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STUDY OF *L*-FUNCTION USING WEAKLY WEIGHTED SHARING DEFINED OVER AN EXTENDED SELBERG CLASS

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Abstract. In this article, the concept of weakly weighted sharing is used to prove the uniqueness results of a meromorphic function and an *L*-function described in Selberg class $\mathbb{S}$. Our result will improve the result due to D.C. Pramanik and Ja. Roy [4].

Keywords: Meromorphic function; *L*-function; Selberg class; Weakly weighted sharing.

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1. INTRODUCTION

We use the basic notations of Nevanilinna theory as described in [1, 8, 9, 10]. We define, $F = \{h_1 : h_1 \text{is a non-constant meromorphic function}\}$, where meromorphic function is always defined in the complex plane. A meromorphic function a is a small function with respect to $h_1 \in F$, if either $a \equiv \infty$ or $T(r,a) = S(r,h_1)$. $S(h_1)$ is the set of all small functions with respect to h_1 that are specified in the complex plane.

In 19th century, A. Selberg introduced a class known as Selberg Class in order to understand the value distribution of L-functions, since then Selberg Class has been an important field of research. For the concept of *L*-function, we encourage the reader to refer [6].

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Definition 1. [9] *For $a \in \mathbb{C} \cup \{\infty\}$, the quantity*

$$\delta(a, h_1) = 1 - \limsup_{r\to\infty} \frac{N(r, a; h_1)}{T(r, h_1)},$$

is called the deficiency of 'a' for the function h_1.

Definition 2. [9] *The quantity*

$$\Theta(a, h_1) = 1 - \limsup_{r\to\infty} \frac{\overline{N}(r, a; h_1)}{T(r, h_1)},$$

for $a \in \mathbb{C} \cup \{\infty\}$ is the ramification index of a for the function h_1.

The first comprehensive review of the principle of weighted sharing to prove the uniqueness of meromorphic functions was proposed by Indrajit Lahiri in 2001. We steer the reader to [2](see p.195, Definition 7) for a discussion on weighted sharing.

The argument of Shanhua Lin and Weichuan Lin in 2006 touched upon the principle of weakly weighted sharing. For the definition, reader can refer [3](see p.274, Definition 4).

We denote with the notation $\overline{N}_L(r, 1; H_1)$, the reduced counting function which are $1-$points of H_1 whose multiplicities are greater than $1-$points of L when H_1 and L share 1 "*IM*". $\overline{N}_L(r, 1; L)$ is described similarly. We denote the reduced counting function for the poles for the function h_1 by $N_1(r, h_1) = \overline{N}(r, \infty; h_1)$. Similarly, $N_1\left(r, \frac{1}{h_1}\right) = \overline{N}(r, 0; h_1)$, $N_1(r, L) = \overline{N}(r, L)$, $N_1\left(r, \frac{1}{L}\right) = \overline{N}(r, 0; L)$.

In 2006, S. Lin and W. Lin (see [3], p. 272, Theorem $1-3$) defined and used the concept of weakly weighted sharing of functions to prove the uniqueness of a meromorphic functions and its derivatives for the first time. By contributing three theorems, they proved the uniqueness of h_1 and $h_1^{(n)}$ when they both share "(a, m)", "$(a, 1)$", "$(a, 0)$" with some relative conditions.

Later in 2011, H-Y. Xu and Y. Hu (see [7], p. 104, Theorem $1-3$) generalized the theorems proved by S. Lin and W. Lin by proving the uniqueness of a non-constant meromorphic function h_1 and $L(h_1) = h_1^{(n_1)} + a_{n_1-1}h_1^{(n_1-1)} + \ldots + a_0 h_1$ where $a_i(\not\equiv 0,\ \infty) \in S(h_1)$ for $0 \le i \le n_1 - 1$, sharing "(a, k_1)", "$(a, 1)$", "$(a, 0)$" with some conditions of suitability.

Recently, in 2019, D. C. Pramanik and Ja. Roy (see [4], p. 44, Theorem 7) considered the issue more broadly with consideration of non-constant homogeneous differential polynomials $P[h_1]$ and *P[g]*, where f and g are two functions having non-constant meromorphic properties and showed that if $P[h_1]$ and $P[g]$ share "(a,l)" with some appropriate conditions, then $P[h_1] = P[g]$.

Inspired by such study, it is normal to inquire that what will be the relation between a non-constant meromorphic function h_1 and an L-function L when they share "(a,l)" and weakly weighted sharing is taken into account where $a \in S(h_1) \cap S(L)$, $a \not\equiv 0$, ∞. As an answer for this, we have proved the result as stated in the Section 3 of this paper.

2. Preliminaries

We highlight only those lemmas that are needed to prove our conclusion.

We consider

$$\Psi = \left(\frac{H_1''}{H_1'} - 2\frac{H_1'}{H_1 - 1}\right) - \left(\frac{H_2''}{H_2'} - 2\frac{H_2'}{H_2 - 1}\right). \tag{2.1}$$

Here, the notations H_1 and H_2 are used and they are considered to be non-constant meromorphic functions. For Second Fundamental Theorem (SFT), we redirect the reader to [1].

If suppose $h_1 \in F$ defined in $\mathbb{C}$ and $p \in \mathbb{Z}^+$, then we have

$$N(r,0;h_1^{(p)}) \leq N(r,0;h_1) + pN_1(r,h_1) + O(log(T(r,h_1)) + logr), \tag{2.2}$$

refer [9].

Suppose,

(1) H_1 and H_2 share "$(1,l)$" then

$$\overline{N}_L(r,1;H_1) \leq \frac{1}{2}\overline{N}(r,0;H_1) + \frac{1}{2}\overline{N}(r,\infty;H_1) + S(r,H_1), \tag{2.3}$$

refer [7], [p. 106, Lemma 4].

(2) H_1 and H_2 share "$(1,0)$" then

$$\overline{N}_L(r,1;H_1) \leq \overline{N}(r,0;H_1) + \overline{N}(r,\infty;H_1) + S(r,H_1), \tag{2.4}$$

refer [7], [p. 106, Lemma 6].

Also, letting a non-negative integer or ∞ as 'l', H_1 and H_2 sharing "$(1,l)$", if Ψ as defined in (2.1) is not equal to zero then we have the following cases.

(1) Whenever $2 \leq l \leq \infty$,

$$T(r,H_1) \leq N_2(r,\infty;H_1)+N_2(r,\infty;H_2)+N_2(r,0;H_1)+N_2(r,0;H_1)+S(r,H_1)+S(r,H_2). \tag{2.5}$$

(2) Whenever $l = 1$,

$$T(r,H_1) \leq N_2(r,\infty;H_1)+N_2(r,\infty;H_2)+N_2(r,0;H_1)+N_2(r,0;H_2)+\overline{N}_L(r,1;H_1)+S(r,H_1)+S(r,H_2). \tag{2.6}$$

(3) Whenever $l = 0$,

$$T(r,H_1) \leq N_2(r,\infty;H_1)+N_2(r,\infty;H_2)+N_2(r,0;H_1)+N_2(r,0;H_1)+2\overline{N}_L(r,1;H_1)+\overline{N}_L(r,1;H_2)+S(r,H_1)+S(r,H_2). \tag{2.7}$$

Similarly, we can define for $T(r,H_2)$, refer [3] [p. 273, Lemma 3].

3. Main Results

Theorem 1. *Considering, $h_1 \in F$ defined in $\mathbb{C}$ and L be an L-function, $a \in S(h_1) \cap S(L)$, where $a \not\equiv 0, \infty$. If h_1 and L share "(a,l)" with one of the conditions mentioned below*

(i) $l \geq 2$ and

(3.1)

$$\min\left\{\frac{2}{p+1}\delta_{p+1}(0,h_1)+\frac{4+2p}{p+1}\Theta(\infty,f), \frac{2}{p+1}\delta_{p+1}(0,L)+\frac{4+2p}{p+1}\Theta(\infty,L)\right\} > \frac{3p+7}{p+1},$$

(ii) $l = 1$ and

(3.2)

$$\min\left\{\frac{5}{2p+2}\delta_{p+1}(0,h_1)+\frac{5p+9}{2p+2}\Theta(\infty,f), \frac{5}{2p+2}\delta_{p+1}(0,L)+\frac{5p+9}{2p+2}\Theta(\infty,L)\right\} > \frac{3p+12}{2p+2},$$

(iii) $l = 0$ and

(3.3)

$$\min\left\{\frac{5}{p+1}\delta_{p+1}(0,h_1)+\frac{5p+7}{p+1}\Theta(\infty,f), \frac{5}{p+1}\delta_{p+1}(0,L)+\frac{5p+7}{p+1}\Theta(\infty,L)\right\} > \frac{4p+11}{p+1},$$

then $f \equiv L$.

Proof. Let

$$H_1 = \frac{h_1^{(p)}}{a}, \ \ H_2 = \frac{L^{(p)}}{a}.$$

Since $h_1^{(p)}$ and $L^{(p)}$ share (a,l), it follows that H_1, H_2 share $(1,l)$ except at the zeros and poles of a.

Suppose that $\Psi \not\equiv 0$. Now we will consider the cases as below:

Case 1: If $2 \leq l \leq \infty$. From (2.5), we obtain

$$T(r,h_1^{(p)}) \leq 2\overline{N}(r,\infty;h_1^{(p)}) + 2\overline{N}(r,\infty;L^{(p)}) + \overline{N}(r,0;h_1^{(p)}) + \overline{N}(r,0;L^{(p)}) + S(r,h_1) + S(r,L). \tag{3.4}$$

From (2.2) and (3.4), we obtain

$$(p+1)T(r,h_1) \leq (2+p)N_1(r,h_1) + (2+p)N_1(r,L) + N_{p+1}\left(r,\frac{1}{h_1}\right) + N_{p+1}\left(r,\frac{1}{L}\right) + S(r,h_1) + S(r,L).$$

So,

$$\begin{aligned} T(r,h_1) \leq{}& \left(\frac{2+p}{p+1}\right)N_1(r,h_1) + \left(\frac{2+p}{p+1}\right)N_1(r,L) + \frac{1}{p+1}N_{p+1}\left(r,\frac{1}{h_1}\right) \\ &+ \frac{1}{p+1}N_{p+1}\left(r,\frac{1}{L}\right) + S(r,h_1) + S(r,L). \end{aligned} \tag{3.5}$$

Similarly,

$$\begin{aligned} T(r,L) \leq{}& \left(\frac{2+p}{p+1}\right)N_1(r,L) + \left(\frac{2+p}{p+1}\right)N_1(r,h_1) + \frac{1}{p+1}N_{p+1}\left(r,\frac{1}{L}\right) \\ &+ \frac{1}{p+1}N_{p+1}\left(r,\frac{1}{h_1}\right) + S(r,L) + S(r,h_1). \end{aligned} \tag{3.6}$$

Now, from (3.5) and (3.6), we obtain

$$\begin{aligned} T(r,h_1) + T(r,L) \leq{}& \left(\frac{4+2p}{p+1}\right)N_1(r,h_1) + \left(\frac{4+2p}{p+1}\right)N_1(r,L) \\ &+ \frac{2}{p+1}N_{p+1}\left(r,\frac{1}{h_1}\right) + \frac{2}{p+1}N_{p+1}\left(r,\frac{1}{L}\right) + S(r,L) + S(r,h_1) \\ \left\{\frac{2}{p+1}\delta_{p+1}(0,h_1) + \left(\frac{4+2p}{p+1}\right)\Theta(\infty,f) - \left(\frac{3p+7}{p+1}\right)\right\}&T(r,h_1) \\ + \left\{\frac{2}{p+1}\delta_{p+1}(0,L) + \left(\frac{4+2p}{p+1}\right)\Theta(\infty,L) - \left(\frac{3p+7}{p+1}\right)\right\}&T(r,L) \\ &\leq S(r,h_1) + S(r,L), \end{aligned}$$

which conflicts our assumption (3.1). Therefore $\Psi \equiv 0$ and now from (2.1) we obtain

$$\frac{1}{H_2-1} = \frac{I_1}{H_1-1} + I_2,$$

where $I_1(\neq 0)$ and I_2 are constants. This gives

$$H_2 = \frac{(I_2+1)H_1 + (I_1-I_2-1)}{I_2H_1 + (I_1-I_2)}, \tag{3.7}$$

$$H_1 = \frac{(I_2-I_1)H_2 + (I_1-I_2-1)}{I_2H_2 + (I_2+1)}. \tag{3.8}$$

Next we consider three subcases:

Subcase 1: $I_2 \neq 0, -1$. Then from (3.8),

$$\overline{N}\left(r, \frac{I_2+1}{I_2}; H_2\right) = \overline{N}(r, \infty; H_1).$$

By using the Second Fundamental Theorem (SFT) of Nevanlinna and (2.2),

$$T(r,H_2) < \overline{N}(r,\infty;H_2) + \overline{N}(r,0;H_2) + \overline{N}\left(r, \frac{I_2+1}{I_2}; H_2\right) + S(r,H_2),$$

$$T(r,L) \leq \frac{1}{(p+1)}\overline{N}(r,\infty;f) + \frac{1}{(p+1)}N_{p+1}\left(r, \frac{1}{L}\right) + N_1(r,L) + S(r,h_1) + S(r,L). \tag{3.9}$$

If $I_1 - I_2 - 1 \neq 0$, then it follows from (3.7) that

$$N\left(r, \frac{-I_1+I_2+1}{I_2+1}; H_1\right) = N(r,0;H_2).$$

Applying SFT of Nevanlinna and (2.2), subsequently we obtain

$$T(r,H_1) < \overline{N}(r,\infty;H_1) + \overline{N}(r,0;H_1) + \overline{N}\left(r, \frac{-I_1+I_2+1}{I_2+1}; H_1\right) + S(r,H_1)$$

$$T(r,f) \leq N_1(r,h_1) + \frac{1}{(p+1)}N_{p+1}\left(r, \frac{1}{h_1}\right) + \frac{1}{(p+1)}\overline{N}(r,0;L) + S(r,h_1). \tag{3.10}$$

From (3.9) and (3.10), we obtain

$$T(r,h_1) + T(r,L) \leq \frac{2}{(p+1)}N_{p+1}\left(r, \frac{1}{h_1}\right) + 2N_1(r,h_1) + \frac{1}{(p+1)}N_{p+1}\left(r, \frac{1}{L}\right) + N_1(r,L) + S(r,h_1) + S(r,L),$$

which again contradicts (3.1).

Therefore $I_1 - I_2 - 1 = 0$. Then from (3.7),

$$\overline{N}\left(r,0;H_1+\frac{1}{I_2}\right)=\overline{N}(r,\infty;H_2).$$

By applying the SFT of Nevanlinna and (2.2) we have

$$T(r,H_1)<\overline{N}(r,\infty;H_1)+\overline{N}(r,0;H_1)+\overline{N}\left(r,0;H_1+\frac{1}{I_2}\right)+S(r,H_1),$$

$$(p+1)T(r,h_1)\leq N_1(r,h_1)+pN_1(r,h_1)+N_{p+1}\left(r,\frac{1}{h_1}\right)+N_1(r,L)$$
$$+S(r,h_1)+S(r,L).$$

So

$$T(r,h_1)\leq N_1(r,h_1)+\frac{1}{(p+1)}N_{p+1}\left(r,\frac{1}{h_1}\right)+\frac{1}{(p+1)}N_1(r,L)$$
$$+S(r,h_1)+S(r,L). \tag{3.11}$$

From (3.9) and (3.11) we obtain

$$T(r,h_1)+T(r,L)\leq\frac{1}{(p+1)}N_{p+1}\left(r,\frac{1}{h_1}\right)+\left(\frac{p+2}{p+1}\right)N_1(r,h_1)$$
$$+\frac{1}{(p+1)}N_{p+1}\left(r,\frac{1}{L}\right)+\left(\frac{2+p}{p+1}\right)N_1(r,L)$$
$$+S(r,h_1)+S(r,L),$$

which violates assumption (3.1).

Subcase 2: $I_2=-1$. Then from (3.7) and (3.8) we obtain

$$H_2=\frac{I_1}{I_1+1-H_1}\ ,\ \ H_1=\frac{(1+I_1)H_2-I_1}{H_2}.$$

If $I_1+1\neq 0$, then

$$\overline{N}(r,I_1+1;H_1)=\overline{N}(r,\infty;H_2)\ ,\ \ \overline{N}\left(r,\frac{I_1}{I_1+1};H_2\right)=\overline{N}(r,0;H_1).$$

By similar argument as in previous subcase, we arrive at a contradiction. Hence $I_1+1=0$, then $H_1H_2=1$.

Subcase 3: $I_2 = 0$. Then (3.7) and (3.8) gives $H_2 = \frac{H_1+I_1-1}{I_1}$ and $H_1 = I_1H_2 + 1 - I_1 \neq 0$, $N(r,0;I_1 - 1 + H_1) = N(r,0;H_2)$ and $N(r, \frac{I_1-1}{I_1}; H_2) = N(r,0;H_1)$. Proceeding in the same direction as in Subcase 1 we obtain a paradox. Therefore $I_1 - 1 = 0$, then $H_1 = H_2$.

Case 2: For $l = 1$, from (2.6) we have

$$T(r,H_1) \leq 2\overline{N}(r,\infty;H_1) + 2\overline{N}(r,\infty;H_2) + N(r,0;H_1) + N(r,0;H_2) + \overline{N}_L(r,1;H_1) + S(r,H_1) + S(r,H_2).$$

Now, by using (2.2) and (2.3), we obtain

$$\begin{aligned} T(r,H_1) &\leq 2\overline{N}(r,\infty;H_1) + 2\overline{N}(r,\infty;H_2) + pN_1(r,h_1) + N_{p+1}\left(r,\frac{1}{h_1}\right) \\ &+ pN_1(r,L) + N_{p+1}\left(r,\frac{1}{L}\right) + \frac{1}{2}\overline{N}(r,0;H_1) + \frac{1}{2}\overline{N}(r,\infty;H_1) \\ &+ S(r,H_1) + S(r,H_2) \end{aligned}$$

$$\begin{aligned} T(r,h_1) &\leq \left(\frac{3p+5}{2p+2}\right)N_1(r,h_1) + \frac{3}{2(p+1)}N_{p+1}\left(r,\frac{1}{h_1}\right) \\ &+ \left(\frac{2+p}{p+1}\right)N_1(r,L) + \frac{1}{(p+1)}N_{p+1}\left(r,\frac{1}{L}\right) \\ &+ S(r,h_1) + S(r,L). \end{aligned} \tag{3.12}$$

Likewise,

$$\begin{aligned} T(r,L) &\leq \left(\frac{3p+5}{2p+2}\right)N_1(r,L) + \frac{3}{2(p+1)}N_{p+1}\left(r,\frac{1}{L}\right) \\ &+ \left(\frac{2+p}{p+1}\right)N_1(r,h_1) + \frac{1}{(p+1)}N_{p+1}\left(r,\frac{1}{h_1}\right) \\ &+ S(r,h_1) + S(r,L). \end{aligned} \tag{3.13}$$

From (3.12) and (3.13), we obtain

$$\begin{aligned} T(r,h_1) + T(r,L) &\leq \left(\frac{3p+5}{2p+2}\right)N_1(r,h_1) + \frac{3}{2(p+1)}N_{p+1}\left(r,\frac{1}{h_1}\right) + \left(\frac{2+p}{p+1}\right)N_1(r,L) \\ &+ \frac{1}{(p+1)}N_{p+1}\left(r,\frac{1}{L}\right) + \left(\frac{3p+5}{2p+2}\right)N_1(r,L) + \frac{3}{2(p+1)}N_{p+1}\left(r,\frac{1}{L}\right) \\ &+ \left(\frac{2+p}{p+1}\right)N_1(r,h_1) + \frac{1}{(p+1)}N_{p+1}\left(r,\frac{1}{h_1}\right) + S(r,h_1) + S(r,L), \end{aligned}$$

$$\left\{\frac{5}{2p+2}\delta_{p+1}(0,h_1)+\frac{5p+9}{2p+2}\Theta(\infty,f)-\frac{3p+12}{2p+2}\right\}T(r,h_1)$$
$$+\left\{\frac{5}{2p+2}\delta_{p+1}(0,L)+\frac{5p+9}{2p+2}\Theta(\infty,L)-\frac{3p+12}{2p+2}\right\}T(r,L)$$
$$\leq S(r,h_1)+S(r,g),$$

that conflicts with our assumption (3.2). Following the procedure of case (i), we obtain the result for this case.

Case 3: $l=0$. Now, from (2.7), we have

$$T(r,H_1)\leq 2\overline{N}(r,\infty;H_1)+2\overline{N}(r,\infty;H_2)+N(r,0;H_1)+N(r,0;H_2)$$
$$+2\overline{N}_L(r,1;H_1)+\overline{N}_L(r,1;H_2)+S(r,H_1)+S(r,H_2). \tag{3.14}$$

Using (2.2), (2.4) and (3.14) we obtain

$$T(r,h_1)\leq\frac{4p+5}{p+1}N_1(r,h_1)+\frac{4}{p+1}N_{p+1}\left(r,\frac{1}{h_1}\right)+\frac{2+p}{p+1}N_1(r,L)$$
$$+\frac{1}{p+1}N_{p+1}\left(r,\frac{1}{L}\right)+S(r,h_1)+S(r,L). \tag{3.15}$$

Similarly,

$$T(r,L)\leq\frac{4p+5}{p+1}N_1(r,L)+\frac{4}{p+1}N_{p+1}\left(r,\frac{1}{L}\right)+\frac{2+p}{p+1}N_1(r,h_1)$$
$$+\frac{1}{p+1}N_{p+1}\left(r,\frac{1}{h_1}\right)+S(r,h_1)+S(r,L). \tag{3.16}$$

From (3.15) and (3.16), we obtain

$$T(r,h_1)+T(r,L)\leq\frac{4p+5}{p+1}N_1(r,h_1)+\frac{4}{p+1}N_{p+1}\left(r,\frac{1}{h_1}\right)+\frac{2+p}{p+1}N_1(r,L)$$
$$+\frac{1}{p+1}N_{p+1}\left(r,\frac{1}{L}\right)+\frac{4p+5}{p+1}N_1(r,L)+\frac{4}{p+1}N_{p+1}\left(r,\frac{1}{L}\right)$$
$$+\frac{2+p}{p+1}N_1(r,h_1)+\frac{1}{p+1}N_{p+1}\left(r,\frac{1}{h_1}\right)+S(r,h_1)+S(r,L)$$

$$\left\{\frac{5}{p+1}\delta_{p+1}(0,h_1)+\left(\frac{5p+7}{p+1}\right)\Theta(\infty,f)-\frac{4p+11}{p+1}\right\}T(r,h_1)$$
$$+\left\{\frac{5}{p+1}\delta_{p+1}(0,L)+\left(\frac{5p+7}{p+1}\right)\Theta(\infty,L)-\frac{4p+11}{p+1}\right\}T(r,L)$$
$$\leq S(r,h_1)+S(r,L),$$

that conflicts with our assumption (3.3).

We obtain the requisite inference for this case in the same way as in Case 1. □

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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