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A NEW METHOD FOR SOLVING BIMATRIX GAME USING LCP UNDER FUZZY ENVIRONMENT

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Abstract: In this paper a new method is proposed for solving the Fully Fuzzy Bimatrix Game (FFBG) using Linear Complementarity approach. In FFBG the player's payoff matrix is represented by trapezoidal fuzzy numbers. The given Fully Fuzzy Bimatrix Game is transformed into the Fully Fuzzy Linear Complementarity Problem (FLCP). Hence the FLCP with trapezoidal fuzzy number is converted into the FLCP with interval number using α - cut. Then we solve the Fuzzy Linear Complementarity Problem with interval numbers. The obtained solution of this FLCP is the solution of the given fuzzy Bimatrix Game. The effectiveness of the proposed method is illustrated by means of a numerical example.

Keywords: Fully Fuzzy Bimatrix Game, Fuzzy Linear Complementarity Problem, Trapezoidal Fuzzy Numbers, Interval Numbers.

2000 AMS Subject Classification: 03E72, 90C05, 90C33

1. Introduction

Many practical problems require decision – making in a competitive situation where there are two or more opposing parties with conflicting interests and where the action of one depends upon the one taken by the opponent. For example, candidates for an election, advertising and marketing campaigns by competing business firms, countries involved in military battles, etc. have their conflicting interests. In a competitive situation the alternatives for each competitor may be either finite or infinite. A competitive situation will be called a Game, if it has the following properties:

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i. There are a finite number of competitors (participants) called players.
ii. Each player has a finite number of strategies (alternatives) available to him.
iii. A play of the game takes place when each player employs his strategy.
iv. Every game result in an outcome, for example, loss or gain or a draw, usually called payoff, to some player.

The conventional game theory is based on known payoffs. In the real situations, usually the payoffs are not known and have to be approximated. In this paper, a method for solving the Bimatrix games with fuzzy payoffs has been proposed.

A theory of rational behaviour of foundations of economics and of the main mechanisms of social organization requires a thorough study of games [5]. One of the most important issues faced by the manager is based on the prediction of the other plans. The real competitive situations may be seen in business, military battles, sport and different cases of contest.

There are different types of games based on the number of players, the number of strategies and the structure of the payoff matrix. When there are two competitors playing a game, it is called a two-person game. In case the number of competitors exceeds two, say n, and then the game is termed as a n-person game. Games having the zero-sum character that the algebraic sum of gains and losses of all the players is zero called zero-sum games. The play does not add a single paisa to the total wealth of all the players it merely results in a new distribution of initial money among them. Zero-sum games with two players are called two-person zero-sum games. In this case the loss or gain of one player is exactly equal to the gain or loss of the other. If the sum of gains or losses is not equal to zero, then the game is of non-zero sum game or Bimatrix Game.

There are lots of methods developed by different authors to encounter with matrix games [2,4,8]. One of the methods is based on LCP [6]. Almost all of the methods are based on the precisely known payoff matrix [9]. However there are lots of situations that the payoffs matrix is not absolutely known and has to be approximated.

In this paper, we proposed the method for solving the Bimatrix game with trapezoidal fuzzy numbers based on Linear Complementarity Problem (LCP). The trapezoidal fuzzy number is transformed into the interval number using the α-cut. Then the interval number Bimatrix game is transformed into the Linear Complementarity Problem with interval payoffs. The solution of this Linear Complementarity Problem is the solution of the given Bimatrix Game. The different level of α-cut is calculated to verify the obtained solution is optimal solution or not.

The sections of this paper are given as follows, Section 2, considers some preliminaries of LCP, trapezoidal Fuzzy numbers, interval numbers and its arithmetic operations. In section 3, Fuzzy Linear Complementarity Problem has been considered. Section 4 deals the method for solving the Linear Complementarity Problems with Fuzzy data. In Section 5, the procedure for converting the Bimatrix Games into LCP has been explained. Section 6, presents a numerical example.

2. Preliminaries

Definition 2.1: Fuzzy Set

A fuzzy set $\tilde{A}$ is defined by $\tilde{A}=\{(x, \mu_A(x)): x\in A, \mu_A(x)\in[0,1]\}$. In the pair $(x, \mu_A(x))$, the first element x belong to the classical set A, the second element $\mu_A(x)$ belong to the closed interval [0, 1] called Membership function.

Definition 2.2: Fuzzy Number

The notion of fuzzy numbers was introduced by Dubois D. and Prade H. A fuzzy subset $\tilde{A}$ of the real line R with membership function $\mu_{\tilde{A}}: R \to [0,1]$ is called a fuzzy number if

i) A fuzzy set $\tilde{A}$ is normal.

ii) $\tilde{A}$is fuzzy convex,

(i.e.) $\mu_{\tilde{A}}[\lambda x_1 + (1-\lambda)x_2] \geq \mu_{\tilde{A}}(x_1) \wedge \mu_{\tilde{A}}(x_2), x_1, x_2 \in R, \forall\lambda \in [0,1]$.

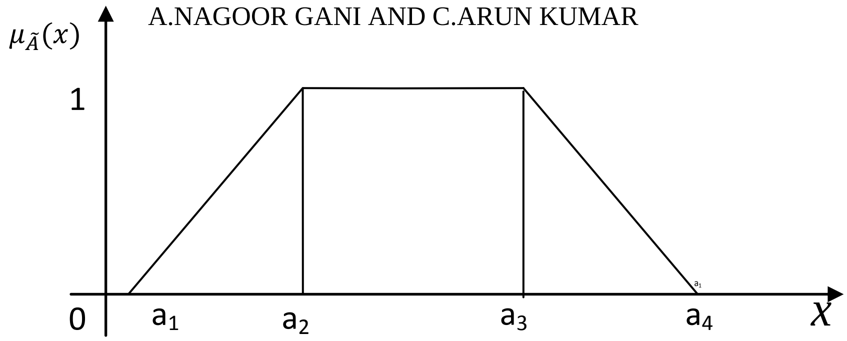
iii) $\mu_{\tilde{A}}$ is upper continuous, and

iv) Supp $\tilde{A}$ is bounded, where supp $\tilde{A} = \{x \in R: \mu_{\tilde{A}}(x) > 0\}$.

Definition 2.3: Trapezoidal Fuzzy Number

A Trapezoidal fuzzy number $\tilde{A}$ is denoted as, $\tilde{A} = (a_1, a_2, a_3, a_4)$ and is defined by the membership function as,

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{(x-a_1)}{(a_2-a_1)} & for\ a_1 \leq x \leq a_2 \\ 1 & for\ a_2 \leq x \leq a_3 \\ \dfrac{(a_4-x)}{(a_4-a_3)} & for\ a_3 \leq x \leq a_4 \\ 0, & otherwise \end{cases}$$



Definition 2.4: Interval Numbers

If $\tilde{A}$ is a trapezoidal fuzzy number, we will let $\tilde{A}_\alpha = [A_\alpha^-, A_\alpha^+]$ be the closed interval which is a α – cut for $\tilde{A}$ where A_α^- and A_α^+ are its left and right end points respectively. Let I and J be two interval numbers defined by ordered pairs of real numbers with lower and upper bounds.

I=[a,b], where a≤b, J=[c,d], where c≤d, when a=b and c=d, these interval numbers degenerate to a scalar real number.

Definition 2.5: Arithmetic Operations on Interval Numbers

The Arithmatic operations on I and J are given [7] below

Addition:

I+J = [a,b]+[c,d] = [a+b,c+d], where a,b,c & d are any real numbers.

Subtraction:

I-J = [a,b]-[c,d] = [a-d,b-c], where a,b,c & d are any real numbers.

Multiplication:

I.J = [a,b].[c,d] = [min(ac,ad,bc,bd), max(ac,ad,bc,bd)], where ac,ad,bc,bd are all arithmetic products.

Scalar Multiplication:

Let λ ∈ R, then λ[a,b] = [λa,λb], λ ≥ 0

λ[a,b] = [λb,λa], λ < 0.

Division:

$\frac{I}{J} = \frac{[a,b]}{[c,d]} = [a,b].\left[\frac{1}{d},\frac{1}{c}\right]$, provided 0 ∉[c,d], where $\frac{1}{d}$ and $\frac{1}{c}$ are quotients.

Definition 2.6: Linear Complementarity Problem

Let M be a given square matrix of order n and q be a column vector in R^n. Throughout this paper we will use the symbols $w_1, w_2, \ldots, w_n$ and $z_1, z_2, \ldots, z_n$ to denote the variables in the problem. In an LCP there is no objective function to be optimized. The problem is to find the unknown variables, $W = (w_1, w_2, \ldots, w_n)^T$ and $Z = (z_1, z_2, \ldots, z_n)^T$ satisfying

W – MZ = q

$W, Z \geq 0$

$w_i z_i = 0$ for all i (2.6.1)

The only data in the problem is the column vector q and the square matrix M. So we will denote the LCP of finding $W \in R^n, Z \in R^n$ satisfying (2.6.1) by the symbol (q, M). It is said to be an LCP of order n. In an LCP of order n there are 2n variables to be obtained.

3. Fuzzy Linear Complementarity Problem (FLCP)

Given a real nxn square matrix M and a nx1 real vector q, then the linear complementarity problem denoted by LCP(q,M) is to find real nx1 vector w, z such that

$w - Mz = q$ (3.1)

$w_j \geq 0, z_j \geq 0$, for j = 1,2,3,………n (3.2)

$w_j z_j = 0$, for j = 1,2,3,………n (3.3)

Here the pair (w_j, z_j) is said to be a pair of complementary variables.

A solution (w, z) to the above system is called a complementary feasible solution, if (w, z) is a basic feasible solution to (3.1) and (3.2) with one of the pair (w_j, z_j) is basic for j = 1, 2, 3,…n.

If $q \geq 0$, then we immediately see that w = q, z = 0 is a solution to the linear complementarity problem. If however, $q \leq 0$, we consider the related system

$W - MZ - ez_0 = q$ (3.4)

$w_j \geq 0, z_j \geq 0, z_0 \geq 0$, j = 1, 2, 3,………n (3.5)

$w_j z_j = 0$, j = 1, 2, 3,………n (3.6)

where z_0 is an artificial variable and e is an n-vector with all components equal to one. Letting z_0 = maximum $\{-q_i / 1 \leq i \leq n\}$, Z = 0, and $w = q + ez_0$, we obtain a starting solution to the above system. Lemke's algorithm attempts to drive z_0 to level zero, thus obtaining a solution to the linear complementarity problem (LCP). But in this paper the above LCP is solved without introducing the artificial variable z_0.

Assume that all parameters in (3.1) - (3.3) are fuzzy and are described by trapezoidal fuzzy numbers. Then, the following fuzzy linear complementarity problem can be obtained by replacing crisp parameters with fuzzy numbers.

$\widetilde{W} - \widetilde{M}\tilde{Z} = \tilde{q}$

$\widetilde{w}_j \geq 0, \tilde{z}_j \geq 0, j = 1,2, \dots, n$

$\widetilde{w}_j \tilde{z}_j = 0, j = 1,2, \dots, n$

The pair $(\tilde{w}_j \tilde{z}_j)$ is said to be a pair of fuzzy complementarity variables.

4. An Algorithm For Solving FLCP Without Introducing Any Artificial Variables

Lemke [6] suggested an algorithm for solving linear complementarity problems. Based on this idea, an algorithm for solving fuzzy linear complementarity problem is developed here. Consider the FLCP $(\tilde{q}, \tilde{M})$ of order n, suppose the fuzzy matrix $\tilde{M}$ satisfies the conditions: there exists a column vector of $\tilde{M}$ in which all the entries are strictly positive. Then a variant of the complementarity pivot algorithm which uses no artificial variable at all can be applied on the FLCP $(\tilde{q}, \tilde{M})$. The original table for this version of the algorithm is

$\tilde{W}$	$\tilde{Z}$	
$\tilde{I}$	$-\tilde{M}$	$\tilde{q}$

We assume that $\tilde{q} \ngeq 0$. Let s be such that $\tilde{M}_{.s} > 0$. So, the column vector associated with $\tilde{Z}_s$ is strictly negative. Hence the variable $\tilde{Z}_s$ can be made to play the same role as that of the artificial variable $\tilde{Z}_0$.

4.1 Algorithm

Step 1: Determine t to satisfy $\left(\frac{\tilde{q}_t}{\tilde{m}_{ts}}\right) = minimum \left\{\frac{\tilde{q}_i}{\tilde{m}_{is}} / i = 1,2, \dots, n\right\}$, and update the table by pivoting at row t and $\tilde{Z}_s$ column. Thus, the right hand side constants vector becomes nonnegative after this pivot step. Hence, $(\tilde{w}_1, \tilde{w}_2, \dots, \tilde{w}_{t-1}, \tilde{Z}_s, \tilde{w}_{t+1}, \dots, \tilde{w}_n)$ is feasible basic vector for the given LCP, and

if s=t, it is fuzzy complementarity feasible basic vector and the solution corresponding to a FLCP $(\tilde{q}, \tilde{M})$, terminate.

if s≠t, the feasible basic vector $(\tilde{w}_1, \tilde{w}_2, \dots, \tilde{w}_{t-1}, \tilde{Z}_s, \tilde{w}_{t+1}, \dots, \tilde{w}_n)$ for the given LCP satisfies the following properties.

(i) It contains exactly one basic variable from the complementarity pair $(\tilde{W}_i, \tilde{Z}_i)$ for n-2 values of i (i≠s).

(ii) It contains both the variables from a fixed complementarity pair $(\tilde{W}_s, \tilde{Z}_s)$ as fuzzy basic variables.

(iii) There exists exactly one fuzzy complementarity pair in which both the variables are contained in this basic vector $(\widetilde{W}_t, \tilde{Z}_t)$.

For carrying out this version of the fuzzy complementarity pivot algorithms, any feasible fuzzy basic vector for the given FLCP satisfying (i), (ii), (iii) is known as an almost complementarity feasible basic vector.

Step 2: In the given canonical table with respect to the initial almost fuzzy complementarity feasible basic vector, the updated column vector of $\widetilde{W}_t$ can be verified to be strictly negative. Hence if $\widetilde{W}_t$ is selected as the entering variable into the initial basic vector, an almost complementarity extreme half line is generated. Hence the initial almost complementarity basic feasible solution of the given FLCP is at the end of an almost complementarity ray.

Step 3: Choose $\tilde{Z}_t$ as the entering variable into the initial almost complementarity feasible basic vector $(\widetilde{w}_1, \widetilde{w}_2, \dots, \widetilde{w}_{t-1}, \tilde{Z}_s, \widetilde{w}_{t+1}, \dots, \widetilde{w}_n)$. In all subsequent steps, the entering variable is uniquely determined by the complementarity pivot rule. The algorithm can terminate in two possible ways:

(i) At some stage one of the variables from the complementarity pair $(\widetilde{W}_s, \tilde{Z}_s)$ drops out of the basic vector or becomes equal to zero in the basic feasible solution of the given FLCP. The basic feasible solution of the given FLCP at that stage is a solution of the FLCP $(\tilde{q}, \widetilde{M})$.

(ii) At some stage of the algorithm, both the variables in the complementarity pair $(\widetilde{W}_s, \tilde{Z}_s)$ may be strictly positive in the basic feasible solution and the pivot column in that stage may turn out to be non-positive, and in this case the algorithm stops with almost complementarity ray.

4.2 Numerical Example

Consider the Fuzzy Linear Complementarity Problems$(\tilde{q}, \widetilde{M})$, with trapezoidal fuzzy number is,

$$\widetilde{M} = \begin{pmatrix} [0.5,0.75,1.25,1.5] & [0,0,0,0] & [0,0,0,0] \\ [1.5,1.75,2.25,2.5] & [0.5,0.75,1.25,1.5] & [0,0,0,0] \\ [1.5,1.75,2.25,2.5] & [1.5,1.75,2.25,2.5] & [0.5,0.75,1.25,1.5] \end{pmatrix},$$

$$\tilde{q} = \begin{pmatrix} [-1.5,-1.25,-0.75,-0.5] \\ [-1.5,-1.25,-0.75,-0.5] \\ [-1.5,-1.25,-0.75,-0.5] \end{pmatrix}$$

For solving the above FLCP, we first reduce it into the following interval number fuzzy linear complementarity problem by taking different α – cuts.

When α = 1, the given FLCP can be written as,

$$\widetilde{M} = \begin{pmatrix} [0.75,1.25] & [0,0] & [0,0] \\ [1.75,2.25] & [0.75,1.25] & [0,0] \\ [1.75,2.25] & [1.75,2.25] & [0.75,1.25] \end{pmatrix},$$

$$\tilde{q} = \begin{pmatrix} [-1.5,-1.25,-0.75,-0.5] \\ [-1.5,-1.25,-0.75,-0.5] \\ [-1.5,-1.25,-0.75,-0.5] \end{pmatrix}$$

The above problem can be written in the simplex table format.

Basic Variables	$\widetilde{w}_1$ $\widetilde{w}_2\widetilde{w}_3\tilde{z}_1\tilde{z}_2\tilde{z}_3$	$\tilde{q}$
$\widetilde{w}_1$	[0.75,1.25] [0,0] [0,0] **[-1.25,-0.75]** [0,0] [0,0]	[-1.25,-0.75]
$\widetilde{w}_2$	[0,0][0.75,1.25][0,0][-2.25,-1.75][-1.25,-0.75][0,0]	[-1.25,-0.75]
$\widetilde{w}_3$	[0,0][0,0][0.75,1.25] [-2.25,-1.75][-1.25,-0.75] [-1.25,-0.75]	[-1.25,-0.75]
$\tilde{z}_1$	[-1.67,-0.6] [0,0] [0,0] [0.6,1.67] [0,0] [0,0]	[0.6,1.67]
$\widetilde{w}_2$	[-3.75,-1.05] [0.75,1.25] [0,0] [-1.2,2] [-1.25,-0.75] [0,0]	[-0.2,3]
$\widetilde{w}_3$	[-3.75,-1.05] [0,0] [0.75,1.25] [-1.2,2] [-2.25,-1.75] [-1.25,-0.75]	[-0.2,3]

Then the solution of the given Fuzzy Linear Complementarity Problem is given by,

$$\widetilde{w}_1 = [0,0], \widetilde{w}_2 = [-0.2,3], \widetilde{w}_3 = [-0.2,3];\ \tilde{z}_1 = [0.6,1.67], \tilde{z}_2 = [0,0], \tilde{z}_3 = [0,0]$$

5. Application

Procedure for convert the Fuzzy Bimatrix Game into Fuzzy Linear Complementarity Problem (FLCP)

Consider a game where in each play of the game, player I picks one out of a possible set of his m choices and independently player II picks one out of a possible set of his N choices. In a play, if player I has picked his choice I, and player II has picked his choice j, then player I loses amount $\tilde{a}'_{ij}$ dollars and player II loses an amount $\tilde{b}'_{ij}$ dollar, where $\tilde{A}' = (\tilde{a}'_{ij})$ and $\tilde{B}' = (\tilde{b}'_{ij})$ are given fuzzy loss matrices of player I and player II respectively. If $\tilde{a}'_{ij} + \tilde{b}'_{ij} = \tilde{0}$ for all i,j the game is known as a fuzzy zero-sum game, in this case it is possible to develop the concept of an optimum strategy for playing the game using Von Neumann's Minimax theorem. Games that are not fuzzy zero-sum games are called fuzzy non zero-sum games or fuzzy Bimatrix games. In fuzzy bimatrix games it is difficult to define an optimum fuzzy strategy. However, in this case, an equilibrium pair of fuzzy strategies can be defined and problem of computing an equilibrium pair of fuzzy strategies can be transformed into an FLCP.

Suppose player I picks his choice i with probability of x_i. The column vector $\tilde{x} = (\tilde{x}_i)$ completely defines player I's fuzzy strategy. Similarly, let the probability vector $\tilde{y} = (\tilde{y}_j)$ be player II's fuzzy strategy. If player I adopts fuzzy strategy $\tilde{x}$ and player II adopts strategy $\tilde{y}$, the expected loss of player I is obviously $\tilde{x}^T\tilde{A}'\tilde{y}$ and that of player II is $\tilde{x}^T\tilde{B}'\tilde{y}$.

The fuzzy strategy pair $(\tilde{\bar{x}}, \tilde{\bar{y}})$ is said to be an equilibrium pair if no player benefits by unilaterally changing his own fuzzy strategy while the other player keeps his fuzzy strategy in the pair $(\tilde{\bar{x}}, \tilde{\bar{y}})$ unchanged, that is, if

$\tilde{\bar{x}}^T\tilde{A}'\tilde{\bar{y}} \le \tilde{x}^T\tilde{A}'\tilde{\bar{y}}$ for all probability vector $\tilde{x}$

$\tilde{\bar{x}}^T\tilde{B}'\tilde{\bar{y}} \le \tilde{\bar{x}}^T\tilde{B}'\tilde{y}$ for all probability vector $\tilde{y}$

Let α, β be arbitrary positive numbers such that $\tilde{a}_{ij} = \tilde{a}'_{ij} + \alpha > 0$ and $\tilde{b}_{ij} = \tilde{b}'_{ij} + \beta > 0$ for all i, j. Let $\tilde{A} = (\tilde{a}_{ij}), \tilde{B} = (\tilde{b}_{ij})$ since $\tilde{x}^T\tilde{A}'\tilde{y} = \tilde{x}^T\tilde{A}\tilde{y} - \alpha$ and $\tilde{x}^T\tilde{B}'\tilde{y} = \tilde{x}^T\tilde{B}\tilde{y} - \beta$ for all probability vector $\tilde{x}$ and $\tilde{y}$. If $(\tilde{\bar{x}}, \tilde{\bar{y}})$ is an equilibrium pair of fuzzy strategies for the game with fuzzy loss matrices$\tilde{A}', \tilde{B}'$ then $(\tilde{\bar{x}}, \tilde{\bar{y}})$ is an equilibrium pair of fuzzy strategies for the game with fuzzy loss matrices $\tilde{A}, \tilde{B}$ and vice versa. So without loss of generality, consider the game in which the fuzzy loss matrices are $\tilde{A}, \tilde{B}$. Since $\tilde{x}$ is a probability vector, the condition

$\tilde{\bar{x}}^T\tilde{A}\tilde{\bar{y}} \le \tilde{x}^T\tilde{A}\tilde{\bar{y}}$ for all probability vectors $\tilde{x}$ is equivalent to the system of constraints

$\tilde{\bar{x}}^T\tilde{A}\tilde{\bar{y}} \le \tilde{A}_i\tilde{\bar{y}}$ for all i=1,2,…,m.

Let $\tilde{e}_r$ denote the column vector in which all elements are equal to$\tilde{1}$. In matrix notation the above system of constraints can be written as $(\tilde{\bar{x}}^T\tilde{A}\tilde{\bar{y}})\tilde{e}_m \le \tilde{A}\tilde{\bar{y}}$. In similar way the condition

$\tilde{\bar{x}}^T\tilde{B}\tilde{\bar{y}} \le \tilde{\bar{x}}^T\tilde{B}\tilde{y}$ by for all probability vectors $\tilde{y}$ is equivalent to $(\tilde{\bar{x}}^T\tilde{B}\tilde{\bar{y}})\tilde{e}_N \le \tilde{B}^T\tilde{\bar{x}}$. Hence the fuzzy strategy pair $(\tilde{\bar{x}}, \tilde{\bar{y}})$ is an equilibrium pair of fuzzy strategies for the game with loss matrices $\tilde{A}, \tilde{B}$ iff

$$(\tilde{\bar{x}}^T\tilde{A}\tilde{\bar{y}})\tilde{e}_m \le \tilde{A}\tilde{\bar{y}}$$

$$(\tilde{\bar{x}}^T\tilde{B}\tilde{\bar{y}})\tilde{e}_N \le \tilde{B}^T\tilde{\bar{x}} \qquad (5.1)$$

Since $\tilde{A}, \tilde{B}$ are strictly positive matrices, $\tilde{\tilde{x}}^T \tilde{A} \tilde{\tilde{y}}$ and $\tilde{\tilde{x}}^T \tilde{B} \tilde{\tilde{y}}$ are strictly positive numbers. Let $\tilde{\tilde{\xi}} = \frac{\tilde{x}}{\tilde{\tilde{x}}^T \tilde{B} \tilde{\tilde{y}}}$ and $\tilde{\tilde{\eta}} = \frac{\tilde{y}}{\tilde{\tilde{x}}^T \tilde{A} \tilde{\tilde{y}}}$

Introducing slack variables corresponding to the inequality constraints, (5.1) is equivalent to

$$\begin{bmatrix} \tilde{\tilde{u}} \\ \tilde{\tilde{v}} \end{bmatrix} - \begin{pmatrix} 0 & \tilde{A} \\ \tilde{B}^T & 0 \end{pmatrix} \begin{bmatrix} \tilde{\tilde{\xi}} \\ \tilde{\tilde{\eta}} \end{bmatrix} = \begin{bmatrix} -\tilde{e}_m \\ -\tilde{e}_N \end{bmatrix}$$

$$\begin{bmatrix} \tilde{\tilde{u}} \\ \tilde{\tilde{v}} \end{bmatrix} \geq 0, \begin{bmatrix} \tilde{\tilde{\xi}} \\ \tilde{\tilde{\eta}} \end{bmatrix} \geq 0$$

$$\begin{bmatrix} \tilde{\tilde{u}} \\ \tilde{\tilde{v}} \end{bmatrix}^T \begin{bmatrix} \tilde{\tilde{\xi}} \\ \tilde{\tilde{\eta}} \end{bmatrix} \geq 0 \qquad (5.2)$$

Conversely, if $(\tilde{\tilde{u}}, \tilde{\tilde{v}}, \tilde{\tilde{\xi}}, \tilde{\tilde{\eta}})$ is a solution of the FLCP (5.2) then the equilibrium pair of fuzzy strategies for the original game is $(\tilde{\tilde{x}}, \tilde{\tilde{y}})$ where, $\tilde{\tilde{x}} = \frac{\tilde{\tilde{\xi}}}{\sum \tilde{\tilde{\xi}}_i}$ and $\tilde{\tilde{y}} = \frac{\tilde{\tilde{\eta}}}{\sum \tilde{\tilde{\eta}}_i}$.

Therefore,

$$\begin{pmatrix} 0 & \tilde{A} \\ \tilde{B}^T & 0 \end{pmatrix} \begin{bmatrix} \tilde{\tilde{\xi}} \\ \tilde{\tilde{\eta}} \end{bmatrix} \geq \begin{pmatrix} \tilde{e}_m \\ \tilde{e}_N \end{pmatrix} \Rightarrow \begin{cases} \tilde{A}\tilde{\tilde{\eta}} \geq \tilde{e}_m \Rightarrow \tilde{A}(\tilde{\tilde{y}} \sum \tilde{\tilde{\eta}}_j) \geq \tilde{e}_m \\ \tilde{B}^T \tilde{\tilde{\xi}} \geq \tilde{e}_N \Rightarrow \tilde{B}^T(\tilde{\tilde{x}} \sum \tilde{\tilde{\xi}}_i) \geq \tilde{e}_N \end{cases}$$

Then we have, $\begin{cases} \tilde{A}\tilde{\tilde{y}} \geq \frac{1}{\sum \tilde{\tilde{\eta}}_j} \tilde{e}_m \\ \tilde{B}^T \tilde{\tilde{x}} \geq \frac{1}{\sum \tilde{\tilde{\xi}}_i} \tilde{e}_N \end{cases}$

By defining, $\tilde{\tilde{x}}^T \tilde{A} \tilde{\tilde{y}} = \frac{1}{\sum \tilde{\tilde{\eta}}_j}, \tilde{\tilde{x}}^T \tilde{B} \tilde{\tilde{y}} = \frac{1}{\sum \tilde{\tilde{\xi}}_i}$

We have, $\begin{cases} \tilde{A}\tilde{\tilde{y}} \geq (\tilde{\tilde{x}}^T \tilde{A} \tilde{\tilde{y}})\tilde{e}_m \\ \tilde{B}^T \tilde{\tilde{x}} \geq (\tilde{\tilde{x}}^T \tilde{B} \tilde{\tilde{y}})\tilde{e}_N \end{cases}$

Thus an equilibrium pair of fuzzy strategies can be computed by solving the transformed FLCP (5.2).

5.1 Numerical Example

Consider a game, in which the fuzzy loss matrices are given by,

$$\tilde{A} = \begin{pmatrix} (1.5,1.75,2.25,2.5) & (1.5,1.75,2.25,2.5) & (0.5,0.75,1.25,1.5) \\ (0.5,0.75,1.25,1.5) & (1.5,1.75,2.25,2.5) & (1.5,1.75,2.25,2.5) \end{pmatrix}$$

$$\tilde{B} = \begin{pmatrix} (0.5,0.75,1.25,1.5) & (2.5,2.75,3.25,3.5) & (1.5,1.75,2.25,2.5) \\ (1.5,1.75,2.25,2.5) & (0.5,0.75,1.25,1.5) & (2.5,2.75,3.25,3.5) \end{pmatrix}$$

The FLCP corresponding to the given fuzzy bimatrix game is,

$\tilde{u}$	$\tilde{v}$	$\tilde{\xi}$	$\tilde{\eta}$	q
$\tilde{I}_m$	0	0	$-\tilde{A}$	$-\tilde{e}_m$
0	$\tilde{I}_N$	$-\tilde{B}^T$	0	$-\tilde{e}_N$

$$\tilde{u} \geq 0, \tilde{v} \geq 0, \tilde{\xi} \geq 0, \tilde{\eta} \geq 0$$

The above trapezoidal fuzzy number payoff matrix can be converted into the interval number payoff matrix using α-cut.

For α = 1, the given problem can be written as,

$$\tilde{A} = \begin{pmatrix} [1.75,2.25] & [1.75,2.25] & [0.75,1.25] \\ [0.75,1.25] & [1.75,2.25] & [1.75,2.25] \end{pmatrix}$$

$$\tilde{B} = \begin{pmatrix} [0.75,1.25] & [2.75,3.25] & [1.75,2.25] \\ [1.75,2.25] & [0.75,1.25] & [2.75,3.25] \end{pmatrix}$$

Hence the following FLCP is the equations based on above mentioned fuzzy loss matrices using (5.2).

$$\begin{pmatrix} \tilde{u}_1 \\ \tilde{u}_2 \\ \tilde{v}_1 \\ \tilde{v}_2 \\ \tilde{v}_3 \end{pmatrix} - \begin{pmatrix} [0,0] & [0,0] & [1.75,2.25] & [1.75,2.25] & 0.75,1.25 \\ [0,0] & [0,0] & [0.75,1.25] & [1.75,2.25] & [1.75,2.25] \\ [0.75,1.25] & [1.75,2.25] & [0,0] & [0,0] & [0,0] \\ [2.75,3.25] & [0.75,1.25] & [0,0] & [0,0] & [0,0] \\ [1.75,2.25] & [2.75,3.25] & [0,0] & [0,0] & [0,0] \end{pmatrix} \begin{pmatrix} \tilde{\xi}_1 \\ \tilde{\xi}_2 \\ \tilde{\eta}_1 \\ \tilde{\eta}_2 \\ \tilde{\eta}_3 \end{pmatrix}$$

$$= \begin{pmatrix} [-1.25, -0.75] \\ [-1.25, -0.75] \\ [-1.25, -0.75] \\ [-1.25, -0.75] \\ [-1.25, -0.75] \end{pmatrix}$$

Here,

$$\widetilde{M} = \begin{pmatrix} [0,0] & [0,0] & [1.75,2.25] & [1.75,2.25] & 0.75,1.25 \\ [0,0] & [0,0] & [0.75,1.25] & [1.75,2.25] & [1.75,2.25] \\ [0.75,1.25] & [1.75,2.25] & [0,0] & [0,0] & [0,0] \\ [2.75,3.25] & [0.75,1.25] & [0,0] & [0,0] & [0,0] \\ [1.75,2.25] & [2.75,3.25] & [0,0] & [0,0] & [0,0] \end{pmatrix}, \widetilde{W} = \begin{pmatrix} \tilde{u}_1 \\ \tilde{u}_2 \\ \tilde{v}_1 \\ \tilde{v}_2 \\ \tilde{v}_3 \end{pmatrix}, \tilde{Z}$$

$$= \begin{pmatrix} \tilde{\xi}_1 \\ \tilde{\xi}_2 \\ \tilde{\eta}_1 \\ \tilde{\eta}_2 \\ \tilde{\eta}_3 \end{pmatrix}, \tilde{q} = \begin{pmatrix} [-1.25, -0.75] \\ [-1.25, -0.75] \\ [-1.25, -0.75] \\ [-1.25, -0.75] \\ [-1.25, -0.75] \end{pmatrix}$$

Then the above problem can be written in the simplex table format

Basic Variable	$\tilde{u}_1$	$\tilde{u}_2$	$\tilde{v}_1$	$\tilde{v}_2$	$\tilde{v}_3$	$\tilde{\xi}_1$	$\tilde{\xi}_2$	$\tilde{\eta}_1$	$\tilde{\eta}_2$	$\tilde{\eta}_3$	$\tilde{q}$
$\tilde{u}_1$	[0.75,1.25]	[0,0]	[0,0]	[0,0]	[0,0]	[0,0]	[0,0]	[-2.25,-1.75]	[-2.25,-1.75]	[-1.25,-0.75]	[-1.25,-0.75]
$\tilde{u}_2$	[0,0]	[0.75,1.25]	[0,0]	[0,0]	[0,0]	[0,0]	[0,0]	[-1.25,-0.75]	[-2.25,-1.75]	[-2.25,-1.75]	[-1.25,-0.75]
$\tilde{v}_1$	[0,0]	[0,0]	[0.75,1.25]	[0,0]	[0,0]	**[-1.25,-0.75]**	[-2.25,-1.75]	[0,0]	[0,0]	[0,0]	[-1.25,-0.75]
$\tilde{v}_2$	[0,0]	[0,0]	[0,0]	[0.75,1.25]	[0,0]	[-3.25,-2.75]	[-1.25,-0.75]	[0,0]	[0,0]	[0,0]	[-1.25,-0.75]
$\tilde{v}_3$	[0,0]	[0,0]	[0,0]	[0,0]	[0.75,1.25]	[-2.25,-1.75]	[-3.25,-2.75]	[0,0]	[0,0]	[0,0]	[-1.25,-0.75]
$\tilde{u}_1$	[0.75,1.25]	[0,0]	[0,0]	[0,0]	[0,0]	[0,0]	[0,0]	[-2.25,-1.75]	[-2.25,-1.75]	[-1.25,-0.75]	[-1.25,-0.75]
$\tilde{u}_2$	[0,0]	[0.75,1.25]	[0,0]	[0,0]	[0,0]	[0,0]	[0,0]	**[-1.25,-0.75]**	[-2.25,-1.75]	[-2.25,-1.75]	[-1.25,-0.75]
$\tilde{\xi}_1$	[0,0]	[0,0]	[-1.67,-0.6]	[0 0]	[0,0]	[0.6,1.67]	[1.4,3]	[0,0]	[0,0]	[0,0]	[0.6,1.67]
$\tilde{v}_2$	[0,0]	[0,0]	[-5.4,-1.65]	[0.75,1.25]	[0,0]	[-1.6,2.65]	[2.6,9]	[0,0]	[0,0]	[0,0]	[0.4,4.65]
$\tilde{v}_3$	[0,0]	[0,0]	[-3.8,-1.05]	[0,0]	[0.75,1.25]	[-1.2,2.05]	[-0.8,4]	[0,0]	[0,0]	[0,0]	[-0.2,3.05]
$\tilde{u}_1$	[0.75,1.25]	[-3.8,-1.05]	[0,0]	[0,0]	[0,0]	[0,0]	[0,0]	[-1.2,2.05]	[0.2,5]	[1.2,6]	[-0.2,3.05]
$\tilde{\eta}_1$	[0,0]	[-1.67,-0.6]	[0,0]	[0,0]	[0,0]	[0,0]	[0,0]	[0.6,1.67]	[1.4,3]	[1.4,3]	[0.6,1.67]
$\tilde{\xi}_1$	[0,0]	[0,0]	[-1.67,-0.6]	[0 0]	[0,0]	[0.6,1.67]	[1.4,3]	[0,0]	[0,0]	[0,0]	[0.6,1.67]
$\tilde{v}_2$	[0,0]	[0,0]	[-5.4,-1.65]	[0.75,1.25]	[0,0]	[-1.6,2.65]	**[2.6,9]**	[0,0]	[0,0]	[0,0]	[0.4,4.65]
$\tilde{v}_3$	[0,0]	[0,0]	[-3.8,-1.05]	[0,0]	[0.75,1.25]	[-1.2,2.05]	[-0.8,4]	[0,0]	[0,0]	[0,0]	[-0.2,3.05]
$\tilde{u}_1$	[0.75,1.25]	[-3.8,-1.05]	[0,0]	[0,0]	[0,0]	[0,0]	[0,0]	[-1.2,2.05]	**[0.2,5]**	[1.2,6]	[-0.2,3.05]
$\tilde{\eta}_1$	[0,0]	[-1.67,-0.6]	[0,0]	[0,0]	[0,0]	[0,0]	[0,0]	[0.6,1.67]	[1.4,3]	[1.4,3]	[0.6,1.67]
$\tilde{\xi}_1$	[0,0]	[0,0]	[-1.42,5.7]	[-1.44,-0.11]	[0,0]	[-2.46,3.53]	[-9.1,2.6]	[0,0]	[0,0]	[0,0]	[-4.77,1.61]
$\tilde{\xi}_2$	[0,0]	[0,0]	[-2.1,-0.18]	[0.08,0.48]	[0,0]	[-0.62,1.02]	[0.29,3.5]	[0,0]	[0,0]	[0,0]	[0.04,1.79]
$\tilde{v}_3$	[0,0]	[0,0]	[-5.48,7.35]	[-1.92,0.38]	[0.75,1.25]	[-5.28,4.53]	[-14.8,6.8]	[0,0]	[0,0]	[0,0]	[-7.4,4.45]

$\tilde{\eta}_2$	[0.15,6.25]	[-19,-0.21]	[0,0]	[0,0]	[0,0]	[0,0]	[0,0]	[-6,10.25]	[0.04,25]	[0.24,30]	[-1,0.25]
$\tilde{\eta}_1$	[-18.75,-0.21]	[-1.37,56.4]	[0,0]	[0,0]	[0,0]	[0,0]	[0,0]	[-30.2,19.7]	[-73.6,2.9]	[-88.6,2.7]	[-0.15,4.67]
$\tilde{\tilde{\xi}}_1$	[0,0]	[0,0]	[-1.42,5.7]	[-1.44,-0.11]	[0,0]	[-2.46,3.53]	[-9.1,2.6]	[0,0]	[0,0]	[0,0]	[-4.77,1.61]
$\tilde{\tilde{\xi}}_2$	[0,0]	[0,0]	[-2.1,-0.18]	[0.08,0.48]	[0,0]	[-0.62,1.02]	[0.29,3.5]	[0,0]	[0,0]	[0,0]	[0.04,1.79]
$\tilde{v}_3$	[0,0]	[0,0]	[-5.48,7.35]	[-1.92,0.38]	[0.75,1.25]	[-5.28,4.53]	[-14.8,6.8]	[0,0]	[0,0]	[0,0]	[-7.4,4.45]

Hence the solution of the above FLCP is,

$$\tilde{u}_1 = [0,0], \tilde{u}_2 = [0,0], \tilde{v}_1 = [0,0], \tilde{v}_2 = [0,0], \tilde{v}_3 = [-7.4,4.45], \tilde{\tilde{\xi}}_1 = [-4.77, 1.67], \tilde{\tilde{\xi}}_2 = [0.04,1.79], \tilde{\eta}_1 = [-0.15,4.67],$$

$$\tilde{\eta}_2 = [-1,0.25], \tilde{\eta}_3 = [0,0].$$

$$\tilde{\tilde{\xi}}_1 + \tilde{\tilde{\xi}}_2 = [-4.73,3.4]$$

$$\tilde{x}_i = \frac{\tilde{\tilde{\xi}}}{\sum \tilde{\tilde{\xi}}_i}$$

$$\tilde{x}_1 = \frac{\tilde{\tilde{\xi}}_1}{\sum_{i=1}^{2} \tilde{\tilde{\xi}}_i} = \frac{[-4.77,1.61]}{[-4.73,3.4]} = [-1.4,1]$$

$$\tilde{x}_2 = \frac{\tilde{\tilde{\xi}}_2}{\sum_{i=1}^{2} \tilde{\tilde{\xi}}_i} = \frac{[0.04,1.79]}{[-4.73,3.4]} = [-0.4,0.5]$$

$$\tilde{y}_1 = \frac{\tilde{\eta}_1}{\sum_{j=1}^{3} \tilde{\eta}_j} = \frac{[-0.15,4.67]}{[-1.15,4.92]} = [-4.1,0.9]$$

$$\tilde{y}_2 = \frac{\tilde{\eta}_2}{\sum_{j=1}^{3} \tilde{\eta}_j} = \frac{[-1,0.25]}{[-1.15,4.92]} = [-0.2,0.87]$$

$$\tilde{y}_3 = \frac{\tilde{\eta}_3}{\sum_{j=1}^{3} \tilde{\eta}_j} = \frac{[0,0]}{[-1.15,4.92]} = [0,0]$$

Hence the equilibrium pair of fuzzy strategies for the given fuzzy bimatrix game is,

$$(\tilde{x}_i; \tilde{y}_j) = ([-1.4,1], [-0.4,0.5]; [-4.1,0.9], [-0.2,0.87], [0,0]).$$

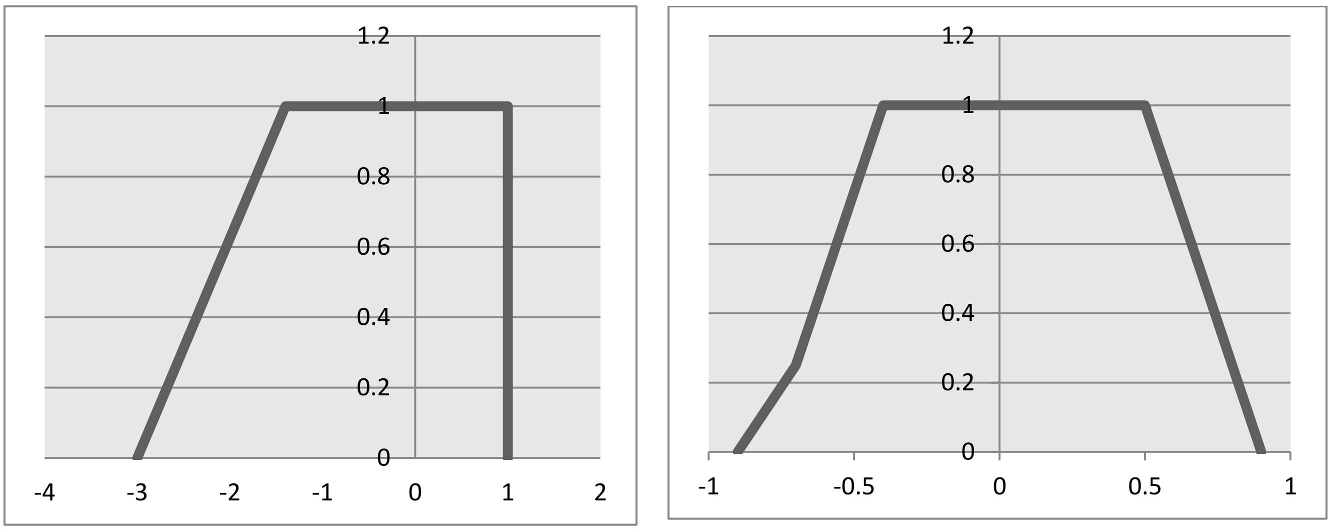
Results obtain by different levels of α – cut

α	$\tilde{x}_i$	$\tilde{y}_j$
0	{[-3,1], [-0.9,0.9]}	{[-5.5,0.98],[-0.39,0.99],[0,0]}
0.25	{[-2.6,1], [-0.7,0.8]}	{[-5.1,0.96],[-0.35,0.96],[0,0]}

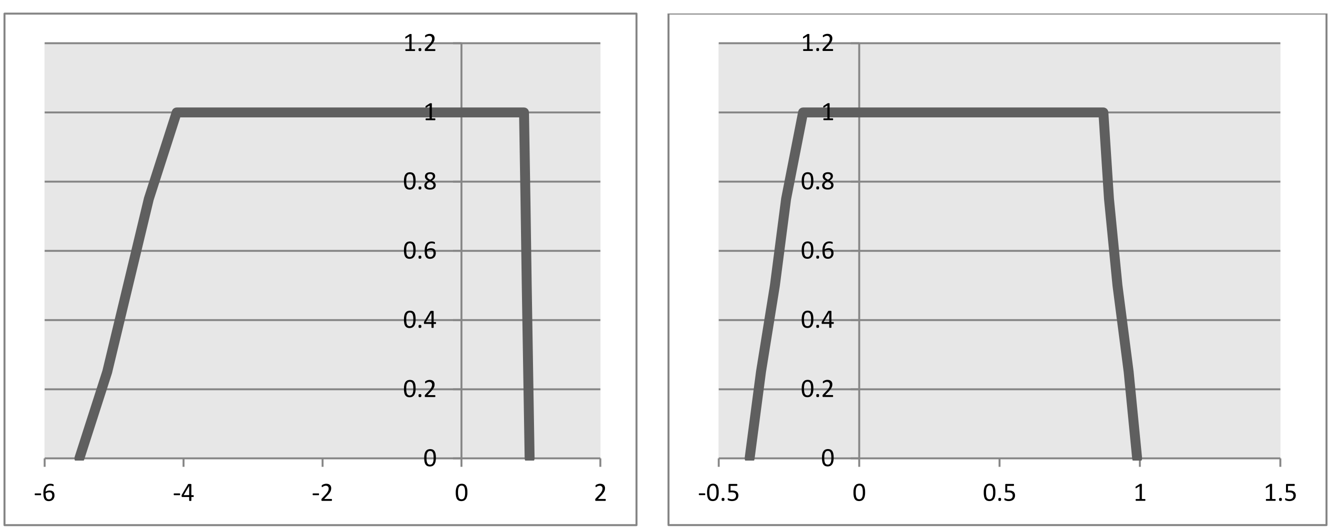
0.50	{[-2.2,1], [-0.6,0.7]}	{[-4.8,0.94],[-0.30,0.92],[0,0]}
0.75	{[-1.8,1], [-0.5,0.6]}	{[-4.5,0.92],[-0.26,0.89],[0,0]}
1	{[-1.4,1], [-0.4,0.5]}	{[-4.1,0.9],[-0.2,0.87],[0,0]}

The Excel Output for the following above Results obtain by different levels of α – cut are given by,

For fuzzy strategy $\tilde{x}_i$



For fuzzy strategy $\tilde{y}_j$



Conclusion

In this paper, a new algorithm for solving the fully fuzzy Bimatrix Game is suggested. A numerical example is given to clarify the developed theory and the proposed algorithm. And the procedure for solving the fully fuzzy Bimatrix Gameusing the linear complementarity approach is suggested. Here the fully fuzzy Bimatrix Game is converted into FLCP and it is solved by complementarity pivoting algorithm without introducing any artificial variables.In FLCP, there is no objective function is to be optimized. Hence, we get anoptimum fuzzy strategy at the minimum number of iterations. In general, probability cannot be a negative value, but here the negative value represent for the interval number spread of the given problem’ssolution output.

REFERENCES

[1] M. S. Bazaraa,H. D. Sherali, and C. M. Shetty.Nonlinear Programming: Theory and Algorithms, John Wiley and Sons, Newyork (1979).

[2] M. S. Bazaraa, H. D. Sherali, J. Jarvis. Linear Programming and Network Flows (2nd Edition), John Wiley & Sons, New York (1990).

[3] R. E. Bellman, L. A. Zadeh. Decision making in a fuzzy environment, Management Science,17(1970),141-164.

[4] R. W. Cottle, G. B. Dantzig. Complementarity pivot theory of mathematical programming, Linear algebra and its Applications 1 (1968), 103-125.

[5] S. T. Liu, C. Kao. Matrix Games with interval data, Computers & Industrial Engineering, 56 (2008), 1697-1700.

[6] K. G. Murthy. Linear Complementarity, Linear and Nonlinear Programming, Internet Edition, http://www.personal.engin.umich.edu/~murty(1997).

[7] A. Nagoorgani, and R. Irene Hepzibah. Multi-objective fuzzy linear programmingProblems with interval numbers, Bulletin of Pure and Applied Sciences, 26E (2) (2007), 51-56.

[8] Nagoor Gani, A., and Arun Kumar, C.,(2012) An Index Method for Solving the Linear Complementarity Problem under Fuzzy Environment, Applied Mathematical Sciences, Vol. 6, no. 42, 2081 – 2089.

[9] Richard W. Cottle, Jong – Shi Pang, Richard E. Stone. The Linear Complementarity Problem, SIAM (2009).

[10] V. Vijay, S. Chandra, C. R. Bector. Matrix games with fuzzy goals and fuzzy payoff’s, Omega, 33 (2005), 425 – 429.