Available online at http://scik.org
J. Math. Comput. Sci. 11 (2021), No. 6, 7360-7371
https://doi.org/10.28919/jmcs/6588
ISSN: 1927-5307

ON BALANCED EDGE PRODUCT CORDIAL GRAPHS

PHAISATCHA INPOONJAI*

Faculty of Science and Agricultural Technology,
Rajamangala University of Technology Lanna Chiang Rai,
99, Sai Khao, Phan, Chiang Rai, 57120, Thailand

Abstract. An edge product cordial labelling is a variant of the well-known cordial labelling. In this paper, a balanced edge product cordial labelling is suggested and some sufficient conditions for balanced edge product cordial graphs are proved. Also, a construction of balanced edge product cordial graphs is presented.

Keywords: edge product cordial labellings; balanced edge product cordial labellings.

2010 AMS Subject Classification: 05C78.

1. Introduction

One considers finite undirected graphs without loops, multiple edges and isolated vertices. If G is a graph, then $V(G)$ and $E(G)$ stand for the vertex set and the edge set of G, respectively. Cardinalities of these sets are called the *order* and the *size* of G. The set of vertices of G adjacent to a vertex $v \in V(G)$ is denoted by $N_G(v)$.

For a graph G, an edge mapping $f : E(G) \to \{0,1\}$ induces a vertex mapping $f^* : V(G) \to \{0,1\}$ defined by

*Corresponding author
E-mail address: phaisatcha_in@outlook.com
Received July 31, 2021

$$f^*(v) = \prod_{u \in N_G(v)} f(vu).$$

One denotes by $e_f(i)$ the number of edges of G having label i under f and $v_f(i)$ the number of vertices of G having label i under f^* for each $i \in \{0,1\}$. A mapping $f : E(G) \to \{0,1\}$ is called an *edge product cordial labelling* of G if

$$|e_f(0) - e_f(1)| \leq 1 \quad \text{and} \quad |v_f(0) - v_f(1)| \leq 1.$$

A graph G is called *edge product cordial* if it admits an edge product cordial labelling.

The *crown* $C_n \bigodot K_1$ is the graph obtained by joining a pendant edge to each vertex of a cycle C_n. The *armed crown* AC_n is the graph obtained by attaching a path P_2 to each vertex of a cycle C_n. The *wheel* W_n is the graph obtained by connecting a vertex to each vertex of a cycle C_{n-1}. All vertices of C_{n-1} called *rim vertices* join to one vertex called an *apex vertex*. The *helm* H_n is the graph obtained by attaching a pendant edge to each rim vertex of a wheel W_n. Herein, let one remind the results on edge product cordial graphs in [3] that will be referred in the next as the following Theorems.

Theorem 1.1. [3] *The crown $C_n \bigodot K_1$ is an edge product cordial graph.*

Theorem 1.2. [3] *The armed crown AC_n is an edge product cordial graph.*

Theorem 1.3. [3] *The helm H_n is an edge product cordial graph.*

Edge product cordial graphs were introduced by Vaidya and Barasara and they investigated several results on this concept in [3]. After, k-edge product cordial graphs were put forward by Azaizeh et al. in [1]. Currently, the graphs admitting an edge product cordial labelling are characterized and the edge product cordiality of broad classes of graphs was studied by Ivančo in [2].

In this paper, a balanced edge product cordial labelling is recommended and some sufficient conditions for graphs admitting a balanced edge product cordial labelling are investigated. Moreover, a construction of balanced edge product cordial graphs, which is created from a connected graph of order at least three, is shown.

2. Balanced Edge Product Cordial Graphs

Now, one will add more definition of an edge product cordial labelling. An edge product cordial labelling $f : E(G) \to \{0,1\}$ of a graph G is called *balanced* if

$$e_f(0) = e_f(1) \quad \text{and} \quad v_f(0) = v_f(1).$$

A graph G is called *balanced edge product cordial* if it admits a balanced edge product cordial labelling.

Then, one is able to prove the following characterization.

Theorem 2.1. *The graph G is balanced edge product cordial if and only if it is edge product cordial having both even order and even size.*

Proof. Let f be a balanced edge product cordial labelling of G. Then, $e_f(0) = e_f(1)$ and $v_f(0) = v_f(1)$. Obviously, it is an edge product cordial labelling. Since $|E(G)| = e_f(0)+e_f(1) = 2e_f(0)$ and $|V(G)| = v_f(0)+v_f(1) = 2v_f(0)$, G has both even size and even order.
On the other hand, let G be a graph of even order and even size and let f be an edge product cordial labelling of G. Suppose that $|e_f(0)-e_f(1)| = 1$, then $e_f(0) = e_f(1)+1$ or $e_f(0) = e_f(1)-1$. As $|E(G)| = e_f(0)+e_f(1) = e_f(1)+1+e_f(1) = 2e_f(1)+1$ or $|E(G)| = e_f(0)+e_f(1) = e_f(1)-1+e_f(1) = 2e_f(1)-1$, the size is odd, a contradiction. Similarly, suppose that $|v_f(0)-v_f(1)| = 1$, then $v_f(0) = v_f(1)+1$ or $v_f(0) = v_f(1)-1$. Since $|V(G)| = v_f(0)+v_f(1) = v_f(1)+1+v_f(1) = 2v_f(1)+1$ or $|V(G)| = v_f(0)+v_f(1) = v_f(1)-1+v_f(1) = 2v_f(1)-1$, the order is odd, a contradiction. This reveals that $e_f(0) = e_f(1)$ and $v_f(0) = v_f(1)$. Therefore, f is a balanced edge product cordial labelling of G. □

Next, using the known results on edge product cordial graphs in [3] and applying Theorem 2.1, one immediately has the following assertions.

Corollary 2.2. *The crown $C_n \odot K_1$ is a balanced edge product cordial graph.*

Proof. Since the crown $C_n \odot K_1$ has $2n$ vertices and $2n$ edges, by Theorem 1.1 and Theorem 2.1, it is a desired graph. □

Corollary 2.3. *The armed crown AC_n is a balanced edge product cordial graph for even n.*

Proof. As the order and the size of the armed crown AC_n are equal to $3n$ and $3n$ is an even number for even n, by Theorem 1.2 and Theorem 2.1, AC_n is a required graph. □

After, one can find some sufficient conditions for some graphs constructed from an edge product cordial graph of both odd order and odd size to be balanced edge product cordial.

Theorem 2.4. *Let f be an edge product cordial labelling of a graph G having both odd order and odd size and let u be a vertex of G such that $f^*(u) = 0$. If $e_f(0) < e_f(1)$ and $v_f(0) < v_f(1)$, then the graph H obtained by joining a pendant edge to a vertex u of G is balanced edge product cordial.*

Proof. Let e_1 be a pendant edge joining a vertex u of G and let w be a pendant vertex incident with e_1. Consider a mapping $g : E(H) \to \{0,1\}$ defined by

$$g(e) = \begin{cases} f(e) & : \quad e \in E(G), \\ 0 & : \quad e = e_1. \end{cases}$$

Clearly, $g(e) = f(e)$ for all $e \in E(G)$, $g(e_1) = 0$, $g^*(v) = f^*(v)$ for all $v \in V(G)$ and $g^*(w) = 0$. Thus, $e_g(0) = e_f(0) + 1 = e_f(1) = e_g(1)$ and $v_g(0) = v_f(0) + 1 = v_f(1) = v_g(1)$. This means that g is a balanced edge product cordial labelling of H. Therefore, H is an expected graph. □

Corollary 2.5. *The graph G obtained by joining a pendant edge to a vertex of a cycle C_n of the armed crown AC_n is balanced edge product cordial for odd n.*

Proof. For odd n, it is clear that the armed crown AC_n has $3n$ vertices and $3n$ edges such that $3n$ is also an odd number. Let v_i be a vertex of C_n of AC_n, let u_i be a vertex of AC_n adjacent to v_i and let w_i be a pendant vertex of AC_n adjacent to u_i for all $i = 1, 2, ..., n$. Consider a mapping $f : E(AC_n) \to \{0,1\}$ defined by

$$f(e) = \begin{cases} 0 & : \quad e \in E(C_n), \\ 0 & : \quad e = v_i u_i, i = 1, 2, ..., \lfloor \frac{n}{2} \rfloor, \\ 1 & : \quad e = v_i u_i, i = \lfloor \frac{n}{2} \rfloor + 1, ..., n, \\ 1 & : \quad e = u_i w_i, i = 1, 2, ..., n. \end{cases}$$

Evidently, $e_f(0) = n + \lfloor \frac{n}{2} \rfloor < n + \lfloor \frac{n}{2} \rfloor + 1 = e_f(1)$. Moreover, $f^*(v_i) = 0$ for all $i = 1,2,...,n$, $f^*(u_i) = 0$ for all $i = 1,2,...,\lfloor \frac{n}{2} \rfloor$, $f^*(u_i) = 1$ for all $i = \lfloor \frac{n}{2} \rfloor + 1,...,n$ and $f^*(w_i) = 1$ for all $i = 1,2,...,n$. Thus, $v_f(0) = n + \lfloor \frac{n}{2} \rfloor < n + \lfloor \frac{n}{2} \rfloor + 1 = v_f(1)$. Since $|e_f(0) - e_f(1)| = 1$ and $|v_f(0) - v_f(1)| = 1$, f is an edge product cordial labelling of AC_n. By applying Theorem 2.4, G is a balanced edge product cordial graph. □

Theorem 2.6. *Let f be an edge product cordial labelling of a graph G having both odd order and odd size. If $e_f(0) > e_f(1)$ and $v_f(0) > v_f(1)$, then the graph H obtained by joining a pendant edge to a vertex of G is balanced edge product cordial.*

Proof. Let e_1 be a pendant edge joining a vertex of G and let u be a pendant vertex incident with e_1. Consider a mapping $g : E(H) \to \{0,1\}$ defined by

$$g(e) = \begin{cases} f(e) & : \quad e \in E(G), \\ 1 & : \quad e = e_1. \end{cases}$$

Obviously, $g(e) = f(e)$ for all $e \in E(G)$, $g(e_1) = 1$, $g^*(v) = f^*(v)$ for all $v \in V(G)$ and $g^*(u) = 1$. Hence, $e_g(0) = e_f(0) = e_f(1) + 1 = e_g(1)$ and $v_g(0) = v_f(0) = v_f(1) + 1 = v_g(1)$. That is, g is a balanced edge product cordial labelling of H. Therefore, H ia a desired graph. □

Corollary 2.7. *The graph G obtained by joining a pendant edge to a vertex of the helm H_n is balanced edge product cordial for even n.*

Proof. For even n, it is obvious that the helm H_n has odd order $2n-1$ and odd size $3n-3$. Let x be an apex vertex of W_n of H_n, let v_i be a rim vertex of W_n of H_n and let u_i be a pendant vertex of H_n adjacent to v_i for all $i = 1,2,...,n-1$. Consider a mapping $f : E(H_n) \to \{0,1\}$ defined by

$$f(e) = \begin{cases} 0 & : \quad e \in E(C_{n-1}), \\ 0 & : \quad e = xv_i, i = 1,2,...,\frac{n}{2}, \\ 1 & : \quad e = xv_i, i = \frac{n}{2}+1,...,n-1, \\ 1 & : \quad e = v_iu_i, i = 1,2,...,n-1. \end{cases}$$

Evidently, $e_f(0) = \frac{3n}{2} - 1 > \frac{3n}{2} - 2 = e_f(1)$. Moreover, $f^*(x) = 0$, $f^*(v_i) = 0$ for all $i = 1,2,...,n-1$ and $f^*(u_i) = 1$ for all $i = 1,2,...,n-1$. Hence, $v_f(0) = n > n-1 = v_f(1)$. As $|e_f(0) - e_f(1)| = 1$ and $|v_f(0) - v_f(1)| = 1$, f is an edge product cordial labelling of H_n. By applying Theorem 2.6, G is a balanced edge product cordial graph. □

Next, the following assertion for balanced edge product cordial graphs is obvious.

Observation 2.8. *A mapping $f : E(G) \to \{0,1\}$ is a balanced edge product cordial labelling of a graph G if and only if $e_f(0) = \frac{|E(G)|}{2}$ and $v_f(0) = \frac{|V(G)|}{2}$.*

Now, one will recall some definitions and a property on graphs that will be used in the next statement. A *matching* in a graph G is a subgraph of G where every vertex has degree 1. In particular, the matching consists of edges that do not share vertices. The largest number of edges in any matching of G is denoted by $\alpha(G)$. An *edge cover* of a graph G is a subgraph of G such that every vertex of G is incident with at least one edge of the subgraph. The smallest number of edges in any edge cover of G is denoted by $\rho(G)$. For any graph G, Ivančo [2] guaranteed that $\alpha(G) + \rho(G) = |V(G)|$, which is proved by Gallai. Hereinafter, balanced edge product cordial graphs can be characterized by using the same proofs as the characterizations of edge product cordial graphs in [2] as follows.

Theorem 2.9. *Let G be a graph of both even order and even size. Then G is a balanced edge product cordial graph if and only if there exists a set $U \subset V(G)$ satisfying*
(i) $|U| = \frac{|V(G)|}{2}$,
(ii) $G[U]$ contains no isolated vertex,
(iii) $\alpha(G[U]) \geq |U| - \frac{|E(G)|}{2}$,
(iv) $|E(G[U])| \geq \frac{|E(G)|}{2}$.

Proof. If G is a balanced edge product cordial graph, then there is a balanced edge product cordial labelling f of G. Set $U := \{v \in V(G) : f^*(v) = 0\}$. Since $|U| = v_f(0)$, by Observation 2.8, $|U| = \frac{|V(G)|}{2}$. Thus, a condition (i) holds. Obviously, a vertex v is an element of U if and only if v is incident with an edge belonging to $A := \{e \in E(G) : f(e) = 0\}$. Therefore, A is an edge over of $G[U]$, that is, (ii) holds. Moreover,

$$\frac{|E(G)|}{2} = e_f(0) = |A| \geq \rho(G[U]) = |U| - \alpha(G[U])$$

which implies (iii). In the same way, one then has

$$|E(G[U])| \geq |A| = e_f(0) = \frac{|E(G)|}{2}$$

and so a condition (iv) holds.

On the other hand, suppose that U is a set of vertices of G which satisfies conditions (i)-(iv). According to (ii), $E(G[U])$ is an edge cover of $G[U]$. Furthermore, by (iii),

$$\rho(G[U]) = |U| - \alpha(G[U]) \leq \frac{|E(G)|}{2}.$$

Since $|E(G[U])| \geq \frac{|E(G)|}{2}$ and $\rho(G[U]) \leq \frac{|E(G)|}{2}$, there exists an edge cover $A \subseteq E(G[U])$ of $G[U]$ such that $|A| = \frac{|E(G)|}{2}$. Consider a mapping $f : E(G) \to \{0,1\}$ defined by

$$f(e) = \begin{cases} 0 & : \quad e \in A, \\ 1 & : \quad e \notin A. \end{cases}$$

Evidently, $e_f(0) = |A|$ and $v_f(0) = |U|$. Hence, by Observation 2.8, f is a balanced edge product cordial labelling of G. □

Here, the following finding for connected graphs is proved.

Corollary 2.10. *Let G be a connected graph of both even order at least 4 and even size. Then G is a balanced edge product cordial graph if and only if there is a set $U \subset V(G)$ such that $|U| = \frac{|V(G)|}{2}$ and $|E(G[U])| \geq \frac{|E(G)|}{2}$.*

Proof. Assume that G is a balanced edge product cordial graph. According to Theorem 2.9, there is a set $U \subset V(G)$ satisfying conditions (i)-(iv). Thus, the statements (i) and (iv) imply $|U| = \frac{|V(G)|}{2}$ and $|E(G[U])| \geq \frac{|E(G)|}{2}$.

On the other hand, suppose that there exists a set $U \subset V(G)$ such that $|U| = \frac{|V(G)|}{2}$ and $|E(G[U])| \geq \frac{|E(G)|}{2}$. Let W be a subset of $V(G)$ such that $|W| = \frac{|V(G)|}{2}$ and $G[W]$ has the largest possible size. If w is an isolated vertex of $G[W]$, then $E(G[W]) = E(G[W - \{w\}])$. Since G

is a connected graph, there is a vertex $u \in V(G) - W$ adjacent to a vertex of $W - \{w\}$. Set $W' := (W - \{w\}) \cup \{u\}$. Clearly, $|W'| = |W|$ and

$$|E(G[W'])| > |E(G[W - \{w\}])| = |E(G[W])|,$$

a contradiction. Therefore, $G[W]$ contains no isolated vertex. As G is a connected graph, $|E(G)| \geq |V(G)| - 1$. Hence,

$$\frac{|E(G)|}{2} \geq \frac{|V(G)|}{2} - 1.$$

Since $G[W]$ contains no isolated vertex, $\alpha(G[W]) \geq 1$. Thus,

$$\alpha(G[W]) \geq 1 \geq \frac{|V(G)|}{2} - \frac{|E(G)|}{2} = |W| - \frac{|E(G)|}{2}.$$

Clearly,

$$|E(G[W])| \geq |E(G[U])| \geq \frac{|E(G)|}{2}.$$

Therefore, the set W satisfies conditions (i)-(iv), by Theorem 2.9, G is a balanced edge product cordial graph. □

Let G be a connected graph with n vertices and n edges, then G has exactly one cycle subgraph. For the next result, a balanced edge product cordial graph, which the order is the same as the size, is then characterized as follows.

Theorem 2.11. *For even integer $n \geq 4$, let G be a connected graph with n vertices and n edges. Then G is a balanced edge product cordial graph if and only if there is a cycle subgraph C_m of G such that $m \leq \frac{n}{2}$.*

Proof. Assume that G is a balanced edge product cordial graph. According to Theorem 2.9, there exists a set $U \subset V(G)$ satisfying conditions (i)-(iv). Hence, the conditions (i) and (iv) imply $|U| = \frac{n}{2}$ and $|E(G[U])| \geq \frac{n}{2}$. By (ii), $E(G[U])$ is an edge cover of $G[U]$. According to (iii), one gets that $\rho(G[U]) \leq \frac{n}{2}$. Thus, there is a set $W \subset V(G)$ such that $|W| = \frac{n}{2}$ and $|E(G[W])| = \frac{n}{2}$. That is, $G[W]$ is a connected graph. Since $G[W]$ has $\frac{n}{2}$ vertices and $\frac{n}{2}$ edges, there is exactly one cycle subgraph C_m of $G[W]$. Moreover, one then has

$$m = |E(C_m)| \leq |E(G[W])| = \frac{n}{2}.$$

Therefore, C_m is a desired cycle subgraph of G.

On the other hand, suppose that there is a cycle subgraph C_m of G such that $m \leq \frac{n}{2}$. Let e_i be an edge of C_m and let v_i be a vertex of C_m for all $i = 1,2,...,m$. One will consider two cases as follows.

(i) For $m = \frac{n}{2}$, let $e_i^{'}$ be an edge of $E(G) - E(C_m)$ and let $v_i^{'}$ be a vertex of $V(G) - V(C_m)$ for all $i = 1,2,...,m$. Consider a mapping $f : E(G) \rightarrow \{0,1\}$ defined by

$$f(e) = \begin{cases} 0 & : \quad e = e_i, i = 1,2,...,m, \\ 1 & : \quad e = e_i^{'}, i = 1,...,m. \end{cases}$$

Clearly, $f^*(v_i) = 0$ and $f^*(v_i^{'}) = 1$ for all $i = 1,2,...,m$. Since $e_f(0) = m = e_f(1)$ and $v_f(0) = m = v_f(1)$, G is a balanced edge product cordial graph.

(ii) For $m < \frac{n}{2}$, since $G - e_i$ is a tree for some $i \in \{1,2,...,m\}$, there exists a subgraph H of G such that $|V(H)| = \frac{n}{2}$ and $|E(H)| = \frac{n}{2}$ containing C_m. Let $e_i^{'}$ be an edge of $E(H) - E(C_m)$ and let $v_i^{'}$ be a vertex of $V(H) - V(C_m)$ for all $i = 1,2,..., \frac{n}{2} - m$. Also, let $e_i^{''}$ be an edge of $E(G) - E(H)$ and $v_i^{''}$ be a vertex of $V(G) - V(H)$ for all $i = 1,2,..., \frac{n}{2}$. Consider a mapping $f : E(G) \rightarrow \{0,1\}$ defined by

$$f(e) = \begin{cases} 0 & : \quad e = e_i, i = 1,2,...,m, \\ 0 & : \quad e = e_i^{'}, i = 1,2,..., \frac{n}{2} - m, \\ 1 & : \quad e = e_i^{''}, i = 1,..., \frac{n}{2}. \end{cases}$$

Evidently, $f^*(v_i) = 0$ for all $i = 1,2,...,m$, $f^*(v_i^{'}) = 0$ for all $i = 1,2,..., \frac{n}{2} - m$ and $f^*(v_i^{''}) = 1$ for all $i = 1,2,..., \frac{n}{2}$. Since $e_f(0) = \frac{n}{2} = e_f(1)$ and $v_f(0) = \frac{n}{2} = v_f(1)$, G admits a balanced edge product cordial labelling. □

Then, by applying Theorem 2.11, some sufficient conditions for a balanced edge product cordial graph, which its order is similar to its size, are found.

Corollary 2.12. *For even integer $n \geq 4$, let G be a connected graph with n vertices and n edges. If G contains a cycle subgraph $C_{n/2}$, then G is a balanced edge product cordial graph.*

Corollary 2.13. *For even integer $n \geq 4$, let G be a connected graph with n vertices and n edges and let k,m be positive integers such that $k+m = \frac{n}{2}$. If G contains a subgraph H obtained by joining k pendant edges to any vertex of a cycle graph C_m, then G is a balanced edge product cordial graph.*

Corollary 2.14. *For even integer $n \geq 4$, let G be a connected graph with n vertices and n edges and let h,k,m be positive integers such that $hk+m = \frac{n}{2}$. If G contains a subgraph H obtained by joining h paths P_k to any vertex of a cycle graph C_m, then G is a balanced edge product cordial graph.*

For the last result, a construction of graphs admitting a balanced edge product cordial labelling is presented.

Theorem 2.15. *For any connected graph G of order $n \geq 3$ and size m, there exists a balanced edge product cordial graph constructed from G.*

Proof. Let v_i be a vertex of G for $i = 1,2,...,n$. Since G is a connected graph, $m \geq n-1$. Thus, one considers 3 cases as follows.

(i) If $m = n-1$, then G is a tree. Hence, there exist at least two pendant vertices v_j, v_k of G for some $j,k \in \{1,2,...,n\}$. Let H be a graph obtained by joining a pendant edge e_i to a vertex v_i of G for all $i = 1,2,...,n$ and adding an edge e_x incident with vertices v_j, v_k. Let u_i be a pendant vertex incident with e_i for all $i = 1,2,...,n$. Consider a mapping $f : E(H) \to \{0,1\}$ defined by

$$f(e) = \begin{cases} 0 & : \quad e \in E(G), \\ 0 & : \quad e = e_x = v_j v_k, \\ 1 & : \quad e = e_i, i = 1,...,n. \end{cases}$$

Clearly, $f^*(v_i) = 0$ and $f^*(u_i) = 1$ for all $i = 1,2,...,n$. Since $e_f(0) = m+1 = e_f(1)$ and $v_f(0) = n = v_f(1)$, H is a balanced edge product cordial graph.

(ii) If $m = n$, then let H be a graph obtained by joining a pendant edge e_i to a vertex v_i of G

for all $i = 1, 2, ..., n$. Let u_i be a pendant vertex incident with e_i for $i = 1, 2, ..., n$. Consider a mapping $f : E(H) \to \{0, 1\}$ defined by

$$f(e) = \begin{cases} 0 & : \quad e \in E(G), \\ 1 & : \quad e = e_i, i = 1, ..., n. \end{cases}$$

Evidently, $f^*(v_i) = 0$ and $f^*(u_i) = 1$ for all $i = 1, 2, ..., n$. As $e_f(0) = m = e_f(1)$ and $v_f(0) = n = v_f(1)$, H admits a balanced edge product cordial labelling.

(iii) If $m > n$, then let G_1 be a graph obtained by joining a pendant edge e_i to a vertex v_i of G for all $i = 1, 2, ..., n$. Let u_i be a pendant vertex incident with e_i of G_1 for all $i = 1, 2, ..., n$. Then, let H be a graph obatained by adding an edge $e_i^{'}$ incident with any two pendant vertices of G_1 for all $i = 1, 2, ..., m-n$. Consider a mapping $f : E(H) \to \{0, 1\}$ defined by

$$f(e) = \begin{cases} 0 & : \quad e \in E(G), \\ 1 & : \quad e = e_i, i = 1, ..., n, \\ 1 & : \quad e = e_i^{'}, i = 1, 2, ..., m-n. \end{cases}$$

Obviously, $f^*(v_i) = 0$ and $f^*(u_i) = 1$ for all $i = 1, 2, ..., n$. Since $e_f(0) = m = e_f(1)$ and $v_f(0) = n = v_f(1)$, f is a balanced edge product cordial labelling of H. Therefore, H is a desired graph. □

ACKNOWLEDGEMENTS

The author would like to thank the academic referee for the careful reading and helpful comments for improving this paper.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

REFERENCES

[1] A. Azaizeh, R. Hasni, A. Ahmad, G.C. Lau, 3-Total edge product cordial labeling of graphs, Far East J. Math. Sci. 96(2) (2015), 193-209.

[2] J. Ivančo, On edge product cordial graphs, Opuscula Math. 39(5) (2019), 691-703.

[3] S.K. Vaidya, C.M. Barasara, Edge product cordial labeling of graphs, J. Math. Comput. Sci. 2(5) (2012), 1436-1450.