Available online at http://scik.org
J. Math. Comput. Sci. 11 (2021), No. 6, 6745-6772
https://doi.org/10.28919/jmcs/6473
ISSN: 1927-5307

OPERATIONS ON BIPOLAR INTERVAL VALUED ANTI INTUITIONISTIC FUZZY GRAPH

P. SUNITHA*, P. SUNDARI†

PG Department of Mathematics, Arignar Anna Govt Arts College, Musiri, Tamil Nadu, India

Abstract: In this study we have made an attempt by introducing the notion of bipolar anti intuitionistic fuzzy graph and bipolar interval valued anti intuitionistic fuzzy graph. Some basic concepts such as composition, Cartesian product, union on bipolar anti intuitionistic fuzzy graph and bipolar interval valued anti intuitionistic fuzzy graph has been discussed.

Keywords: bipolar anti intuitionistic fuzzy graph; interval valued; Cartesian product; union.

2010 AMS Subject Classification: 03F55, 03E72, 05C99.

1. INTRODUCTION

Fuzzy set was introduced by Lofti Asker Zadeh in 1965. A Kaufmann's initial definition of fuzzy graphs[5] was based on Zadeh's fuzzy relations[11]. Attanassov [2] extended the idea of a fuzzy set and introduced the concept of an intuitionistic fuzzy set. Attanassov[3] also introduced the concept of intuitionistic fuzzy graphs and intuitionistic fuzzy relations. Shannon and Attanassov

*Corresponding author
E-mail address: psunitha33@yahoo.com
†M. Phil Scholar
Received July 7, 2021

[10] investigated the properties of intuitionistic fuzzy relations and intuitionistic fuzzy graphs. Parvathi[7] also defined operations of intuitionistic fuzzy graphs. Mohammed Jabarulla [6] has explained the concept of anti-intuitionistic fuzzy graph. Zhang [12,13] introduced the concept of bipolar fuzzy set. Sunil Mathew [9] has discussed in detail about bipolar fuzzy graph and interval valued fuzzy graph. Asad Alnaser[1] has explained the notions of intuitionistic bipolar fuzzy graph and it's matrices. Sonia Mandal has explained the concept of product of bipolar anti intuitionistic fuzzy graph [8]. Using these notions we have tried to introduce the concept of bipolar anti intuitionistic fuzzy graph and bipolar interval valued anti intuitionistic fuzzy graph.

2. PRELIMINARIES

Definition 2.1 [4]:

An intuitionistic fuzzy graph is of the form $G = < V, E >$ where

(i) $V = \{v1, v2, \ldots, vn\}$ such that $\eta_1: V \to [0,1]$ and $\zeta_1: V \to [0,1]$ denote the degree of membership and non-membership of the element $v_i \in V$, respectively, and $0 \leq \eta_1(vi) + \zeta_1(vi) \leq 1$ for every $vi \in V, (i = 1,2, \ldots \ldots . n)$,

(ii) $E \subseteq V X V$ where $\eta_2: V X V \to [0,1]$ and $\zeta_2: V X V \to [0,1]$ are such that

$$\eta_2(vi, vj) \leq min\,[\eta_1(vi), \eta_1(vj)]\; and\; \zeta_2(vi, vj) \leq max\,[\zeta_1(vi), \zeta_1(vj)]$$

and $0 \leq \eta_2(vi, vj) + \zeta_2(vi, vj) \leq 1$ for every $(vi, vj) \in E, (i, j = 1,2, \ldots \ldots . n)$

Definition 2.2 [6]:

An anti-intuitionistic fuzzy graph is of the form $G = < V, E >$ where

(i) $V = \{v1, v2, \ldots, vn\}$ such that $\mu_1: V \to [0,1]$ and $\gamma_1: V \to [0,1]$ denote the degree of membership and non-membership of the element $v_i \in V$, respectively, and $0 \leq \mu_1(vi) + \gamma_1(vi) \leq 1$ for every $vi \in V, (i = 1,2, \ldots \ldots . n)$,

(ii) $E \subseteq V X V$ where $\mu_2: V X V \to [0,1]$ and $\gamma_2: V X V \to [0,1]$ are such that

$$\mu_2(vi, vj) \geq max\,[\mu_1(vi), \mu_1(vj)]\; and\; \gamma_2(vi, vj) \geq min\,[\gamma_1(vi), \gamma_1(vj)]$$

and $0 \leq \mu_2(vi, vj) + \gamma_2(vi, vj) \leq 1$ for every $(vi, vj) \in E, (i, j = 1,2, \ldots \ldots . n)$

Definition 2.3[8]

Let X be a non-empty set. A bipolar fuzzy set B in X is a set of triples $B = \{x, \eta_B^P(x),\ \eta_B^N(x))$ $/x \in X\}$, where η_B^P: $X \rightarrow [0, 1]$ and η_B^N : $X \rightarrow [-1, 0]$.

We often write $B = (\eta_B^P, \eta_B^N)$ for the bipolar fuzzy set $B = \{(x, \eta_B^P(x), \eta_B^N(x))\backslash x \in X\}$. We also use the notation $B = \{x, m^+(x), m^{-(x)}\backslash x \in X\}$ or $B = m^+, m^-$ for a bipolar fuzzy set.

Definition 2.4 [8]

Let $A = (m_1^+, m_2^-)$ and $B = (m_2^+, m_2^-)$ be bipolar fuzzy sets in X. We define the bipolar fuzzy sets $A \cap B$ and $A \cup B$ as follows: for all $x \in X$,

$$(A \cap B) = \left(\eta_A^P(x) \wedge \eta_B^P(x), \eta_A^N(x) \vee \eta_B^N(x)\right).$$

$$(A \cup B) = \left(\eta_A^P(x) \vee \eta_B^P(x), \eta_A^N(x) \wedge \eta_B^N(x)\right).$$

Definition 2.5 [8]

A bipolar fuzzy graph with an underlying set V is defined to be a pair $G = (A, B)$, where $A = (\eta_A^P, \eta_A^N)$ is a bipolar fuzzy set in V and $B = (\eta_B^P, \eta_B^N)$ is a bipolar fuzzy set in E such that

$$(\eta_B^P)(xy) \leq \eta_A^p(x) \wedge \eta_A^P(y)\ and\ (\eta_B^N)(xy) \geq \eta_A^N(x) \vee \eta_A^N(y)\ for\ all\ x\,y\ \in\ E\ ,\ \eta^P(xy) = \eta_B^N(xy) = 0$$ for all $xy \in \varepsilon\backslash E$.

3. MAIN RESULTS

3.1 BIPOLAR ANTI INTUITIONISTIC FUZZY GRAPH

Definition 3.1.1

A bipolar anti intuitionistic fuzzy graph (BAIFG) with an underlying set V is defined to be a pair $G = (A, B)$, where $A = (\eta_A^P, \eta_A^N, \zeta_A^P, \zeta_A^N)$ is a bipolar anti intuitionistic fuzzy set in V and $B = (\eta_B^P, \eta_B^N, \zeta_B^P, \zeta_B^N)$ is a bipolar anti intuitionistic fuzzy set in E such that

$$(\eta_B^P)(xy) \geq \eta_A^p(x) \vee \eta_A^P(y)\ ,\ (\zeta_B^P)(xy) \geq \zeta_A^P(x) \wedge \zeta_A^P(y)\quad and,$$

$$(\eta_B^N)(xy) \leq \eta_A^N(x) \wedge \eta_A^N(y), (\zeta_B^N)(xy) \leq \zeta_A^N(x) \vee \zeta_A^N(y)\ \ for\ all\ x\,y\ \in\ E$$

Example 3.1.2

Consider the BAIFG G where $V = \{a, b, c\}$, $E = \{ab, bc, ac\}$ such that

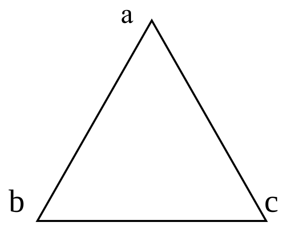

Figure 1: Bipolar anti intuitionistic fuzzy graph (BAIFG)

	a	b	c
η_A^P	0.4	0.5	0.6
η_A^N	-0.3	-0.4	-0.5
ζ_A^P	0.5	0.3	0.1
ζ_A^N	-0.4	-0.3	-0.3

	ab	bc	ac
η_B^P	0.6	0.7	0.7
η_B^N	-0.5	-0.5	-0.5
ζ_B^P	0.4	0.2	0.1
ζ_B^N	-0.4	-0.4	-0.3

Table 1: *Bipolar anti intuitionistic fuzzy sets in* V *and* E *of* G

Definition 3.1.3

A bipolar anti intuitionistic fuzzy graph (BAIFG) is said to be strong- bipolar anti intuitionistic fuzzy graph if

$$(\eta_B^P)(xy) = \eta_A^p(x) \vee \eta_A^P(y)\ ,\ (\zeta_B^P)(xy) = \zeta_A^P(x) \wedge \zeta_A^P(y) \quad and,$$

$$(\eta_B^N)(xy) = \eta_A^N(x) \vee \eta_A^N(y), (\zeta_B^N)(xy) = \zeta_A^N(x) \wedge \zeta_A^N(y)\ \ for\ all\ x\ y\ \in\ E$$

Definition 3.1.4

A bipolar anti intuitionistic fuzzy graph is said to be complete- bipolar anti intuitionistic fuzzy graph if $(\eta_B^P)(xy) = \eta_A^p(x) \vee \eta_A^P(y)\ ,\ (\zeta_B^P)(xy) = \zeta_A^P(x) \wedge \zeta_A^P(y) \quad and$

$$(\eta_B^N)(xy) = \eta_A^N(x) \vee \eta_A^N(y), (\zeta_B^N)(xy) = \zeta_A^N(x) \wedge \zeta_A^N(y)\ \ for\ all\ x, y\ \in\ V$$

Definition 3.1.5

The degree of a vertex in a bipolar anti intuitionistic fuzzy graph is defined to be

$d(x) = \left(d_\eta^P(x), d_\eta^N(x), d_\zeta^P(x), d_\zeta^P(x)\right)$ where

$$d_\eta^P(x) = \sum_{x \neq y} \eta^P(xy),\ d_\eta^N(x) = \sum_{x \neq y} \eta^N(xy),$$

$$d_\zeta^P(x) = \sum_{x \neq y} \zeta^P(xy),\ d_\zeta^N(x) = \sum_{x \neq y} \zeta^N(xy)$$

Definition 3.1.6

Let $A_1 = (\eta_{A1}^P, \eta_{A1}^N, \zeta_{A1}^P, \zeta_{A1}^N)$ and $A_2 = (\eta_{A2}^P, \eta_{A2}^N, \zeta_{A2}^P, \zeta_{A2}^N)$ be bipolar anti intuitionistic fuzzy subsets of V_1 and V_2 and $B_1 = (\eta_{B1}^P, \eta_{B1}^N, \zeta_{B1}^P, \zeta_{B1}^N)$and $B_2 = (\eta_{B2}^P, \eta_{B2}^N, \zeta_{B2}^P, \zeta_{B2}^N)$ be bipolar anti intuitionistic fuzzy subsets of E_1 and E_2 respectively. Then we denote the Cartesian product of two bipolar anti intuitionistic fuzzy graphs G_1 and G_2 of the graphs G_1^* and G_2^* by $G_1 \times G_{2=}(A_1 \times A_2, B_1 \times B_2)$ and defined by

(i)
$$(\eta_{A1}^P \times \eta_{A2}^P)(x_1, x_2) = \eta_{A1}^P(x_1) \vee \eta_{A2}^P(x_2)$$
$$(\eta_{A1}^N \times \eta_{A2}^N)(x_1, x_2) = \eta_{A1}^N(x_1) \wedge \eta_{A2}^N(x_2)$$
$$(\zeta_{A1}^P \times \zeta_{A2}^P)(x_1, x_2) = \zeta_{A1}^P(x_1) \wedge \zeta_{A2}^P(x_2)$$
$$(\zeta_{A1}^N \times \zeta_{A2}^N)(x_1, x_2) = \zeta_{A1}^N(x_1) \vee \zeta_{A2}^N(x_2) \text{for all } (x_1, x_2) \in V \times V.$$

(ii)
$$(\eta_{B1}^P \times \eta_{B2}^P)((x, x_2)(x, y_2)) = \eta_{A1}^P(x) \vee \eta_{B2}^P(x_2 y_2)$$
$$(\eta_{B1}^N \times \eta_{B2}^N)((x, x_2)(x, y_2)) = \eta_{A1}^N(x) \wedge \eta_{B2}^N(x_2 y_2)$$
$$(\zeta_{B1}^P \times \zeta_{B2}^P)((x, x_2)(x, y_2)) = \zeta_{A1}^P(x) \wedge \zeta_{B2}^P(x_2 y_2)$$
$$(\zeta_{B1}^N \times \zeta_{B2}^N)((x, x_2)(x, y_2)) = \zeta_{A1}^N(x) \vee \zeta_{B2}^N(x_2 y_2)$$

for all $x \in V_1$, for all $x_2 y_2 \in E_2$.

(iii)
$$(\eta_{B1}^P \times \eta_{B2}^P)\big((x_1, z)(y_1, z)\big) = \eta_{B1}^P(x_1 y_1) \vee \eta_{A2}^P(z)$$
$$(\eta_{B1\times}^N \eta_{B2}^N)((x_1, z)(y_1, z)) = \eta_{B1}^N(x_1 y_1) \wedge \eta_{A2}^N(z)$$
$$(\zeta_{B1}^P \times \zeta_{B2}^P)((x_1, z)(y_1, z)) = \zeta_{A2}^P(z) \wedge \zeta_{B1}^P(x_1 y_1)$$
$$(\zeta_{B1}^N \times \zeta_{B2}^N)((x_1, z)(y_1, z)) = \zeta_{A2}^N(z) \vee \zeta_{B1}^N(x_1 y_1)$$

for all $z \in V_2$, for all $x_1 y_1 \in E_1$.

Example 3.1.7: Let G_1 and G_2 be BAIFG where $V_1 = \{a, b\}$, $E_1 = \{ab\}$ and $V_2 = \{c, d\}$, $E_2 = \{cd\}$ respectively. Then we denote the Cartesian product of two BAIFG as follows

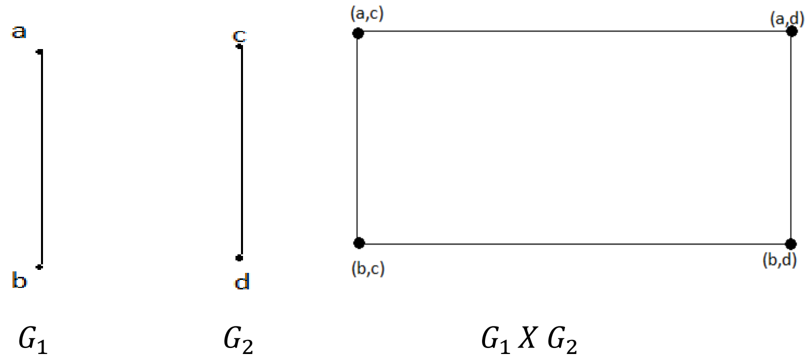

Figure2. Cartesian product of two BAIFG

	a	***b***	***c***	***d***	***(a,c)***	***(a,d)***	***(b,c)***	***(b,d)***
η_A^P	0.4	0.5	0.6	0.3	0.6	0.4	0.6	0.5
η_A^N	-0.3	-0.4	-0.5	-0.5	-0.5	-0.5	-0.5	-0.5
ζ_A^P	0.5	0.3	0.1	0.5	0.1	0.5	0.1	0.3
ζ_A^N	-0.4	-0.3	-0.3	-0.3	-0.3	-0.3	-0.3	-0.3

	ab	***cd***	***(a,c)(a,d)***	***(b,c)(b,d)***	***(a,c)(b,c)***	***(a,d)(b,d)***
η_B^P	0.6	0.7	0.7	0.7	0.6	0.6
η_B^N	-0.5	-0.5	-0.5	-0.5	-0.5	-0.5
ζ_B^P	0.4	0.2	0.2	0.2	0.1	0.4
ζ_B^N	-0.4	-0.3	-0.3	-0.3	-0.3	-0.3

Table 2: *Bipolar anti intuitionistic fuzzy sets in V and E of G_1, G_2, $G_1\ X\ G_2$*

Proposition 3.1.8

If G_1 and G_2 are bipolar anti intuitionistic fuzzy graphs, then $G_1 \times G_2$ is a bipolar anti intuitionistic fuzzy graph.

Proof:

Let $x \in V_1$ and $x_2y_2 \in E_2$. Then

$$(\eta_{B1}^P \times \eta_{B2}^P)\big((x,x_2)(x,y_2)\big) = \eta_{A1}^P(x) \vee \eta_{B2}^P(x_2y_2)$$

$$\geq \vee \{\eta_{A1}^{P}(x), \eta_{A2}^{P}(x_2) \vee \eta_{A2}^{P}(y_2)\}$$

$$= \vee \{\eta_{A1}^{P}(x) \vee \eta_{A2}^{P}(x_2)), \eta_{A1}^{P}(x) \vee \eta_{A2}^{P}(y_2)\}$$

$$= (\eta_{A1}^{P} \times \eta_{A2}^{P})(x, x_2) \vee (\eta_{A1}^{P} \times \eta_{A2}^{P})(x, y_2).$$

$$(\eta_{B1}^{N} \times \eta_{B2}^{N})((x, x_2)(x, y_2)) = \eta_{A1}^{N}(x) \wedge \eta_{B2}^{N}(x_2 y_2)$$

$$\leq \wedge \{\eta_{A1}^{N}(x), (\eta_{A2}^{N}(x_2) \wedge \eta_{A2}^{N}(y_2)\}$$

$$= \wedge \{\eta_{A1}^{N}(x) \wedge \eta_{A2}^{N}(x_2), \eta_{A1}^{N}(x) \wedge \eta_{A2}^{N}(y_2)\}$$

$$= (\eta_{A1}^{N} \times \eta_{A2}^{N})(x, x_2) \wedge (\eta_{A2}^{N} \times \eta_{A1}^{N})(x, y_2).$$

$$(\zeta_{B1}^{P} \times \zeta_{B2}^{P})\big((x, x_2)(x, y_2)\big) = \zeta_{A1}^{P}(x) \wedge \zeta_{B2}^{P}(x_2 y_2)$$

$$\geq \wedge \{\zeta_{A1}^{P}(x), \zeta_{A2}^{P}(x_2) \wedge \zeta_{A2}^{P}(y_2)\}$$

$$= \wedge \{\zeta_{A1}^{P}(x) \wedge \zeta_{A2}^{P}(x_2)), \zeta_{A1}^{P}(x) \wedge \zeta_{A2}^{P}(y_2)\}$$

$$= (\zeta_{A1}^{P} \times \zeta_{A2}^{P})(x, x_2) \wedge (\zeta_{A1}^{P} \times \zeta_{A2}^{P})(x, y_2).$$

$$(\zeta_{B1}^{N} \times \zeta_{B2}^{N})((x, x_2)(x, y_2)) = \zeta_{A1}^{N}(x) \vee \zeta_{B2}^{N}(x_2 y_2)$$

$$\leq \vee \{\zeta_{A1}^{N}(x), (\zeta_{A2}^{N}(x_2) \vee \zeta_{A2}^{N}(y_2)\}$$

$$= \vee \{\zeta_{A1}^{N}(x) \vee \zeta_{A2}^{N}(x_2), \zeta_{A1}^{N}(x) \vee \zeta_{A2}^{N}(y_2)\}$$

$$= (\zeta_{A1}^{N} \times \zeta_{A2}^{N})(x, x_2) \vee (\zeta_{A2}^{N} \times \zeta_{A1}^{N})(x, y_2).$$

Let $z \in V_{2,} and\ x_1 y_1 \in E_{1.}$ Thus

$$(\eta_{B1}^{P} \times \eta_{B2}^{P})((x_1, z)(y_1, z) = \eta_{B1}^{P}(x_1 y_1) \vee \eta_{A2}^{P}(z)$$

$$\geq \vee \{\big(\eta_{A1}^{P}(x_1) \vee \eta_{A1}^{P}(y_1)\big), \eta_{A2}^{P}(z)\}$$

$$= \vee \{\eta_{A1}^{P}(x_1) \vee \eta_{A2}^{P}(z), \eta_{A1}^{P}(y_1) \vee \eta_{A2}^{P}(z)\}$$

$$= (\eta_{A1}^{P} \times \eta_{A2}^{P})(x_1, z) \vee \big(\eta_{A1}^{P} \times \eta_{A2}^{P}(y_1, z)\big)$$

$$(\eta_{B1}^{N} \times \eta_{B2}^{N})\big((x_1, z)(y_1, z)\big) = \eta_{B1}^{N}(x_1 y_1) \wedge \eta_{A2}^{N}(z)$$

$$\leq \wedge \{\eta_{A1}^{N}(x_1) \wedge \eta_{A1}^{N}(y_1), \eta_{A2}^{N}(z)\}$$

$$= \wedge \{\eta_{A1}^{N}(x_1) \wedge \eta_{A2}^{N}(z), \eta_{A1}^{N}(y_1) \wedge \eta_{A2}^{N}(z)\}$$

$$= (\eta_{A1}^{N} \times \eta_{A2}^{N})(x_1, z) \wedge (\eta_{A1}^{N} \times \eta_{A2}^{N})(y_1, z).$$

$$(\zeta_{B1}^{P} \times \zeta_{B2}^{P})((x_1, z)(y_1, z) = \zeta_{B1}^{P}(x_1 y_1) \wedge \zeta_{A2}^{P}(z)$$

$$\geq \wedge \{(\zeta_{A1}^P(x_1) \wedge \zeta_{A1}^P(y_1)), \zeta_{A2}^P(z)\}$$

$$= \wedge \{\zeta_{A1}^P(x_1) \wedge \zeta_{A2}^P(z), \zeta_{A1}^P(y_1) \wedge \zeta_{A2}^P(z)\}$$

$$= (\zeta_{A1}^P \times \zeta_{A2}^P)(x_1, z) \wedge (\zeta_{A1}^P \times \zeta_{A2}^P(y_1, z))$$

$$(\zeta_{B1}^N \times \zeta_{B2}^N)((x_1, z)(y_1, z)) = \zeta_{B1}^N(x_1 y_1) \vee \zeta_{A2}^N(z)$$

$$\leq \vee \{\zeta_{A1}^N(x_1) \vee \zeta_{A1}^N(y_1), \zeta_{A2}^N(z)\}$$

$$= \vee \{\zeta_{A1}^N(x_1) \vee \zeta_{A2}^N(z), \zeta_{A1}^N(y_1) \vee \zeta_{A2}^N(z)\}$$

$$= (\zeta_{A1}^N \times \zeta_{A2}^N)(x_1, z) \vee (\zeta_{A1}^N \times \zeta_{A2}^N)(y_1, z).$$

Let $E = \{(x, x_2)(x, y_2) \backslash x \in V_1, x_2 y_2 \in E_2\} \cup \{(x_1, z)(y_1, z) \backslash z \in V_2, x_1 y_1 \in E_1\}$ and

Let $E^{(0)} = E \cup (x_1, x_2)(y_1, y_2) \backslash x_1 y_1 \in E_1$ and $x_2, y_2 \in V_2, x_2 \neq y_2\}$.

Definition 3.1.9

Let $A_1 = (\eta_{A1}^P, \eta_{A1}^N, \zeta_{A1}^P, \zeta_{A1}^N)$ and $A_2 = (\eta_{A2}^P, \eta_{A2}^N, \zeta_{A2}^P, \zeta_{A2}^N)$ be bipolar anti intuitionistic fuzzy subsets of V_1 and V_2 and $B_1 = (\eta_{B1}^P, \eta_{B1}^N, \zeta_{B1}^P, \zeta_{B1}^N)$ and $B_2 = (\eta_{B2}^P, \eta_{B2}^N, \zeta_{B2}^P, \zeta_{B2}^N)$ be bipolar anti intuitionistic fuzzy subsets of E_1 and E_2 respectively. The composition of two bipolar anti intuitionistic fuzzy graphs G_1 and G_2 of the graphs G_1^* and G_2^* denoted by $G_1[G_2] = (A_1 \circ A_2, B_1 \circ B_2)$ is defined by

(i) $(\eta_{A1}^P \circ \eta_{A2}^P)(x_1, x_2) = \eta_{A1}^P(x_1) \vee \eta_{A2}^P(x_2)$

$(\eta_{A1}^N \circ \eta_{A2}^N)(x_1, x_2) = \eta_{A1}^N(x_1) \wedge \eta_{A2}^N(x_2)$

$(\zeta_{A1}^P \circ \zeta_{A2}^P)(x_1, x_2) = \zeta_{A1}^P(x_1) \wedge \zeta_{A2}^P(x_2)$

$(\zeta_{A1}^N \circ \zeta_{A2}^N)(x_1, x_2) = \zeta_{A1}^N(x_1) \vee \zeta_{A2}^N(x_2)$ for all $(x_1, x_2) \in V \times V$.

(ii) $(\eta_{B1}^P \circ \eta_{B2}^P)((x, x_2)(x, y_2)) = \eta_{A1}^P(x) \vee \eta_{B2}^P(x_2 y_2)$

$(\eta_{B1}^N \circ \eta_{B2}^N)((x, x_2)(x, y_2)) = \eta_{A1}^N(x) \wedge \eta_{B2}^N(x_2 y_2)$

$(\zeta_{B1}^P \circ \zeta_{B2}^P)((x, x_2)(x, y_2)) = \zeta_{A1}^P(x) \wedge \zeta_{B2}^P(x_2 y_2)$

$(\zeta_{B1}^N \circ \zeta_{B2}^N)((x, x_2)(x, y_2)) = \zeta_{A1}^N(x) \vee \zeta_{B2}^N(x_2 y_2)$

for all $x \in V_1$, for all $x_2 y_2 \in E_2$.

(iii) $(\eta_{B1}^P \circ \eta_{B2}^P)((x_1, z)(y_1, z)) = \eta_{B1}^P(x_1 y_1) \vee \eta_{A2}^P(z)$

$$(\eta_{B1}^N \circ \eta_{B2}^N)((x_1,z)(y_1,z)) = \eta_{B1}^N(x_1y_1) \wedge \eta_{A2}^N(z)$$

$$(\zeta_{B1}^P \circ \zeta_{B2}^P)((x_1,z)(y_1,z)) = \zeta_{A2}^P(z) \wedge \zeta_{B1}^P(x_1y_1)$$

$$(\zeta_{B1}^N \circ \zeta_{B2}^N)((x_1,z)(y_1,z)) = \zeta_{A2}^N(z) \vee \zeta_{B1}^N(x_1y_1)$$

for all $z \in V_2$, for all $x_1y_1 \in E_1$.

(iv) $(\eta_{B1}^P \circ \eta_{B2}^P)\big((x_1,x_2)(y_1y_2)\big) = \vee \{\eta_{A2}^P(x_2), \eta_{A2}^P(y_2), \eta_{B1}^P(x_1y_1)\}$

$(\eta_{B1}^N \circ \eta_{B2}^N)\big((x_1,x_2)(y_1,y_2)\big) = \wedge \{\eta_{A2}^N(x_2), \eta_{A2}^N(y_2), \eta_{B1}^N(x_1y_1)\}$

$(\zeta_{B1}^P \circ \zeta_{B2}^P)\big((x_1,x_2)(y_1y_2)\big) = \wedge \{\zeta_{A2}^P(x_2), \zeta_{A2}^P(y_2), \zeta_{B1}^P(x_1y_1)\}$

$(\zeta_{B1}^N \circ \zeta_{B2}^N)\big((x_1,x_2)(y_1,y_2)\big) = \vee \{\zeta_{A2}^N(x_2), \zeta_{A2}^N(y_2), \zeta_{B1}^N(x_1y_1)\}$

for all $(x_1,x_2)(y_1y_2) \in E^0 \backslash E$.

Example 3.1.10: Let G_1 and G_2 be BAIFG where $V_1 = \{a,b\}$, $E_1 = \{ab\}$ and $V_2 = \{c,d\}$, $E_2 = \{cd\}$ respectively. Then we denote the Composition of two BAIFG as follows

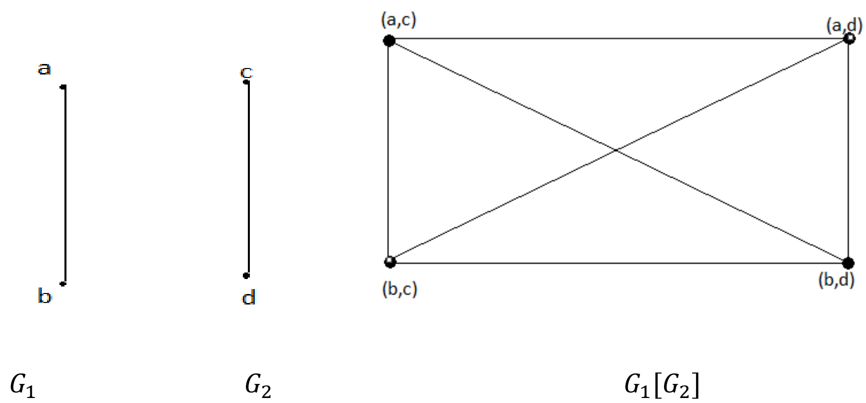

Figure 3. Composition of two BAIFG

	a	*b*	*c*	*d*	(a,c)	(a,d)	(b,c)	(b,d)
η_A^P	0.4	0.5	0.6	0.3	0.6	0.4	0.6	0.5
η_A^N	-0.3	-0.4	-0.5	-0.5	-0.5	-0.5	-0.5	-0.5
ζ_A^P	0.5	0.3	0.1	0.5	0.1	0.5	0.1	0.3
ζ_A^N	-0.4	-0.3	-0.3	-0.3	-0.3	-0.3	-0.3	-0.3

	ab	cd	(a,c)(a,d)	(b,c)(b,d)	(a,c)(b,c)	(a,d)(b,d)	(a,c)(b,d)	(a,d)(b,c)
η_B^P	0.6	0.7	0.7	0.7	0.6	0.6	0.6	0.6
η_B^N	-0.5	-0.5	-0.5	-0.5	-0.5	-0.5	-0.5	-0.5
ζ_B^P	0.4	0.2	0.2	0.2	0.1	0.4	0.1	0.1
ζ_B^N	-0.4	-0.3	-0.3	-0.3	-0.3	-0.3	-0.3	-0.3

Table 3: *Bipolar anti intuitionistic fuzzy sets in V and E of G_1, $G_2, G_1[G_2]$*

Proposition 3.1.11

If G_1 and G_2 are bipolar anti intuitionistic fuzzy graphs, then $G_1[G_2]$ is a bipolar anti intuitionistic fuzzy graph.

Proof: Let $x \in V_1$ and $x_2y_2 \in E_2$. Then

$$\left(\eta_{B1}^p \circ \eta_{B2}^p\right)(x,x_2)(x,y_2) = \eta_{A1}^P(x) \vee \eta_{B2}^P(x_2y_2)$$
$$\geq \vee \{\eta_{A1}^P(x), \eta_{A2}^P(x_2) \vee \eta_{A2}^P(y_2)\}$$
$$= \vee\, \eta_{A1}^P(x) \vee \eta_{A2}^P(x_2), \eta_{A1}^P(x) \vee \eta_{A2}^P(y_2)\}$$
$$= (\eta_{A1}^P \circ \eta_{A2}^P)(x,x_2) \vee (\eta_{A1}^P \circ \eta_{A2}^P(x,y_2)),$$

$$(\eta_{B1}^N \circ \eta_{B2}^N)\big((x,x_2)(x,y_2)\big) = \eta_{A1}^N(x) \wedge \eta_{B2}^N(x_2y_2)$$
$$\leq \wedge \{\eta_{A1}^N(x), \eta_{A2}^N(x_2) \wedge \eta_{A2}^N(y_2)\}$$
$$= \wedge \{\eta_{A1}^N(x) \wedge \eta_{A2}^N(x_2), \eta_{A1}^N(x) \wedge \eta_{A2}^N(y_2)\}$$
$$= (\eta_{A1}^N \circ \eta_{A2}^N)(x,x_2) \wedge (\eta_{A1}^N \circ \eta_{A2}^N)(x,y_2).$$

$$(\zeta_{B1}^P \circ \zeta_{B2}^P)\big((x,x_2)(x,y_2)\big) = \zeta_{A1}^P(x) \wedge \zeta_{B2}^P(x_2y_2)$$
$$\geq \wedge \{\zeta_{A1}^P(x), \zeta_{A2}^P(x_2) \wedge \zeta_{A2}^P(y_2)\}$$
$$= \wedge \left\{\zeta_{A1}^P(x) \wedge \zeta_{A2}^P(x_2)\big), \zeta_{A1}^P(x) \wedge \zeta_{A2}^P(y_2)\right\}$$
$$= (\zeta_{A1}^P \circ \zeta_{A2}^P)(x,x_2) \wedge (\zeta_{A1}^P \circ \zeta_{A2}^P)(x,y_2).$$

$$(\zeta_{B1}^N \circ \zeta_{B2}^N)((x,x_2)(x,y_2)) = \zeta_{A1}^N(x) \vee \zeta_{B2}^N(x_2y_2)$$
$$\leq \vee \{\zeta_{A1}^N(x), (\zeta_{A2}^N(x_2) \vee \zeta_{A2}^N(y_2)\}$$
$$= \vee \{\zeta_{A1}^N(x) \vee \zeta_{A2}^N(x_2), \zeta_{A1}^N(x) \vee \zeta_{A2}^N(y_2)\}$$
$$= (\zeta_{A1}^N \circ \zeta_{A2}^N)(x,x_2) \vee (\zeta_{A2}^N \circ \zeta_{A1}^N)(x,y_2).$$

Let $z \in V_{2,} and\ x_1y_1 \in E_{1.}$ Thus

$$(\eta_{B1}^P \circ \eta_{B2}^P)((x_1,z)(y_1,z) = \eta_{B1}^P(x_1y_1) \vee \eta_{A2}^P(z)$$
$$\geq \vee \{(\eta_{A1}^P(x_1) \vee \eta_{A1}^P(y_1)), \eta_{A2}^P(z)\}$$
$$= \vee \{\eta_{A1}^P(x_1) \vee \eta_{A2}^P(z), \eta_{A1}^P(y_1) \vee \eta_{A2}^P(z)\}$$
$$= (\eta_{A1}^P \circ \eta_{A2}^P)(x_1,z) \vee (\eta_{A1}^P \circ \eta_{A2}^P(y_1,z))$$

$$(\eta_{B1}^N \circ \eta_{B2}^N)((x_1,z)(y_1,z)) = \eta_{B1}^N(x_1y_1) \wedge \eta_{A2}^N(z)$$
$$\leq \wedge \{\eta_{A1}^N(x_1) \wedge \eta_{A1}^N(y_1), \eta_{A2}^N(z)\}$$
$$= \wedge \{\eta_{A1}^N(x_1) \wedge \eta_{A2}^N(z), \eta_{A1}^N(y_1) \wedge \eta_{A2}^N(z)\}$$
$$= (\eta_{A1}^N \circ \eta_{A2}^N)(x_1,z) \wedge (\eta_{A1}^N \circ \eta_{A2}^N)(y_1,z).$$

$$(\zeta_{B1}^P \circ \zeta_{B2}^P)((x_1,z)(y_1,z) = \zeta_{B1}^P(x_1y_1) \wedge \zeta_{A2}^P(z)$$
$$\geq \wedge \{(\zeta_{A1}^P(x_1) \wedge \zeta_{A1}^P(y_1)), \zeta_{A2}^P(z)\}$$
$$= \wedge \{\zeta_{A1}^P(x_1) \wedge \zeta_{A2}^P(z), \zeta_{A1}^P(y_1) \wedge \zeta_{A2}^P(z)\}$$
$$= (\zeta_{A1}^P \circ \zeta_{A2}^P)(x_1,z) \wedge (\zeta_{A1}^P \circ \zeta_{A2}^P(y_1,z))$$

$$(\zeta_{B1}^N \circ \zeta_{B2}^N)((x_1,z)(y_1,z)) = \zeta_{B1}^N(x_1y_1) \vee \zeta_{A2}^N(z)$$
$$\leq \vee \{\zeta_{A1}^N(x_1) \vee \zeta_{A1}^N(y_1), \zeta_{A2}^N(z)\}$$
$$= \vee \{\zeta_{A1}^N(x_1) \vee \zeta_{A2}^N(z), \zeta_{A1}^N(y_1) \vee \zeta_{A2}^N(z)\}$$
$$= (\zeta_{A1}^N \circ \zeta_{A2}^N)(x_1,z) \vee (\zeta_{A1}^N \circ \zeta_{A2}^N)(y_1,z).$$

Let $(x_1,x_2)(y_1,y_2) \in E^{(0)} \backslash E$. Then $x_1y_1 \in E_1, x_2 \neq y_{2.}$Thus,

$$(\eta_{B1}^p \circ \eta_{B2}^p)((x_1,x_2)(y_1,y_2)) = \vee \{\eta_{A2}^P(x_2).\eta_{A2}^P(y_2), \eta_{B2}^P(x_1y_1)\}$$
$$\geq \vee \{\eta_{A2}^P(x_2), \eta_{A2}^P(y_2), \eta_{A1}^P(x_1) \vee \eta_{A1}^P(y_1)\}$$
$$= \vee \{\eta_{A1}^P(x_1) \vee \eta_{A2}^P(x_2), \eta_{A1}^P(y_1) \vee \eta_{A2}^P(y_2)\}$$
$$= (\eta_{A1}^P \circ \eta_{A2}^P)(x_1,x_2) \vee (\eta_{A1}^P \circ \eta_{A2}^P)(y_1,y_2)$$

$$(\eta_{B1}^N \circ \eta_{B2}^N)((x_1,x_2)(y_1,y_2)) = \wedge \{\eta_{A2}^N(x_2).\eta_{A2}^N(y_2), \eta_{B2}^N(x_1y_1)\}$$
$$\leq \wedge \{\eta_{A2}^N(x_2), \eta_{A2}^N(y_2), \eta_{A1}^N(x_1) \wedge \eta_{A1}^N(y_1)\}$$
$$= \wedge \{\eta_{A1}^N(x_1) \wedge \eta_{A2}^N(x_2), \eta_{A1}^N(y_1) \wedge \eta_{A2}^N(y_2)\}$$

$$= (\eta^N_{A1} \circ \eta^N_{A2})(x_1, x_2) \wedge (\eta^N_{A1} \circ \eta^N_{A2})(y_1, y_2)$$

$$(\zeta^p_{B1} \circ \zeta^p_{B2})\big((x_1, x_2)(y_1, y_2)\big) = \wedge \{\zeta^P_{A2}(x_2).\, \zeta^P_{A2}(y_2), \zeta^P_{B2}(x_1 y_1)\}$$

$$\geq \wedge \{\zeta^P_{A2}(x_2), \zeta^P_{A2}(y_2), \zeta^P_{A1}(x_1) \wedge \zeta^P_{A1}(y_1)\}$$

$$= \wedge \{\zeta^P_{A1}(x_1) \wedge \zeta^P_{A2}(x_2), \zeta^P_{A1}(y_1) \wedge \zeta^P_{A2}(y_2)\}$$

$$= (\zeta^P_{A1} \circ \zeta^P_{A2})(x_1, x_2) \wedge (\zeta^P_{A1} \circ \zeta^P_{A2})(y_1, y_2)$$

$$(\zeta^N_{B1} \circ \zeta^N_{B2})\big((x_1, x_2)(y_1, y_2)\big) = \vee \{\zeta^N_{A2}(x_2).\, \zeta^N_{A2}(y_2), \zeta^N_{B2}(x_1 y_1)\}$$

$$\leq \vee \{\zeta^N_{A2}(x_2), \zeta^N_{A2}(y_2), \zeta^N_{A1}(x_1) \vee \zeta^N_{A1}(y_1)\}$$

$$= \vee \{\zeta^N_{A1}(x_1) \vee \zeta^N_{A2}(x_2), \zeta^N_{A1}(y_1) \vee \zeta^N_{A2}(y_2)\}$$

$$= (\zeta^N_{A1} \circ \zeta^N_{A2})(x_1, x_2) \vee (\zeta^N_{A1} \circ \zeta^N_{A2})(y_1, y_2)$$

Definition 3.1.12

Let $A_1 = (\eta^P_{A1}, \eta^N_{A1}, \zeta^P_{A1}, \zeta^N_{A1})$ and $A_2 = (\eta^P_{A2}, \eta^N_{A2}, \zeta^P_{A2}, \zeta^N_{A2})$ be bipolar anti intuitionistic fuzzy subsets of V_1 and V_2 and $B_1 = (\eta^P_{B1}, \eta^N_{B1}, \zeta^P_{B1}, \zeta^N_{B1})$ and $B_2 = (\eta^P_{B2}, \eta^N_{B2}, \zeta^P_{B2}, \zeta^N_{B2})$ be bipolar anti intuitionistic fuzzy subsets of E_1 and E_2 respectively. Then we denote the union of two bipolar anti intuitionistic fuzzy graphs G_1 and G_2 of the graphs G_1^* and G_2^* by $G_1 \cup G_2 = (A_1 \cup A_2, B_1 \cup B_2)$ and defined as follows:

$$(\eta^P_{A1} \cup \eta^P_{A2})(x) = \begin{cases} \eta^P_{A1}(x) & if\ x \in V_1 \cap \bar{V}_2 \\ \eta^P_{A2}(x) & if\ x \in V_2 \cap \bar{V}_1 \\ \eta^P_{A1}(x) \wedge \eta^P_{A2}(x) & if\ x \in V_1 \cap V_2 \end{cases}$$

$$(\eta^N_{A1} \cup \eta^N_{A2})(x) = \begin{cases} \eta^N_{A1}(x) & if\ x \in V_1 \cap \bar{V}_2 \\ \eta^N_{A2}(x) & if\ x \in V_2 \cap \bar{V}_1 \\ \eta^N_{A1}(x) \vee \eta^N_{A2}(x) & if\ x \in V_1 \cap V_2 \end{cases}$$

$$(\zeta^P_{A1} \cup \zeta^P_{A2})(x) = \begin{cases} \zeta^P_{A1}(x) & if\ x \in V_1 \cap \bar{V}_2 \\ \zeta^P_{A2}(x) & if\ x \in V_2 \cap \bar{V}_1 \\ \zeta^P_{A1}(x) \vee \zeta^P_{A2}(x) & if\ x \in V_1 \cap V_2 \end{cases}$$

$$(\zeta^N_{A1} \cup \zeta^N_{A2})(x) = \begin{cases} \zeta^N_{A1}(x) & if\ x \in V_1 \cap \bar{V}_2 \\ \zeta^N_{A2}(x) & if\ x \in V_2 \cap \bar{V}_1 \\ \zeta^N_{A1}(x) \wedge \zeta^N_{A2}(x) & if\ x \in V_1 \cap V_2 \end{cases}$$

$$(\eta_{B1}^P \cup \eta_{B2}^P)(xy) = \begin{cases} \eta_{B1}^P(xy) & if\ xy \in E_1 \cap \bar{E}_2 \\ \eta_{B2}^P(xy) & if\ xy \in E_2 \cap \bar{E}_1 \\ \eta_{B1}^P(xy) \wedge \eta_{B2}^P(xy) & if\ xy \in E_1 \cap E_2 \end{cases}$$

$$(\eta_{B1}^N \cup \eta_{B2}^N)(xy) = \begin{cases} \eta_{B1}^N(xy) & if\ xy \in E_1 \cap \bar{E}_2 \\ \eta_{B2}^N(xy) & if\ xy \in E_2 \cap \bar{E}_1 \\ \eta_{B1}^N(xy) \vee \eta_{B2}^N(xy) & if\ xy \in E_1 \cap E_2 \end{cases}$$

$$(\zeta_{B1}^P \cup \zeta_{B2}^P)(xy) = \begin{cases} \zeta_{B1}^P(xy) & if\ xy \in E_1 \cap \bar{E}_2 \\ \zeta_{B2}^P(xy) & if\ xy \in E_2 \cap \bar{E}_1 \\ \zeta_{B1}^P(xy) \vee \zeta_{B2}^P(xy) & if\ xy \in E_1 \cap E_2 \end{cases}$$

$$(\zeta_{B1}^N \cup \zeta_{B2}^N)(xy) = \begin{cases} \zeta_{B1}^N(xy) & if\ xy \in E_1 \cap \bar{E}_2 \\ \zeta_{B2}^N(xy) & if\ xy \in E_2 \cap \bar{E}_1 \\ \zeta_{B1}^N(xy) \wedge \zeta_{B2}^N(xy) & if\ xy \in E_1 \cap E_2 \end{cases}$$

Example 3.1.13:

Let G_1 and G_2 be BAIFG where $V_1 = \{a, b, c, d\}$, $E_1 = \{ab, bc, ac, cd\}$ and $V_2 = \{c, d, e\}$, $E_2 = \{cd, de\}$ respectively. Then we denote the union of two BAIFG as follows

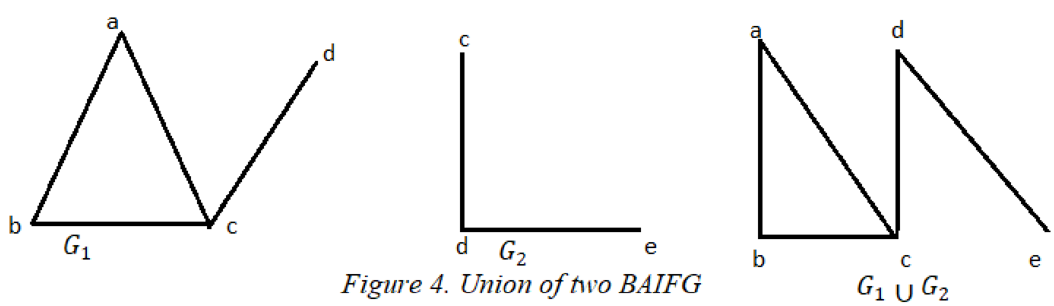

Figure 4. Union of two BAIFG

	a	***b***	***c***	***d***		***ab***	***bc***	***ac***	***cd***
η_A^P	0.4	0.5	0.6	0.3	η_B^P	0.6	0.7	0.7	0.7
η_A^N	-0.3	-0.4	-0.5	-0.5	η_B^N	-0.5	-0.5	-0.5	-0.5
ζ_A^P	0.5	0.3	0.1	0.5	ζ_B^P	0.4	0.2	0.1	0.2
ζ_A^N	-0.4	-0.3	-0.3	-0.3	ζ_B^N	-0.4	-0.4	-0.3	-0.3

Table 3: *Bipolar anti intuitionistic fuzzy sets in V and E of G_1*

	c	d	e		cd	de
η_A^P	0.4	0.7	0.2	η_B^P	0.7	0.8
η_A^N	-0.4	-0.2	-0.3	η_B^N	-0.4	-0.3
ζ_A^P	0.3	0.1	0.6	ζ_B^P	0.2	0.1
ζ_A^N	-0.2	-0.5	-0.4	ζ_B^N	-0.3	-0.5

Table 4: *Bipolar anti intuitionistic fuzzy sets in* V *and* E *of* G_2

	a	b	c	d	e
η_A^P	0.4	0.5	0.4	0.3	0.4
η_A^N	-0.3	-0.4	-0.4	-0.2	-0.4
ζ_A^P	0.5	0.3	0.3	0.5	0.3
ζ_A^N	-0.4	-0.3	-0.3	-0.5	-0.2

Table 5 *Bipolar anti intuitionistic fuzzy sets in* V *of* $G_1 \cup G_2$

	ab	bc	ac	de	cd
η_B^P	0.6	0.7	0.7	0.8	0.7
η_B^N	-0.5	-0.5	-0.5	-0.3	-0.4
ζ_B^P	0.4	0.2	0.1	0.1	0.2
ζ_B^N	-0.4	-0.4	-0.3	-0.5	-0.3

Table 6 *Bipolar anti intuitionistic fuzzy sets in* E *of* $G_1 \cup G_2$

Proposition 3.1.14

If G_1 and G_2 are bipolar anti intuitionistic fuzzy graphs, then $G_1 \cup G_2$ is a bipolar anti intuitionistic fuzzy graph.

Proof:

Let $xy \in E_1 \cap E_2$. Then

$$(\eta_{B1}^P \cup \eta_{B2}^P)(xy) = \eta_{B1}^P(xy) \wedge \eta_{B2}^P(xy)$$

$$\geq \wedge \{\eta_{A1}^P(x) \vee \eta_{A1}^P(y), \eta_{A2}^P(x) \vee \eta_{A2}^P(y)\}$$

$$= \vee \{\eta_{A1}^P(x) \wedge \eta_{A2}^P(x), \eta_{A2}^P(y) \wedge \eta_{A2}^P(y)\}$$

$$= (\eta^P_{A1} \cup \eta^P_{A2}(x) \vee (\eta^P_{A1} \cup \eta^P_{A2})(y).$$

$$(\eta^N_{B1} \cup \eta^N_{B2})(xy) = (\eta^N_{B1})(xy) \vee \eta^N_{B2}(xy)$$

$$\leq \vee \{\eta^N_{A1}(x) \wedge \eta^N_{A1}(y), \eta^N_{A2}(x) \wedge \eta^N_{A2}(y)\}$$

$$= \wedge \{\eta^N_{A1}(x) \vee \eta^N_{A2}(x), \eta^N_{A1}(y) \vee \eta^N_{A2}(y)\}$$

$$= (\eta^N_{A1} \cup \eta^N_{A2})(x) \wedge (\eta^N_{A1} \cup \eta^N_{A2})(y).$$

$$(\zeta^P_{B1} \cup \zeta^P_{B2})(xy) = (\zeta^P_{B1})(xy) \vee \zeta^P_{B2}(xy)$$

$$\leq \vee \{\zeta^P_{A1}(x) \wedge \zeta^P_{A1}(y), \zeta^P_{A2}(x) \wedge \zeta^P_{A2}(y)\}$$

$$= \wedge \{\zeta^P_{A1}(x) \vee \zeta^P_{A2}(x), \zeta^N_{A1}(y) \vee \zeta^P_{A2}(y)\}$$

$$= (\zeta^P_{A1} \cup \zeta^P_{A2})(x) \wedge (\zeta^P_{A1} \cup \zeta^N_{A2})(y).$$

$$(\zeta^N_{B1} \cup \zeta^N_{B2})(xy) = (\zeta^N_{B1})(xy) \wedge \zeta^N_{B2}(xy)$$

$$\geq \wedge \{\zeta^N_{A1}(x) \vee \zeta^N_{A1}(y), \zeta^N_{A2}(x) \vee \zeta^N_{A2}(y)\}$$

$$= \vee \{\zeta^N_{A1}(x) \wedge \zeta^N_{A2}(x), \zeta^N_{A1}(y) \wedge \zeta^N_{A2}(y)\}$$

$$= (\zeta^N_{A1} \cup \zeta^N_{A2})(x) \wedge (\zeta^N_{A1} \cup \zeta^N_{A2})(y).$$

If $xy \in E_1 \cap \bar{E}_2$,

$$(\eta^P_{B1} \cup \eta^P_{B2})(xy) = \eta^P_{B1}(xy) \geq \eta^P_{A1}(x) \vee \eta^P_{A1}(y)$$

$$= (\eta^P_{A1} \cup (\eta^P_{A2})(x)) \vee (\eta^P_{A1} \cup \eta^P_{A2})(y)$$

Similarly, for $xy \in E_1 \cap \bar{E}_2$, we have

$$(\eta^N_{B1} \cup \eta^N_{B2})(xy) \leq (\eta^N_{A1} \cup \eta^N_{A2}(x) \wedge (\eta^N_{A1} \cup \eta^N_{A2})(y)$$

$$(\zeta^P_{B1} \cup \zeta^P_{B2})(xy) \geq (\zeta^P_{A1} \cup \zeta^P_{A2})(x) \wedge (\zeta^P_{A1} \cup \zeta^N_{A2})(y)$$

$$(\zeta^N_{B1} \cup \zeta^N_{B2})(xy) \leq (\zeta^N_{A1} \cup \zeta^N_{A2})(x) \vee (\zeta^N_{A1} \cup \zeta^N_{A2})(y)$$

If $xy \in E_2 \cap \bar{E}_1$, then

$$(\eta^P_{B1} \cup \eta^P_{B2})(xy) \leq (\eta^P_{A1} \cup \eta^P_{A2})(x) \wedge (\eta^P_{A1} \cup \eta^P_{A2})(y)$$

$$(\eta^N_{B1} \cup \eta^N_{B2})(xy) \geq (\eta^N_{A1} \cup \eta^N_{A2})(x) \vee (\eta^N_{A1} \cup \eta^N_{A2})(y).$$

$$(\zeta^P_{B1} \cup \zeta^P_{B2})(xy) \leq (\zeta^P_{A1} \cup \zeta^P_{A2})(x) \vee (\zeta^P_{A1} \cup \zeta^P_{A2})(y)$$

$$(\zeta^N_{B1} \cup \zeta^N_{B2})(xy) \geq (\zeta^N_{A1} \cup \zeta^N_{A2})(x) \wedge (\zeta^N_{A1} \cup \zeta^N_{A2})(y)$$

Thus $G_1 \cup G_2$ is a bipolar anti intuitionistic fuzzy graph.

3.2 BIPOLAR INTERVAL VALUED ANTI INTUITIONISTIC FUZZY GRAPH

Definition 3.2.1

A bipolar interval valued anti intuitionistic fuzzy graph **(BIVAIFG)** with an underlying set V is defined to be a pair $G = (A,B)$, where $A = \left(\eta_{A_L}^P, \eta_{A_U}^P, \zeta_{A_L}^P, \zeta_{A_U}^P, \eta_{A_L}^N, \eta_{A_U}^N, \zeta_{A_L}^N, \zeta_{A_U}^N\right)$ is a bipolar interval valued anti intuitionistic fuzzy set **(BIVAIFS)** in V and $B = \left(\eta_{B_L}^P, \eta_{B_U}^P, \zeta_{B_L}^P, \zeta_{B_U}^P, \eta_{B_L}^N, \eta_{B_U}^N, \zeta_{B_L}^N, \zeta_{B_U}^N\right)$ is a bipolar interval valued anti intuitionistic fuzzy set in E such that

$$\left(\eta_{B_L}^P\right)(xy) \geq \eta_{A_L}^p(x) \vee \eta_{A_L}^P(y)\ ,\ \left(\zeta_{B_L}^P\right)(xy) \geq \zeta_{A_L}^P(x) \wedge \zeta_{A_L}^P(y)\ ,$$

$$\left(\eta_{B_U}^P\right)(xy) \geq \eta_{A_U}^p(x) \vee \eta_{A_U}^P(y)\ ,\left(\zeta_{B_U}^P\right)(xy) \geq \zeta_{A_U}^P(x) \wedge \zeta_{A_U}^P(y),$$

$$\left(\eta_{B_L}^N\right)(xy) \leq \eta_{A_L}^N(x) \wedge \eta_{A_L}^N(y), \left(\zeta_{B_L}^N\right)(xy) \leq \zeta_{A_L}^N(x) \vee \zeta_{A_L}^N(y)\ ,$$

$$\left(\eta_{B_U}^N\right)(xy) \leq \eta_{A_U}^N(x) \wedge \eta_{A_U}^N(y), \left(\zeta_{B_U}^N\right)(xy) \leq \zeta_{A_U}^N(x) \vee \zeta_{A_U}^N(y)\ \ for\ all\ xy\ \in\ E$$

Example 3.2.2: Consider the graph shown in figure 1 where

$a = (0.1,0.5,-0.5,-0.1,0.3,0.5,-0.5,-0.3); b = (0.2,0.5,-0.6,-0.2,0.1,0.4,-0.4,-0.1)$
$c = (0.3,0.6,-0.6,-0.4,0.1,0.4,-0.4,-0.2)$ we get the values of the edges as

$$ab = (0.3,0.6,-0.7,-0.3,0.2,0.4,-0.4,-0.2)$$

$$bc = (0.4,0.7,-0.6,-0.5,0.2,0.4,-0.4,-0.1)$$

$$ac = (0.4,0.6,-0.6,-0.4,0.2,0.4,-0.4,-0.3)$$

Definition 3.2.3

Let $A_1 = \left(\eta_{A1_L}^P, \eta_{A1_U}^P, \zeta_{A1_L}^P, \zeta_{A1_U}^P, \eta_{A1_L}^N, \eta_{A1_U}^N, \zeta_{A1_L}^N, \zeta_{A1_U}^N\right)$ and

$$A_2 = \left(\eta_{A2_L}^P, \eta_{A2_U}^P, \zeta_{A2_L}^P, \zeta_{A2_U}^P, \eta_{A2_L}^N, \eta_{A2_U}^N, \zeta_{A2_L}^N, \zeta_{A2_U}^N\right)$$

be BIVAIF subset of V_1 and V_2 and $B_1 = \left(\eta_{B1_L}^P, \eta_{B1_U}^P, \zeta_{B1_L}^P, \zeta_{B1_U}^P, \eta_{B1_L}^N, \eta_{B1_U}^N, \zeta_{B1_L}^N, \zeta_{B1_U}^N\right)$ and $B_2 = \left(\eta_{B2_L}^P, \eta_{B2_U}^P, \zeta_{B2_L}^P, \zeta_{B2_U}^P, \eta_{B2_L}^N, \eta_{B2_U}^N, \zeta_{B2_L}^N, \zeta_{B2_U}^N\right)$ be BIVAIF subsets of E_1 and E_2 respectively. Then we denote the Cartesian product of two bipolar interval valued anti intuitionistic fuzzy graphs G_1 and G_2 of the graphs G_1^* and G_2^* by $G_1 \times G_2 = (A_1 \times A_2, B_1 \times B_2)$ and defined by

(i)

$$\left(\eta^P_{A1_L} \times \eta^P_{A2_L}\right)(x_1, x_2) = \eta^P_{A1_L}(x_1) \vee \eta^P_{A2_L}(x_2)$$

$$\left(\eta^P_{A1_U} \times \eta^P_{A2_U}\right)(x_1, x_2) = \eta^P_{A1_U}(x_1) \vee \eta^P_{A2_U}(x_2)$$

$$\left(\eta^N_{A1_L} \times \eta^N_{A2_L}\right)(x_1, x_2) = \eta^N_{A1_L}(x_1) \wedge \eta^N_{A2_L}(x_2)$$

$$\left(\eta^N_{A1_U} \times \eta^N_{A2_U}\right)(x_1, x_2) = \eta^N_{A1_U}(x_1) \wedge \eta^N_{A2_U}(x_2)$$

$$\left(\zeta^P_{A1_L} \times \zeta^P_{A2_L}\right)(x_1, x_2) = \zeta^P_{A1_L}(x_1) \wedge \zeta^P_{A2_L}(x_2)$$

$$\left(\zeta^P_{A1_U} \times \zeta^P_{A2_U}\right)(x_1, x_2) = \zeta^P_{A1_U}(x_1) \wedge \zeta^P_{A2_U}(x_2)$$

$$\left(\zeta^N_{A1\,L} \times \zeta^N_{A2_L}\right)(x_1, x_2) = \zeta^N_{A1_L}(x_1) \vee \zeta^N_{A2_L}(x_2)$$

$$\left(\zeta^N_{A1_U} \times \zeta^N_{A2_U}\right)(x_1, x_2) = \zeta^N_{A1_U}(x_1) \vee \zeta^N_{A2_U}(x_2) \text{for all } (x_1, x_2) \in V \times V.$$

(ii)

$$(\eta^P_{B1_L} \times \eta^P_{B2_L})((x, x_2)(x, y_2)) = \eta^P_{A1_L}(x) \vee \eta^P_{B2_L}(x_2 y_2)$$

$$(\eta^P_{B1_U} \times \eta^P_{B2_U})((x, x_2)(x, y_2)) = \eta^P_{A1_U}(x) \vee \eta^P_{B2_U}(x_2 y_2)$$

$$(\eta^N_{B1_L} \times \eta^N_{B2_L})((x, x_2)(x, y_2)) = \eta^N_{A1_L}(x) \wedge \eta^N_{B2_L}(x_2 y_2)$$

$$(\eta^N_{B1_U} \times \eta^N_{B2_U})((x, x_2)(x, y_2)) = \eta^N_{A1_U}(x) \wedge \eta^N_{B2_U}(x_2 y_2)$$

$$(\zeta^P_{B1_L} \times \zeta^P_{B2_L})((x, x_2)(x, y_2)) = \zeta^P_{A1_L}(x) \wedge \zeta^P_{B2_L}(x_2 y_2)$$

$$(\zeta^P_{B1_U} \times \zeta^P_{B2_U})((x, x_2)(x, y_2)) = \zeta^P_{A1_U}(x) \wedge \zeta^P_{B2_U}(x_2 y_2)$$

$$(\zeta^N_{B1_L} \times \zeta^N_{B2_L})((x, x_2)(x, y_2)) = \zeta^N_{A1_L}(x) \vee \zeta^N_{B2_L}(x_2 y_2)$$

$$(\zeta^N_{B1_U} \times \zeta^N_{B2_U})((x, x_2)(x, y_2)) = \zeta^N_{A1_U}(x) \vee \zeta^N_{B2_U}(x_2 y_2)$$

for all $x \in V_1$, for all $x_2 y_2 \in E_2$.

(iii)

$$\left(\eta^P_{B1_L} \times \eta^P_{B2_L}\right)\left((x_1, z)(y_1, z)\right) = \eta^P_{B1_L}(x_1 y_1) \vee \eta^P_{A2_L}(z)$$

$$\left(\eta^P_{B1_U} \times \eta^P_{B2_U}\right)\left((x_1, z)(y_1, z)\right) = \eta^P_{B1_U}(x_1 y_1) \vee \eta^P_{A2_U}(z)$$

$$(\eta^N_{B1_L} \times \eta^N_{B2_L})((x_1, z)(y_1, z)) = \eta^N_{B1_L}(x_1 y_1) \wedge \eta^N_{A2_L}(z)$$

$$(\eta^N_{B1_U} \times \eta^N_{B2_U})((x_1, z)(y_1, z)) = \eta^N_{B1_U}(x_1 y_1) \wedge \eta^N_{A2_U}(z)$$

$$(\zeta^P_{B1_L} \times \zeta^P_{B2_L})((x_1,z)(y_1,z)) = \zeta^P_{A2_L}(z) \wedge \zeta^P_{B1_L}(x_1y_1)$$

$$(\zeta^P_{B1_U} \times \zeta^P_{B2_U})((x_1,z)(y_1,z)) = \zeta^P_{A2_U}(z) \wedge \zeta^P_{B1_U}(x_1y_1)$$

$$(\zeta^N_{B1_L} \times \zeta^N_{B2_L})((x_1,z)(y_1,z)) = \zeta^N_{A2_L}(z) \vee \zeta^N_{B1_L}(x_1y_1)$$

$$(\zeta^N_{B1_U} \times \zeta^N_{B2_U})((x_1,z)(y_1,z)) = \zeta^N_{A2_U}(z) \vee \zeta^N_{B1_U}(x_1y_1)$$

for all $z \in V_2$, for all $x_1y_1 \in E_1$.

Example 3.2.4: Consider the graph shown in figure 2 where

$a = (0.1,0.5,-0.5,-0.1,0.3,0.5,-0.5,-0.3); b = (0.2,0.5,-0.6,-0.2,0.1,0.4,-0.4,-0.1)$

$c = (0.3,0.6,-0.6,-0.4,0.1,0.4,-0.4,-0.2); d = (0.2,0.5,-0.6,-0.3,0.3,0.6,-0.4,-0.2)$

we get the values of the vertices in $G_1 \times G_2$ as

$$(a,c) = (0.3,0.6,-0.6,-0.4,0.1,0.4,-0.4,-0.2)$$

$$(a,d) = (0.2,0.5,-0.6,-0.3,0.3,0.5,-0.4,-0.2)$$

$$(b,c) = (0.3,0.6,-0.6,-0.4,0.1,0.4,-0.4,-0.1)$$

$$(b,d) = (0.2,0.5,-0.6,-0.2,0.1,0.4,-0.4,-0.1)$$

and edges in $G_1 \times G_2$ as

$$(a,c)(a,d) = (0.4,0.6,-0.6,-0.5,0.2,0.5,-0.4,-0.3)$$

$$(b,c)(b,d) = (0.4,0.6,-0.6,-0.5,0.\,,0.4,-0.4,-0.1)$$

$$(a,c)(b,c) = (0.3,0.6,-0.7,-0.4,0.1,0.4,-0.4,-0.2)$$

$$(a,d)(b,d) = (0.3,0.6,-0.7,-0.3,0.2,0.4,-0.4,-0.2)$$

Proposition 3.2.5

If G_1 and G_2 are BIVAIF graphs, then $G_1 \times G_2$ is a BIVAIF graph.

Proof:

Let $x \in V_1$ and $x_2y_2 \in E_2$. Then

$$(\eta^P_{B1_L} \times \eta^P_{B2_L})((x,x_2)(x,y_2)) = \eta^P_{A1_L}(x) \vee \eta^P_{B2_L}(x_2y_2)$$

$$\geq \vee \{\eta^P_{A1_L}(x), \eta^P_{A2_L}(x_2) \vee \eta^P_{A2_L}(y_2)\}$$

$$= \vee \left\{\eta^P_{A1_L}(x) \vee \eta^P_{A2_L}(x_2)\right), \eta^P_{A1_L}(x) \vee \eta^P_{A2_L}(y_2)\}$$

$$= \left(\eta^P_{A1_L} \times \eta^P_{A2_L}\right)(x, x_2) \vee (\eta^P_{A1_L} \times \eta^P_{A2_L})(x, y_2).$$

$$(\eta^P_{B1_U} \times \eta^P_{B2_U})((x, x_2)(x, y_2)) = \eta^P_{A1_U}(x) \vee \eta^P_{B2_U}(x_2 y_2)$$

$$\geq \vee \{\eta^P_{A1_U}(x), \eta^P_{A2_U}(x_2) \vee \eta^P_{A2_U}(y_2)\}$$

$$= \vee \left\{\eta^P_{A1_U}(x) \vee \eta^P_{A2_U}(x_2)\right), \eta^P_{A1_U}(x) \vee \eta^P_{A2_U}(y_2)\}$$

$$= \left(\eta^P_{A1_U} \times \eta^P_{A2_U}\right)(x, x_2) \vee (\eta^P_{A1_U} \times \eta^P_{A2_U})(x, y_2).$$

$$(\eta^N_{B1_L} \times \eta^N_{B2_L})((x, x_2)(x, y_2)) = \eta^N_{A1_L}(x) \wedge \eta^N_{B2_L}(x_2 y_2)$$

$$\leq \wedge \{\eta^N_{A1_L}(x), (\eta^N_{A2_L}(x_2) \wedge \eta^N_{A2_L}(y_2)\}$$

$$= \wedge \{\eta^N_{A1_L}(x) \wedge \eta^N_{A2_L}(x_2), \eta^N_{A1_L}(x) \wedge \eta^N_{A2_L}(y_2)\}$$

$$= (\eta^N_{A1_L} \times \eta^N_{A2_L})(x, x_2) \wedge \left(\eta^N_{A1_L} \times \eta^N_{A2_L}\right)(x, y_2).$$

$$(\eta^N_{B1_U} \times \eta^N_{B2_U})((x, x_2)(x, y_2)) = \eta^N_{A1_U}(x) \wedge \eta^N_{B2_U}(x_2 y_2)$$

$$\leq \wedge \{\eta^N_{A1_U}(x), (\eta^N_{A2_U}(x_2) \wedge \eta^N_{A2_U}(y_2)\}$$

$$= \wedge \{\eta^N_{A1_U}(x) \wedge \eta^N_{A2_U}(x_2), \eta^N_{A1_U}(x) \wedge \eta^N_{A2_U}(y_2)\}$$

$$= (\eta^N_{A1_U} \times \eta^N_{A2_U})(x, x_2) \wedge \left(\eta^N_{A1_U} \times \eta^N_{A2_U}\right)(x, y_2).$$

$$(\zeta^P_{B1_L} \times \zeta^P_{B2_L})((x, x_2)(x, y_2)) = \zeta^P_{A1_L}(x) \wedge \zeta^P_{B2_L}(x_2 y_2)$$

$$\geq \wedge \{\zeta^P_{A1_L}(x), \zeta^P_{A2_L}(x_2) \wedge \zeta^P_{A2_L}(y_2)\}$$

$$= \wedge \left\{\zeta^P_{A1_L}(x) \wedge \zeta^P_{A2_L}(x_2)\right), \zeta^P_{A1_L}(x) \wedge \zeta^P_{A2_L}(y_2)\}$$

$$= \left(\zeta^P_{A1_L} \times \zeta^P_{A2_L}\right)(x, x_2) \wedge (\zeta^P_{A1_L} \times \zeta^P_{A2_L})(x, y_2).$$

$$(\zeta^P_{B1_U} \times \zeta^P_{B2_U})((x, x_2)(x, y_2)) = \zeta^P_{A1_U}(x) \wedge \zeta^P_{B2_U}(x_2 y_2)$$

$$\geq \wedge \{\zeta^P_{A1_U}(x), \zeta^P_{A2_U}(x_2) \wedge \zeta^P_{A2_U}(y_2)\}$$

$$= \wedge \left\{\zeta^P_{A1_U}(x) \wedge \zeta^P_{A2_U}(x_2)\right), \zeta^P_{A1_U}(x) \wedge \zeta^P_{A2_U}(y_2)\}$$

$$= \left(\zeta^P_{A1_U} \times \zeta^P_{A2_U}\right)(x, x_2) \wedge (\zeta^P_{A1_U} \times \zeta^P_{A2_U})(x, y_2).$$

$$(\zeta^N_{B1_L} \times \zeta^N_{B2_L})((x,x_2)(x,y_2)) = \zeta^N_{A1_L}(x) \vee \zeta^N_{B2_L}(x_2y_2)$$

$$\leq \vee \{\zeta^N_{A1_L}(x), (\zeta^N_{A2_L}(x_2) \vee \zeta^N_{A2_L}(y_2)\}$$

$$= \vee \{\zeta^N_{A1_L}(x) \vee \zeta^N_{A2_L}(x_2), \zeta^N_{A1_L}(x) \vee \zeta^N_{A2_L}(y_2)\}$$

$$= (\zeta^N_{A1_L} \times \zeta^N_{A2_L})(x,x_2) \vee \left(\zeta^N_{A1_L} \times \zeta^N_{A2_L}\right)(x,y_2).$$

$$(\zeta^N_{B1_U} \times \zeta^N_{B2_U})((x,x_2)(x,y_2)) = \zeta^N_{A1_U}(x) \vee \zeta^N_{B2_U}(x_2y_2)$$

$$\leq \vee \{\zeta^N_{A1_U}(x), (\zeta^N_{A2_U}(x_2) \vee \zeta^N_{A2_U}(y_2)\}$$

$$= \vee \{\zeta^N_{A1_U}(x) \vee \zeta^N_{A2_U}(x_2), \zeta^N_{A1_U}(x) \vee \zeta^N_{A2_U}(y_2)\}$$

$$= (\zeta^N_{A1_U} \times \zeta^N_{A2_U})(x,x_2) \vee \left(\zeta^N_{A1_U} \times \zeta^N_{A2_U}\right)(x,y_2).$$

Let $z \in V_{2,} and\ x_1y_1 \in E_{1.}$ Thus

$$\left(\eta^P_{B1_L} \times \eta^P_{B2_L}\right)\left((x_1,z)(y_1,z)\right) = \eta^P_{B1_L}(x_1y_1) \vee \eta^P_{A2_L}(z)$$

$$\geq \vee \{\left(\eta^P_{A1_L}(x_1) \vee \eta^P_{A1_L}(y_1)\right), \eta^P_{A2_L}(z)\}$$

$$= \vee \{\eta^P_{A1_L}(x_1) \vee \eta^P_{A2_L}(z), \eta^P_{A1_L}(y_1) \vee \eta^P_{A2_L}(z)\}$$

$$= \left(\eta^P_{A1_L} \times \eta^P_{A2_L}\right)(x_1,z) \vee \left(\eta^P_{A1_L} \times \eta^P_{A2_L}(y_1,z)\right)$$

$$\left(\eta^P_{B1_U} \times \eta^P_{B2_U}\right)\left((x_1,z)(y_1,z)\right) = \eta^P_{B1_U}(x_1y_1) \vee \eta^P_{A2_U}(z)$$

$$\geq \vee \{\left(\eta^P_{A1_U}(x_1) \vee \eta^P_{A1_U}(y_1)\right), \eta^P_{A2_U}(z)\}$$

$$= \vee \{\eta^P_{A1_U}(x_1) \vee \eta^P_{A2_U}(z), \eta^P_{A1_U}(y_1) \vee \eta^P_{A2_U}(z)\}$$

$$= \left(\eta^P_{A1_U} \times \eta^P_{A2_U}\right)(x_1,z) \vee \left(\eta^P_{A1_U} \times \eta^P_{A2_U}(y_1,z)\right)$$

$$(\eta^N_{B1_L} \times \eta^N_{B2_L})((x_1,z)(y_1,z)) = \eta^N_{B1_L}(x_1y_1) \wedge \eta^N_{A2_L}(z)$$

$$\leq \wedge \{\eta^N_{A1_L}(x_1) \wedge \eta^N_{A1_L}(y_1), \eta^N_{A2_L}(z)\}$$

$$= \wedge \{\eta^N_{A1_L}(x_1) \wedge \eta^N_{A2_L}(z), \eta^N_{A1_L}(y_1) \wedge \eta^N_{A2_L}(z)\}$$

$$= \left(\eta^N_{A1_L} \times \eta^N_{A2_L}\right)(x_1,z) \wedge \left(\eta^N_{A1_L} \times \eta^N_{A2_L}\right)(y_1,z).$$

$$(\eta^N_{B1_U} \times \eta^N_{B2_U})((x_1,z)(y_1,z)) = \eta^N_{B1_U}(x_1y_1) \wedge \eta^N_{A2_U}(z)$$

$$\leq \wedge \{\eta^N_{A1_U}(x_1) \wedge \eta^N_{A1_U}(y_1), \eta^N_{A2_U}(z)\}$$

$$= \wedge \{\eta^N_{A1_U}(x_1) \wedge \eta^N_{A2_U}(z), \eta^N_{A1_U}(y_1) \wedge \eta^N_{A2_U}(z)\}$$

$$= \left(\eta^N_{A1_U} \times \eta^N_{A2_U}\right)(x_1,z) \wedge \left(\eta^N_{A1_U} \times \eta^N_{A2_U}\right)(y_1,z).$$

$$(\zeta^P_{B1_L} \times \zeta^P_{B2_L})((x_1,z)(y_1,z)) = \zeta^P_{A2_L}(z) \wedge \zeta^P_{B1_L}(x_1y_1)$$

$$\geq \wedge \{\left(\zeta^P_{A1_L}(x_1) \wedge \zeta^P_{A1_L}(y_1)\right), \zeta^P_{A2_L}(z)\}$$

$$= \wedge \{\zeta^P_{A1_L}(x_1) \wedge \zeta^P_{A2_L}(z), \zeta^P_{A1_L}(y_1) \wedge \zeta^P_{A2_L}(z)\}$$

$$= \left(\zeta^P_{A1_L} \times \zeta^P_{A2_L}\right)(x_1,z) \wedge \left(\zeta^P_{A1_L} \times \zeta^P_{A2_L}(y_1,z)\right)$$

$$(\zeta^P_{B1_U} \times \zeta^P_{B2_U})((x_1,z)(y_1,z)) = \zeta^P_{A2_U}(z) \wedge \zeta^P_{B1_U}(x_1y_1)$$

$$\geq \wedge \{\left(\zeta^P_{A1_U}(x_1) \wedge \zeta^P_{A1_U}(y_1)\right), \zeta^P_{A2_U}(z)\}$$

$$= \wedge \{\zeta^P_{A1_U}(x_1) \wedge \zeta^P_{A2_U}(z), \zeta^P_{A1_U}(y_1) \wedge \zeta^P_{A2_U}(z)\}$$

$$= \left(\zeta^P_{A1_U} \times \zeta^P_{A2_U}\right)(x_1,z) \wedge \left(\zeta^P_{A1_U} \times \zeta^P_{A2_U}(y_1,z)\right)$$

$$(\zeta^N_{B1_L} \times \zeta^N_{B2_L})((x_1,z)(y_1,z)) = \zeta^N_{A2_L}(z) \vee \zeta^N_{B1_L}(x_1y_1)$$

$$\leq \vee \{\zeta^N_{A1_L}(x_1) \vee \zeta^N_{A1_L}(y_1), \zeta^N_{A2_L}(z)\}$$

$$= \vee \{\zeta^N_{A1_L}(x_1) \vee \zeta^N_{A2_L}(z), \zeta^N_{A1_L}(y_1) \vee \zeta^N_{A2_L}(z)\}$$

$$= \left(\zeta^N_{A1_L} \times \zeta^N_{A2_L}\right)(x_1,z) \vee \left(\zeta^N_{A1_L} \times \zeta^N_{A2_L}\right)(y_1,z).$$

$$(\zeta^N_{B1_U} \times \zeta^N_{B2_U})((x_1,z)(y_1,z)) = \zeta^N_{A2_U}(z) \vee \zeta^N_{B1_U}(x_1y_1)$$

$$\leq \vee \{\zeta^N_{A1_U}(x_1) \vee \zeta^N_{A1_U}(y_1), \zeta^N_{A2_U}(z)\}$$

$$= \vee \{\zeta^N_{A1_U}(x_1) \vee \zeta^N_{A2_U}(z), \zeta^N_{A1_U}(y_1) \vee \zeta^N_{A2_U}(z)\}$$

$$= \left(\zeta^N_{A1_U} \times \zeta^N_{A2_U}\right)(x_1,z) \vee \left(\zeta^N_{A1_U} \times \zeta^N_{A2_U}\right)(y_1,z).$$

Definition 3.2.6

Let $A_1 = \left(\eta^P_{A1_L}, \eta^P_{A1_U}, \ \zeta^P_{A1_L}, \zeta^P_{A1_U}, \eta^N_{A1_L}, \eta^N_{A1_U}, \zeta^N_{A1_L}, \zeta^N_{A1_U}\right)$ and

$$A_2 = \left(\eta^P_{A2_L}, \eta^P_{A2_U}, \zeta^P_{A2_L}, \zeta^P_{A2_U}, \eta^N_{A2_L}, \eta^N_{A2_U}, \zeta^N_{A2_L}, \zeta^N_{A2_U}\right)$$

be BIVAIF subset of V_1 and V_2 and $B_1 = \left(\eta^P_{B1_L}, \eta^P_{B1_U}, \zeta^P_{B1_L}, \zeta^P_{B1_U}, \eta^N_{B1_L}, \eta^N_{B1_U}, \zeta^N_{B1_L}, \zeta^N_{B1_U}\right)$ and $B_2 = \left(\eta^P_{B2_L}, \eta^P_{B2_U}, \zeta^P_{B2_L}, \zeta^P_{B2_U}, \eta^N_{B2_L}, \eta^N_{B2_U}, \zeta^N_{B2_L}, \zeta^N_{B2_U}\right)$ be BIVAIF subsets of E_1 and E_2 respectively. The composition of two BIVAIF graphs G_1 and G_2 of the graphs G_1^* and G_2^* denoted by $G_1[G_2] = (A_1 \circ A_2, B_1 \circ B_2)$ is defined by

(i)

$$\left(\eta^P_{A1_L} \circ \eta^P_{A2_L}\right)(x_1, x_2) = \eta^P_{A1_L}(x_1) \vee \eta^P_{A2_L}(x_2)$$

$$\left(\eta^P_{A1_U} \circ \eta^P_{A2_U}\right)(x_1, x_2) = \eta^P_{A1_U}(x_1) \vee \eta^P_{A2_U}(x_2)$$

$$\left(\eta^N_{A1_L} \circ \eta^N_{A2_L}\right)(x_1, x_2) = \eta^N_{A1_L}(x_1) \wedge \eta^N_{A2_L}(x_2)$$

$$\left(\eta^N_{A1_U} \circ \eta^N_{A2_U}\right)(x_1, x_2) = \eta^N_{A1_U}(x_1) \wedge \eta^N_{A2_U}(x_2)$$

$$\left(\zeta^P_{A1_L} \circ \zeta^P_{A2_L}\right)(x_1, x_2) = \zeta^P_{A1_L}(x_1) \wedge \zeta^P_{A2_L}(x_2)$$

$$\left(\zeta^P_{A1_U} \circ \zeta^P_{A2_U}\right)(x_1, x_2) = \zeta^P_{A1_U}(x_1) \wedge \zeta^P_{A2_U}(x_2)$$

$$\left(\zeta^N_{A1_L} \circ \zeta^N_{A2_L}\right)(x_1, x_2) = \zeta^N_{A1_L}(x_1) \vee \zeta^N_{A2_L}(x_2)$$

$$\left(\zeta^N_{A1_U} \circ \zeta^N_{A2_U}\right)(x_1, x_2) = \zeta^N_{A1_U}(x_1) \vee \zeta^N_{A2_U}(x_2)$$

for all $(x_1, x_2) \in V \times V$.

(ii)

$$(\eta^P_{B1_L} \circ \eta^P_{B2_L})((x, x_2)(x, y_2)) = \eta^P_{A1_L}(x) \vee \eta^P_{B2_L}(x_2 y_2)$$

$$(\eta^P_{B1_U} \circ \eta^P_{B2_U})((x, x_2)(x, y_2)) = \eta^P_{A1_U}(x) \vee \eta^P_{B2_U}(x_2 y_2)$$

$$(\eta^N_{B1_L} \circ \eta^N_{B2_L})((x, x_2)(x, y_2)) = \eta^N_{A1_L}(x) \wedge \eta^N_{B2_L}(x_2 y_2)$$

$$(\eta^N_{B1_U} \circ \eta^N_{B2_U})((x, x_2)(x, y_2)) = \eta^N_{A1_U}(x) \wedge \eta^N_{B2_U}(x_2 y_2)$$

$$(\zeta^P_{B1_L} \circ \zeta^P_{B2_L})((x, x_2)(x, y_2)) = \zeta^P_{A1_L}(x) \wedge \zeta^P_{B2_L}(x_2 y_2)$$

$$(\zeta^P_{B1_U} \circ \zeta^P_{B2_U})((x, x_2)(x, y_2)) = \zeta^P_{A1_U}(x) \wedge \zeta^P_{B2_U}(x_2 y_2)$$

$$(\zeta^N_{B1_L} \circ \zeta^N_{B2_L})((x,x_2)(x,y_2)) = \zeta^N_{A1_L}(x) \vee \zeta^N_{B2_L}(x_2y_2)$$

$$(\zeta^N_{B1_U} \circ \zeta^N_{B2_U})((x,x_2)(x,y_2)) = \zeta^N_{A1_U}(x) \vee \zeta^N_{B2_U}(x_2y_2)$$

for all $x \in V_1$, for all $x_2y_2 \in E_2$.

(iii)

$$\left(\eta^P_{B1_L} \circ \eta^P_{B2_L}\right)\left((x_1,z)(y_1,z)\right) = \eta^P_{B1_L}(x_1y_1) \vee \eta^P_{A2_L}(z)$$

$$\left(\eta^P_{B1_U} \circ \eta^P_{B2_U}\right)\left((x_1,z)(y_1,z)\right) = \eta^P_{B1_U}(x_1y_1) \vee \eta^P_{A2_U}(z)$$

$$(\eta^N_{B1_L} \circ \eta^N_{B2_L})((x_1,z)(y_1,z)) = \eta^N_{B1_L}(x_1y_1) \wedge \eta^N_{A2_L}(z)$$

$$(\eta^N_{B1_U} \circ \eta^N_{B2_U})((x_1,z)(y_1,z)) = \eta^N_{B1_U}(x_1y_1) \wedge \eta^N_{A2_U}(z)$$

$$(\zeta^P_{B1_L} \circ \zeta^P_{B2_L})((x_1,z)(y_1,z)) = \zeta^P_{A2_L}(z) \wedge \zeta^P_{B1_L}(x_1y_1)$$

$$(\zeta^P_{B1_U} \circ \zeta^P_{B2_U})((x_1,z)(y_1,z)) = \zeta^P_{A2_U}(z) \wedge \zeta^P_{B1_U}(x_1y_1)$$

$$(\zeta^N_{B1_L} \circ \zeta^N_{B2_L})((x_1,z)(y_1,z)) = \zeta^N_{A2_L}(z) \vee \zeta^N_{B1_L}(x_1y_1)$$

$$(\zeta^N_{B1_U} \circ \zeta^N_{B2_U})((x_1,z)(y_1,z)) = \zeta^N_{A2_U}(z) \vee \zeta^N_{B1_U}(x_1y_1)$$

for all $z \in V_2$, for all $x_1y_1 \in E_1$.

(v)

$$\left(\eta^P_{B1_L} \circ \eta^P_{B2_L}\right)\left((x_1,x_2)(y_1y_2)\right) = \vee \left\{\eta^P_{A2_L}(x_2), \eta^P_{A2_L}(y_2), \eta^P_{B1_L}(x_1y_1)\right\}$$

$$\left(\eta^P_{B1_U} \circ \eta^P_{B2_U}\right)\left((x_1,x_2)(y_1y_2)\right) = \vee \left\{\eta^P_{A2_U}(x_2), \eta^P_{A2_U}(y_2), \eta^P_{B1_U}(x_1y_1)\right\}$$

$$\left(\eta^N_{B1_L} \circ \eta^N_{B2_L}\right)\left((x_1,x_2)(y_1,y_2)\right) = \wedge \{\eta^N_{A2_L}(x_2), \eta^N_{A2_L}(y_2), \eta^N_{B1_L}(x_1y_1)\}$$

$$\left(\eta^N_{B1_U} \circ \eta^N_{B2_U}\right)\left((x_1,x_2)(y_1,y_2)\right) = \wedge \{\eta^N_{A2_U}(x_2), \eta^N_{A2_U}(y_2), \eta^N_{B1_U}(x_1y_1)\}$$

$$\left(\zeta^P_{B1_L} \circ \zeta^P_{B2_L}\right)\left((x_1,x_2)(y_1y_2)\right) = \wedge \left\{\zeta^P_{A2_L}(x_2), \zeta^P_{A2_L}(y_2), \zeta^P_{B1_L}(x_1y_1)\right\}$$

$$\left(\zeta^P_{B1_U} \circ \zeta^P_{B2_U}\right)\left((x_1,x_2)(y_1y_2)\right) = \wedge \left\{\zeta^P_{A2_U}(x_2), \zeta^P_{A2_U}(y_2), \zeta^P_{B1_U}(x_1y_1)\right\}$$

$$\left(\zeta^N_{B1_L} \circ \zeta^N_{B2_L}\right)\left((x_1,x_2)(y_1,y_2)\right) = \vee \{\zeta^N_{A2_L}(x_2), \zeta^N_{A2_L}(y_2), \zeta^N_{B1_L}(x_1y_1)\}$$

$$\left(\zeta^N_{B1_U} \circ \zeta^N_{B2_U}\right)\left((x_1,x_2)(y_1,y_2)\right) = \vee \{\zeta^N_{A2_U}(x_2), \zeta^N_{A2_U}(y_2), \zeta^N_{B1_U}(x_1y_1)\}$$

for all $(x_1,x_2)(y_1y_2) \in E^0 \backslash E$.

Example 3.2.7: Consider the graph shown in figure 3 where

$a = (0.1,0.5,-0.5,-0.1,0.3,0.5,-0.5,-0.3)$; $b = (0.2,0.5,-0.6,-0.2,0.1,0.4,-0.4,-0.1)$

$c = (0.3,0.6,-0.6,-0.4,0.1,0.4,-0.4,-0.2)$; $d = (0.2,0.5,-0.6,-0.3,0.3,0.6,-0.4,-0.2)$

we get the values of the vertices in $G_1 \times G_2$ as

$$(a,c) = (0.3,0.6,-0.6,-0.4,0.1,0.4,-0.4,-0.2)$$

$$(a,d) = (0.2,0.5,-0.6,-0.3,0.3,0.5,-0.4,-0.2)$$

$$(b,c) = (0.3,0.6,-0.6,-0.4,0.1,0.4,-0.4,-0.1)$$

$$(b,d) = (0.2,0.5,-0.6,-0.2,0.1,0.4,-0.4,-0.1)$$

and edges in $G_1 \times G_2$ as

$$(a,c)(a,d) = (0.4,0.6,-0.6,-0.5,0.2,0.5,-0.4,-0.3)$$

$$(b,c)(b,d) = (0.4,0.6,-0.6,-0.5,0.,0.4,-0.4,-0.1)$$

$$(a,c)(b,c) = (0.3,0.6,-0.7,-0.4,0.1,0.4,-0.4,-0.2)$$

$$(a,d)(b,d) = (0.3,0.6,-0.7,-0.3,0.2,0.4,-0.4,-0.2)$$

$$(a,c)(b,d) = (0.3,0.6,-0.7,-0.4,0.1,0.4,-0.4,-0.4)$$

$$(a,d)(b,c) = (0.3,0.6,-0.7,-0.3,0.2,0.4,-0.4,-0.4)$$

Proposition 3.2.8

If G_1 and $G_2$are BIVAIF graphs, then $G_1[G_2]$ is a BIVAIF graph.

Proof:

Let $x \in V_1$ and $x_2y_2 \in E_2$. Then

$$\left(\eta^{p}_{B1_L} \circ \eta^{p}_{B2_L}\right)(x,x_2)(x,y_2) = \eta^{P}_{A1_L}(x) \vee \eta^{P}_{B2_L}(x_2y_2)$$

$$\geq \vee \left\{\eta^{P}_{A1_L}(x), \eta^{P}_{A2_L}(x_2) \vee \eta^{P}_{A2_L}(y_2)\right\}$$

$$= \vee\, \eta^{P}_{A1_L}(x) \vee \eta^{P}_{A2_L}(x_2), \eta^{P}_{A1_L}(x) \vee \eta^{P}_{A2_L}(y_2)\}$$

$$= (\eta^{P}_{A1_L} \circ \eta^{P}_{A2_L})(x,x_2) \vee (\eta^{P}_{A1_L} \circ \eta^{P}_{A2_L}(x,y_2))$$

Similarly $\left(\eta^{p}_{B1_U} \circ \eta^{p}_{B2_U}\right)(x,x_2)(x,y_2) \geq (\eta^{P}_{A1_U} \circ \eta^{P}_{A2_U})(x,x_2) \vee (\eta^{P}_{A1_U} \circ \eta^{P}_{A2_U}(x,y_2))$

$$\left(\eta^{N}_{B1_L} \circ \eta^{N}_{B2_L}\right)\left((x,x_2)(x,y_2)\right) = \eta^{N}_{A1_L}(x) \wedge \eta^{N}_{B2_L}(x_2y_2)$$

$$\leq \wedge \left\{\eta^N_{A1_L}(x), \eta^N_{A2_L}(x_2) \wedge \eta^N_{A2_L}(y_2)\right\}$$

$$= \wedge \left\{\eta^N_{A1_L}(x) \wedge \eta^N_{A2_L}(x_2), \eta^N_{A1_L}(x) \wedge \eta^N_{A2_L}(y_2)\right\}$$

$$= \left(\eta^N_{A1_L} \circ \eta^N_{A2_L}\right)(x, x_2) \wedge \left(\eta^N_{A1_L} \circ \eta^N_{A2_L}\right)(x, y_2).$$

$$\left(\zeta^P_{B1_L} \circ \zeta^P_{B2_L}\right)\left((x, x_2)(x, y_2)\right) = \zeta^P_{A1_L}(x) \wedge \zeta^P_{B2_L}(x_2 y_2)$$

$$\geq \wedge \{\zeta^P_{A1_L}(x), \zeta^P_{A2_L}(x_2) \wedge \zeta^P_{A2_L}(y_2)\}$$

$$= \wedge \left\{\zeta^P_{A1_L}(x) \wedge \zeta^P_{A2_L}(x_2)\right), \zeta^P_{A1_L}(x) \wedge \zeta^P_{A2_L}(y_2)\}$$

$$= \left(\zeta^P_{A1_L} \circ \zeta^P_{A2_L}\right)(x, x_2) \wedge (\zeta^P_{A1_L} \circ \zeta^P_{A2_L})(x, y_2).$$

$$\left(\zeta^N_{B1_L} \circ \zeta^N_{B2_L}\right)((x, x_2)(x, y_2)) = \zeta^N_{A1_L}(x) \vee \zeta^N_{B2_L}(x_2 y_2)$$

$$\leq \vee \{\zeta^N_{A1_L}(x), (\zeta^N_{A2_L}(x_2) \vee \zeta^N_{A2_L}(y_2)\}$$

$$= \vee \{\zeta^N_{A1_L}(x) \vee \zeta^N_{A2_L}(x_2), \zeta^N_{A1_L}(x) \vee \zeta^N_{A2_L}(y_2)\}$$

$$= (\zeta^N_{A1_L} \circ \zeta^N_{A2_L})(x, x_2) \vee \left(\zeta^N_{A2_L} \circ \zeta^N_{A1_L}\right)(x, y_2).$$

Similarly the other results can be derived.

Let $z \in V_{2,} and\ x_1 y_1 \in E_{1.}$ Thus

$$\left(\eta^P_{B1_L} \circ \eta^P_{B2_L}\right)((x_1, z)(y_1, z) = \eta^P_{B1_L}(x_1 y_1) \vee \eta^P_{A2_L}(z)$$

$$\geq \vee \{\left(\eta^P_{A1_L}(x_1) \vee \eta^P_{A1_L}(y_1)\right), \eta^P_{A2_L}(z)\}$$

$$= \vee \{\eta^P_{A1_L}(x_1) \vee \eta^P_{A2_L}(z), \eta^P_{A1_L}(y_1) \vee \eta^P_{A2_L}(z)\}$$

$$= \left(\eta^P_{A1_L} \circ \eta^P_{A2_L}\right)(x_1, z) \vee \left(\eta^P_{A1_L} \circ \eta^P_{A2_L}(y_1, z)\right)$$

$$(\eta^N_{B1_L} \circ \eta^N_{B2})\left((x_1, z)(y_1, z)\right) = \eta^N_{B1_L}(x_1 y_1) \wedge \eta^N_{A2_L}(z)$$

$$\leq \wedge \{\eta^N_{A1_L}(x_1) \wedge \eta^N_{A1_L}(y_1), \eta^N_{A2_L}(z)\}$$

$$= \wedge \{\eta^N_{A1_L}(x_1) \wedge \eta^N_{A2_L}(z), \eta^N_{A1_L}(y_1) \wedge \eta^N_{A2_L}(z)\}$$

$$= \left(\eta^N_{A1_L} \circ \eta^N_{A2_L}\right)(x_1, z) \wedge \left(\eta^N_{A1_L} \circ \eta^N_{A2_L}\right)(y_1, z).$$

$$\left(\zeta^P_{B1_L}\circ\zeta^P_{B2_L}\right)((x_1,z)(y_1,z) \ =\zeta^P_{B1_L}(x_1y_1)\wedge\zeta^P_{A2_L}(z)$$

$$\geq\wedge\left\{\left(\zeta^P_{A1_L}(x_1)\wedge\zeta^P_{A1_L}(y_1)\right),\zeta^P_{A2_L}(z)\right\}$$

$$=\wedge\{\zeta^P_{A1_L}(x_1)\wedge\zeta^P_{A2_L}(z),\zeta^P_{A1_L}(y_1)\wedge\zeta^P_{A2_L}(z)\}$$

$$=\left(\zeta^P_{A1_L}\circ\zeta^P_{A2_L}\right)(x_1,z)\wedge\left(\zeta^P_{A1_L}\circ\zeta^P_{A2_L}(y_1,z)\right)$$

$$(\zeta^N_{B1_L}\circ\zeta^N_{B2_L})\big((x_1,z)(y_1,z)\big)=\zeta^N_{B1_L}(x_1y_1)\vee\zeta^N_{A2_L}(z)$$

$$\leq\vee\{\zeta^N_{A1_L}(x_1)\vee\zeta^N_{A1_L}(y_1),\zeta^N_{A2_L}(z)\}$$

$$=\vee\{\zeta^N_{A1_L}(x_1)\vee\zeta^N_{A2_L}(z),\zeta^N_{A1_L}(y_1)\vee\zeta^N_{A2_L}(z)\}$$

$$=\left(\zeta^N_{A1_L}\circ\zeta^N_{A2_L}\right)(x_1,z)\vee\left(\zeta^N_{A1_L}\circ\zeta^N_{A2_L}\right)(y_1,z).$$

Let $(x_1,x_2)(y_1,y_2)\in E^{(0)}\backslash E$. Then $x_1y_1\in E_1, x_2\neq y_2$. Thus,

$$\left(\eta^p_{B1_L}\circ\eta^p_{B2_L}\right)\big((x_1,x_2)(y_1,y_2)\big)=\vee\{\eta^P_{A2_L}(x_2).\eta^P_{A2_L}(y_2),\eta^P_{B2}(x_1y_1)\}$$

$$\geq\vee\left\{\eta^P_{A2_L}(x_2),\eta^P_{A2_L}(y_2),\eta^P_{A1_L}(x_1)\vee\eta^P_{A1_L}(y_1)\right\}$$

$$=\vee\left\{\eta^P_{A1_L}(x_1)\vee\eta^P_{A2_L}(x_2),\eta^P_{A1_L}(y_1)\vee\eta^P_{A2_L}(y_2)\right\}$$

$$=\left(\eta^P_{A1_L}\circ\eta^P_{A2_L}\right)(x_1,x_2)\vee\left(\eta^P_{A1_L}\circ\eta^P_{A2_L}\right)(y_1,y_2)$$

$$\left(\eta^N_{B1_L}\circ\eta^N_{B2_L}\right)\big((x_1,x_2)(y_1,y_2)\big)=\wedge\{\eta^N_{A2_L}(x_2).\eta^N_{A2_L}(y_2),\eta^N_{B2_L}(x_1y_1)\}$$

$$\leq\wedge\left\{\eta^N_{A2_L}(x_2),\eta^N_{A2_L}(y_2),\eta^N_{A1_L}(x_1)\wedge\eta^N_{A1_L}(y_1)\right\}$$

$$=\wedge\left\{\eta^N_{A1_L}(x_1)\wedge\eta^N_{A2_L}(x_2),\eta^N_{A1_L}(y_1)\wedge\eta^N_{A2_L}(y_2)\right\}$$

$$=\left(\eta^N_{A1_L}\circ\eta^N_{A2_L}\right)(x_1,x_2)\wedge\left(\eta^N_{A1_L}\circ\eta^N_{A2_L}\right)(y_1,y_2)$$

$$\left(\zeta^p_{B1_L}\circ\zeta^p_{B2_L}\right)\big((x_1,x_2)(y_1,y_2)\big)=\wedge\{\zeta^P_{A2_L}(x_2).\zeta^P_{A2_L}(y_2),\zeta^P_{B2_L}(x_1y_1)\}$$

$$\geq\ \wedge\left\{\zeta^P_{A2_L}(x_2),\zeta^P_{A2_L}(y_2),\zeta^P_{A1_L}(x_1)\wedge\zeta^P_{A1_L}(y_1)\right\}$$

$$=\wedge\left\{\zeta^P_{A1_L}(x_1)\wedge\zeta^P_{A2_L}(x_2),\zeta^P_{A1_L}(y_1)\wedge\zeta^P_{A2_L}(y_2)\right\}$$

$$=\left(\zeta^P_{A1_L}\circ\zeta^P_{A2_L}\right)(x_1,x_2)\wedge\left(\zeta^P_{A1_L}\circ\zeta^P_{A2_L}\right)(y_1,y_2)$$

$$\left(\zeta^N_{B1_L} \circ \zeta^N_{B2_L}\right)\left((x_1, x_2)(y_1, y_2)\right) = \vee \{\zeta^N_{A2_L}(x_2). \zeta^N_{A2_L}(y_2), \zeta^N_{B2_L}(x_1 y_1)\}$$

$$\leq \vee \left\{\zeta^N_{A2_L}(x_2), \zeta^N_{A2_L}(y_2), \zeta^N_{A1_L}(x_1) \vee \zeta^N_{A1_L}(y_1)\right\}$$

$$= \vee \left\{\zeta^N_{A1_L}(x_1) \vee \zeta^N_{A2_L}(x_2), \zeta^N_{A1_L}(y_1) \vee \zeta^N_{A2_L}(y_2)\right\}$$

$$= \left(\zeta^N_{A1_L} \circ \zeta^N_{A2_L}\right)(x_1, x_2) \vee \left(\zeta^N_{A1_L} \circ \zeta^N_{A2_L}\right)(y_1, y_2)$$

Similarly all other cases can be derived.

CONCLUSION

In this paper we introduce the notion of bipolar anti intuitionistic fuzzy graph and discuss the various operations such as composition, Cartesian product and their union of two bipolar anti intuitionistic fuzzy graph. Also we have introduced the notion of bipolar interval valued anti intuitionistic fuzzy graph and the various operations such as composition, Cartesian product of two bipolar interval valued anti intuitionistic fuzzy graph.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

REFERENCES

[1] A.M.A. Alnaser, W.A. AlZoubi, M.O. Massadeh, Bipolar intuitionistic fuzzy graph and it's matrices, Appl. Math. Inform. Sci. 14 (2020), 205-214.

[2] K.T. Atanassov, Intuitionistic fuzzy sets, Physica-Verlag, Heidelberg, (2012).

[3] K. T. Atanassov, Intuitionistic fuzzy sets VII ITKR's session, Sofia, June 1983(Deposed in Central Sci-Techn. Library of Bulg. Acad. of. Sci., 1967/84) (in Bulgarian).

[4] Atanassov, G. Pasi, R. Yager, V. Atanassova, Intuitionistic fuzzy graph interpretations of multi-person multi-criteria decision making, Eusflat Conf., 177-182, (2003).

[5] A. Kauffmann, Introduction to the theory of fuzzy sets, Vol. 1, Academic Press, Inc., Orlando, Florida, (1973).

[6] M.M. Jabarulla, K.A. Banu, Anti intuitionistic fuzzy graph, Adv. Appl. Math. Sci. 20 (2021), 597-604.

[7] R. Parvathi, M.G. Karunambigai, K.T. Atanassov, Operations on intuitionistic fuzzy graphs, in: 2009 IEEE International Conference on Fuzzy Systems, IEEE, Jeju Island, 2009: pp. 1396–1401.

[8] S. Mandal, M. Pal, Product of bipolar intuitionistic fuzzy graphs and their degree. TWMS J. Appl. Eng. Math. 9(2) (2019), 327-338.

[9] S. Mathew, J.N. Mordeson, D.S. Malik, Fuzzy graph theory. Springer, Berlin (2018).

[10] A. Shannon, K.T. Atanassov, A first step to a theory of the intuitionistic fuzzy graphs. In: Proceeding of the First Workshop on Fuzzy Based Expert Systems pp.59-61, 1994.

[11] L.A. Zadeh, Fuzzy sets, Inform and Control 8 (1965) 338–353.

[12] W.R. Zhang, Bipolar fuzzy sets, Proceedings of FUZZ-IEEE, 835–840, (1998).

[13] W.R. Zhang, Bipolar fuzzy sets and relations: a computational framework for cognitive modeling and multiagent decision analysis, Proceedings of IEEE Conf., pp. 305–309, (1994).