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SPHERICAL INVOLUTES OF THE FIXED POLE CURVE $\left(C^{*}\right)$ ON THE TIMELIKE BERTRAND CURVE COUPLE

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Abstract: In this paper, it has been showed that every single indicatrix of tangents and indicatrix of binormals of the curve, α^{*} are spherical involutes of the fixed pole curve, $\left(C^{*}\right)$ by finding a transition link of the timelike Bertrand curve couple through Frenet frames.

Keywords: Lorentz Space, Timelike Bertrand Curve Couple, Spherical Involutes

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1.Preliminaries

Let Minkowski 3-space IR_1^3 be the vector space IR^3 equipped with the Lorentzian inner product g given by

$$g\left(X,X\right)=x_1^2+x_2^2-x_3^2$$

where $X=\left(x_1,x_2,x_3\right)\in IR^3$. A vector $X=\left(x_1,x_2,x_3\right)\in IR^3$ is said to be timelike if $g\left(X,X\right)<0$, spacelike if $g\left(X,X\right)>0$ and lightlike (or null) if $g\left(X,X\right)=0$. Similarly, an arbitrary curve $\alpha=\alpha\left(s\right)$ in IR_1^3 where s is a arclength parameter, can locally be timelike, spacelike or null (lightlike), if all of its velocity vectors $\alpha'\left(s\right)$ are respectively timelike, spacelike or null (lightlike) for every $s\in IR$. The norm of a vector $X\in IR_1^3$ is defined by $\left[2\right]$

$$\left\|X\right\|=\sqrt{\left|g\left(X,X\right)\right|}.$$

We denote by $\left\{T\left(s\right),N\left(s\right),B\left(s\right)\right\}$ the moving Frenet frame along the curve α. Let α be a timelike curve with curvature κ and torsion τ. Let frenet vector fields of α be $\left\{T,N,B\right\}$.

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In this trihedron, T is timelike vector field, N and B are spacelike vector fields. Then Frenet formulas are given by [3]

$$T' = \kappa N \ \ , N' = \kappa T - \tau B \ \ , B' = \tau N . \tag{1}$$

Let α be a timelike vector, the frenet vectors T timelike, N and B are spacelike vector, respectively, such that

$$T \times N = -B,\ \mathrm{N} \times B = T,\ B \times T = -N$$

and the frenet instantaneous rotation vector is given by [5]

$$W = \tau T - \kappa B, \quad \|W\| = \sqrt{\left|\kappa^2 - \tau^2\right|} .$$

Let φ be the angle between W and $-B$ vectors and if W is a spacelike vector, we can write

$$\begin{cases} \kappa = \|W\| \cosh\varphi \\ \\ \tau = \|W\| \sinh\varphi \end{cases} , \qquad \mathrm{C} = \sinh\varphi T - \cosh\varphi B \tag{2}$$

and if W is a timelike vector, we can write

$$\begin{cases} \kappa = \|W\| \sinh\varphi \\ \\ \tau = \|W\| \cosh\varphi \end{cases} , \qquad C = \cosh\varphi T - \sinh\varphi B . \tag{3}$$

Let $X = (x_1, x_2, x_3)$ and $Y = (y_1, y_2, y_3)$ be the vectors in IR_1^3. The cross product of X and Y is defined by [1]

$$X \wedge Y = (x_3 y_2 - x_2 y_3, x_1 y_3 - x_3 y_1, x_1 y_2 - x_2 y_1).$$

The curvatures drawn by unit speed non-null curve, $\alpha : I \to IR_1^3$ at the point $\alpha(s)$ with the frenet vectors T, N, B and the unit Darboux vector C over the Lorentzian unit sphere $S_1^{\ 2}$ or Hyperbolic unit sphere $H_0^{\ 2}$ are named respectively as indicatrix of tangents, indicatrix of principal normals , indicatrix of binormals and fixed pole curve. These curvatures are indicated in order as $(T),(N),(B)$ and (C) [4].

Let $\alpha : I \to IR_1^3$ and $\alpha^* : I \to IR_1^3$ be two timelike curves and if the tangent of α, $\alpha(s)$ posses through the point $\alpha^*(s)$ and $\langle T^*(s), T(s) \rangle = 0$, then the curve α^* is said to be the involute of α [6].

2. Spherical Involutes of the Fixed Pole Curve $\left(C^*\right)$ on the Timelike Bertrand Curve Couple

Definition 2.1: Let $\{T, N, B\}$ and $\{T^*, N^*, B^*\}$ be respectively the frenet frames of the timelike curves, $\alpha : I \to IL^3$ and $\alpha^* : I \to IL^3$ at points $\alpha(s)$ and $\alpha^*(s)$. If the principal normal vectors N and N^* are linearly dependent, then the pair $\left(\alpha, \alpha^*\right)$ is said to be timelike Bertrand curve couple.

Theorem 2.1: There is a connection between timelike Bertrand curve couple and Frenet frames that are written as followings

$$\begin{cases} T^* = -\cosh\theta T + \sinh\theta B \\ N^* = N \\ B^* = -\sinh\theta T + \cosh\theta B. \end{cases}$$

Here, the angle θ is the angle between T and T^*.

Proof: By taking the derivative of $\alpha^*(s) = \alpha(s) + \lambda N(s)$ with respect to arc lenght s and using the equation , we get

$$T^* \frac{ds^*}{ds} = T(1+\lambda\kappa) - \lambda\tau B \, . \tag{4}$$

The inner products of the above equation with respect to T and B are respectively defined as

$$\begin{cases} -\cosh\theta \dfrac{ds^*}{ds} = 1 + \lambda\kappa \\ -\sinh\theta \dfrac{ds^*}{ds} = \lambda\tau \end{cases}$$

and substituting these present equations in (4) we obtain

$$T^* = -\cosh\theta T + \sinh\theta B \, . \tag{5}$$

Here, by finding the derivative of (4) and using (1) we get

$$N^* = N .$$

Firstly, we can write

$$B^* = -\sinh\theta T + \cosh\theta B \tag{6}$$

by availing the equation $B^* = -\left(T^* \times N^*\right)$.

By the derivative of $\alpha_{T^*}\left(s_{T^*}\right) = T^*\left(s\right)$ with respect to arc-lenght s_{T^*} parameter, we get

$$T_{T^*} = \frac{dT^*}{ds}.\frac{ds}{ds_{T^*}} .$$

Afterwards, by some algebraic manipulations and substituting $\left(5\right)$ in T_{T^*}, the following result can be achieved

$$T_{T^*} = \mp N . \tag{7}$$

Similarly, by taking the derivative of $\alpha_{B^*}\left(s_{B^*}\right) = B^*\left(s\right)$ with respect to arc-lenght s_{B^*} parameter, we get

$$T_{B^*} = \frac{dB^*}{ds}.\frac{ds}{ds_{B^*}} .$$

By using the equation $\left(6\right)$, we write down

$$T_{B^*} = \mp N . \tag{8}$$

Lastly, by taking the derivative of $\alpha_{C^*}\left(s_{C^*}\right) = C^*\left(s\right)$ with respect to arc-lenght s_{C^*} parameter, we obtain

$$T_{C^*} = \frac{dC^*}{ds}.\frac{ds}{ds_{C^*}} .$$

If W^* is spacelike, then by considering $\left(2\right)$ and with some algebraic operation we get

$$T_{C^*} = \cosh\varphi^* T^* - \sinh\varphi^* B^*, \tag{9}$$

if W^* is timelike, then then by considering $\left(3\right)$ and with some algebraic operation we get

$$T_{C^*} = \sinh\varphi^* T^* - \cosh\varphi^* B^*. \tag{10}$$

Theorem 2.2: Let (α,α^*) be timelike Bertrand curve couple. Tach of the indicatrix of tangents (T^*) and the indicatrix of binormals (B^*) of α^* curve is a spherical involute of the fixed pole curve (C^*).

Proof: In order to show that (T^*) and (B^*) of the α^* curve is each a spherical involute of (C^*), we need to prove the following statements

$$\langle\frac{dT^*}{ds},\frac{dC^*}{ds}\rangle=0$$

and

$$\langle\frac{dB^*}{ds},\frac{dC^*}{ds}\rangle=0\,.$$

If W^* is spacelike, by taking into account (7) and (9) we write down

$$\langle\frac{dT^*}{ds},\frac{dC^*}{ds}\rangle=\langle N,\cosh\varphi^* T^*-\sinh\varphi^* B^*\rangle\,.$$

Next, by using (5) and (6) and doing required manipulations, the following result can be obtained

$$\langle\frac{dT^*}{ds},\frac{dC^*}{ds}\rangle=0\,.$$

Furthermore, by exploiting the equations $(5),(6),(8)$ and (9), we do the similar calculations to get

$$\langle\frac{dB^*}{ds},\frac{dC^*}{ds}\rangle=0\,.$$

If W^* is timelike, we use (7) and (10) to reach

$$\langle\frac{dT^*}{ds},\frac{dC^*}{ds}\rangle=\langle N,\sinh\varphi^* T^*-\cosh\varphi^* B^*\rangle\,.$$

Here, by substitution of (5) and (6) in the above the formula, we write

$$\langle\frac{dT^*}{ds},\frac{dC^*}{ds}\rangle=0\,.$$

Once again, when the given relations $(5),(6),(8)$ and (10) are taken into consideration, we get

$$\langle \frac{dB^*}{ds}, \frac{dC^*}{ds} \rangle = 0 .$$

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