) = \frac{1}{(s+\lambda+\eta)} \sum_{j=0}^{\infty} (F(s))^j \left[ \mu_q \rho_{1,1}(s) + \mu_q \sum_{m=2}^{\infty} \rho_{1,m}(s) \tau_1^{2-m} (\Omega_{2-m}(s) - \Omega_m(s)) \right] \tag{11}$$

where

$$\rho_1 = s+\lambda+\eta+\mu_v$$

$$\Omega_\rho(s) = \left(\frac{\rho_1 - \sqrt{\rho_1^2 - \sigma_1^2}}{\sigma_1}\right) \frac{1}{\sqrt{\rho_1^2 - \sigma_1^2}}$$

Inverting equation (11) and substituting in equation (9) we get

$$\begin{aligned} \rho_{0,n}(t) = {} & \lambda \tau_1^{n-1} * \chi_{n-1,n+1}(t) * e^{-(\lambda+\eta)t} * \sum_{j=0}^{\infty} (F(t))^{*j} \\ & * (\mu_q \rho_{1,1}(t) + \mu_q \sum_{m=2}^{\infty} \tau_1^{2-m} \rho_{1,m}(t) * \chi_{2-m,m}(t)) \\ & + \mu_q \sum_{m=2}^{\infty} \tau_1^{n-m+1} \rho_{1,m}(t) \chi_{n-m+1,n+m-1}(t) * \sum_{m=2}^{\infty} \tau_1^{2-m} \rho_{1,m}(t) * \rho_{1,m}(t) \end{aligned} \tag{12}$$

Hence $\rho_{0,n}(t)$ is derived in terms of $\rho_{1,n}(t), n = 1,2,\cdots$

Calculation for $\rho_{2,0}(t)$:

Taking LT on both sides by eqn. (5), we have

$$\rho_{2,0}(s) = \frac{\sigma}{s+\lambda}\sum_{n=1}^{\infty}\rho_{1,n}(s)\frac{1}{s+\lambda} + \frac{\eta}{s+\lambda}\rho_{0,0}(s)$$

Hence $\rho_{2,0}(t)$ is the form of $\rho_{1,n}(t), n = 1,2,\cdots$

Calculation for $\rho_{1,n}(t)$:

Define

$$U(z,t) = \sum_{n=2}^{\infty}\rho_{1,n}(t)z^n$$
$$U'(z,t) = \sum_{n=2}^{\infty}\rho_{1,n}^{'}(t)z^n$$

After substituting equation (4) and simplifying we get

$$U'(z,t) - (-(\lambda+\mu+\sigma+\lambda z+\frac{\mu\rho}{z})U(z,t) = \lambda z^2\rho_{1,1}(t) - \mu p\rho_{1,2}(t)z + \eta\sum_{n=2}^{\infty}\rho_{0,n}(t)z^n$$

Integrating and substituting modified Bessel functions we get

$$U(z,t) = \int_0^t (\lambda z^2\rho_{1,1}(x) - \mu p\rho_{1,2}(x)z)e^{(-(\lambda+\mu+\sigma)+\lambda z+\frac{\mu p}{z})(t-x)}dx$$
$$+\eta\int_0^t (\sum_{n=2}^{\infty}\rho_{0,n}(x)z^n)e^{(-(\lambda+\mu+\sigma)+\lambda z+\frac{\mu p}{z})(t-x)}dx$$

where $\sigma = 2\sqrt{\lambda\mu}$ and $\tau = \sqrt{\frac{\lambda}{\mu}}$

Equating coefficient of $z^n$ on both sides and simplifying we get

$$\rho_{1,n}(t) = \lambda\tau^{n-2}(\rho_{1,1}(t) * e^{-(\lambda+\mu+\sigma)t}I_{n-2}(\sigma(t)))$$
$$-\mu p\tau^{n-1}(\rho_{1,2}(t) * e^{-(\lambda+\mu+\sigma)t}I_{n-1}(\sigma(t)))$$
$$+\eta[\sum_{k=2}^{\infty}\rho_{0,k}(t) * e^{-(\lambda+\mu+\sigma)t}\tau^{n-k}\rho_{n-k}(\sigma(t))], \text{for } n = 2,3,\cdots$$

Hence $\rho_{1,2}(t)$ may be obtained in the form of $\rho_{1,n}(t)$ and $\rho_{1,1}(t)$ after some simplifications and it is given by

$$\begin{aligned}\rho_{1,2}(t) = \sum_{j=0}^{\infty}(F_1(t))^{*j} * \Big[R(t) * \rho_{1,1}(t) * Q(t) \\ * \sum_{n=1}^{\infty}\rho_{1,n}(t) + w(t) * \sum_{m=2}^{\infty}\tau_1^{2-m}b_{1,m}(t) * \chi_{2-m,m}(t) \\ + \mu q\eta \sum_{k=2}^{\infty}\sum_{m=2}^{\infty}\rho_{1,m}(t) * \tau_1^{k-m+1}\chi_{k-m+1,k+m+1}(t) * u_{2-k}(t)\Big]\end{aligned} \tag{13}$$

Evaluation of $\rho_{1,1}(t)$:

Taking Laplace transform for equation (3) we get

$$\widehat{\rho}_{1,1}(s) = \frac{\mu p}{s+\lambda+\mu+\sigma}\widehat{\rho}_{1,2}(s) + \frac{\eta\rho_{0,1}(s)}{s+\lambda+\mu+\sigma}$$

Substituting the value of $\widehat{\rho}_{1,2}(s)$ and $\widehat{\rho}_{0,1}(s)$ and after some simplification by inverting it we get

$$\begin{aligned}\rho_{1,1}(t) = \sum_{j=0}^{\infty}(F_2(t))^{*j}(\mu p e^{-(\lambda+\mu+\sigma)t} * \sum_{j=0}^{\infty}(F_1(t))^{*j} + \lambda e^{-(\lambda+\mu+\sigma(t)}) \sum_{n=2}^{\infty}\rho_{1,n}(t) \\ + \lambda\eta e^{-(\lambda+\mu+\sigma)t} * \mu q e^{-\lambda t} * e^{-(\lambda+\eta)t}\sum(F(t))^{*j} + \mu p e^{-(\lambda+\mu+\sigma)t}\sum_{j=0}^{\infty}(F_1(t))^{*j}w(t) \\ + \eta\mu q e^{-(\lambda+\mu+\sigma)t} * (\lambda e^{-(\lambda+\eta)t} * \chi_{0,2}(t) \\ * \sum_{j=0}^{\infty}(F(t))^{*j} + \delta(t)) * \sum_{m=2}^{\infty}\rho_{1,m}(t)\tau_1^{2-m} * \chi_{2-m,m}(t)\end{aligned} \tag{14}$$

Hence all time dependent probabilities are obtained in the form of $\rho_{1,n}(t)$ while may be found by normality conditions.

## 4. TRANSIENT PROBABILITIES IN NON-BERNOULLI WORKING VACATION SCHEDULE

Kolmogorov differential equations are

$$\rho'_{0,0} = -(\lambda+\eta)\rho_{0,0} + \mu_\rho\rho_{0,1} \tag{15}$$

$$\rho'_{0,n} = -(\lambda+\eta+\mu_\rho)\rho_{0,n} + \lambda\rho_{0,n-1} + \mu_\rho\rho_{0,n+1} \tag{16}$$

$$\rho'_{1,1} = -(\lambda+\mu+\sigma)\rho_{1,1} + \lambda i_{2,0} + \mu b_{1,2} + \eta v_{0,1} \tag{17}$$

(18) $$\rho'_{1,n} = -(\lambda+\mu+\sigma)\rho_{1,n} + \lambda b_{1,n-1} + \mu b_{1,n+1} + \eta v_{0,n}$$

(19) $$\rho'_{2,0} = -\lambda\rho_{2,0} + \mu\rho_{1,1} + \eta\rho_{0,0}$$

Evaluation of $\rho_{0,n}(t)$:

Let $G(z,t) = \sum_{n=1}^{\infty} \rho_{0,n}(t)z^n$

then

(20) $$G'(z,t) = \sum_{n=1}^{\infty} \rho'_{0,n}(t)z^n$$

Substituting equations (18) in equation (20), simplifying we get

$$G'(z,t) - ((-\lambda+\eta+\mu_\rho) + \lambda z + \frac{\mu_\rho}{z})G(z,t) = \lambda z\rho_{0,0}(t) - \mu_\rho\rho_{0,1}(t)$$

Interesting after some simplification we get $\rho_{0,n}(t)$ in terms of $\rho_{0,0}(t)$ and $\rho_{1,n}(t)$

(21) $$\rho_{0,n}(t) = \lambda\int_0^t z\rho_{0,0}(t)e^{-(\lambda+\eta+\mu_\rho)(t-y)}\tau_1^{n-1}(\rho_{n-1}(\sigma_1(t-y)) - \rho_{n+1}(\sigma_1(t-x)))dx$$

To find $\rho_{2,0}(t)$:

Taking LT of eqn. (19), thus we have

$$\rho_{2,0}(s) = \frac{\sigma}{s+\lambda} + \frac{\mu}{s+\lambda}\rho_{1,1}(s) + \frac{\eta}{s+\lambda}\rho_{0,0}(s)$$

Inverting we get

$$\rho_{2,n}(t) = e^{-\lambda t} + \mu e^{-\lambda t} * \rho_{1,1}(t) + \eta e^{-\lambda t} * \rho_{0,0}(t)$$

Evaluation of $\rho_{1,n}(t)$:

Define

$$H(z,t) = \sum_{n=2}^{\infty} \rho_{1,n}(t)z^n$$

$$\text{then } H'(z,t) = \sum_{n=2}^{\infty} \rho_{1,n}^{\prime(t)z^n}$$

Substituting equation (18) in above equation and simplifying we get

(22) $$H'(z,t) - (-(\lambda+\mu+\sigma) + \lambda z + \frac{\mu}{z})H(z,t) = \lambda z^2\rho_{1,1}(t) - \mu\rho_{1,2}(t)z + \eta\sum_{n=2}^{\infty}\rho_{0,n}(t)z^n$$

Integrating equation (22) and applying modified Bessel functions we get

$$H(z,t) = \int_0^t (\lambda z^2 \rho_{1,1}(y) - \mu \rho_{1,2}(x) z) e^{(-(\lambda+\mu+\sigma)+\lambda z+\frac{\mu}{z})(t-x)} dx$$
$$+ \eta \int_0^t (\sum_{n=2}^{\infty} \rho_{0,n}(y) z^n) e^{(-(\lambda+\mu+\sigma)+\lambda z+\frac{\mu p}{z})(t-x)} dx$$

Equating coefficients of $z^n$ on both sides we get

$$\rho_{1,n}(t) = \lambda \tau^{n-2} [\rho_{1,1}(t) * e^{-(\lambda+\mu+\sigma)t} \rho_{n-2}(\sigma(t))$$
$$- \mu \tau^{n-1} (\rho_{1,2}(t) * e^{-(\lambda+\mu+\sigma)t} \rho_{n-1}(\sigma(t))]$$
$$+ \eta [\sum_{k=2}^{\infty} \rho_{0,k}(t) * e^{-(\lambda+\mu+\sigma)t} \tau^{n-k} I_{n-k}(\sigma(t))] \tag{23}$$

for $n \geq 2$

Hence $\rho_{1,2}(t)$ is obtained in the form of $\rho_{1,1}(t)$ and $\rho_{1,n}(t)$

To find $\rho_{0,0}(t)$:

Taking LT of eqn.(15), thus we have

$$\widehat{\rho}_{0,0}(s) = \frac{\mu_\rho}{s+\lambda+\eta} \widehat{\rho}_{0,1}(s)$$

Inverting the above equation we get

$$\rho_{0,0}(s) = \mu_\rho e^{-(\lambda+\eta)t} * \rho_{0,1}(s) \tag{24}$$

Evaluation of $\rho_{1,1}(t)$:

Taking Laplace transform for equation (17) we get

$$\widehat{\rho}_{1,1}(s) = \frac{\lambda}{s+\lambda+\mu+\sigma} \widehat{\rho}_{2,0}(s) + \frac{\mu}{s+\lambda+\mu+\sigma} \widehat{\rho}_{1,2}(s) + \frac{\eta}{s+\lambda+\mu+\sigma} \widehat{\rho}_{0,n}(s)$$

Substituting the value of $\widehat{\rho}_{2,0}(s)$, $\widehat{\rho}_{1,2}(s)$ and $\widehat{\rho}_{1,2}(s)$ in the above equation and after some simplification by inverting it we get $\rho_{1,1}(s)$ expressed in terms of $\rho_{1,n}(s)$. Also $\rho_{1,n}(s)$ can be explicitly determined using normality condition.

## 5. CONCLUSION

$M/M/1$ with disaster us analysed under Bernoulli and Non-Bernoulli working vacation schedule. This model can be extended further by including customer impatience in the system and also in multi server queueing model.

## CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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