Available online at http://scik.org
J. Math. Comput. Sci. 11 (2021), No. 6, 7821-7831
https://doi.org/10.28919/jmcs/6587
ISSN: 1927-5307

ON SOFT QUASI R_0-SPACES

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Abstract. Separation axioms is one of the important topic of research in general topology. In 1961, Davis introduced a separation axiom called R_0 spaces which is a weaker than both T_0 and T_1. In the present paper, we studied this axiom in soft bitopological spaces obtain some of its properties.

Keywords: soft sets; soft topology; soft quasi open; soft quasi closed; soft quasi R_0-spaces; soft bitopological spaces.

2010 AMS Subject Classification: Primary 54C40, 54D10, Secondary 06D72.

1. Introduction

It appears to be easy to understand that a mathematical theory is based on various abstract thoughts. In such cases one has full freedom to establish certain environment depending on neglecting relevent facts, for example in physics we often neglect the frictional effect of air on a free falling body but this fact is fully impossible in real life. Similarily as we know in vacuum 1 kg of cotton and 1 kg of iron from a height touches the ground at the same time. However this will be impossible in our real life problem due to the presence of frictional forces. Similarily other branches like economics, engineering, social sciences are full of uncertainties.

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Received July 31, 2021

As we know, we had four mathematical tools to deal with uncertainties, namely probability theory, fuzzy set theory, rough set theory and theory of interval in mathematics. In order to overcome the choice of degree of membership in fuzzy set theory when the facts are concerned with uncertainties, Molodtsov [12] introduced the concept of soft set theory and investigated its applications in game theory, smoothness of function, operation research, Perron integration, probability theory and theory of measurement. However afterwards Maji et al. [11] defined various operations on soft set to study some of the fundamental properties. Pei, Miao [14] and Chen [2] stressed that error in the paper of Maji et al. [11] and accordingly introduced new notions and properties. Now, in present days investigation of different properties and applications of soft set theory have attracted many researchers from various backgrounds. It is pertinent to mention that Shabir and Naz [16] in the year 2011 introduced the term of soft topology and studied some introductory results. In the same year, Cagman et al. [1] introduced soft topology into different perspective. Till then various researchers have studied various foundation results in soft topology [6, 5, 16, 20]. It is relevent to note that Kandil et al. [7, 8, 9] intoduced the concept of soft ideal and studied some weaker notions and properties. Recently, Yuksel et al. [18] applied soft set theory to determine prostrate cancer risk. Senel and Cagman [15] introduced soft topological subspace and studied some properties. Kelly [10] introduced the concept of bitopological spaces. The study of quasi open sets was initiated by Dutta [4]. The present paper extended and studied the concept of soft quasi open sets in soft bitopological spaces and utilizes soft quasi open set to introduce and investigate the properties of soft quasi R_0 bitopological spaces.

2. PRELIMINARIES

Let U is an initial universe set, E be a set of parameters, P(U) denote the power set of U and $A \subseteq E$.

Definition 2.1. [12] A pair (F,A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For all $e \in A$, F(e) may be considered as the set of e-approximate elements of the soft set (F,A).

Definition 2.2. [11] For two soft sets (F,A) and (G,B) over a common universe U, we say that (F,A) is a soft subset of (G,B), denoted by $F(A) \subseteq G(B)$, if:

(1) $A \subseteq B$ and

(2) $F(e) \subseteq G(e), \forall\, e \in E$.

Definition 2.3. [11] Two soft sets (F,A) and (G,B) over a common universe U are said to be soft equal denoted by (F,A) = (G,B), if $(F,A) \subseteq (G,B)$ and $(G,B) \subseteq (F,A)$.

Definition 2.4. [11] The complement of a soft set (F,A) denoted by $(F,A)^c$, is defined by $(F,A)^c = (F^c,A)$ where $F^c : A \to P(U)$ is a mapping given by $F^c(e) = U - F(e), \forall\, e \in E$.

Definition 2.5. [11] Let a soft set (F,A) over U.

(1) Null soft set denoted by ϕ, if $\forall\, e \in A$, $F(e) = \phi$.

(1) Absolute soft set denoted by $\tilde{U}$ if $\forall\, e \in A$, F(e) = U.

Clearly, $\tilde{U}^c = \phi$ and $\phi^c = \tilde{U}$.

Definition 2.6. [16] The difference of two soft sets (F,A) and (G,B) over the common universe U denoted by $(F,A) - (G,B)$ is the soft set (H,C), where $H(e) = F(e) - G(e), \forall\, e \in C$.

Definition 2.7. [11] Union of two soft sets (F,A) and (G,B) over the common universe U, is the soft set (H,C), where $C = A \cup B$ and $\forall\, e \in C$,

$$H(e) = \begin{cases} F(e) & \text{if } e = A - B \\ G(e) & \text{if } e = B - A \\ F(e) \cup G(e) & \text{if } e \in A \cap B \end{cases} \tag{1}$$

Definition 2.8. [14] Intersection of two soft sets (F,A) and (G,B) over a common universe U, is the soft set (H,C), where $C = A \cap B$ and $H(e) = F(e) \cap G(e)$, $\forall\, e \in E$.

Let X and Y be an initial universe sets and E and K be the nonempty sets of parameters, S(X,E) denotes the family of all soft sets over X and S(Y,K) denotes the family of all soft sets over Y.

Definition 2.9. [19] Let $\{(F_j,E) : j \in J\}$ be a nonempty family of soft sets over a common universe U. Then:

(1) Intersection of this family, denoted by $\cap_{j\in J}$, is defined by $\cap_{j\in J}(F_J,E) = (\cap_{j\in J}F_J,E)$, where $(\cap_{j\in J}F_J)(e) = \cap_{j\in J}(F_J(e)), \forall\, e\in E$.

(2) Union of this family, denoted by $\cup_{j\in J}$, is defined by $\cup_{j\in J}(F_J,E) = (\cup_{j\in J}F_J,E)$, where $(\cup_{j\in J}F_J)(e) = \cup_{j\in J}(F_J(e)), \forall\, e\in E$.

Definition 2.10. [16] A subfamily τ of S(X,E) is called a soft topology on X if:

(1) $\tilde{\phi}, \tilde{X}$ belongs to τ.

(2) The union of any number of soft sets in τ belongs to τ.

(3) The intersection of any two soft sets in τ belongs to τ.

The triplet (X,τ,E) is called a soft topological space. The members of τ are called soft open sets in X and their complements called soft closed sets in X.

Lemma 2.1. [16] *Let (X,τ,E) be a soft topological space. Then the collection $\tau_\alpha = \{F(\alpha) : (F,E)\in\tau\}$ for each $\alpha\in E$, defines a topology on X.*

Definition 2.11. [16] In a soft topological space (X,τ,E) the intersection of all soft closed super sets of (F,E) is called the soft closure of (F,E). It is denoted by Cl(F,E).

Definition 2.12. [19] In a soft topological space (X,τ,E) the union of all soft open subsets of (F,E) is called soft interior of (F,E). It is denoted by Int(F,E).

Lemma 2.2. [1, 16, 19] *Let (X,τ,E) be a soft topological space and let $(F,E),(G,E)\in S(X,E)$. Then:*

(1) *(F,E) is soft closed if and only if (F,E)= Cl(F,E).*

(2) *If $(F,E)\subseteq(G,E)$, then $Cl(F,E)\subseteq Cl(G,E)$.*

(3) *(F,E) is soft open if and only if (F,E) = Int (F,E).*

(4) *If $(F,E)\subseteq(G,E)$, then $Int(F,E)\subseteq Int(G,E)$.*

(5) $(Cl(F,E))^c = Int((F,E)^c)$.

(6) $(Int(F,E))^c = Cl((F,E)^c)$.

Definition 2.13. [6] Let (X,τ,E) be a soft topological space over X and Y be a nonempty subset of X. Then $\tau_Y = \{(F_Y,E) : (F,E)\in\tau\}$ is said to be the soft relative topology on Y and (Y,τ_Y,E) is called a soft subspace of (X,τ,E).

Lemma 2.3. [6] *Let (Y,τ_Y,E) be a soft subspace of a soft topological space (X,τ,E) and (F,E) be a soft open set in Y. If $\tilde{Y} \in \tau$ then $(F,E) \in \tau$.*

Lemma 2.4. [16] *Let (Y,τ_Y,E) be a soft topological subspace of a soft topological space (X,τ,E) and (F,E) be a soft set over X, then:*

(1) *(F,E) is soft open in Y if and only if $(F,E) = \tilde{Y} \cap (G,E)$ for some soft open set (G,E) in X.*

(2) *(F,E) is soft closed in Y if and only if $(F,E) = \tilde{Y} \cap (G,E)$ for some soft closed set (G,E) in X.*

Lemma 2.5. [15] *Let (X,τ,E) be a soft topological space and (Y,τ_Y,E) be a soft subspace of (X,τ,E), then a soft closed set (F_Y,E) of Y is soft closed in X if and only if $\tilde{Y}$ is soft closed in X.*

Lemma 2.6. [19] *The soft set $(F,E) \in S(X,E)$ is called a soft point if there exists $x \in X$ and $e \in E$ such that $F(e) = \{x\}$ and $F(e^c) = \phi$ for each $e^c \in E - \{e\}$ and the soft point (F,E) is denoted by x_e. We denote the family of all soft points over X by SP(X,E).*

Definition 2.14. [19] The soft point x_e is said to be in the soft set (G,E), denoted by $x_e \in (G,E)$ if $x_e \subseteq (G,E)$.

Lemma 2.7. [3, 13] *Let $(F,E),(G,E) \in S(X,E)$ and $x_e \in SP(X,E)$. Then we have:*

(1) *$x_e \in (F,E)$ if and only if $x_e \notin (F,E)^c$.*

(2) *$x_e \in (F,E) \cup (G,E)$ if and only if $x_e \in (F,E)$ or $x_e \in (G,E)$.*

(3) *$x_e \in (F,E) \cap (G,E)$ if and only if $x_e \in (F,E)$ and $x_e \in (G,E)$.*

(4) *$(F,E) \subseteq (G,E)$ if and only if $x_e \in (F,E)$ implies $x_e \in (G,E)$.*

Definition 2.15. [17] Let (X,τ,E) be a soft topological space, (F,E) be a soft set and x_e be a soft point over X, then (F,E) is called soft neighborhood of x_e, if there exists soft open set (G,E) such that $x_e \in (G,E) \subseteq (F,E)$.

3. Soft Quasi Open Sets

Definition 3.1. A soft bitopological space (X,τ_1,τ_2,E) is a quadral with a non empty set of parameter E and with two soft topologies τ_1 and τ_2 defined on X.

Definition 3.2. Let (X,τ_1,τ_2,E) be a soft bitopological space over X and Y be a non-empty subset of X. Then $\tau_{1Y} = \{((F,E) \cap \tilde{Y}) : (F,E) \in \tau_1\}$ and $\tau_{2Y} = \{((F,E) \cap \tilde{Y}) : (F,E) \in \tau_2\}$ are said to be the soft relative topology on Y and $(Y,\tau_{1Y},\tau_{2Y},E)$ is called a soft relative bitopological space.

Definition 3.3. A subset (F, E) of a soft bitopological space (X,τ_1,τ_2,E) is said to be soft quasi open if it is a union of a τ_1-soft open and a τ_2-soft open set.

Remark 3.1. In a soft bitopological space (X,τ_1,τ_2,E), every τ_1-soft open (resp. τ_2-soft open) set is soft quasi open.

However, the converse need not be true . For,

Example 3.1. Let $X=\{a,b\}$, $E=\{e_1,e_2\}$, $\tau_1=\{\tilde{\phi},\tilde{X},\{\{a\},\phi\}\}$, $\tau_2=\{\tilde{\phi},\tilde{X},\{\phi,\{b\}\}\}$ be soft topology on X ,then $\{\{$a$\}$,$\{$b$\}\}$ is a soft quasi open set in a soft bitopological space (X,τ_1,τ_2,E) but it is neither τ_1-soft open nor τ_2-soft open.

Definition 3.4. The complement of soft quasi open set in a soft bitopological space is called soft quasi closed. The family of all soft quasi open (resp. soft quasi closed) sets in (X,τ_1,τ_2,E) will be denoted by qO(X) (resp. qC(X)).

Theorem 3.1. *Any union of soft quasi open sets in a soft bitopological space is soft quasi open.*

Proof. Evident □

Remark 3.2. The intersection of two soft quasi open sets in a soft bitopological space may not be soft quasi open. For,

Example 3.2. Let $X=\{a,b,c\}$, $E=\{e_1,e_2\}$, $\tau_1=\{\tilde{\phi},\tilde{X},\{\{a\},\{b\}\}\}$, $\tau_2=\{\tilde{\phi},\tilde{X},\{\{c\},\{b\}\}\}$be soft topology on X ,then $\{\{$a$\}$,$\{$b$\}\}$, $\{\{$c$\}$,$\{$b$\}\}$ are soft quasi open sets in soft bitopological space (X,τ_1,τ_2,E) but their intersection $\{\phi,\{b\}\}$ is not soft quasi open .

Definition 3.5. Let (A,E) be a soft subset of a soft bitopological space (X,τ_1,τ_2,E), then the smallest soft quasi closed set which contains (A,E) is called soft quasi closure of (A,E). It is denoted by qCl(A,E) .

Theorem 3.2. *Let (X,τ_1,τ_2,E) be a soft bitopological space and (A,E) be a soft subset of X then,*

(1) *A soft point $x_e \in qCl(A,E)$ if and only if for any soft quasi open set (U,E) containing x_e, $(U,E)\cap(A,E) \neq \phi$.*

(2) *(A,E) is soft quasi closed if and only if (A,E)= qCl(A,E).*

(3) $qCl(A,E) = \tau_1 - Cl(A,E) \cap \tau_2 - Cl(A,E)$.

Proof. Evident □

Theorem 3.3. *Let (X,τ_1,τ_2,E) be a soft bitopological space. If (O,E) is soft biopen and (A,E) is soft quasi open in X, then $(O,E)\cap(A,E)$ is soft quasi open.*

Proof. Since (A,E) is soft quasi open by Definition 3.3, let (W,E) be τ_1-soft open and (V,E) be a τ_2-soft open set such that $(A,E) = (W,E)\cup(V,E)$. Then, $(O,E)\cap(A,E) = ((O,E)\cap(W,E))\cup((O,E)\cap(V,E))$. Since (O,E) is soft biopen, $(O,E)\cap(W,E)$ is τ_1-soft open and $(O,E)\cap(V,E)$ is τ_2-soft open. Therefore by Definition 3.3, $(O,E)\cap(A,E)$ is soft quasi open. □

Theorem 3.4. *Let $(Y,(\tau_1)_Y,(\tau_2)_Y,E)$ be a soft subspace of a soft bitopological space (X,τ_1,τ_2,E). If (A,E) is soft quasi open in X then $(A,E)\cap\tilde{Y}$ is soft quasi open in Y.*

Proof. Since (A,E) is soft quasi open in X , by virtue of Definition 3.3, there is a τ_1-soft open set (W,E) and a τ_2-soft open set (V,E) such that $(A,E) = (W,E)\cup(V,E)$. Then $(A,E)\cap\tilde{Y} = ((W,E)\cap\tilde{Y})\cup((V,E)\cap\tilde{Y})$. It is clear that $(W,E)\cap\tilde{Y}$ is $(\tau_1)_Y$-soft open and $(V,E)\cap\tilde{Y}$ is $(\tau_2)_Y$-soft open. Consequently by Definition 3.3, it results that $(A,E)\cap\tilde{Y}$ is soft quasi open in Y. □

Remark 3.3. Let $(Y,(\tau_1)_Y,(\tau_2)_Y,E)$ be a soft subspace of a soft bitopological space (X,τ_1,τ_2,E). If (A,E) is soft quasi open in Y and Y is even if τ_1-soft open (resp. τ_2-soft open) then (A,E) may not be soft quasi open in X.

Theorem 3.5. *Let $(Y,(\tau_1)_Y,(\tau_2)_Y,E)$ be a soft biopen subspace of a soft bitopological space (X,τ_1,τ_2,E). If (A,E) is soft quasi open in Y, then (A,E) is soft quasi open in X.*

Proof. Let $(W,E)_Y$ be $(\tau_1)_Y$-soft open and $(V,E)_Y$ be $(\tau_2)_Y$-soft open such that $(A,E) = (W,E)_Y\cup(V,E)_Y$. Since $(W,E)_Y \subseteq \tilde{Y}, (V,E)_Y \subseteq \tilde{Y}$ and $\tilde{Y}$ is soft biopen, $(W,E)_Y$ is $(\tau_1)_Y$-soft open and $(V,E)_Y$ is $(\tau_2)_Y$-soft open .Consequently, (A,E) is soft quasi open in X. □

4. Soft Quasi R_0-Space

Definition 4.1. A soft bitopological space (X,τ_1,τ_2,E) is soft quasi R_0-space if for each soft quasi open set (F,E) and a soft point $x_e \in (F,E)$ implies $qC1\{x_e\} \subseteq (F,E)$.

Definition 4.2. A soft bitopological space (X,τ_1,τ_2,E) is termed as pairwise soft-R_0, if for every τ_1-soft open set (O,E) and a soft point $x_e \in (O,E)$ implies that $\tau_j - Cl\{x_e\} \subseteq (O,E), i, j = 1,2,\ldots i \neq j$.

Theorem 4.1. *Every pairwise soft R_0-space (X,τ_1,τ_2,E) is soft quasi R_0.*

Proof. Let (G,E) be a soft quasi open set and a soft point $x_e \in (G,E)$. Then by Definition 3.3, there exists a τ_1-soft open set (W,E) and a τ_2-soft open set (V,E) such that $(G,E) = (W,E) \cup (V,E)$. If $x_e \in (W,E)$ then τ_2–$Cl\{x_e\} \subseteq (W,E)$, since X is pairwise soft R_0. And so, $q(Cl\{x_e\}) \subseteq (W,E)$, in view of Definition 3.2(3), Consequently, $q(Cl\{x_e\}) \subseteq (G,E)$. The case if $x_e \in (V,E)$ is similar. Hence, the space (X,τ_1,τ_2,E) is soft quasi R_0. □

Definition 4.3. In a soft bitopological space (X,τ_1,τ_2,E) the soft quasi kernel of a soft point x_e of X denoted by $q(Ker\{x_e\})$ is defined as follows:

$q(Ker\{x_e\}) = \cap\{(G,E) : (G,E)$ is soft quasi open, $x_e \in (G,E)\}$.

Lemma 4.1. $q(Ker\{x_e\}) = \{y_e : x_e \in qCl\{y_e\}\}$. *First, we prove,*

Proof. Results from Definition 4.3 and Theorem 3.2(1). □

The following theorem now obtain several characterization of soft quasi R_0-spaces.

Theorem 4.2. *In a soft bitopological space (X,τ_1,τ_2,E), the following statements are equivalent:*

(1) (X,τ_1,τ_2,E) *is soft quasi R_0-spaces.*

(2) *For $x_e \in X$, $qCl\{x_e\} \subseteq qKer\{x_e\}$.*

(3) *For $x_e, y_e \in X, y_e \in qKer\{x_e\} \Leftrightarrow x_e \in qKer\{y_e\}$.*

(4) *For $x_e, y_e \in X, y_e \in qCl\{x_e\} \Leftrightarrow x_e \in qCl\{y_e\}$.*

(5) *For any soft quasi closed set (F,E) and a soft point $x_e \notin (F,E)$, there exists a soft quasi open set (G,E) such that $x_e \notin (G,E)$ and $(F,E) \subseteq (G,E)$.*

(6) *For any soft quasi closed set* $(F,E), (F,E) = \cap\{(G,E) : (G,E)$ *is soft quasi open,* $(F,E) \subseteq (G,E)\}$.

(7) *Each soft quasi open set (G,E) is a union of soft quasi closed sets contained in (G,E).*

(8) *For each soft quasi closed set (F,E) and a soft point* $x_e \notin (F,E)$ *implies* $qCl\{x_e\} \cap (F,E) = \phi$.

Proof. **(1)⇒(2)** For $x_e \in X$, $qKer\{x_e\} = \cap\{(G,E) : (G,E)$ is soft quasi open, $x_e \in (G,E)\}$. Since X is soft quasi R_0 each soft quasi open set (G,E) containing x_e contains $qCl\{x_e\}$. Hence, $qCl\{x_e\} \subseteq qKer\{x_e\}$.

(2)⇒(3) If $y_e \in qKer\{x_e\}$, then $x_e \in qCl\{y_e\}$. Now by (2), $x_e \in qKer\{y_e\}$. Analogously, $x_e \in qKer\{y_e\}$ implies $y_e \in qKer\{x_e\}$.

(3)⇒(4) For $x_e, y_e \in X$, $y_e \in qCl\{x_e\} \Leftrightarrow x_e \in qKer\{y_e\}$. But by (3) $x_e \in qKer\{y_e\} \Leftrightarrow y_e \in qKer\{x_e\}$. Since, $y_e \in qKer\{x_e\} \Leftrightarrow x_e \in qCl\{y_e\}$, (4) holds.

(4)⇒(5) Let (F,E) be a soft quasi closed set and $x_e \notin (F,E)$. Then $y_e \subseteq (F,E) \Rightarrow qCl\{y_e\} \subseteq (F,E) \Rightarrow x_e \notin qCl\{y_e\} \Rightarrow y_e \notin qCl\{x_e\}$ by (4). Therefore there exists a soft quasi open set $(G,E)_{y_e}$ such that $y_e \in (G,E)_{y_e}$ and $x_e \notin (G,E)_{y_e}$. Let $(G,E) = \cup_{y_e \in (F,E)}(G,E)_{y_e}$. Then (G,E) is soft quasi open such that $x_e \notin (G,E)$ and $(F,E) \subseteq (G,E)$.

(5)⇒(6) Let (F,E) be a soft quasi closed set and suppose that $(H,E) = \cap\{(G,E) : (G,E)$ is soft quasi open, $(F,E) \subseteq (G,E)\}$. Clearly, $(F,E) \subseteq (H,E)$. Let $x_e \notin (F,E)$ then by (1), there exists a soft quasi open set (G,E), such that $x_e \notin (G,E)$ and $(F,E) \subseteq (G,E)$. Therefore, $x_e \notin (H,E)$. Hence, (F,E)=(H,E).

(6)⇒(7) Obvious.

(7)⇒(8) Let (F,E) be a soft quasi closed set and $x_e \notin (F,E)$. Then $(F,E)^c = (G,E)$ (say) is soft quasi open and contains x_e. By (7), there exists a soft quasi closed set (H,E) such that $x_e \in (H,E) \subseteq (G,E)$. Therefore, $qCl\{x_e\} \subseteq (G,E)$. Hence, $qCl\{x_e\} \subseteq (G,E) \cap (F,E) = \phi$.

(8)⇒(1) Obvious. □

Theorem 4.3. *Let* (X, τ_1, τ_2, E) *be a soft quasi* R_0*-space and* $x_e, y_e \in X$. *Then either* $qCl\{x_e\} = qCl\{y_e\}$ *or* $qCl\{x_e\} \cap qCl\{y_e\} = \phi$.

Proof. Suppose that $qCl\{x_e\} \cap qCl\{y_e\} \neq \phi$, and let $z_e \in qCl\{x_e\} \cap qCl\{y_e\}$. Now, $z_e \in qCl\{x_e\} \Rightarrow qCl\{z_e\} \subseteq qCl\{x_e\}$. By Theorem 4.2(4), $z_e \in qCl\{x_e\} \Rightarrow x_e \in qCl\{z_e\} \Rightarrow qCl\{x_e\} \subseteq$

$qCl\{z_e\}$. And so, $qCl\{x_e\} = qCl\{z_e\}$. Similarily, $qCl\{y_e\} = qCl\{z_e\}$. Consequently, $qCl\{x_e\} = qCl\{y_e\}$. □

Theorem 4.4. *Let (X, τ_1, τ_2, E) be a soft quasi R_0-space and $x_e, y_e \in X$. Then either $qKer\{x_e\} = qKer\{y_e\}$ or $qKer\{x_e\} \cap qKer\{y_e\} = \phi$.*

Proof. Evident. □

Theorem 4.5. *Let (X, τ_1, τ_2, E) be a soft bitopological space and $(Y, (\tau_1)_Y, (\tau_2)_Y, E)$ be a subspace of (X, τ_1, τ_2, E). If X is soft R_0 then so is Y.*

Proof. Let (X, τ_1, τ_2, E) be a soft quasi R_0-space and let $(Y, (\tau_1)_Y, (\tau_2)_Y, E)$ be a subspace of X. Let $(F,E)_Y$ be a soft quasi closed set in Y and x_e be a soft point of Y such that $x_e \notin (F,E)_Y$. There exists a $(\tau_1)_Y$- soft closed set (A,E) and a $(\tau_2)_Y$-soft closed set (B,E) such that $(F,E)_Y = (A,E) \cap (B,E)$. Now there is a $(\tau_1)_Y$-soft closed set (F,E) such that $(A,E) = (F,E) \cap \tilde{Y}$ and a $(\tau_2)_Y$ soft closed set (H,E) such that $(A,E) = (F,E) \cap \tilde{Y}$. Therefore $(F,E)_Y = ((F,E) \cap (H,E)) \cap \tilde{Y}$. Now, the soft point $x_e \in Y$ and $x_e \notin (F,E)_Y$ implies that $x_e \notin (F,E) \cap (H,E)$. It is clear that $(F,E) \cap (H,E)$ is soft quasi closed in X. By hypothesis the space X is soft quasi R_0, therefore by Theorem 4.2 (5), there exists a soft quasi open set (G,E) in X such that $(F,E) \cap (H,E) \subseteq (G,E)$ and $x_e \notin (G,E)$. Thus by Theorem 3.4 , $\tilde{Y} \cap (G,E) = (G,E)_Y$ is a soft quasi open set in Y such that $x_e \notin (G,E)_Y$ and $(F,E)_Y \subseteq (G,E)_Y$. Consequently by Theorem 4.2(5) the subspace $(Y, (\tau_1)_Y, (\tau_2)_Y, E)$ is soft quasi R_0. □

5. Conclusion

In this paper, soft quasi R_0 spaces have been introduced, it is shown with the help of example that union of soft quasi open sets in a soft bitopological space is soft quasi open but intersection of two soft quasi open sets in a soft bitopological space may not be soft quasi open. Several characterizations and properties of soft quasi R_0 spaces have been studied.

Conflict of Interests

The author(s) declare that there is no conflict of interests.

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