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A FUZZY LEAKAGE INVENTORY MODEL WITH SHORTAGE USING TRIANGULAR AND TRAPEZOIDAL FUZZY NUMBERS

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Abstract. Inventory refers to any kind of economically valued resources that are in various stages of being made ready for sale. With the development of the Economic Order Quantity (EOQ) model by Ford Harris, many models under different conditions and assumptions are proposed. Leakage is a common phenomenon whose occurrence will reduce the profitability of the firm by increasing the minimum operational cost. In majority of real-life inventory problems we face many uncertainties in the key parameters of the corresponding model. These impreciseness and uncertainties in crisp model are improved by using fuzzy set theory. So, in this context, a fuzzy leakage inventory order level is developed by taking holding cost and shortage cost as Triangular as well as Trapezoidal Fuzzy Numbers. Defuzzification is done using Signed Distance Method. A relevant numerical example is illustrated along with sensitivity analysis to justify the proposed notion that the optimal values are improved in fuzzy environment as compared to that of in crisp environment.

Keywords: leakage; triangular fuzzy number; trapezoidal fuzzy number; defuzzification; signed distance method.

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1. INTRODUCTION

Inventory refers to the itemized catalog or list of tangible goods or property, or the intangible attributes or qualities. This refers to the value of materials and goods held by an organization (i) to support production (raw materials, subassemblies, work in process), (ii) for support activities (repair, maintenance, consumables), or (iii) for sale or customer service (merchandise, finished goods, spare parts).

In an inventory system, goods in stock may suffer leakage. Here, leakage refers to the loss in quantity, for example, leakage in case of liquid stock (oil/fuel) or breakages in case of solid goods (glass/bottles), damages made by insects or animals (food items) and transit loss (business) due to malpractices of sales by personnel involved, etc. The occurrence of such leakages will affect the profitability of the organization by way of increasing minimum operational cost, decreasing the optimal quantity to be maintained. Since leakages will exist till it is detected by the management and measures to control it are taken up, it is temporary in nature. Therefore, the solution of inventory problem is a set of specific values of variables under discussion that minimizes the total cost of the system.

The first quantitative treatment of inventory was the simple EOQ model. This model was developed by Harris[16], which was later investigated and applied in academics and industries. Later on, many researchers developed many inventory models in crisp environment under different parameters and assumptions. But these assumptions do not suit the real world environment and hence there is a great deal of uncertainty and variability. The modern concept of uncertainty evolved with the publication of a seminar paper by Zadeh[8] where he introduced a theory on sets, known as fuzzy set theory, with boundaries that are not specific and precise. Zimmerman[5] also discussed the concept of fuzzy set theory and its applications in different fields like operations research.

In literature, there are many papers in fuzzified problems on EOQ model and many had studied various cases of fuzzy inventory models. Dhivya and Pandian[9] in their paper drew

the attention of the contributions of various researchers in various classes of fuzzy inventory models. Jayjayanti[10] gave a brief note on study of fuzziness in inventory management by different authors. Urgeletti[4] developed EOQ model in fuzzy nature and used triangular fuzzy number. Park[14] proposed a single product inventory model with fuzzy parameters on the basis of Harris[16] model. Dutta and Pawan[3] developed fuzzy inventory model without shortages using trapezoidal fuzzy number and used signed distance method for defuzzification. Kweimei and Jing[17] fuzzified the order quantity and shortage quantity into triangular fuzzy numbers in an inventory model with backorder. Jaggi et. al[2] developed a fuzzy inventory model for deteriorating items with time varying demand and shortages. Syed and Aziz[7] in their paper developed an inventory model without shortages, representing both the ordering and holding costs by triangular fuzzy numbers and calculating the optimal order quantity using Signed Distance Method of defuzzification. Nabendu and Sanjukta[13] attempted to study inventory with shortages by considering the associated costs involved as different fuzzy numbers. Chandrasiri[1] studied the economic order quantity inventory model without shortages using triangular fuzzy number. Rexlin[15] estimated the fuzzy optimal order quantity and fuzzy total cost of an inventory system with shortage. Uthayakumar and Karuppasamy[11] developed his fuzzy inventory model to reduce the healthcare cost without sacrificing customer service by taking ordering cost, holding cost and order quantity all as triangular fuzzy numbers.

Based on the deterministic leakage inventory model developed by Tomba and Geeta[6], the present fuzzy leakage inventory model is developed by considering holding cost and shortage cost as fuzzy numbers.

2. PRELIMINARIES

2.1. Fuzzy Set: If X is a collection of objects denoted generically by x , then a fuzzy set \tilde{A} in X is defined as a set of ordered pairs $\tilde{A} = \{ (x, \mu_{\tilde{A}}(x)) : x \in X \}$, where $\mu_{\tilde{A}} : X \rightarrow [0, 1]$ is a mapping called the membership function of the fuzzy set \tilde{A} . $\mu_{\tilde{A}}(x)$ is called the membership grade or degree of membership of x in the fuzzy set \tilde{A} .

2.2. Fuzzy Number: A Fuzzy Number is a fuzzy set \tilde{A} with membership function $\mu_{\tilde{A}} : X \rightarrow [0, 1]$ having the following properties:

- (i) \tilde{A} is normal, i.e., there exists $x \in X$ such that $\mu_{\tilde{A}}(x) = 1$.
- (ii) \tilde{A} is piece-wise continuous.
- (iii) Support (\tilde{A}) is bounded, where $\text{Support}(\tilde{A}) = \{ x \in X : \mu_{\tilde{A}}(x) > 0 \}$.
- (iv) \tilde{A} is a convex fuzzy set, i.e., $\mu_{\tilde{A}}(\lambda x_1 + (1 - \lambda)x_2) \geq \min [\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)]$, where $x_1, x_2 \in X$ and $\forall \lambda \in [0, 1]$

2.3. Triangular Fuzzy Number: A fuzzy number $\tilde{M} = (a, b, c)$ where a, b, c are real numbers such that $a < b < c$, is called a Triangular Fuzzy Number if its membership function is given by

$$\mu_{\tilde{M}}(x) = \begin{cases} 0 & \text{if } x < a \\ \frac{x-a}{b-a} & \text{if } a \leq x \leq b \\ \frac{c-x}{c-b} & \text{if } b \leq x \leq c \\ 0 & \text{if } x > c \end{cases}$$

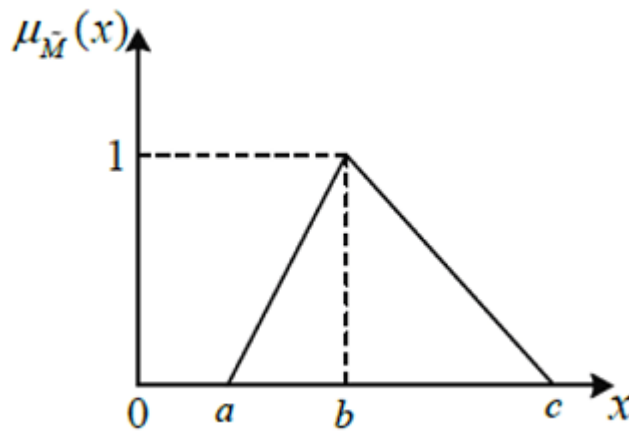
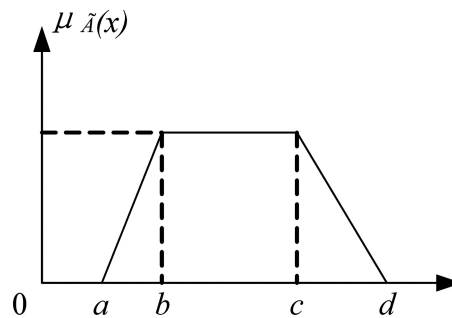


Figure 1: Triangular Fuzzy Number $\tilde{M}=(a,b,c)$

2.4. Trapezoidal Fuzzy Number: A fuzzy number $\tilde{A} = (a, b, c, d)$ where a, b, c, d are real numbers such that $a < b < c < d$, is called a Trapezoidal Fuzzy Number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a} & \text{if } a \leq x \leq b \\ 1 & \text{if } b \leq x \leq c \\ \frac{d-x}{d-c} & \text{if } c \leq x \leq d \\ 0 & \text{otherwise.} \end{cases}$$

Figure 2: Trapezoidal Fuzzy Number $\tilde{A}=(a,b,c,d)$

2.5. Function Principle: The basic arithmetic operations of fuzzy numbers are based on function principle introduced by Shan-Huo Chen[12].

Suppose $\tilde{A} = (a_1, b_1, c_1)$ and $\tilde{B} = (a_2, b_2, c_2)$ are two triangular fuzzy numbers. Then

$$(1) \tilde{A} + \tilde{B} = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$$

$$(2) -\tilde{B} = (-c_2, -b_2, -a_2)$$

$$(3) \tilde{A} - \tilde{B} = (a_1 - c_2, b_1 - b_2, c_1 - a_2)$$

$$(4) \tilde{A} \times \tilde{B} = (a_1 a_2, b_1 b_2, c_1 c_2)$$

$$(5) \frac{1}{\tilde{B}} = \tilde{B}^{-1} = \left(\frac{1}{c_2}, \frac{1}{b_2}, \frac{1}{a_2}\right)$$

$$(6) \frac{\tilde{A}}{\tilde{B}} = \left(\frac{a_1}{c_2}, \frac{b_1}{b_2}, \frac{c_1}{a_2}\right)$$

$$(7) k\tilde{A} = k(a_1, b_1, c_1) = \begin{cases} (ka_1, kb_1, kc_1) & \text{if } k > 0 \\ (kc_1, kb_1, ka_1) & \text{if } k < 0 \end{cases}$$

Suppose $\tilde{A} = (a_1, b_1, c_1, d_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2)$ are two trapezoidal fuzzy numbers. Then

$$(1) \tilde{A} + \tilde{B} = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$$

$$(2) -\tilde{B} = (-d_2, -c_2, -b_2, -a_2)$$

$$(3) \tilde{A} - \tilde{B} = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2)$$

$$(4) \tilde{A} \times \tilde{B} = (a_1 a_2, b_1 b_2, c_1 c_2, d_1 d_2)$$

$$(5) \frac{1}{\tilde{B}} = \tilde{B}^{-1} = \left(\frac{1}{d_2}, \frac{1}{c_2}, \frac{1}{b_2}, \frac{1}{a_2}\right)$$

$$(6) \frac{\tilde{A}}{\tilde{B}} = \left(\frac{a_1}{d_2}, \frac{b_1}{c_2}, \frac{c_1}{b_2}, \frac{d_1}{a_2}\right)$$

$$(7) k\tilde{A} = k(a_1, b_1, c_1, d_1) = \begin{cases} (ka_1, kb_1, kc_1, kd_1) & \text{if } k > 0 \\ (kd_1, kc_1, kb_1, ka_1) & \text{if } k < 0 \end{cases}$$

2.6. Signed Distance Method: Defuzzification is the conversion of a fuzzy quantity to a crisp quantity. In this paper, defuzzification is done by using Signed Distance Method.

If \tilde{A} is a Triangular Fuzzy Number, then the signed distance of \tilde{A} is defined as

$$d(\tilde{A}, 0) = \frac{1}{2} \int_0^1 [A_L(\alpha) + A_R(\alpha)] d\alpha$$

where, $A_L(\alpha) = a + (b - a)\alpha$,

$A_R(\alpha) = c - (c - b)\alpha$, $\alpha \in [0, 1]$.

If \tilde{A} is a Trapezoidal Fuzzy Number, then the signed distance of \tilde{A} is defined as

$$d(\tilde{A}, 0) = \frac{1}{2} \int_0^1 [A_L(\alpha) + A_R(\alpha)] d\alpha$$

where, $A_L(\alpha) = a + (b - a)\alpha$, $A_R(\alpha) = d - (d - c)\alpha$, $\alpha \in [0, 1]$.

3. NOTATIONS AND ASSUMPTIONS

3.1. Notations: The following notations are used in developing the model in the respective environments:

C_h = holding cost or carrying cost per unit quantity per unit time

C_s = shortage cost per unit quantity per unit time

d = demand rate per unit time

d_1 = leakage rate per unit time

t = time interval between runs

z = ordered quantity per run

Q = total demand per production run

TC = average total cost per unit time

TC^* = minimum total cost per unit time

z^* = optimal order quantity

\tilde{C}_h = fuzzy holding or carrying cost per unit quantity of time

\tilde{C}_s = fuzzy shortage cost per unit quantity per unit time

$\tilde{T}C$ = fuzzy average total cost

3.2. Assumptions: In this paper, the following assumptions are considered

- (i) Total demand is considered uniform at a rate of d units per unit time.
- (ii) Leakage rate is d_1 units per unit time.
- (iii) Shortages are allowed.
- (iv) Holding cost and shortage cost are taken as fuzzy numbers.
- (v) Lead time is zero.
- (vi) Production is instantaneous and finite.

4. MATHEMATICAL FORMULATION OF INVENTORY PROBLEMS IN DIFFERENT ENVIRONMENTS

4.1. Leakage Inventory Model with Shortage in Crisp sense.

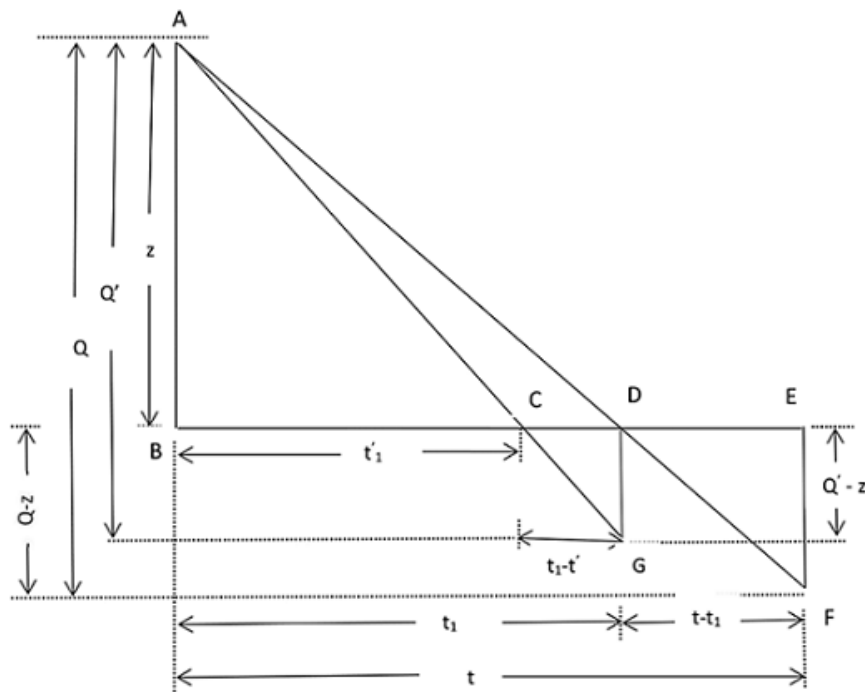


Figure 3: Graphical representation of the inventory system

When the model has shortage and z is the order to which the inventory is raised in the beginning of a run of time interval t , then the inventory is reduced to zero in time t_1 . Then,

$$t_1 = z/d \tag{1}$$

$$Q = dt \tag{2}$$

Shortage increase from 0 to $Q-z$ in the remaining time $(t-t_1)$.

The inventory also has leakage at a rate of d_1 units per unit time, so the inventory reduces to zero in time t_1' instead of time t_1 .

$$\therefore t_1' = \frac{z}{d+d_1} \tag{3}$$

Shortages then increase from 0 to $Q'-z$ in time $(t_1 - t_1')$, where Q' is the quantity produced per run having leakage in demand pattern.

$$\therefore Q' = (d+d_1)t_1 \tag{4}$$

With the occurrence of leakage, the penalty cost created

$$= C_h \times \text{area of } \Delta ACD = \frac{1}{2}C_1z(t_1 - t_1')$$

Then, holding cost (due to leakage) per production run

$$= C_h \times \text{area of } \Delta ABD + C_h \times \text{area of } \Delta ACD = \frac{1}{2}C_hz^2\left(\frac{d+2d_1}{d(d+d_1)}\right) \tag{5}$$

With the occurrence of leakage, the shortage cost is increased.

Then, cost increased in the original shortage cost = $C_s \times \text{area of } \Delta CDG$

$$= \frac{1}{2}C_s(t_1 - t_1') \times (Q' - z) \tag{6}$$

\therefore Shortage cost (due to leakage) for one run = $C_s \times \text{area of } \Delta DEF + C_s \times \text{area of } \Delta CDG$

$$= \frac{C_s}{2d} \left[(Q-z)^2 + \frac{d_1^2z^2}{d(d+d_1)} \right] \tag{7}$$

\therefore Average total cost, $TC(z) =$

$$\frac{1}{2}C_hz^2\left(\frac{d+2d_1}{Q(d+d_1)}\right) + \frac{C_s}{2Q} \left[(Q-z)^2 + \frac{d_1^2z^2}{d(d+d_1)} \right] \tag{8}$$

The optimum order quantity z^* and average minimum total cost TC^* are obtained by equating the partial derivatives of TC to zero and solving the resulting equations.

Optimum order quantity,

$$z^* = \frac{C_s d^2 t (d + d_1)}{C_h d (d + 2d_1) + C_s (d^2 + dd_1 + d_1^2)} \quad (9)$$

Minimum total cost per unit time,

$$TC_{min}^* = \frac{C_s dt \{ C_h d (d + 2d_1) + C_s d_1^2 \}}{2 \{ C_h d (d + 2d_1) + C_s (d^2 + dd_1 + d_1^2) \}} \quad (10)$$

When $d_1 \rightarrow 0$,

$$z^* \rightarrow \frac{C_s dt}{(C_h + C_s)} = z^+ \quad (11)$$

$$TC_{min}^* \rightarrow \frac{C_h C_s dt}{2(C_h + C_s)} = TC_{min}^+ \quad (12)$$

Equations (11) and (12) represent the general model without leakages.

4.2. Leakage Inventory Model with Shortage in Fuzzy sense.

The above model is now considered in fuzzy environment under two cases:

Case I: When the holding cost and shortage cost are represented by Triangular Fuzzy Numbers.

Let \tilde{C}_h : fuzzy carrying or holding cost per unit quantity per unit time,

\tilde{C}_s : fuzzy shortage cost per unit per unit time.

The total cost given in eqn.(8) is fuzzified as fuzzy average total cost,

$$\tilde{TC} = \frac{\tilde{C}_h z^2 (d + 2d_1)}{2Q(d + d_1)} + \frac{\tilde{C}_s}{2Q} \left\{ (Q - z)^2 + \frac{d_1^2 z^2}{d(d + d_1)} \right\} \quad (13)$$

Suppose $\tilde{C}_h = (a_1, b_1, c_1)$ and $\tilde{C}_s = (a_2, b_2, c_2)$ are triangular fuzzy numbers. Then, from eqn.(13),

we get

$$\tilde{TC} = (a_1, b_1, c_1) \times \frac{z^2 (d + 2d_1)}{2Q(d + d_1)} + (a_2, b_2, c_2) \times \frac{1}{2Q} \left\{ (Q - z)^2 + \frac{d_1^2 z^2}{d(d + d_1)} \right\} = (a, b, c) \text{ (say)}$$

Then,

$$\begin{aligned}
 a &= \left\{ \frac{z^2(d+2d_1)}{2Q(d+d_1)} \right\} a_1 + \frac{1}{2Q} \left\{ (Q-z)^2 + \frac{d_1^2 z^2}{d(d+d_1)} \right\} a_2 \\
 b &= \left\{ \frac{z^2(d+2d_1)}{2Q(d+d_1)} \right\} b_1 + \frac{1}{2Q} \left\{ (Q-z)^2 + \frac{d_1^2 z^2}{d(d+d_1)} \right\} b_2 \\
 c &= \left\{ \frac{z^2(d+2d_1)}{2Q(d+d_1)} \right\} c_1 + \frac{1}{2Q} \left\{ (Q-z)^2 + \frac{d_1^2 z^2}{d(d+d_1)} \right\} c_2
 \end{aligned}$$

Defuzzification of fuzzy number \tilde{TC} is done by using Signed Distance method. So,

$$\begin{aligned}
 TC_L(\alpha) &= a + (b-a)\alpha \\
 &= \left\{ \frac{z^2(d+2d_1)}{2Q(d+d_1)} \right\} \{ a_1 + (b_1 - a_1)\alpha \} + \frac{1}{2Q} \left\{ (Q-z)^2 + \frac{d_1^2 z^2}{d(d+d_1)} \right\} \{ a_2 + (b_2 - a_2)\alpha \}
 \end{aligned}$$

$$\begin{aligned}
 TC_R(\alpha) &= c - (c-b)\alpha \\
 &= \left\{ \frac{z^2(d+2d_1)}{2Q(d+d_1)} \right\} \{ c_1 - (c_1 - b_1)\alpha \} + \frac{1}{2Q} \left\{ (Q-z)^2 + \frac{d_1^2 z^2}{d(d+d_1)} \right\} \{ c_2 - (c_2 - b_2)\alpha \}
 \end{aligned}$$

$$\begin{aligned}
 \therefore TC_L(\alpha) + TC_R(\alpha) &= \left\{ \frac{z^2(d+2d_1)}{2Q(d+d_1)} \right\} \{ (a_1 + c_1) + (2b_1 - a_1 - c_1)\alpha \} + \\
 &\frac{1}{2Q} \left\{ (Q-z)^2 + \frac{d_1^2 z^2}{d(d+d_1)} \right\} \{ (a_2 + c_2) + (2b_2 - a_2 - c_2)\alpha \}
 \end{aligned}$$

$$\begin{aligned}
 \text{Then, } d(\tilde{TC}, 0) &= \frac{1}{2} \int_0^1 [TC_L(\alpha) + TC_R(\alpha)] d\alpha \\
 &= \frac{1}{2} \int_0^1 \left[\left\{ \frac{z^2(d+2d_1)}{2Q(d+d_1)} \right\} \{ (a_1 + c_1) + (2b_1 - a_1 - c_1)\alpha \} + \right. \\
 &\left. \frac{1}{2Q} \left\{ (Q-z)^2 + \frac{d_1^2 z^2}{d(d+d_1)} \right\} \{ (a_2 + c_2) + (2b_2 - a_2 - c_2)\alpha \} \right] d\alpha
 \end{aligned}$$

$$= \frac{1}{2} \left[\frac{z^2(d+2d_1)}{2Q(d+d_1)} \left(\frac{2b_1 + a_1 + c_1}{2} \right) + \frac{1}{2Q} \left\{ (Q-z)^2 + \frac{d_1^2 z^2}{d(d+d_1)} \right\} \left(\frac{2b_2 + a_2 + c_2}{2} \right) \right] \quad (14)$$

Differentiating $d(\tilde{TC}, 0)$ w.r.t. z and equating to zero, we get

$$z = z^* = \frac{Qd(d+d_1)(2b_2 + a_2 + c_2)}{d(d+2d_1)(2b_1 + a_1 + c_1) + (d^2 + dd_1 + d_1^2)(2b_2 + a_2 + c_2)} \quad (15)$$

which is the fuzzy optimal order quantity.

Also, $\frac{d^2}{dz^2}(d(\tilde{TC}, 0)) > 0$ when $z = z^*$. This shows that $d(\tilde{TC}, 0)$ is minimum when $z = z^*$. From

(14) using (15), we get

fuzzy minimum total cost

$$\begin{aligned}
 d^*(\tilde{TC}, 0) &= \frac{1}{2} \left[\frac{z^{*2}(d+2d_1)}{2Q(d+d_1)} \left(\frac{2b_1 + a_1 + c_1}{2} \right) + \frac{1}{2Q} \left\{ (Q-z^*)^2 + \frac{d_1^2 z^{*2}}{d(d+d_1)} \right\} \left(\frac{2b_2 + a_2 + c_2}{2} \right) \right] \\
 &= \frac{Q}{8} (2b_2 + a_2 + c_2) \left\{ \frac{d_1^2 (2b_2 + a_2 + c_2) + d(d+2d_1)(2b_1 + a_1 + c_1)}{d(d+2d_1)(2b_1 + a_1 + c_1) + (d^2 + dd_1 + d_1^2)(2b_2 + a_2 + c_2)} \right\} \quad (16)
 \end{aligned}$$

When $d_1 \rightarrow 0$,

$$z^* \rightarrow \frac{Q(2b_2 + a_2 + c_2)}{(2b_1 + a_1 + c_1) + (2b_2 + a_2 + c_2)} \quad (17)$$

$$\begin{aligned} d^*(\tilde{TC}, 0) &\rightarrow \frac{1}{2} \left[\frac{z^{*2}d}{2Qd} \left(\frac{2b_1 + a_1 + c_1}{2} \right) + \frac{1}{2Q} \left[(Q - z^*)^2 + \left(\frac{2b_2 + a_2 + c_2}{2} \right) \right] \right] \\ &= \frac{Q}{8} \left\{ \frac{(2b_2 + a_2 + c_2)(2b_1 + a_1 + c_1)}{(2b_1 + a_1 + c_1) + (2b_2 + a_2 + c_2)} \right\} \end{aligned} \quad (18)$$

Equations (17) and (18) represent fuzzy model without leakages.

Case II: When the holding cost and shortage cost are represented by Trapezoidal Fuzzy Numbers

Suppose $\tilde{C}_h = (a_1, b_1, c_1, d_1)$ and $\tilde{C}_s = (a_2, b_2, c_2, d_2)$ are trapezoidal fuzzy numbers. Then from (13) we get

$$\tilde{TC} = (a_1, b_1, c_1, d_1) \times \frac{z^2(d+2d_1)}{2Q(d+d_1)} + (a_2, b_2, c_2, d_2) \times \frac{1}{2Q} \left\{ (Q - z)^2 + \frac{d_1^2 z^2}{d(d+d_1)} \right\} = (a, b, c, d) \text{ (say)}$$

Then,

$$\begin{aligned} a &= \left\{ \frac{z^2(d+2d_1)}{2Q(d+d_1)} \right\} a_1 + \frac{1}{2Q} \left\{ (Q - z)^2 + \frac{d_1^2 z^2}{d(d+d_1)} \right\} a_2 \\ b &= \left\{ \frac{z^2(d+2d_1)}{2Q(d+d_1)} \right\} b_1 + \frac{1}{2Q} \left\{ (Q - z)^2 + \frac{d_1^2 z^2}{d(d+d_1)} \right\} b_2 \\ c &= \left\{ \frac{z^2(d+2d_1)}{2Q(d+d_1)} \right\} c_1 + \frac{1}{2Q} \left\{ (Q - z)^2 + \frac{d_1^2 z^2}{d(d+d_1)} \right\} c_2 \\ d &= \left\{ \frac{z^2(d+2d_1)}{2Q(d+d_1)} \right\} d_1 + \frac{1}{2Q} \left\{ (Q - z)^2 + \frac{d_1^2 z^2}{d(d+d_1)} \right\} d_2 \end{aligned}$$

Defuzzification of fuzzy number \tilde{TC} is done by using Signed Distance method. So,

$$\begin{aligned} TC_L(\alpha) &= a + (b - a)\alpha \\ &= \left\{ \frac{z^2(d+2d_1)}{2Q(d+d_1)} \right\} \{ a_1 + (b_1 - a_1)\alpha \} + \frac{1}{2Q} \left\{ (Q - z)^2 + \frac{d_1^2 z^2}{d(d+d_1)} \right\} \{ a_2 + (b_2 - a_2)\alpha \} \\ TC_R(\alpha) &= d - (d - c)\alpha \\ &= \left\{ \frac{z^2(d+2d_1)}{2Q(d+d_1)} \right\} \{ d_1 - (d_1 - c_1)\alpha \} + \frac{1}{2Q} \left\{ (Q - z)^2 + \frac{d_1^2 z^2}{d(d+d_1)} \right\} \{ d_2 - (d_2 - c_2)\alpha \} \end{aligned}$$

$$\begin{aligned} \therefore TC_L(\alpha) + TC_R(\alpha) &= \left\{ \frac{z^2(d+2d_1)}{2Q(d+d_1)} \right\} \{ (a_1 + d_1) + (b_1 - a_1 - d_1 + c_1)\alpha \} + \\ &\frac{1}{2Q} \left\{ (Q - z)^2 + \frac{d_1^2 z^2}{d(d+d_1)} \right\} \{ (a_2 + d_2) + (b_2 - a_2 - d_2 + c_2)\alpha \} \end{aligned}$$

$$\begin{aligned} \text{Then, } d(\tilde{TC}, 0) &= \frac{1}{2} \int_0^1 [TC_L(\alpha) + TC_R(\alpha)] d\alpha \\ &= \frac{1}{2} \int_0^1 \left[\left\{ \frac{z^2(d+2d_1)}{2Q(d+d_1)} \right\} \{ (a_1 + d_1) + (b_1 - a_1 - d_1 + c_1)\alpha \} + \right. \end{aligned}$$

$$\begin{aligned} & \frac{1}{2Q} \left\{ (Q - z)^2 + \frac{d_1^2 z^2}{d(d+d_1)} \right\} \{ (a_2 + d_2) + (b_2 - a_2 - d_2 + c_2)\alpha \} d\alpha \\ &= \frac{1}{8Q} \left[\frac{z^2(d+2d_1)}{(d+d_1)} (a_1 + b_1 + c_1 + d_1) + \left\{ (Q - z)^2 + \frac{d_1^2 z^2}{d(d+d_1)} \right\} (a_2 + b_2 + c_2 + d_2) \right] \end{aligned} \quad (19)$$

Differentiating $d(\tilde{T}C, 0)$ w.r.t. z and equating to zero, we get

$$z = z^* = \frac{Qd(d+d_1)(a_2 + b_2 + c_2 + d_2)}{d(d+2d_1)(a_1 + b_1 + c_1 + d_1) + (d^2 + dd_1 + d_1^2)(a_2 + b_2 + c_2 + d_2)} \quad (20)$$

which is the fuzzy optimal order quantity.

Also, $\frac{d^2}{dz^2}(d(\tilde{T}C, 0)) > 0$ when $z = z^*$.

This shows that $d(\tilde{T}C, 0)$ is minimum when $z = z^*$.

From (19) using (20), we get

Fuzzy minimum total cost, $d^*(\tilde{T}C, 0)$

$$\begin{aligned} &= \frac{1}{8Q} \left[\frac{z^{*2}(d+2d_1)}{(d+d_1)} (a_1 + b_1 + c_1 + d_1) + \left\{ (Q - z^*)^2 + \frac{d_1^2 z^{*2}}{d(d+d_1)} \right\} (a_2 + b_2 + c_2 + d_2) \right] \\ &= \frac{Q(a_2 + b_2 + c_2 + d_2)}{8} \left\{ \frac{d_1^2(a_2 + b_2 + c_2 + d_2) + d(d+2d_1)(a_1 + b_1 + c_1 + d_1)}{d(d+2d_1)(a_1 + b_1 + c_1 + d_1) + (d^2 + dd_1 + d_1^2)(a_2 + b_2 + c_2 + d_2)} \right\} \end{aligned} \quad (21)$$

When $d_1 \rightarrow 0$,

$$z^* \rightarrow \frac{Qd(a_2 + b_2 + c_2 + d_2)}{(a_1 + b_1 + c_1 + d_1) + (a_2 + b_2 + c_2 + d_2)} \quad (22)$$

$$d^*(\tilde{T}C, 0) \rightarrow \frac{Q}{8} \left\{ \frac{(a_1 + b_1 + c_1 + d_1)(a_2 + b_2 + c_2 + d_2)}{(a_1 + b_1 + c_1 + d_1) + (a_2 + b_2 + c_2 + d_2)} \right\} \quad (23)$$

Equations (22) and (23) represent fuzzy model without leakages.

5. NUMERICAL EXAMPLE WITH SENSITIVITY ANALYSIS

To illustrate the developed model, we consider an example when

$d = 25$ items per day

$C_h = ₹ \frac{8}{15}$ per item per day

$C_s = ₹ 10$ per item per day

$t = 30$ days

leakage rate d_1 is 0.5 items per day to 2.5 items per day

Table 1. When parameters are in Crisp environment

d_1	z^*	C_{min}^*
0.5	711.06	194.73
1.0	709.61	201.97
1.5	707.72	211.42
2.0	705.41	222.94
2.5	702.72	236.37

Table 2. When parameters are considered as Triangular Fuzzy Numbers

d_1	$C_h=(0,0.5,1), C_s=(9,10,11)$		$C_h=(0,0.6,1), C_s=(9,10.5,11)$	
	z^*	C_{min}^*	z^*	C_{min}^*
0.5	713.35	183.23	710.83	200.74
1.0	711.94	190.31	709.38	208.18
1.5	710.01	199.62	707.49	217.89
2.0	707.80	211.03	705.18	229.71
2.5	705.13	224.36	702.49	243.48

Table 3. When parameters are considered as Trapezoidal Fuzzy Numbers

d_1	$C_h=(0,0.4,0.5,1), C_s=(8,9,10,11)$		$C_h=(0,0.3,0.5,1), C_s=(8,9,10.5,11)$	
	z^*	C_{min}^*	z^*	C_{min}^*
0.5	713.35	174.07	715.61	165.52
1.0	711.94	180.00	714.22	172.17
1.5	710.08	189.64	712.39	181.00
2.0	707.79	200.48	710.13	191.87
2.5	705.13	213.14	707.48	204.61

From the above three tables, we observe that economic order quantity obtained in fuzzy environment is very much close to the crisp economic order quantity and the minimum total cost in fuzzy sense is less than crisp minimum total cost.

6. CONCLUSION

In this paper, an inventory leakage model with shortage is studied in fuzzy sense considering the holding cost and shortage cost as fuzzy numbers (triangular and trapezoidal fuzzy numbers). Signed Distance method is opted for defuzzification. Sensitivity analysis indicates that the optimum values obtained in fuzzy environment are close to that of the result from crisp method and are more accurate to real life situations due to the many uncertainties prevailing in practical life.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

REFERENCES

- [1] A.M.P. Chandrasiri, Fuzzy inventory model without shortages using triangular fuzzy numbers and signed distance method, *Int. J. Sci. Res.* 5 (7)(2016), 1179-1182.
- [2] C.K. Jaggi, S. Pareek, A. Sharma, Nidhi, Fuzzy inventory model for deteriorating items with time varying demand and shortages, *Am. J. Oper. Res.* 2(6)(2012), 81-92.
- [3] D. Dutta, Pavan Kumar, Fuzzy Inventory Model Without Shortage Using Trapezoidal Fuzzy Number With Sensitivity Analysis, *IOSR J. Math.* 4(3)(2012), 32-37.
- [4] G. Urgeletti Tinarelli, Inventory Control: Models and problems, *Eur. J. Oper. Res.* 14(1)(1983), 1-12.
- [5] H.J. Zimmerman, Using fuzzy sets in operational research, *Eur. J. Oper. Res.* 13(1983), 201-206.
- [6] I. Tomba, O. Geeta, Some deterministic leakage inventory models, *Bull. Pure Appl. Sci.* 27E(2)(2008), 267-276.
- [7] J.K. Syed, L.A. Aziz, Fuzzy Inventory Model without Shortages using Signed Distance Method, *Appl. Math. Inf. Sci.* 1(2)(2007), 203-209.
- [8] L.A. Zadeh, Fuzzy sets, *Inform. Control*, 8(3)(1965), 338-353
- [9] M. Dhivya Lakshmi, P. Pandian, A review on inventory models in fuzzy environment, *Int. J. Pure Appl. Math.* 119(9)(2018), 113-123.

- [10] R. Jayjayanti, A note on fuzziness in inventory management problems, *Adv. Fuzzy Math.* 12(3)(2017), 663-676.
- [11] R. Uthayakumar, S.K. Karuppasamy, A fuzzy inventory model with lot size dependent ordering cost in health-care industries, *Oper. Res. Appl.* 3(1)(2016), 17-29.
- [12] C. Shan-Huo, Backorder fuzzy inventory model under function principle, *Inform. Sci.* 95(1996), 71-79.
- [13] S. Nabendu, M. Sanjukta, A fuzzy inventory model with shortages using different fuzzy numbers, *Am. J. Math. Stat.* 5(5)(2015), 238-248.
- [14] S. Park Kyung, Fuzzy set theoretical interpretation of economic order quantity, *IEEE Trans. Syst, Man Cybern.* 17(6)(1987), 1082-1084.
- [15] S. Rexlin Jeyakumari , Optimization for Fuzzy Inventory Model for Allowable Shortage, *Int. J. Comput. Sci. Mob. Comput.* 5(8)(2016), 114-122.
- [16] W. Harris Ford, *Operations and Cost (Factory Management Services)*, AW Shaw Co., Chicago, (1915).
- [17] W. Kweimei, S. Yao Jing, Fuzzy inventory with backorder for fuzzy order quantity and fuzzy shortage quantity, *Eur. J. Oper. Res.* 150(2)(2003), 320-352.