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AXISYMMETRIC HYDROMAGNETIC STABILITY OF A STREAMING RESISTIVE HOLLOW JET UNDER OBLIQUE VARYING MAGNETIC FIELD

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Abstract: The axisymmetric magneto hydrodynamic stability of streaming resistive hollow jet under oblique varying magnetic fields has been discussed. The stability criterion is established in its general form, studied analytically and the results are confirmed numerically. The destabilizing effect of the capillary force is found in small domain in the axisymmetric perturbation. We study three different cases for discussing the electromagnetic force influence on the resistive hollow jet instability. The first case is the finite resistivity; it has no any influence on the capillary instability of a resistive hollow jet. The second case is the infinite resistivity, it is destabilizing according to restrictions. The third case is being zero resistivity, it exerts an influence giving a sort of rigidity to the conducting fluid and that influence causes also the bending and twisting of the lines force in the fluid.

Keywords: Resistive fluid – Magnetohydrodynamic – Hollow Jet.

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Introduction

The stability results of liquid cylinder have been documented since lone time ago by the pioneering works of Rayleigh (1945) and summarized by Chandrasekhar (1981) (Noble prize winner 1986) for its crucial applications in science. The mirror case "hollow jet" which is a gas cylinder immersed into infinite liquid has also received considerable attention in the present area, in particular in the last two decades. Rayleigh (1945) referred to the capillary stability of such model, Chandrasekhar (1981) indicated the stability criterion of a hollow jet. Drazin and Reid (1980) reported the dispersion relation, see also Cheng (1985). Kendal (1986) performed neat experiments with modern equipment to produce a finite hollow jet. Radwan (1991) studied the hydrodynamic and magnetohydrodynamic stabilityof an ideal hollow jet. The instability of double perturbed finite hollow jet "annular fluid jet" has been examined in magnetohydrodynamic version by Radwan (1996). Recently, Radwan (1998) developed the stability of a compressible hollow jet. The stability of different cylindrical models under the action of selfgravitating force in addition to other forces has been elaborated by Radwan and Hasan (2008) and (2009). Hasan (2011) has discussed the stability of oscillating streaming fluid cylinder subject to combined effect of the capillary, selfgravitating and electrodynamic forces for all axisymmetric and non-axisymmetric perturbation modes. Samia et. al. (2011) has discussed the stability of streaming fluid cylinder subject to combined effect of the capillary, selfgravitating and magnetohydrodynamic forces for all axisymmetric and non-axisymmetric perturbation modes. In all previous works, the fluid is assumed to be perfectly conducting and the resistivity of the magnetized fluid is neglected. In the present work, the axisymmetric MHD stability of a resistive hollow jet cylinder with oblique varying magnetic field is discussed.

1. Formulation of the problem

Consider a gas cylinder (of negligible motion) of radius R_o surrounded by a non-viscous, incompressible and resistive liquid moving with uniform velocity

$$\underline{u}_o = (0, 0, U) \tag{1}$$

The interior cylinder is being a gas with constant pressure P_o^g and pervaded by an oblique varying magnetic field

$$\underline{H}_o^g = \left(0, \frac{\beta\, r\, H_0}{R_0}, \alpha\, H_o\right) \tag{2}$$

The liquid is penetrated by the magnetic field

$$\underline{H}_0 = (0,0,H_o) \tag{3}$$

where H_o is the intensity of the magnetic field in the liquid and β, α are some parameters satisfying certain restrictions of $\underline{H}_o^g$. The components of equations (1) – (3) are considered along the cylindrical coordinates (r, φ, z) with the z-axis is coinciding with the axis of the hollow jet. The model is acted by the inertia, pressure gradient, capillary and electromagnetic forces.

The hydromagnetic fundamental equations appropriate for studying the stability of the fluid model under consideration are the combination of the pure hydrodynamic equations and those of Maxwell concerning the electromagnetic theory. These equations may be given as follows.

In the gas cylinder

$$\nabla \Lambda \underline{H}^g = 0 \qquad \text{(there is no current)} \tag{4}$$

$$\nabla . \underline{H}^g = 0 \tag{5}$$

In the liquid region,

$$\rho\left(\frac{\partial}{\partial t} + (\underline{u}.\nabla)\right)\underline{u} = -\nabla P + \mu\,(\nabla \Lambda \underline{H})\Lambda \underline{H} \tag{6}$$

$$\nabla . \underline{u} = 0 \tag{7}$$

$$\nabla . \underline{H} = 0 \tag{8}$$

$$\frac{\partial \underline{H}}{\partial t} = \nabla \Lambda (\underline{u} \Lambda \underline{H}) - \nabla \Lambda(\eta \nabla \Lambda \underline{H}) \tag{9}$$

Along the gas – liquid interface,

$$P_s = -T\,(\nabla . \underline{N}) \tag{10}$$

With

$$(\nabla \cdot \underline{N}) = r_1^{-1} + r_2^{-1} \tag{11}$$

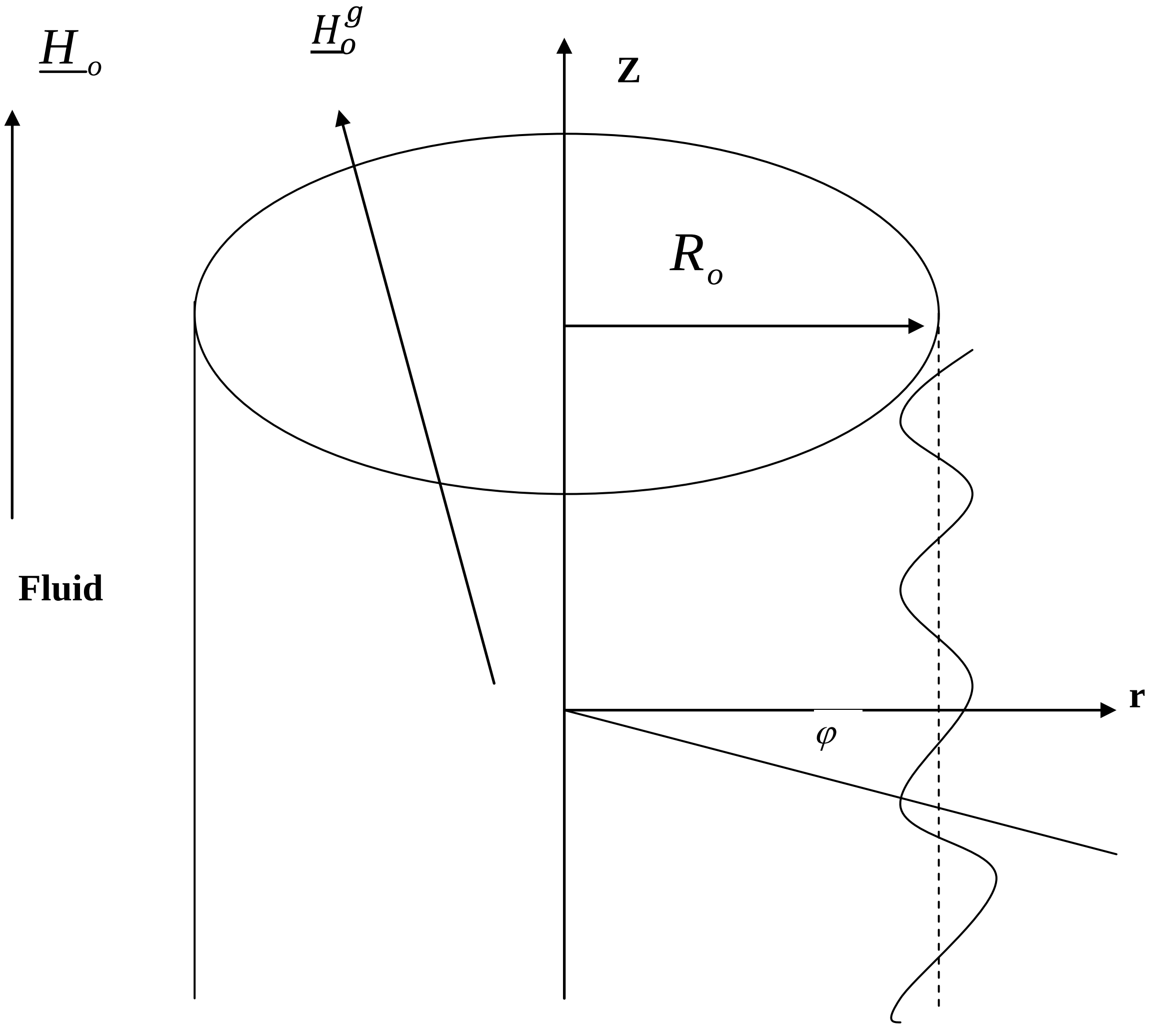

Figure (1)

Sketch for MHD Hollow Jet

Here $\underline{H}$ and $\underline{H}^g$ are the magnetic field intensities in the gas and liquid regions, P_s the curvature pressure due to the capillary force, T the surface tension coefficient, $\underline{N}$ the outward unit vector normal to gas - liquid interface and indicates as r (the radial cylindrical coordinate) does, μ the magnetic permeability coefficient, while ρ, $\underline{u}$ and P are the liquid mass density, velocity vector and kinetic pressure. Upon using some vector identities in order to represent the electromagnetic force $\mu\left(\nabla \Lambda \underline{H}\right) \Lambda \underline{H}$ into magnetic pressure $(\mu/2)\,\nabla\left(\underline{H}\cdot\underline{H}\right)$ and magnetic tension $\mu\left(\underline{H}\cdot\nabla\right)\underline{H}$: equation (6) may be rewritten as

$$\rho\left(\frac{\partial \underline{u}}{\partial t}+\left(\underline{u}.\nabla\right)\underline{u}\right)-\mu\left(\underline{H}\cdot\nabla\right)\underline{H}=-\nabla\Pi \quad (12)$$

with

$$\Pi = P+\frac{\mu}{2}\left(\underline{H}\cdot\underline{H}\right) \quad (13)$$

where Π is the total hydromagnetic pressure which is the sum of the liquid kinetic pressure and magnetic pressure.

The basic equations (6), (7) and (8) are solved, with taking into account equations (1) – (3), in the unperturbed state and we have obtained

$$\Pi_o = P_o+(\mu/2)\,H_o^2 = const. \quad (14)$$

$$P_{os} = -\frac{T}{R_o} \quad (15)$$

Upon applying the balance of the pressures at $r = R_o$ we could determine the value of the constant in equation (14) and we have finally, obtained

$$P_o = -\frac{T}{R_o}+P_o^g+\frac{\mu H_o^2}{2}\left(\alpha^2+\beta^2-1\right) \quad (16)$$

Here P_o^g is the gas constant pressure in the initial state, $\left(-\frac{T}{R_o}\right)$ the contribution of the capillary force, while the third term in the right side of equation (16) is being the net magnetic pressure due to the effect of electromagnetic forces acting in the gas and liquid regions.

One has to refer here that in absence of the magnetic field as $H_o = 0$ the constant gas pressure P_o^g must be greater than $\left(-\frac{T}{R_o}\right)$ in order that $P_o > 0$, otherwise the model collapses and there will not be a gas pervades into the liquid region.

In the general case, such that $P_o \geq 0$, the gas kinetic pressure P_o^g in the initial state must satisfy the restriction:

$$P_o^g \geq \frac{T}{R_o}+\frac{\mu H_o^2}{2}\left(1-\alpha^2-\beta^2\right) \quad (17)$$

Otherwise the model collapses and it will be a homogeneous liquid medium.

2. Perturbation Analysis

For a small departure from the unperturbed state, based on the normal mode analysis technique, every variable quantity Q (r, 0, z, t) could be expanded as

$$Q(r, 0, z, t) = Q_o(r) + Q_1(r, 0, z, t) + \dots\dots\dots \tag{18}$$

Here Q (r, 0, z, t) stands for $\underline{u}, P, \underline{H}, \underline{H}^g, \underline{N}$ and P_s with Q_o (r) is the value of Q (r, 0, z, t) in the unperturbed state, while Q_1(r, 0, z, t) is a small increment of Q (r, 0, z, t) due to perturbation. Based on the expansion (18), the perturbed radial distance of the gas cylinder may be expressed as

$$r = R_o + R_1 \tag{19}$$

with

$$R_1 = \varepsilon_0 \exp(ikz + \sigma t) \tag{20}$$

is the elevation of the surface wave measured from the unperturbed level. Here k (real number) is the longitudinal wave number and σ is the growth rate. From the point of view of the expansions (18) – (20) for the basic equations (4) – (14), the relevant linearized perturbation equations are given as follows.

In the gas region

$$\nabla \Lambda \underline{H}_1^g = 0 \tag{21}$$

$$\nabla \cdot \underline{H}_1^g = 0 \tag{22}$$

In the liquid region

$$\nabla . \underline{u}_1 = 0 \tag{23}$$

$$\nabla . \underline{H}_1 = 0 \tag{24}$$

$$\left(\frac{\partial \underline{u}_1}{\partial t} + (\underline{u}_o . \nabla)\underline{u}_1\right) - \frac{\mu}{\rho}\left(\underline{\mathrm{H}}_o \cdot \nabla\right) \underline{H}_1 = -\nabla \Pi_1 \tag{25}$$

$$\Pi_1 = \frac{P_1}{\rho} + \frac{\mu}{\rho}\left(\underline{H}_o \cdot \underline{H}_1\right) \tag{26}$$

$$\sigma \underline{u}_1 + i\,k\,U\,\underline{u}_1 - \frac{i\,\mu\,k\,H_0}{\rho}\,\underline{H}_1 = -\nabla\,\Pi_1 \tag{27}$$

$$(\sigma + i\,k\,U)\,\underline{u}_1 - \frac{i\,\mu\,k\,H_0}{\rho}\,\underline{H}_1 = -\nabla\,\Pi_1 \tag{28}$$

Where $\underline{u}_1$ and $\underline{H}_1$ have the components

$$\underline{u}_1 = (u_{1r}, u_{1\varphi}, u_{1z}), \qquad \underline{H}_1 = (H_{1r}, H_{1\varphi}, H_{1z}) \tag{29}$$

Equation (28) may be rewritten as

$$(\sigma + i\,k\,U)\,u_{1r} - \frac{i\,\mu\,k\,H_0}{\rho}\,H_{1r} = -\frac{\partial \Pi_1}{\partial r} \tag{30}$$

$$(\sigma + i\,k\,U)\,u_{1\varphi} - \frac{i\,\mu\,k\,H_0}{\rho}\,H_{1\varphi} = 0 \tag{31}$$

$$(\sigma + i\,k\,U)\,u_{1z} - \frac{i\,\mu\,k\,H_0}{\rho}\,H_{1z} = -ik\Pi_1 \tag{32}$$

Also,

$$\frac{\partial \underline{H}_1}{\partial t} + (\underline{u}_0 \cdot \nabla)\,\underline{H}_1 = (\underline{H}_0 \cdot \nabla)\,\underline{u}_1 + \eta\,\nabla^2 \underline{H}_1 \tag{33}$$

$$\sigma\,\underline{H}_1 + i\,k\,U\,\underline{H}_1 = (\underline{H}_0 \cdot \nabla)\,\underline{u}_1 + \eta\,\nabla^2 \underline{H}_1 \tag{34}$$

$$(\sigma + i\,k\,U)\underline{H}_1 = (\underline{H}_0 \cdot \nabla)\,\underline{u}_1 + \eta\nabla^2 \underline{H}_1 \tag{35}$$

Equation (35) may be rewritten as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial H_{1r}}{\partial r}\right) - \left(\frac{\sigma+ikU}{\eta} + k^2\right)H_{1r} = -\frac{ikH_0 u_{1r}}{\eta} \tag{36}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial H_{1\varphi}}{\partial r}\right) - \left(\frac{\sigma+ikU}{\eta} + k^2\right)H_{1\varphi} = -\frac{ikH_0 u_{1\varphi}}{\eta} \tag{37}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial H_{1z}}{\partial r}\right) - \left(\frac{\sigma+ikU}{\eta} + k^2\right)H_{1z} = -\frac{ikH_0 u_{1z}}{\eta} \tag{38}$$

Along the gas – liquid interface

$$P_{1s} = \frac{T}{R_0^2}\left(R_1 + R_0^2\,\frac{\partial^2 R_1}{\partial z^2}\right) \tag{39}$$

$$P_{1s} = \frac{T}{R_0^2}(1 - x^2)R_1 \tag{40}$$

By taking the divergence of equation (28) and using (23) and (24) we get

$$\nabla^2 \Pi_1 = 0 \tag{41}$$

Based on the space-time dependence (19) and based on the linear perturbation technique, every perturbed quantity $Q_1(r, 0, z; t)$ may be expressed as

$$Q_1(r, 0, z; t) = Q_1^*(r) \exp(ikz + \sigma t) \tag{42}$$

Consequently, the non-singular solution of equation (41) is given by

$$\Pi_1 = A\, K_0(kr)\, R_1 \tag{43}$$

where $K_0(kr)$ is the modified Bessel function of second kind of order 0 and A is an arbitrary constant of integration. By substitution from (43) into (30)-(32) we get

$$u_{1r} = \frac{i\mu H_0 k H_{1r}}{\rho(\sigma + ikU)} - \frac{AkK_0'(kr)R_1}{(\sigma + ikU)} \tag{44}$$

$$u_{1\varphi} = \frac{i\mu H_0 k H_{1\varphi}}{\rho(\sigma + ikU)} \tag{45}$$

$$u_{1z} = \frac{i\mu H_0 k H_{1z}}{\rho(\sigma + ikU)} - \frac{iAkK_0(kr)R_1}{(\sigma + ikU)} \tag{46}$$

Substituting (44)-(46) into (36)-(38), we have

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial H_{1r}}{\partial r}\right) - q^2 H_{1r} = \frac{ik^2 H_0 A K_0'(kr) R_1}{\eta(\sigma + ikU)} \tag{47}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial H_{1\varphi}}{\partial r}\right) - q^2 H_{1\varphi} = 0 \tag{48}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial H_{1z}}{\partial r}\right) - q^2 H_{1z} = -\frac{k^2 H_0 A K_0(kr) R_1}{\eta(\sigma + ikU)} \tag{49}$$

Where q is given by

$$q^2 = k^2 + \frac{(\sigma + ikU)^2 + \Omega^2}{\eta(\sigma + ikU)}, \qquad \Omega = H_0 k\sqrt{\frac{\mu}{\rho}} \tag{50}$$

Equations (47)-(49) are second order differential equations for the magnetic field components $H_{1r}, H_{1\varphi}$ and H_{1z}, for instance, the equation pertaining H_{1z} is given as

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dH_{1z}^*}{dr}\right) - q^2 H_{1z}^* = -\frac{k^2 H_0 A K_0(kr) R_1^*}{\eta(\sigma+ikU)} \tag{51}$$

Equation (51) is an ordinary second order differential equation which has an auxiliary solution of equation (36), apart from the singular solution, is given by

$$H_{1z} = \left(BqH_0K_0(qr) + \frac{Ak^2H_0K_0(kr)}{(\sigma+ikU)^2+\Omega^2}\right)R_1 \tag{52}$$

where B is an arbitrary constant to be determined. In similar steps, the components $H_{1\varphi}$ and H_{1r} of the magnetic field H_1 are given by

$$H_{1\varphi} = 0 \tag{53}$$

$$H_{1r} = -ikH_0\left(BK_0'(qr) + \frac{AkK_0'(kr)}{(\sigma+ikU)^2+\Omega^2}\right)R_1 \tag{54}$$

By the aid of equation (49) and equations (52)-(54) the components of the perturbed velocity vector $\underline{u}_1$ could be identified in terms of the cylindrical modified Bessel function, e.g., the radial component u_{1r} of $\underline{u}_1$ is given by

$$u_{1r} = \left(\frac{B\Omega^2K_0'(qr)}{(\sigma+ikU)} - \frac{Ak(\sigma+ikU)K_0'(kr)}{(\sigma+ikU)^2+\Omega^2}\right)R_1 \tag{55}$$

For the gas medium of equation (21)–(22) means that $\underline{H}_1^g$ may be derived from a scalar function ψ_1^g, say called magnetic potential.

$$\underline{H}_1^g = \nabla\psi_1^g \tag{56}$$

Taking the divergence of equation (56) and using equation (22), we get

$$\nabla^2\psi_1^g = 0 \tag{57}$$

From the view point of space-time dependence (19), the non-singular solution of equation (57) is given by

$$\underline{H}_1^g = iH_0C\,\nabla(I_0(kr)\exp(ikz+\sigma t)) \tag{58}$$

3. Boundary conditions

The solution of the relevant perturbations equations (28),(35) and (39) represented by equation (42) and the solution of the unperturbed system of equations represented by equation (16) must satisfy appropriate boundary conditions. Under the present circumstances for the problem at hand, these boundary conditions are given as follows.

i) " the normal component of the velocity $\underline{u}$ of the liquid must be compatible with the velocity of the perturbed boundary gas - liquid (19) at r = R_o". This condition is being

$$\frac{dr}{dt} = u_{1r} \qquad i.e \qquad \frac{\partial r}{\partial t} + (\underline{u}\,.\nabla)\underline{r} = u_{1r} \quad \text{at } r = R_o \tag{59}$$

From equations (19) and (20) we have

$$\frac{\partial r}{\partial t} + (\underline{u}\,.\nabla)\underline{r} = (\sigma + i\,k\,U)R_1 \tag{60}$$

Combining equation (60) and (55) we obtain

$$\left(\frac{B\Omega^2 K_0'(y)}{(\sigma+ikU)} - \frac{Ak(\sigma+ikU)K_0'(x)}{(\sigma+ikU)^2+\Omega^2}\right) = (\sigma + ikU) \tag{61}$$

where $y = qR_0$ and $x = kR_0$ are the resistive and ordinary dimensionless longitudinal wave numbers.

ii) "The normal component of the magnetic field must be continuous across the gas - liquid interface (19) at r = R_o". Mathematically this condition is given by

$$\underline{N_o}.\underline{H_1} + \underline{N_1}\,.\underline{H_o} = \underline{N_o}\,.\,\underline{H_1^g} + \underline{N_1}\,.\underline{H_o^g} \tag{62}$$

with

$$\underline{N_o} = (1,0,0) \quad , \quad \underline{N_1} = (0,0,-i\,k)R_1 \tag{63}$$

$$\underline{H_0^g} = (0, \beta\, H_0, \alpha\, H_0)$$

$$\underline{H_0} = (0,0,H_0)$$

By substitution into the condition (62) we get

$$\frac{Ax^2K_0'(x)}{(\sigma+ikU)^2+\Omega^2} + BxK_0'(y) + CxI_0'(x) = (\alpha - 1)x \tag{64}$$

iii) The tangential component of the magnetic field must be continuous across the interface (19) at r = R_0. This condition is given by

$$\underline{N}_0 \wedge \left(\underline{H}_1^g - \underline{H}_1\right) = \underline{N}_1 \wedge \left(\underline{H}_0^g - \underline{H}_0\right) \tag{65}$$

the condition (65) yields:

$$\alpha = 1,$$

$$\frac{AkxK_0(x)}{(\sigma+ikU)^2+\Omega^2} + ByK_0(y) + CxI_0(x) = 0 \tag{66}$$

Solving (61), (64) and (66) for *A, B* and *C*, we get

$$A = \frac{(\sigma+ikU)^2+\Omega^2\left(LK_0(x)-MK_0'(y)\right)}{kK_0(x)K_0'(x)\left(L-\Omega^2K_0'(y)\right)} \tag{67}$$

$$B = \frac{(\sigma+ikU)^2\left(-1+x(\alpha-1)I_0(x)K_0'(x)\right)}{\left(L-\Omega^2K_0'(y)\right)} \tag{68}$$

$$C = \frac{\left(N-xLK_0(x)\right)}{\left(kI_0(x)K_0'(x)L-\Omega^2K_0'(y)\right)} \tag{69}$$

Where *L*, *M* and *N* are defined as

$$L = (\sigma + ikU)^2K_0'(x)\left(xI_0(x)K_0'(y) - yK_0(y)K_0'(x)\right) \tag{70}$$

$$M = \Omega^2K_0'(x)\left(K_0(x) + x(\alpha-1)I_0(x)K_0'(x)\right) \tag{71}$$

$$N = x\Omega^2K_0'(y)\left(K_0(x) - K_0'(x)\right)$$

$$-x(\alpha-1)I_0(x)(K_0')^2(x)\left(y(\sigma+ikU)^2K_0(y) - x\Omega^2K_0'(y)\right) \tag{72}$$

Now, we have to apply the balance of pressure or the compatibility condition.

iv) " this compatibility condition states that the normal component of the total stress tensor concerning the kinetic and magnetic pressures must be discontinuous by the curvature pressure due to the capillary force, across the gas – liquid interface (19) at r = R_o".

Mathematically, this reads

$$\left[P_1 + \mu\left(\underline{H}_0 \cdot \underline{H}_1\right) + R_1\frac{\partial P_0}{\partial r}\right] - \left[\mu\left(\underline{H}_0 \cdot \underline{H}_1\right) + \frac{\mu}{2}R_1\frac{\partial}{\partial r}\left(\underline{H}_0 \cdot \underline{H}_0\right)\right]^g = P_{1s} \tag{73}$$

Where

$$P_{1s} = \frac{T}{R_0^2}\left(R_1 + R_0^2 \frac{\partial^2 R_1}{\partial z^2}\right)$$

Upon substituting into the condition (73) about the different variables, the following eigenvalue relation is obtained:

$$AK_0(x) + C\mu H_0^2 x\alpha I_0(x) - \frac{\mu}{R_0}\beta^2 H_0^2 = \frac{T}{R_0^2}(1 - x^2) \quad (74)$$

Where *A* and *C* are still given by equations (67) and (69). Therefore, the dispersion relation is given by

$$\left[\frac{(\sigma+ikU)^2+\Omega^2\left(LK_O(x)-MK_0'(y)\right)}{kK_0(x)K_0'(x)\left(L-\Omega^2K_0'(y)\right)}\right]K_0(x)$$

$$+\frac{\mu\alpha H_0^2 xI_0(x)\left(N-xLK_0(x)\right)}{\left(kI_0(x)K_0'(x)L-\Omega^2K_0'(y)\right)} - \frac{\mu}{R_0}\beta^2 H_0^2 = \frac{T}{R_0^2}(1 - x^2) \quad (75)$$

4. General Discussions

Equation (75) is the MHD stability criterion of a resistive hollow jet " a gas jet immersed into a liquid with liquid inertia is predominant over that of the gas" pervaded by an oblique magnetic field. It contains the most information about the instability of the present model whether it is ordinary or marginally (as σ = 0). The relation (75) relates the temporal amplification σ or rather the oscillation frequency ω as σ = *i*ω is imaginary with transverse wave number *m*, the ordinary and resistive longitudinal dimensionless wave numbers *x* or *y*, the modified Bessel function I_0 and K_0 of argument *x* or *y* and with the parameters μ, R_0, $\underline{H}_0$, *T* and ρ of the problem. For reported MHD problems of ideal fluids, see Chandrasekhar (1981) and Radwan (1988, 1996), it is a general feature that the eigenvalue relation was obtained as a linear combination of capillary eigenvalue relation and magnetodynamic eigenvalue relation. Here in the present problem that the behavior is not the case because the fluid is dissipation as the liquid is considered to be a resistive. This later situation is the case as the liquid is viscous, see Radwan (1991).

5. Limiting Cases

The problem under consideration is, for some extent, a general problem, so from its results we have taken the following recent reported stability criteria as limiting cases with appropriate choices.

A lot of simplifications, *like* $U = 0$, $H_0 = 0$, $\beta = 0$ and $\eta = 0$ are essential to get the following dispersion relation from (75)

$$\sigma^2 = -\frac{T}{\rho\, R_0^3}\left(\frac{x\, K_0'(x)}{K_0(x)}\right)(1 - x^2) \tag{76}$$

Which is obtained by Rayleigh (1945) for a naïve model. However, for $U = 0$, $\eta = 0$, $\beta = 0$ while $H_0 \neq 0$, the relation (75) reduces to

$$\sigma^2 = -\frac{T}{\rho\, R_0^3}\left(\frac{x\, K_0'(x)}{K_0(x)}\right)(1 - x^2) - \left(\frac{\mu H_0^2}{\rho R_0^2}\right)\left(\frac{x}{I_0(x) K_0(x)}\right) \tag{77}$$

Which coincides with Radwan's results (1991) as we consider the fluid is non-streaming for m = 0.

6. Stability discussions

In order to discuss the stability state and identify the instability regions of the present model, we have to do as firstly as the liquid is perfectly conducting. In such a case, the dispersion relation (75) degenerates to

$$(\sigma + ikU)^2 = \frac{T}{\rho R_0^3}\left(\frac{x K_0'(x)}{K_0(x)}\right)(x^2 - 1) + \frac{\mu H_0^2}{\rho R_0^2}\left(-x^2 + (-\beta^2 + x^2\alpha^2)\frac{I_0(x) K_0'(x)}{K_0(x) I_0'(x)}\right) \tag{78}$$

where use has been made of the relations

$$K_0'(x) = -K_1(x), \qquad I_0'(x) = I_1(x) \tag{79}$$

together with the Wronskian relation

$$W_0\big(I_0(x), K_0(x)\big) = I_0(x)K_0'(x) - K_0(x)I_0'(x) = -\frac{1}{x} \tag{80}$$

The dispersion relation (78) is a simple linear combination of the dispersion relation of a hollow jet acted by the capillary force and that one acted upon by the electromagnetic force. It relates the growth rate σ with $I_0(x)$, $K_0(x)$ and their derivatives, the natural quantity $\sqrt{\frac{\rho R_0^3}{T}}$ as well as $\sqrt{\frac{\rho R_0^2}{\mu H_0^2}}$ as a unit of time and with parameters α, β, ρ, H_0 and R_0 of the problem. It is found more convenient to study some behaviors of the modified Bessel functions in order to determine the stability states.

We consider the recurrence relations (see Abrmowize and stegun [1])

$$I'_m(x) = \frac{1}{2}\left(I_{m-1}(x) + I_{m+1}(x)\right) \tag{81}$$

$$K'_m(x) = -\frac{1}{2}\left(K_{m-1}(x) + K_{m+1}(x)\right) \tag{82}$$

For every non-zero real value of x for all $m \geq 0$, we have

$$I_0(x) > 0, \qquad\qquad K'_0(x) < 0 \tag{83}$$

with $I_m(x)$ is monotonic increasing and positive definite while $K_m(x)$ is monotonic decreasing but never negative. If the influence of the electromagnetic force is predominant over that of the capillary force $T = 0$, then the dispersion relation (78) yields

$$(\sigma + ikU)^2 = \frac{\mu H_0^2}{\rho R_0^2}\left(-x^2 + (-\beta^2 + x^2\alpha^2)\frac{I_0(x)K'_0(x)}{K_0(x)I'_0(x)}\right) \tag{84}$$

Discussing this relation it is found that the influence of the electromagnetic force in the liquid region surrounded the gas jet is represented by the term $(-x^2)$ followed by the natural quantity $\left(\frac{\mu H_0^2}{\rho R_0^2}\right)$. It has a stabilizing effect and that effect is independent of the kind of perturbation whether it is axisymmetric or not. In such a case, the general eigenvalue relation (75) yields

$$(\sigma + ikU)^2\left[1 + \frac{\mu H_0^2}{\rho R_0^2}\left(\frac{xyK_0(y)}{GK_0(x)(\sigma + ikU)^2} - \frac{x\beta^2 K_1(x)}{K_0(x)(\sigma + ikU)^2}\right)\right]$$

$$= \frac{T}{\rho R_0^3}\left[1 + \frac{\mu H_0^2}{\rho R_0^2}\frac{x^2 K_1(y)}{GK_1(x)(\sigma+ikU)^2}\right]\frac{xK_1(x)}{K_0(x)}(1 - x^2) \tag{85}$$

where

$$x = kR_0, \qquad y^2 = x^2 + \frac{R_0^2}{\eta(\sigma+ikU)}\left[(\sigma+ikU)^2 + \frac{\mu H_0^2 x^2}{\rho R_0^2}\right] \tag{86}$$

$$G = -xI_0(x)K_0'(y) + yK_0(x)I_0'(y) \tag{87}$$

For infinite resistivity, we have

$$\eta \to \infty, \qquad y \to x, \qquad G \to 1 \tag{88}$$

It is found that inserting the approximation (87) into (85), we obtain the relation (76) in absence of transverse component of the magnetic field of the gas. This means that when the fluid is of infinite resistivity, the electromagnetic forces interior of the gas cylinder have influence on the capillary instability of the hollow jet. If we assume that the fluid is perfectly conducting, then we have

$$\eta \to 0, \qquad y \to \infty, \tag{89}$$

and consequently the relation (85) reduces to (77). Discussing the relation (85) for the states which are different from the two previous cases as $\eta \to \infty$, $\eta \to 0$ it is found that the magnetic field is stabilizing for all short and long wavelengths while the surface tension is stabilizing for all short wavelengths i.e. in the domain $1 \le x < \infty$ but it is destabilizing for all long wavelengths $0 < x < 1$. The influence of the resistivity, in the present general case, could be identified by determining the relationship between σ and η through which we may check whether the resistivity is destabilizing or not. In order to do so, it is found more convenient to formulate the dispersion relation (85) in the dimensionless form

$$n^2\left[1 + \frac{\xi^2 x y K_0(y)}{n^2 G K_0(x)} - \frac{\xi^2 x \beta^2 K_1(x)}{n^2 K_0(x)}\right] = \left[1 + \frac{\xi^2 x^2 K_1(y)}{n^2 G K_1(x)}\right]\left[\frac{xK_1(x)(1-x^2)}{K_0(x)}\right] \tag{90}$$

with

$$y^2 = x^2 + \frac{1}{n\zeta}(n^2 + x^2\xi^2) \tag{91}$$

where

$$n = (\sigma+ikU)\sqrt{\frac{\rho R_0^3}{T}}, \qquad \xi = \frac{H_0}{H_s} \tag{92}$$

$$H_s = \sqrt{\frac{T}{\mu R_0}}, \qquad \zeta = \eta\sqrt{\frac{\rho}{TR_0}} \tag{93}$$

7. Numerical Discussions

The stability equation (90) has been formulated in its non – dimensional form. The latter has been inserted in the computer for a given set of values of ξ, x and y, then equation (90) can be solved as a quadratic for n^2; and equation (91) can then be used to determine $\zeta = \eta\sqrt{\frac{\rho}{TR_0}}$. Keeping ξ and x fixed and allowed y to run through a sequence of values, we can derive (n, ζ) relation (to be associated with the chosen values of ξ and x. Interpolating among the (n, ζ) relations for different values of x, but the same values of ξ, we can obtain the (n, x) relation appropriate to some fixed value of ζ and chosen value of ξ. In this manner, the dispersion relation for any set of initial conditions can be deduced. The numerical results of n could be tabulated and presented graphically corresponding to continuous regular values of x for different values of η. By means of these numerical data and discussions, we find out that the axisymmetric capillary unstable domain is increasing or decreasing. From which we may judge whether the resistivity has a destabilizing or stabilizing influence on the capillary instability of a hollow jet and also we may determine exactly the stability restrictions. It is expected that the resistivity decreases the destabilizing character of the model. The dispersion relation (90) takes the form

$$\sigma^* = \left(\left[-\frac{\xi^2 xyK_0(y)}{GK_0(x)} + \frac{\xi^2 x\beta^2 K_1(x)}{K_0(x)}\right] + \left[1 + \frac{\xi^2 x^2 K_1(y)}{n^2 GK_1(x)}\right]\left[\frac{xK_1(x)(1-x^2)}{K_0(x)}\right]\right)^{1/2} + U^* \qquad (94)$$

where

$$\sigma^* = \sigma\left(\frac{\rho R_0^3}{T}\right)^{1/2} \quad \text{and} \quad U^* = -ikU\left(\frac{\rho R_0^3}{T}\right)^{1/2},$$

then equation (94) has been computed in the computer for all short and long wavelengths $0 \leq x \leq 3$.

It is found that there are many features of interest in this numerical analysis as we see in the following.

(i) For β = 0.5, $U^* = 0$ and ξ = 0.5, see figure (2).
Corresponding to ζ = 0.5, 1.0, 1.5, 2.0, 2.5, and 3.0, it is found that the unstable domains are $0 < x < 1.0312$, $0 < x < 1.0304$, $0 < x < 1.0283$, $0 < x < 1.0247$, $0 < x < 1.0188$ and $0 < x < 1.0042$ while the neighboring stable domains are $1.0312 \leq x < \infty$, $1.0304 \leq x < \infty$, $1.0283 \leq x < \infty$, $1.0247 \leq x < \infty$, $1.0188 \leq$

$x < \infty$ and $1.0042 \leq x < \infty$ where the equalities correspond to the marginal stability states.

(ii) For β = 1.5, $U^* = 0$ and ξ = 0.5, see figure (3).

Corresponding to ζ = 0.5, 1.0, 1.5, 2.0, 2.5, and 3.0, it is found that the unstable domains are $0 < x < 1.1063$, $0 < x < 1.1307$, $0 < x < 1.1428$, $0 < x < 1.2334$, $0 < x < 1.2287$ and $0 < x < 1.2199$ while the neighboring stable domains are $1.1063 \leq x < \infty$, $1.1307 \leq x < \infty$, $1.1428 \leq x < \infty$, $1.2334 \leq x < \infty$, $1.2287 \leq x < \infty$ and $1.2199 \leq x < \infty$ where the equalities correspond to the marginal stability states.

(iii) For β = 3.0, $U^* = 0$ and ξ = 0.5, see figure (4).

Corresponding to ζ = 0.5, 1.0, 1.5, 2.0, 2.5, and 3.0, it is found that the unstable domains are $0 < x < 1.4254$, $0 < x < 1.4419$, $0 < x < 1.5163$, $0 < x < 1.6349$, $0 < x < 1.6198$ and $0 < x < 1.6498$ while the neighboring stable domains are $1.4254 \leq x < \infty$, $1.4419 \leq x < \infty$, $1.5163 \leq x < \infty$, $1.6349 \leq x < \infty$, $1.6198 \leq x < \infty$ and $1.6498 \leq x < \infty$ where the equalities correspond to the marginal stability states.

(iv) For β = 5.0, $U^* = 0$ and ξ = 0.5, see figure (5).

Corresponding to ζ = 0.5, 1.0, 1.5, 2.0, 2.5, and 3.0, it is found that the unstable domains are $0 < x < 1.8345$, $0 < x < 1.9328$, $0 < x < 2.0189$, $0 < x < 2.1411$, $0 < x < 2.331$ and $0 < x < 2.418$ while the neighboring stable domains are $1.8345 \leq x < \infty$, $1.9328 \leq x < \infty$, $2.0189 \leq x < \infty$, $2.1411 \leq x < \infty$, $2.331 \leq x < \infty$ and $2.418 \leq x < \infty$ where the equalities correspond to the marginal stability states.

(v) For β = 0.5, $U^* = 0$ and ξ = 1.5, see figure (6).

Corresponding to ζ = 0.5, 1.0, 1.5, 2.0, 2.5, and 3.0, it is found that the unstable domains are $0 < x < 1.0332$, $0 < x < 1.122$, $0 < x < 1.205$, $0 < x < 1.2392$, $0 < x < 1.2449$ and $0 < x < 1.247$ while the neighboring stable domains are $1.0332 \leq x < \infty$, $1.122 \leq x < \infty$, $1.205 \leq x < \infty$, $1.2392 \leq x < \infty$, $1.2449 \leq x < \infty$ and $1.247 \leq x < \infty$ where the equalities correspond to the marginal stability states.

(vi) For β = 1.5, $U^* = 0$ and ξ = 1.5, see figure (7).

Corresponding to ζ = 0.5, 1.0, 1.5, 2.0, 2.5, and 3.0, it is found that the unstable domains are $0 < x < 1.6253$, $0 < x < 2.4316$, $0 < x < 2.5355$, $0 < x < 2.534$, $0 < x < 2.5338$ and $0 < x < 2.5338$ while the neighboring stable domains are $1.6253 \leq x < \infty$, $2.4316 \leq x < \infty$, $2.5355 \leq x < \infty$, $2.534 \leq x < \infty$, $2.5338 \leq x < \infty$ and $2.5338 \leq x < \infty$ where the equalities correspond to the marginal stability states.

(vii) For β = 0.5, $U^* = 0.5$ and ξ = 0.5, see figure (8).

Corresponding to ζ = 0.5, 1.0, 1.5, 2.0, 2.5, and 3.0, it is found that the unstable domains are $0 < x < 1.0395$, $0 < x < 1.0395$, $0 < x < 1.0389$, $0 < x < 1.0378$, $0 < x < 1.0361$ and $0 < x < 1.0321$ while the neighboring stable domains are $1.0395 \leq x < \infty$, $1.0395 \leq x < \infty$, $1.0389 \leq x < \infty$, $1.0378 \leq x < \infty$, $1.0361 \leq x < \infty$ and $1.0321 \leq x < \infty$ where the equalities correspond to the marginal stability states.

(viii) For β = 1.5, $U^* = 0.5$ and ξ = 0.5, see figure (9).

Corresponding to ζ = 0.5, 1.0, 1.5, 2.0, 2.5, and 3.0, it is found that the unstable domains are $0 < x < 1.1287$, $0 < x < 1.1399$, $0 < x < 1.1463$, $0 < x < 1.2403$, $0 < x < 1.2385$ and $0 < x < 1.2353$ while the neighboring stable domains are $1.1287 \leq x < \infty$, $1.1399 \leq x < \infty$, $1.1463 \leq x < \infty$, $1.2403 \leq x < \infty$, $1.2385 \leq x < \infty$ and $1.2353 \leq x < \infty$ where the equalities correspond to the marginal stability states.

σ^*

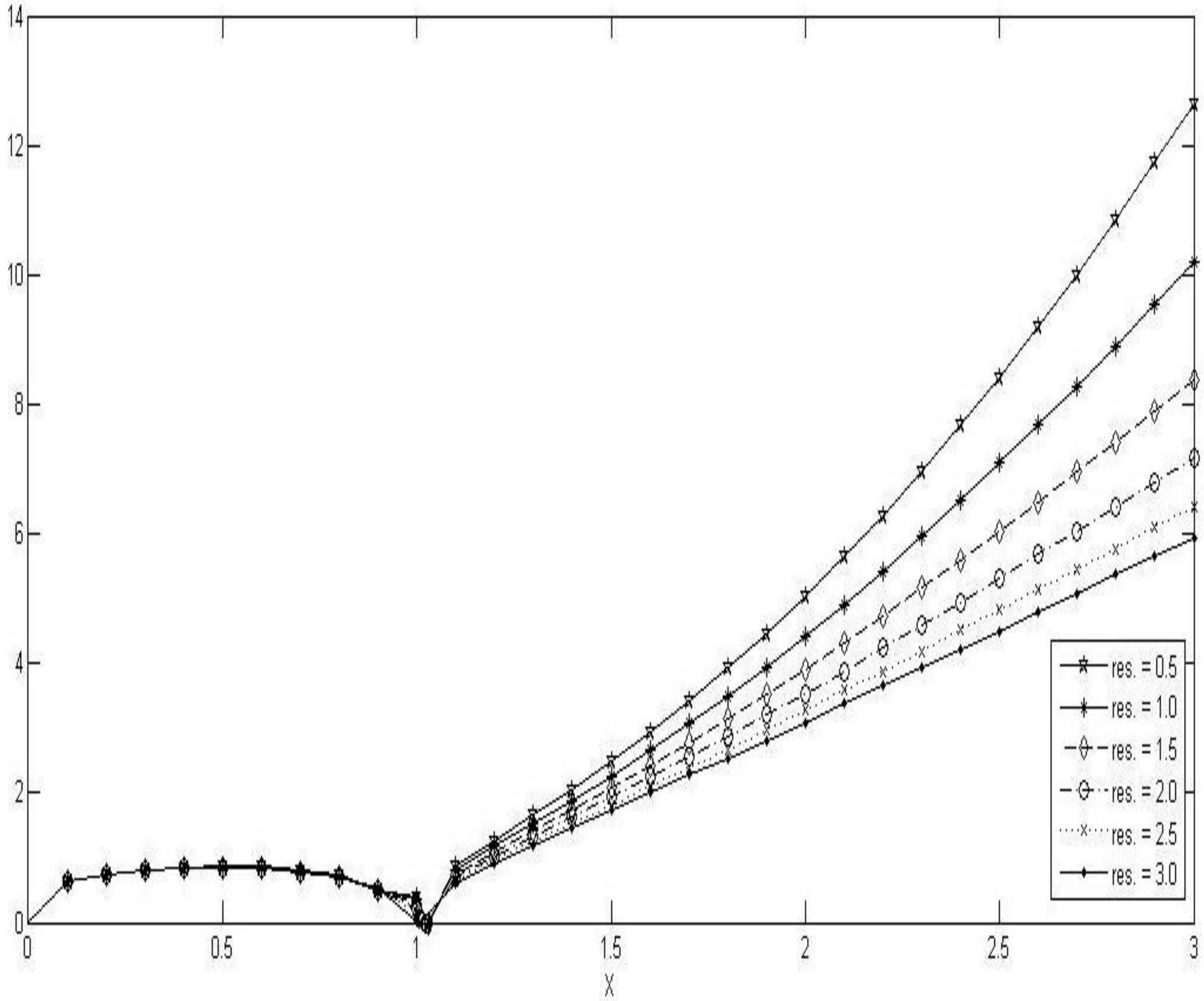

Figure (2)

Stable and unstable domains for β = 0.5, $U^* = 0$ and ξ = 0.5

σ^*

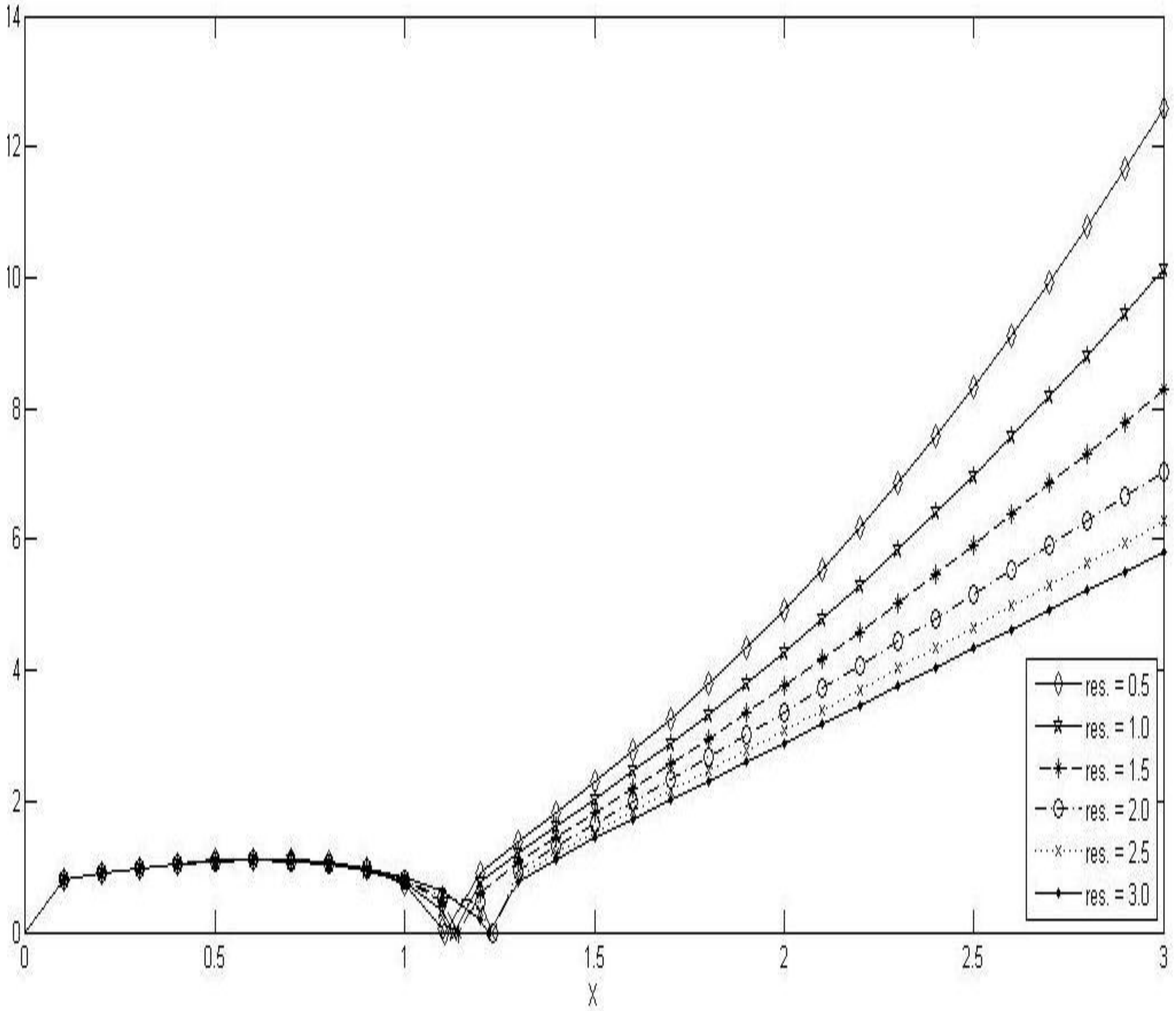

Figure (3)

Stable and unstable domains for β = 1.5, $U^* = 0$ and ξ = 0.5

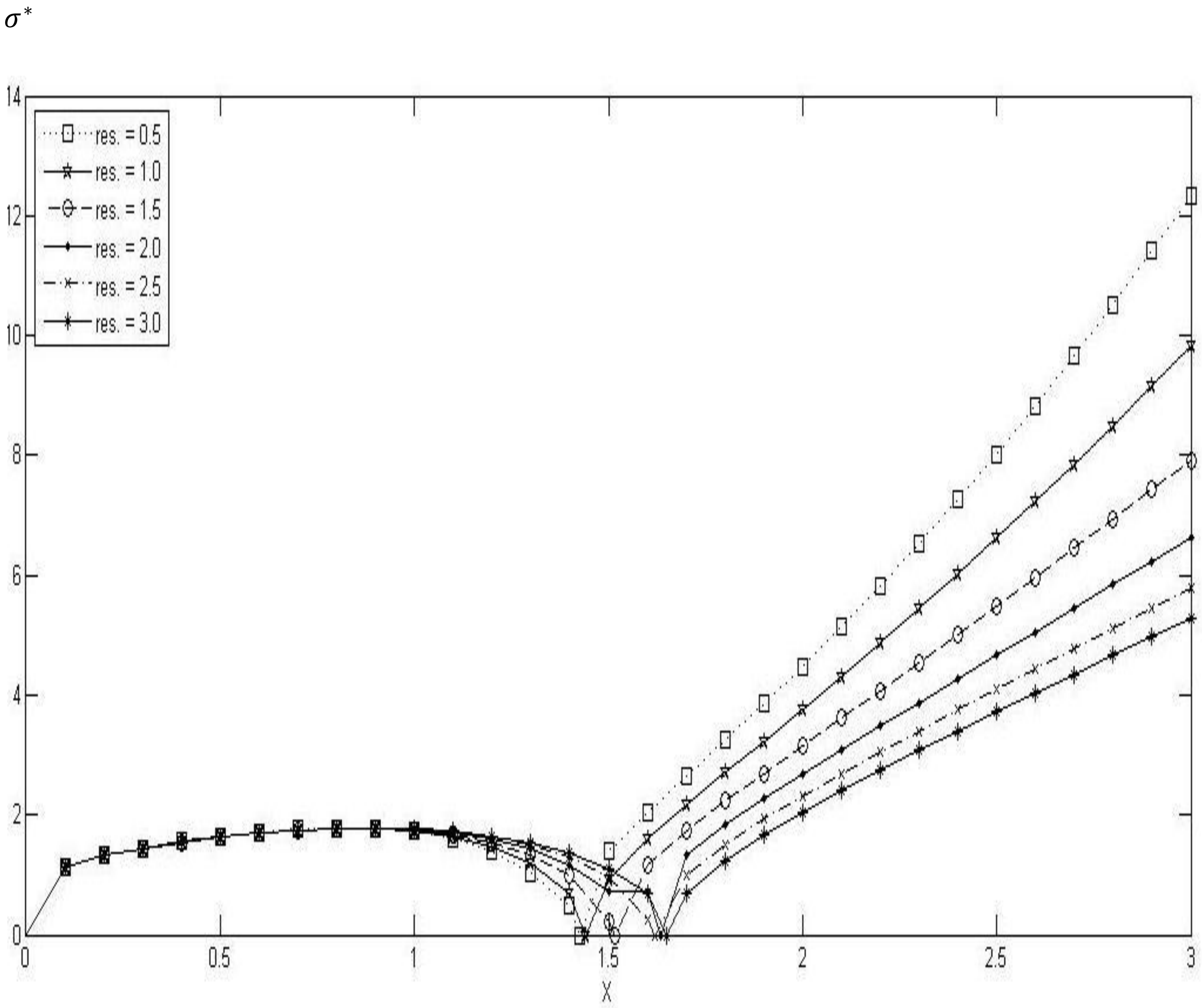

Figure (4)

Stable and unstable domains for β = 3, $U^* = 0$ and ξ = 0.5

σ^*

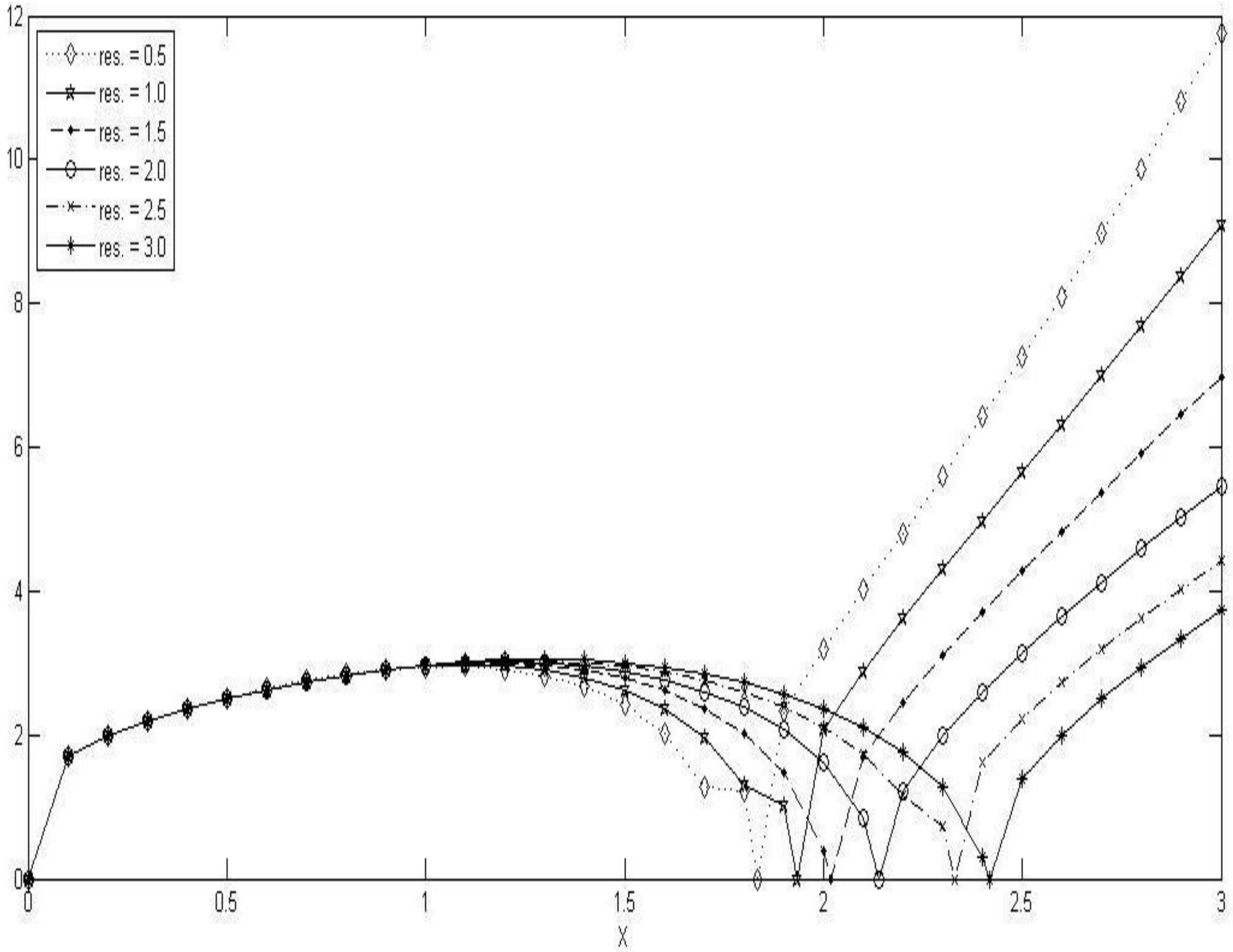

Figure (5)

Stable and unstable domains for β = 5, $U^* = 0$ and ξ = 0.5

σ^*

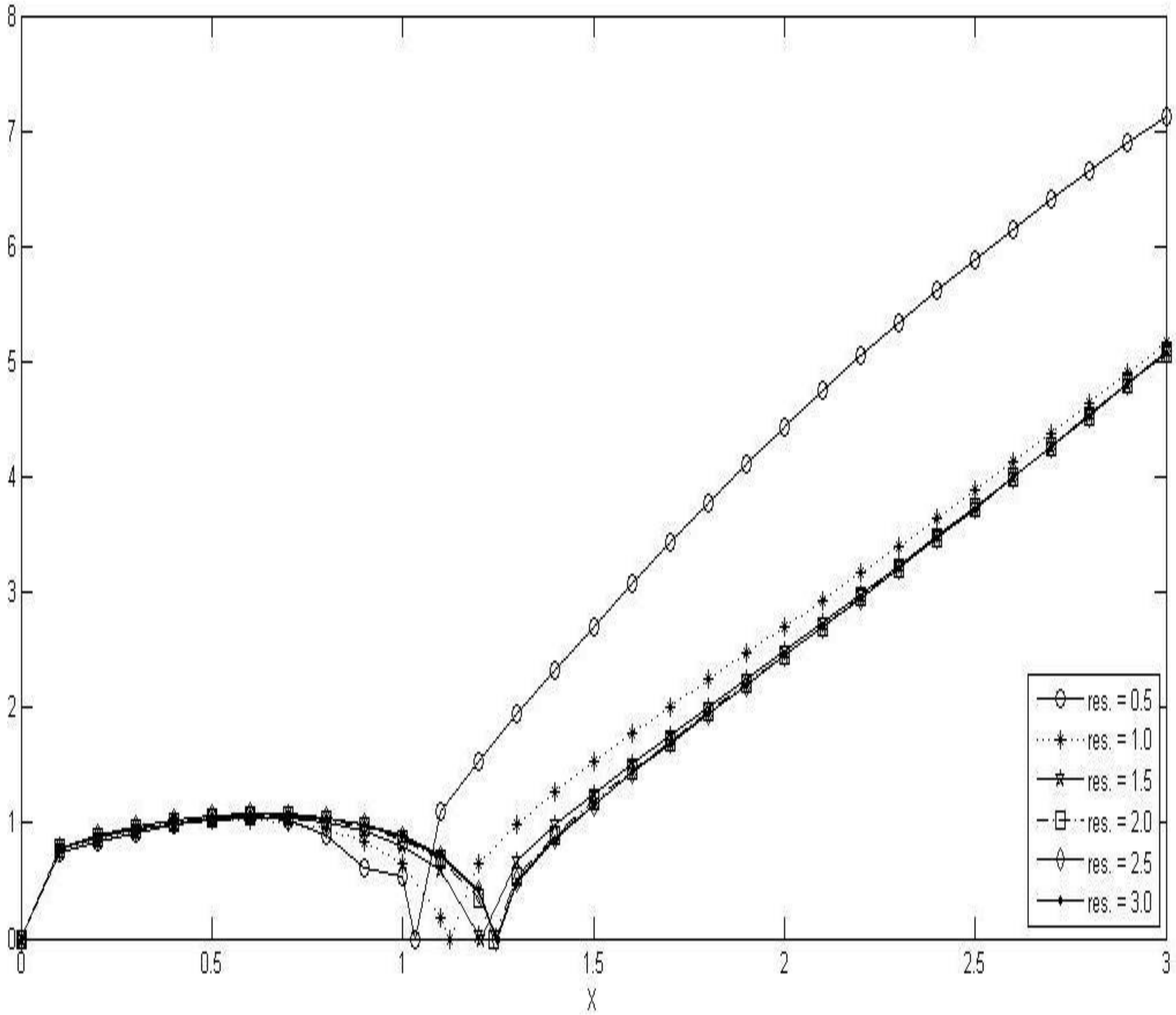

Figure (6)

Stable and unstable domains for β = 0.5, $U^{*} = 0$ and ξ = 1.5

σ^*

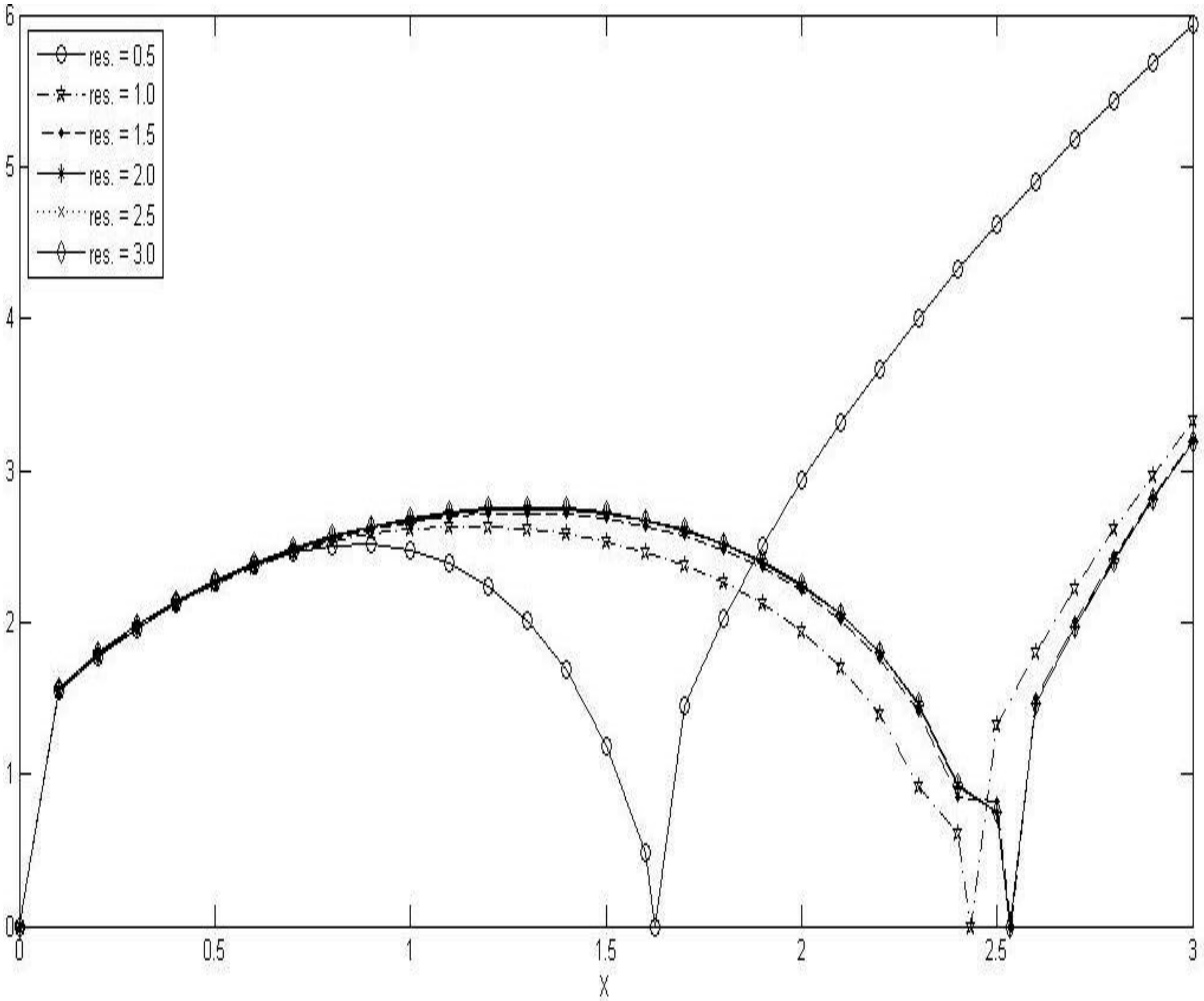

Figure (7)

Stable and unstable domains for β = 1.5, $U^* = 0$ and ξ = 1.5

σ^*

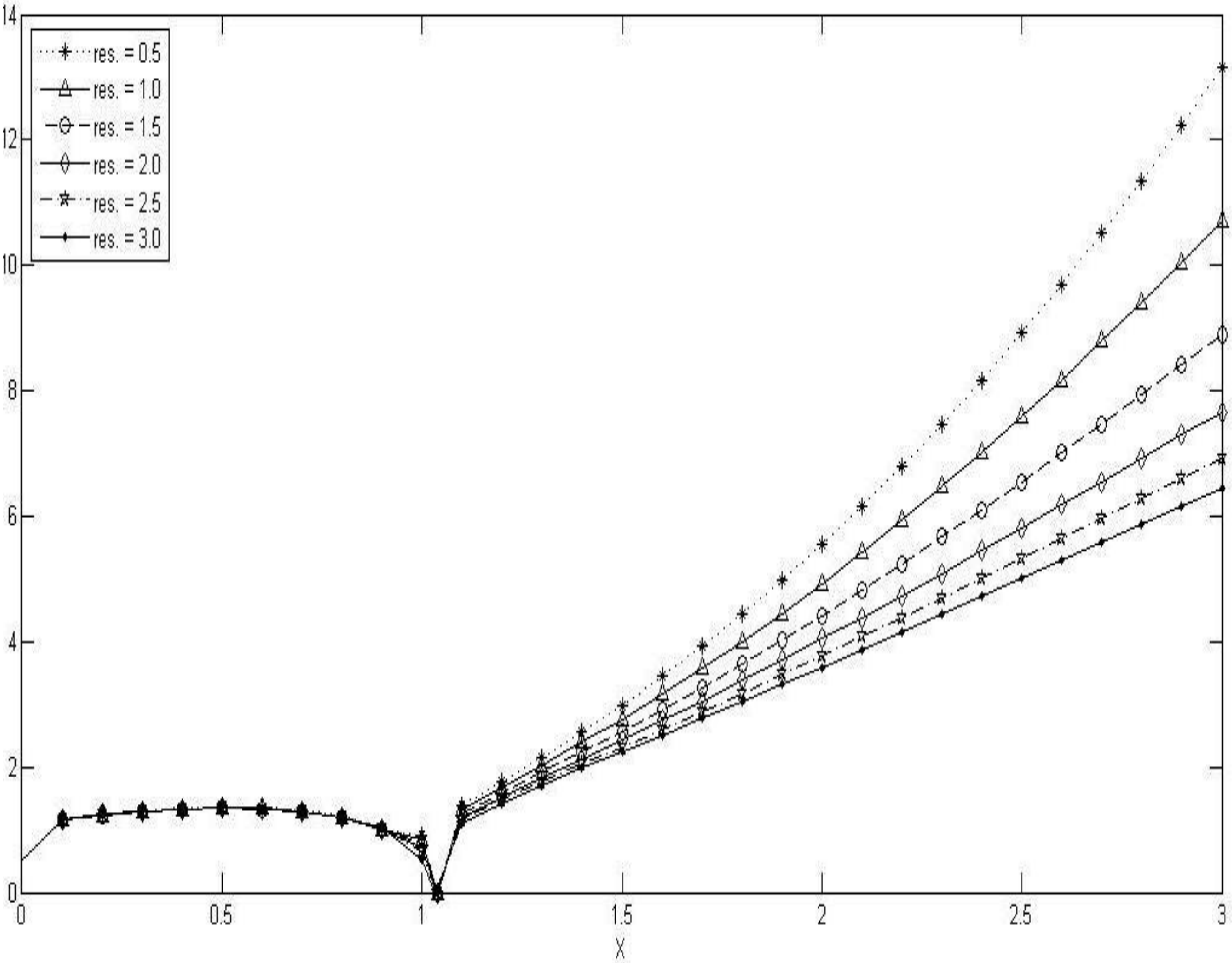

Figure (8)

Stable and unstable domains for β = 0.5, $U^* = 0.5$ and ξ = 0.5

σ^*

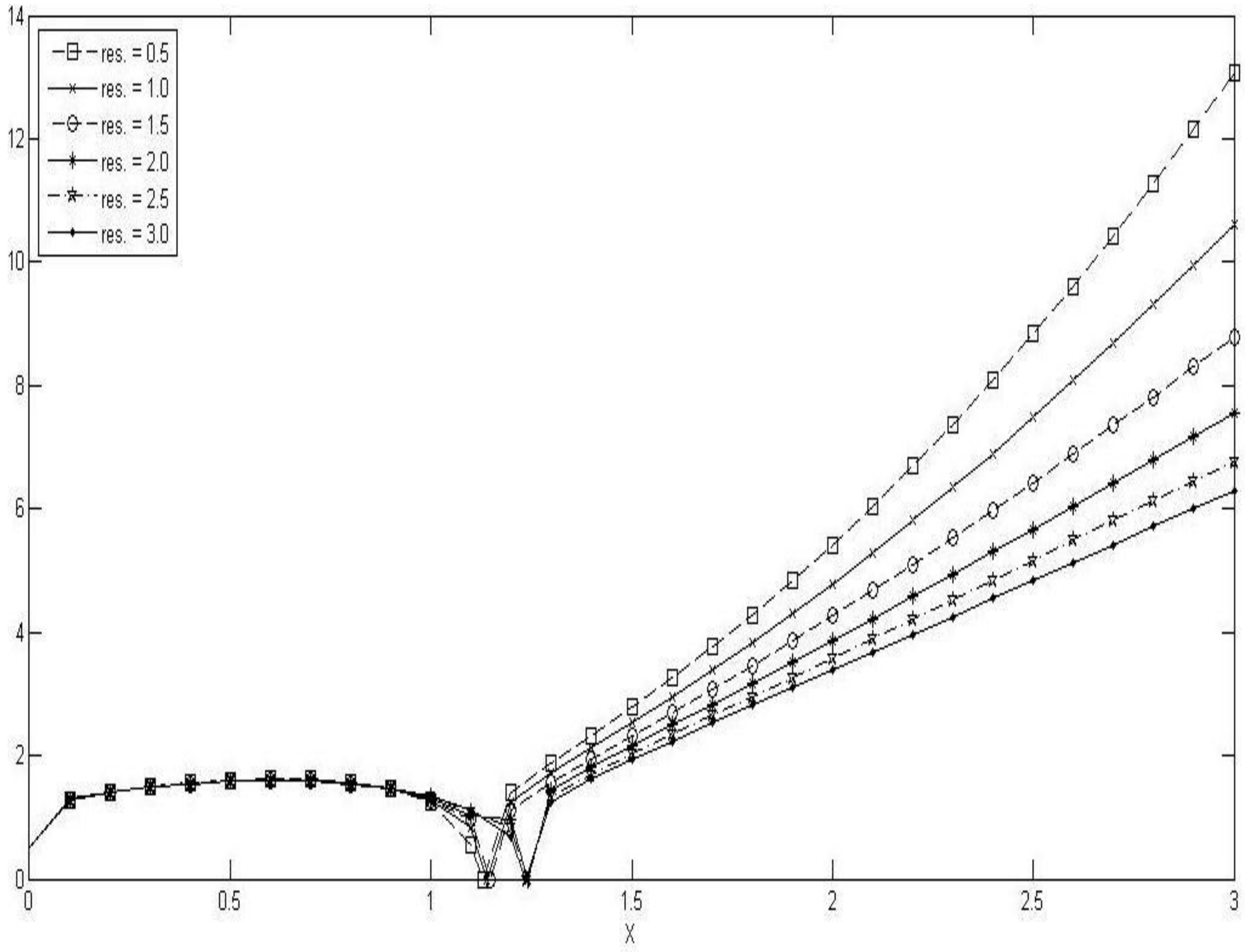

Figure (9)

Stable and unstable domains for β = 1.5, $U^* = 0.5$ and ξ = 0.5

8. Conclusion

From the foregoing discussions of MHD stability a resistive hollow jet endowed with surface tension we may formulate the following conclusions.

i) The hollow jet is capillary unstable only in the axisymmetric mode while it is stable in the rest, due to the fact that the gradient pressure force is predominant over that of the curvature pressure.

ii) If the fluid is with infinite resistivity ($\eta \to \infty$), the electromagnetic force is stabilizing influence on the capillary instability of the cylindrical resistive hollow jet.

iii) If the fluid is with finite resistivity, the electromagnetic force is stabilizing or destabilizing according to the restrictions. In fact, as we see from the figures, it is found that the resistivity factor has a stabilizing tendency, so it decreases the MHD unstable domains and simultaneously it increase those of stability.

iv) If the fluid is perfectly conducting ($\eta \to 0$), the magnetic field exerts an influence that endows the fluid a sort of rigidity. The magnetic field has a strong stabilizing influence, which may cause shrinking the capillary destabilizing effect. Moreover, above a certain high value of the acting basic magnetic field the capillary instability is completely suppressed and stability arises.

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