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ON INFRA GENERALIZED $^{\#}\alpha$-CLOSED SETS IN INFRA TOPOLOGICAL SPACES

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Abstract: In this paper, the relatively new notions of Infra generalized $^{\#}\alpha$- closed set, Infra generalized $^{\#}\alpha$- continuous functions, Infra generalized $^{\#}\alpha$- irresolute mappings are introduced and explored some of its characteristics.

Keywords: infra generalized $^{\#}\alpha$- closed sets; infra generalized $^{\#}\alpha$-continuous functions; infra generalized $^{\#}\alpha$-irresolute mapping.

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1. Introduction

Adel.M.AL.Odhari [1] introduced the concept of Infra topological spaces. In 1970, Levine [4] initiated the notion of generalized closed set. The concept of generalization of closed mapping in topological spaces was introduced by Noiri [6] in 1973. In 1996, D. Andrijevic [2] introduced and studied the class of b-open sets. In 1994, associated topologies of generalized α-closed sets and α-generalized closed sets was introduced by Maki [5]. A.Al-Omari and M.S.M. Naorami [8] made an analytical study and gave the idea of generalized b-closed sets in topological spaces. Later the view of of generalized $^{\#}\alpha$-closed sets were set forth by K. Nono

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[7] in the year 2004. Infra generalized b-closed sets was introduced by K. Vaiyomathi and F. Nirmala Irudayam [10]in 2017. A new class of genralized continuous mapping was introduced by K. Balachandran, P. Sundram and H. Maki [3] in 1991. Infra generalized b-continuous functions was derived by Vaiyomathi [11] in 2017. In this paper, a new form of Infra $g^{\#}\alpha$-closed sets, Infra $g^{\#}\alpha$-continuous functions and Infra $g^{\#}\alpha$-irresolute mappings are introduced and explored some of their properties.

2. PRELIMINARIES

Throughout this paper, (X,τ_{iX})(or X) represent a Infra topological space on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space X, icp(A) and iip(A) denote the Infra closure point of A and the Infra interior point of A and also icp_{α}(A), icp_b(A) denote iαcp(A), ibcp(A) respectively.

The following recalls requisite definitions in Infra topological spaces that will be necessitated in the sequel of our work.

Definition 2.1. [1]*Let X be any arbitrary set. An Infra topological space on X is a collection τ_{iX} subsets of X such that the following axioms are satisfying:*

(1) ϕ, X $\in \tau_{iX}$.
(2) The intersection of the elements of any sub collecction of τ_{iX} in X. Terminology, the ordered pair (X,τ_{iX}) is called Infra-topological space. We simply say X is an Infra space.

Definition 2.2. [1]*Let (X,τ_{iX}) be an infra-topological space and A $\subset$ X. A is called an infra open set (ios) if A $\subset \tau_{iX}$.*

Definition 2.3. [1]*Let (X,τ_{iX}) be an infra topological space. A subset B $\subset$ X is called infra-closed set (ics) in X if X-B is infra-open set in X.*

Definition 2.4. [1]*Let (X,τ_{iX}) be an infra topological space and A $\subset$ X. The Infra Closure Point (ICP) of A is a set denoted by icp(A) and given by : icp(A)= $\cap$ B_i:A$\subset B_i$, $X - B_i \in \tau_{iX}\}$.(i.e) icp(A) is the intersection of all infra closed set containing the set A.*

Definition 2.5. [1]*Let (X,τ_{iX}) be an infra topological space and $A \subset X$. The Infra Interior Point (IIP) of A is a set denoted by iip(A) and given by: iip(A) $=\cup\{O_i : O_i \subset A,\ O_i \in \tau_{iX}\}$ (i.e) iip(A) is the union of all infra open set contained in the set A.*

Definition 2.6. [9]*Let (X,τ_{iX}) be an infra topological space. A is called infra semi-open if $A \subset$ icp(iip(A)) and infra semi-closed set if iip(icp(A))$\subseteq$ A.*

Definition 2.7. [9]*Let (X,τ_{iX}) be an infra topological space. A is called infra pre-open if $A \subset$ iip(icp(A)) and infra pre-closed set if icp(iip(A))$\subseteq$ A.*

Definition 2.8. [9]*Let (X,τ_{iX}) be an infra topological space. A is called infra α-open if $A \subset$ iip(icp(iip)(A)) and infra α-closed set if icp(iip(icp)(A))$\subseteq$ A.*

Definition 2.9. [9]*Let (X,τ_{iX}) be an infra topological space. A is called infra β-open if $A \subset$ icp(iip(icp)(A)) and infra β-closed set if iip(icp(iip)(A))$\subseteq$ A.*

Definition 2.10. [10]*Let (X,τ_{iX}) be an infra topological space. A is called infra b-open if $A \subset$ iip(icp(A)) $\cup$ icp(iip(A)) and infra b-closed set if iip(icp(A))$\cup$icp*
(iip(A))$\subseteq$ A.

Definition 2.11. *A subset A of a space (X,τ) is called*

(1) a infra generalized- closed set (briefly ig-closed) [10] *if icp(A) $\subseteq$ U whenever A $\subseteq$ U and U is infra open.*
(2) a infra α generalized- closed set (briefly iαg-closed) if $icp_{\alpha}(A) \subseteq$ U whenever A $\subseteq$ U and U is infra semi- open.
(3) a infra generalized semi- closed set (briefly igs-closed) [10] *if iscp(A) $\subseteq$ U whenever A $\subseteq$ U and U is infra open.*
(4) an infra α generalized- closed set (briefly iαg-closed) [10] *if iαcp(A) $\subseteq$ U whenever A $\subseteq$ U and U is infra open.*
(5) an infra generalized α- closed set (briefly igα-closed) [10] *if iαcp(A) $\subseteq$ U whenever A $\subseteq$ U and U is infra α- open.*
(6) a infra generalized pre- closed set (briefly igp-closed) [10] *if ipcp(A) $\subseteq$ U whenever A $\subseteq$ U and U is infra open.*

(7) *a infra generalized* β*- closed set (briefly ig*β*- closed)* [10] *if* $ii\beta cp(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is infra open.*

(8) *a infra generalized b- closed set (briefly igb- closed)* [10] *if* $icp_b(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is infra open.*

(9) *a infra generalized sp- closed set (briefly igsp- closed)* [10] *if ispcp(A)* $\subseteq U$ *whenever* $A \subseteq U$ *and U is infra open.*

(10) *a infra generalized* $^*b$*- closed set (briefly ig*$^*b$*- closed)* [10] *if* $icp_b(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is infra g- open.*

Definition 2.12. *A subset A of a space* (X, τ_{iX}) *is called*

(1) *Infra generalized- continuous*[11] *if* $f^{-1}(V)$ *is Infra generalized- closed in X, for every Infra closed set V of Y.*

(2) *Infra* α*generalized- continuous*[11] *if* $f^{-1}(V)$ *is Infra* α*generlaized- closed in X, for every Infra closed set V of Y.*

(3) *Infra generalized b- continuous*[11] *if* $f^{-1}(V)$ *is Infra generalized b- closed in X, for every Infra closed set V of Y.*

(4) *Infra generalized p- continuous*[11] *if* $f^{-1}(V)$ *is Infra generalized p- closed in X, for every Infra closed set V of Y.*

(5) *Infra generalized s- continuous*[11] *if* $f^{-1}(V)$ *is Infra generalized s- closed in X, for every Infra closed set V of Y.*

(6) *Infra generalized* β*- continuous*[11] *if* $f^{-1}(V)$ *is Infra generalized* β*- closed in X, for every Infra closed set V of Y.*

(7) *Infra generalized sp- continuous*[11] *if* $f^{-1}(V)$ *is Infra generalized sp- closed in X, for every Infra closed set V of Y.*

(8) *Infra generalized* $^*b$*- continuous*[11] *if* $f^{-1}(V)$ *is Infra generalized* $^*b$*- closed in X, for every Infra closed set V of Y.*

Definition 2.13. *A subset A of a space* (X, τ_{iX}) *is called*

(1) *Infra generalized- irresolute*[11] *if* $f^{-1}(V)$ *is Infra generalized- closed in X, for every Infra generalized- closed set V of Y.*

(2) *Infra αgeneralized- irresolute*[11] *if $f^{-1}(V)$ is Infra αgenerlaized- closed in X, for every Infra αgenerlaized- closed set V of Y.*

(3) *Infra generalized p- irresolute*[11] *if $f^{-1}(V)$ is Infra generalized p- closed in X, for every Infra generalized p- closed set V of Y.*

(4) *Infra generalized b- irresolute*[11] *if $f^{-1}(V)$ is Infra generalized b- closed in X, for every Infra generalized b- closed set V of Y.*

(5) *Infra generalized s- irresolute*[11] *if $f^{-1}(V)$ is Infra generalized s- closed in X, for every Infra generalized s- closed set V of Y.*

(6) *Infra generalized β- irresolute*[11] *if $f^{-1}(V)$ is Infra generalized β- closed in X, for every Infra generalized β- closed set V of Y.*

(7) *Infra generalized sp- irresolute*[11] *if $f^{-1}(V)$ is Infra generalized sp- closed in X, for every Infra generalized sp- closed set V of Y.*

(8) *Infra generalized *b- irresolute*[11] *if $f^{-1}(V)$ is Infra generalized *b- closed in X, for every Infra generalized *b- closed set V of Y.*

3. Characteristics of Infra Generalized $^{\#}$ α-Closed Sets in Infra Topological Spaces

In this section, we introduce the notion of Infra g$^{\#}$ α-closed sets and study some of its basic properties.

Definition 3.1. *Let (X,τ_{iX}) be a Infra topological space. A subset A of X is called an Infra generalized $^{\#}$ α- closed set (briefly ig$^{\#}\alpha$- closed) if $icp_{\alpha}(A) \subseteq U$ whenever $A \subseteq U$ and U is Infra g- open.*

Theorem 3.2. *Every Infra-closed set is Infra g-closed set.*

Proof: Let A be a Infra-closed set in X. Let U be Infra open set, such that $A \subseteq U$. Since A is Infra closed, icp(A) = A $\subseteq U$. Therefore icp(A) $\subseteq U$. Hence A is Infra g-closed set in X.

Remark 3.3. *The converse of the above theorem need not be true as seen from the following example.*

Example 3.4. *Let X=* $\{a,b,c,d\}$ *with the topology* $\tau = \{X, \phi, \{a\}, \{d\}\}$. *Let* $A = \{b\}$. *Here A is Infra g-closed set but not Infra-closed set of (X,*τ_{iX}*).*

Theorem 3.5. *Every Infra-closed set is Infra* $g^{\#}\alpha$*- closed set.*

Proof: Let A be a Infra-closed set in X. Let U be Infra g- open set, such that $A \subseteq U$. *Since A is Infra closed,* $icp_{\alpha}(A) \subseteq icp(A) \subseteq U$. *Therefore* $icp_{\alpha}(A) \subseteq U$. *Hence A is Infra* $g^{\#}\alpha$*- closed set in X.*

Remark 3.6. *The converse of the above theorem need not be true as seen from the following example.*

Example 3.7. *Let X=* $\{a,b,c,d\}$ *with the topology* $\tau = \{X, \phi, \{a\}, \{a,b\}, \{a,d\}\}$. *Let* $A = \{b\}$. *Here A is Infra* $g^{\#}\alpha$*- closed set but not a Infra-closed set of (X,*τ_{iX}*).*

Theorem 3.8. *Every Infra* α*-closed set is Infra* $g^{\#}\alpha$*- closed set.*

Proof: Let A be a Infra α*-closed set in X. Let U be Infra g- open set, such that* $A \subseteq U$. *Since A is Infra* α*-closed set. We have,* $icp_{\alpha}(A) = A \subseteq U$. *Then* $icp_{\alpha}(A) \subseteq U$. *Hence A is Infra* $g^{\#}\alpha$*- closed set in X.*

Remark 3.9. *The converse of the above theorem need not be true as seen from the following example.*

Example 3.10. *Let X=* $\{a,b,c,d\}$ *with the topology* $\tau = \{X, \phi, \{a\}, \{a,b,d\}\}$. *Let* $A = \{a,b,c\}$. *Here A is Infra* $g^{\#}\alpha$*- closed set but not a Infra* α*-closed set of (X,*τ_{iX}*).*

Theorem 3.11. *Every Infra* $g^{\#}\alpha$*-closed set is Infra gs-closed set.*

Proof: Let A be a Infra $g^{\#}\alpha$*-closed set in X. Let U be Infra open set, such that* $A \subseteq U$. *Since every Infra open set is Infra g-open and A is Infra* $g^{\#}\alpha$*-closed, we have,* $iscp(A) \subseteq icp_{\alpha}(A) \subseteq U$. *Then* $iscp(A) \subseteq U$. *Hence A is Infra gs-closed set in X.*

Remark 3.12. *The converse of the above theorem need not be true as seen from the following example.*

Example 3.13. *Let* $X = \{a,b,c,d\}$ *with the topology* $\tau = \{X, \phi, \{a\}, \{a,b\}, \{a,d\}\}$. *Let A=*$\{a,b,d\}$. *Here A is Infra gs-closed set but not a Infra* $g^{\#}\alpha$*-closed set of (X,*τ_{iX}*).*

Theorem 3.14. *Every Infra $g^{\#}\alpha$-closed set is Infra gp-closed set.*

Proof: *Let A be a Infra $g^{\#}\alpha$-closed set in X. Let U be Infra open set, such that $A \subseteq U$. Since every Infra open set is Infra g-open and A is Infra $g^{\#}\alpha$-closed, we have, $picp(A) \subseteq icp_{\alpha}(A) \subseteq U$. Then $picp(A) \subseteq U$. Hence A is Infra gp-closed set in X.*

Remark 3.15. *The converse of the above theorem need not be true as seen from the following example.*

Example 3.16. *Let $X = \{a,b,c,d\}$ with the topology $\tau = \{X, \phi, \{a\},\{d\}\}$. Let A=$\{a,c,d\}$. Here A is Infra gp-closed set but not a Infra $g^{\#}\alpha$-closed set of (X,τ_{iX}).*

Theorem 3.17. *Every Infra $g^{\#}\alpha$-closed set is Infra αg-closed set.*

Proof: *Let A be a Infra $g^{\#}\alpha$-closed set in X. Let U be Infra open set, such that $A \subseteq U$. Since every Infra open set is Infra g-open and A is Infra $g^{\#}\alpha$-closed, we have, $icp_{\alpha}(A) = A \subseteq U$. Therefore, $icp_{\alpha}(A) \subseteq U$. Hence A is Infra αg-closed set in X.*

Remark 3.18. *The converse of the above theorem need not be true as seen from the following example.*

Example 3.19. *Let $X = \{a,b,c,d\}$ with the topology $\tau = \{X, \phi, \{b\},\{a,b\},\{b,d\}\}$. Let A=$\{a,b,d\}$. Here A is Infra αg- closed set but not a Infra $g^{\#}\alpha$-closed set of (X,τ_{iX}).*

Theorem 3.20. *Every Infra $g^{\#}\alpha$-closed set is Infra gβ-closed set.*

Proof: *Let A be a Infra $g^{\#}\alpha$-closed set in X. Let U be Infra open set, such that $A \subseteq U$. Since every Infra open set is Infra g-open and A is Infra $g^{\#}\alpha$-closed, we have, $\beta icp(A) \subseteq icp_{\alpha}(A) \subseteq U$. Then $\beta icp(A) \subseteq U$. Hence A is Infra gβ-closed set in X.*

Remark 3.21. *The converse of the above theorem need not be true as seen from the following example.*

Example 3.22. *Let $X = \{a,b,c\}$ with the topology $\tau = \{X, \phi, \{a\},\{b\}\}$. Let A=$\{a,b\}$. Here A is Infra g$\beta$-closed set but not a Infra $g^{\#}\alpha$-closed set of (X,τ_{iX}).*

Theorem 3.23. *Every Infra $g^{\#}\alpha$-closed set is Infra gb-closed set.*

Proof: *Let A be a Infra $g^{\#}\alpha$-closed set in X. Let U be Infra open set, such that $A \subseteq U$. Since* *every Infra open set is Infra g-open and A is Infra $g^{\#}\alpha$-closed, we have, $icp_b(A) \subseteq icp_\alpha \subseteq U$. Then $icp_b(A) \subseteq U$. Hence A is Infra gb-closed set in X.*

Remark 3.24. *The converse of the above theorem need not be true as seen from the following example.*

Example 3.25. *Let $X = \{a,b,c\}$ with the topology τ = $\{X, \phi, \{b\},\{c\}, \{a,b\}\}$. Let A=$\{b,c\}$. Here A is Infra gb-closed set but not a Infra $g^{\#}\alpha$-closed set of (X,τ_{iX}).*

Theorem 3.26. *Every Infra $g^{\#}\alpha$-closed set is Infra g^*b-closed set.*

Proof: Let A be a Infra $g^{\#}\alpha$-closed set in X. Let U be Infra g-open set, such that $A \subseteq U$. Since A is Infra $g^{\#}\alpha$-closed, we have, $icp_b(A) \subseteq icp_\alpha \subseteq U$. Then $icp_b(A) \subseteq U$. Hence A is Infra g^*b-closed set in X.

Remark 3.27. *The converse of the above theorem need not be true as seen from the following example.*

Example 3.28. *Let $X = \{a,b,c\}$ with the topology τ = $\{X, \phi, \{b\},\{c\}, \{a,c\}\}$. Let A=$\{b\}$. Here A is Infra g^*b-closed set but not a Infra $g^{\#}\alpha$-closed set of (X,τ_{iX}).*

Theorem 3.29. *Every Infra $g^{\#}\alpha$-closed set is Infra gsp-closed set.*

Proof: Let A be a Infra $g^{\#}\alpha$-closed set in X. Let U be Infra open set, such that $A \subseteq U$. Since every Infra open set is Infra g-open and A is Infra $g^{\#}\alpha$-closed, we have, $\beta icp(A) \subseteq icp_\alpha(A) \subseteq U$. Then $\beta icp(A) \subseteq U$. Hence A is Infra gsp-closed set in X.

Remark 3.30. *The converse of the above theorem need not be true as seen from the following example.*

Example 3.31. *Let $X = \{a,b,c\}$ with the topology τ = $\{X, \phi, \{a\},\{b\}, \{b,c\}\}$. Let A=$\{a,b\}$. Here A is Infra gsp-closed set but not a Infra $g^{\#}\alpha$-closed set of (X,τ_{iX}).*

Theorem 3.32. *Let $A \subseteq X$. If A is Infra $g^{\#}\alpha$-closed in (X,τ_{iX}), then $icp_\alpha(A)$ - A contains no non-empty Infra g-closed set.*

Proof: Let F be any Infra g-closed set such that $F \subseteq icp_\alpha(A) - A$. Then $A \subseteq X - F$ and $X - F$

is Infra g-open in (X, τ). Since A is Infra $g^{\#}\alpha$-closed in X, $icp_{\alpha}(A) \subseteq X - F$, therefore $F \subseteq X - icp_{\alpha}(A)$. Thus $F \subseteq (icp_{\alpha}(A) - A) \cap (X - icp_{\alpha}(A)) = \phi$.

Theorem 3.33. *Let A be any Infra $g^{\#}\alpha$-closed set in (X, τ_{iX}). If $A \subseteq B \subseteq icp_{\alpha}(A)$, then B is also a Infra $g^{\#}\alpha$-closed set.*
Proof: Let $B \subseteq U$ where U is Infra $g^{\#}\alpha$-open (X, τ). Then $A \subseteq U$. Also since A is Infra $g^{\#}\alpha$-closed, $icp_{\alpha}(A) \subseteq U$. Since $B \subseteq icp_{\alpha}(A)$, $icp_{\alpha}(B) \subseteq icp_{\alpha}(A) \subseteq U$. This implies, $icp_{\alpha}(B) \subseteq U$. Thus B is a Infra $g^{\#}\alpha$-closed set.

Theorem 3.34. *If A and B are Infra $g^{\#}\alpha$-closed, then $A \cap B$ is Infra $g^{\#}\alpha$-closed set.*
Proof: Given that A and B are Infra $g^{\#}\alpha$-closed sets in X. Let $A \cap B \subseteq U$, U is Infra g-open set in X. Since A is Infra $g^{\#}\alpha$-closed, $icp_{\alpha}(A) \subseteq U$, whenever $A \subseteq U$, U is Infra g-open in X. Since B is Infra $g^{\#}\alpha$-closed, $icp_{\alpha}(B) \subseteq U$, whenever $B \subseteq U$, U is Infra $g^{\#}\alpha$-open in X. By the fact[9], *$icp_{\alpha}(A \cap B) = icp_{\alpha}(A) \cap icp_{\alpha}(B)$. It follows that $icp_{\alpha}(A \cap B) \subseteq U$, whenever $A \cap B \subseteq U$, U is Infra g-open in X. Hence $A \cap B$ is Infra $g^{\#}\alpha$-closed.*

Example 3.35. *Let X = $\{a, b, c, d\}$ with the topology $\tau = \{X, \phi, \{b\}, \{a, b\}, \{b,d\}\}$. Let A=$\{a,d\}$, B=$\{c,d\}$ are Infra $g^{\#}\alpha$- closed set. Then $A \cap B = \{d\}$ is also an Infra $g^{\#}\alpha$-closed set.*

Theorem 3.36. *If $A \subseteq Y \subseteq X$ and A is Infra $g^{\#}\alpha$-closed in X then A is Infra$g^{\#}\alpha$-closed relative to Y.*
Proof: Given that $A \subseteq Y \subseteq X$ and A is a Infra $g^{\#}\alpha$-closed set in X. We have to prove that A is Infra $g^{\#}\alpha$-closed set relative to Y. Let us assume that $A \subseteq Y \cap U$, where U is Infra g-open in X. Since, A is Infra $g^{\#}\alpha$-closed set, $A \subseteq U$, which implies $icp_{\alpha}(A) \subseteq U$. From this, we get $Y \cap icp_{\alpha}(A) \subseteq Y \cap U$. Hence, A is Infra $g^{\#}\alpha$-closed set relative to Y.

4. PROPERTIES OF INFRA $g^{\#}\alpha$-CONTINUOUS FUNCTIONS

In this section we set forth the concept of Infra $g^{\#}\alpha$-continuous function. The relationship between Infra $g^{\#}\alpha$- continuous function and other defined Infra continuous functions are explored.

Definition 4.1. *Let f: $(X,\tau_{iX}) \to (Y,\tau_{iX})$ be a Infra topological space X into a Infra topological space Y is called $g^{\#}\alpha$-continuous, if the inverse image of every Infra closed set in Y is Infra $g^{\#}\alpha$- closed set in X.*

Theorem 4.2. *If a map f:$(X,\tau_{iX}) \to (Y,\tau_{iX})$ from a Infra topological space X into a Infra topological space Y is Infra continuous, then it is Infra $g^{\#}\alpha$-continuous.*

Proof: Let f: $(X,\tau_{iX}) \to (Y,\tau_{iX})$ be Infra continuous. Let F be any Infra closed set in Y. Then the inverse image $f^{-1}(F)$ is Infra closed in X. Since, every Infra closed set is Infra $g^{\#}\alpha$- closed set, thus $f^{-1}(F)$ is Infra $g^{\#}\alpha$- closed in X. Hence f is Infra $g^{\#}\alpha$- continuous.

Remark 4.3. *The converse of the above theorem need not be true as seen from the following example.*

Example 4.4. *Let X = Y = $\{a, b, c, d\}$ with the Infra topologies $\tau = \{X, \phi, \{a\}, \{a,b\}, \{a,d\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{a,b,d\}\}$, with the identity mapping. Then for the closed set F = $\{c\}$ in Y, $f^{-1}(\{c\}) = \{c\}$ implies f is not Infra continuous, since $f^{-1}(\{c\})$ is not Infra closed in X.*

Theorem 4.5. *If a map f:$(X,\tau_{iX}) \to (Y,\tau_{iX})$ from a Infra topological space X into a Infra topological space Y is Infra continuous, then it is Infra g-continuous.*

Proof: Let f: $(X,\tau_{iX}) \to (Y,\tau_{iX})$ be Infra continuous. Let F be any Infra closed set in Y. Then the inverse image $f^{-1}(F)$ is Infra closed in X. Since, every Infra closed set is Infra g-closed set, thus $f^{-1}(F)$ is Infra g-closed in X. Hence f is Infra g-continuous.

Remark 4.6. *The converse of the above theorem need not be true as seen from the following example.*

Example 4.7. *Let X = Y = $\{a, b, c, d\}$ with the Infra topologies $\tau = \{X, \phi, \{a\}, \{a,b\}, \{a,d\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{d\}\}$, with the identity mapping. Then for the closed set F = $\{a,b,c\}$ in Y, $f^{-1}(\{a,b,c\}) = \{a,b,c\}$ implies f is not Infra continuous, since $f^{-1}(\{a,b,c\})$ is not Infra closed in X.*

Theorem 4.8. *If a map f:$(X,\tau_{iX}) \to (Y,\tau_{iX})$ from a Infra topological space X into a Infra topological space Y is Infra α-continuous, then it is Infra $g^{\#}\alpha$-continuous.*

Proof: *Let f: $(X,\tau_{iX}) \to (Y,\tau_{iX})$ be Infra α-continuous. Let F be any Infra α-closed set in Y. Then the inverse image $f^{-1}(F)$ is Infra α-closed in X. Since, every Infra α-closed set is Infra $g^{\#}\alpha$- closed, thus $f^{-1}(F)$ is Infra $g^{\#}\alpha$- closed in X. Hence f is Infra $g^{\#}\alpha$- continuous.*

Remark 4.9. *The converse of the above theorem need not be true as seen from the following example.*

Example 4.10. *Let X = Y = $\{a, b, c, d\}$ with the Infra topologies $\tau = \{X, \phi, \{a\}, \{a,b\}, \{a,d\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{d\}\}$, with the mapping defined by f(a) = a, f(b) = b, f(c) = c, f(d) = d. For the closed set F = $\{a,b,c\}$ in Y, $f^{-1}(\{a,b,c\}) = \{a,b,c\}$ implies f is not Infra α-continuous, since $f^{-1}(\{a,b,c\})$ is not Infra α-closed in X.*

Theorem 4.11. *If a map f:$(X,\tau_{iX}) \to (Y,\tau_{iX})$ from a Infra topological space X into a Infra topological space Y is Infra $g^{\#}\alpha$-continuous, then it is Infra gs-continuous.*

Proof: *Let f: $(X,\tau_{iX}) \to (Y,\tau_{iX})$ be Infra $g^{\#}\alpha$-continuous. Let F be any Infra $g^{\#}\alpha$-closed set in Y. Then the inverse image $f^{-1}(F)$ is Infra $g^{\#}\alpha$-closed in X. Since, every Infra $g^{\#}\alpha$-closed set is Infra gs-closed, thus $f^{-1}(F)$ is Infra gs-closed in X. Hence f is Infra gs-continuous.*

Remark 4.12. *The converse of the above theorem need not be true as seen from the following example.*

Example 4.13. *Let X = Y = $\{a, b, c, d\}$ with the Infra topologies $\tau = \{X, \phi, \{a\},\{d\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{a,b\}, \{a,d\}\}$, with the identity mapping. For the closed set F = $\{c,d\}$ in Y, $f^{-1}(\{c,d\}) = \{c,d\}$ implies f is not Infra $g^{\#}\alpha$-continuous, since $f^{-1}(\{c,d\})$ is not Infra $g^{\#}\alpha$-closed in X.*

Theorem 4.14. *If a map f:$(X,\tau_{iX}) \to (Y,\tau_{iX})$ from a Infra topological space X into a Infra topological space Y is Infra $^{\#}g\alpha b$-continuous, then it is Infra gp-continuous.*

Proof: *Let f: $(X,\tau_{iX}) \to (Y,\tau_{iX})$ be Infra $g^{\#}\alpha$-continuous. Let F be any Infra $g^{\#}\alpha$-closed set in Y. Then the inverse image $f^{-1}(F)$ is Infra $g^{\#}\alpha$-closed in X. Since, every Infra $g^{\#}\alpha$-closed set is Infra gp-closed, thus $f^{-1}(F)$ is Infra gp-closed in X. Hence f is Infra gp-continuous.*

Remark 4.15. *The converse of the above theorem need not be true as seen from the following example.*

Example 4.16. *Let X = Y = $\{a, b, c\}$ with the Infra topologies $\tau = \{X, \phi, \{a\}, \{b\}, \{b,c\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{c\}\}$, with the mapping defined by f(a) = a, f(b) = b, f(c) = c. For the Infra closed set F = $\{a,b\}$ in Y, $f^{-1}(\{a,b\}) = \{a,b\}$ implies f is not Infra $g^{\#}\alpha$-continuous, since $f^{-1}(\{a,b\})$ is not Infra $g^{\#}\alpha$-closed in X.*

Theorem 4.17. *If a map f:$(X,\tau_{iX}) \to (Y,\tau_{iX})$ from a Infra topological space X into a Infra topological space Y is Infra $g^{\#}\alpha$-continuous, then it is Infra αg-continuous.*

Proof: Let f: $(X,\tau_{iX}) \to (Y,\tau_{iX})$ be Infra $g^{\#}\alpha$-continuous. Let F be any Infra $g^{\#}\alpha$-closed set in Y. Then the inverse image $f^{-1}(F)$ is Infra $g^{\#}\alpha$-closed in X. Since, every Infra $g^{\#}\alpha$-closed set is Infra αg-closed, thus $f^{-1}(F)$ is Infra αg-closed in X. Hence f is Infra αg-continuous.

Remark 4.18. *The converse of the above theorem need not be true as seen from the following example.*

Example 4.19. *Let X = Y = $\{a, b, c\}$ with the Infra topologies $\tau = \{X, \phi, \{b\}, \{c\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b\}\}$, with the mapping defined by f(a) = a, f(b) = b, f(c) = c. Then for the closed set F = $\{b,c\}$ in Y, $f^{-1}(\{b,c\}) = \{b,c\}$ implies f is not Infra $g^{\#}\alpha$-continuous, since $f^{-1}(\{b,c\})$ is not Infra $g^{\#}\alpha$-closed in X.*

Theorem 4.20. *If a map f:$(X,\tau_{iX}) \to (Y,\tau_{iX})$ from a Infra topological space X into a Infra topological space Y is Infra $g^{\#}\alpha$-continuous, then it is Infra $g\beta$-continuous.*

Proof: Let f: $(X,\tau_{iX}) \to (Y,\tau_{iX})$ be Infra $g^{\#}\alpha$-continuous. Let F be any Infra $g^{\#}\alpha$-closed set in Y. Then the inverse image $f^{-1}(F)$ is Infra $g^{\#}\alpha$-closed in X. Since, every Infra $g^{\#}\alpha$-closed set is Infra $g\beta$-closed, thus $f^{-1}(F)$ is Infra $g\beta$-closed in X. Hence f is Infra $g\beta$-continuous.

Remark 4.21. *The converse of the above theorem need not be true as seen from the following example.*

Example 4.22. *Let X = Y = $\{a, b, c,d\}$ with the Infra topologies $\tau = \{X, \phi, \{b\}, \{a,b\}, \{b,d\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{a,b\}, \{a,d\}\}$, with the mapping defined by f(a) = a, f(b) = b, f(c) = c, f(d) = d. Then for the closed set F = $\{b,c\}$ in Y, $f^{-1}(\{b,c\}) = \{b,c\}$ implies f is not Infra $g^{\#}\alpha$-continuous, since $f^{-1}(\{b,c\})$ is not Infra $g^{\#}\alpha$-closed in X.*

Theorem 4.23. *If a map f:$(X,\tau_{iX}) \to (Y,\tau_{iX})$ from a Infra topological space X into a Infra topological space Y is Infra $g^{\#}\alpha$-continuous, then it is Infra gb-continuous.*

Proof: Let f: $(X,\tau_{iX}) \to (Y,\tau_{iX})$ be Infra $g^{\#}\alpha$-continuous. Let F be any Infra $g^{\#}\alpha$-closed set in Y. Then the inverse image $f^{-1}(F)$ is Infra $g^{\#}\alpha$-closed in X. Since, every Infra $g^{\#}\alpha$-closed set is Infra gb-closed, thus $f^{-1}(F)$ is Infra gb-closed in X. Hence f is Infra gb-continuous.

Remark 4.24. *The converse of the above theorem need not be true as seen from the following example.*

Example 4.25. *Let X = Y = $\{a, b, c, d\}$ with the Infra topologies $\tau = \{X, \phi, \{b\}, \{a,b\}, \{b,d\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{a,b\},\{a,d\}\}$, with the mapping defined by f(a) = a, f(b) = b, f(c) = c, f(d) = d. Then for the closed set F = $\{b,c\}$ in Y, $f^{-1}(\{b,c\}) = \{b,c\}$ implies f is not Infra $g^{\#}\alpha$-continuous, since $f^{-1}(\{b,c\})$ is not Infra $g^{\#}\alpha$-closed in X.*

Theorem 4.26. *If a map f:$(X,\tau_{iX}) \to (Y,\tau_{iX})$ from a Infra topological space X into a Infra topological space Y is Infra $g^{\#}\alpha$-continuous, then it is Infra $g^{*}b$-continuous.*

Proof: Let f: $(X,\tau_{iX}) \to (Y,\tau_{iX})$ be Infra $g^{\#}\alpha$-continuous. Let F be any Infra $g^{\#}\alpha$-closed set in Y. Then the inverse image $f^{-1}(F)$ is Infra $g^{\#}\alpha$-closed in X. Since, every Infra $g^{\#}\alpha$-closed set is Infra $g^{*}b$-closed, thus $f^{-1}(F)$ is Infra $g^{*}b$-closed in X. Hence f is Infra $g^{*}b$-continuous.

Remark 4.27. *The converse of the above theorem need not be true as seen from the following example.*

Example 4.28. *Let X = Y = $\{a, b, c, d\}$ with the Infra topologies $\tau = \{X, \phi, \{b\}, \{a\}, \{d\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{a,b\},\{a,d\}\}$, with the mapping defined by f(a) = a, f(b) = b, f(c) = c, f(d) = d. Then for the closed set F = $\{c,d\}$ in Y, $f^{-1}(\{c,d\}) = \{c,d\}$ implies f is not Infra $g^{\#}\alpha$-continuous, since $f^{-1}(\{c,d\})$ is not Infra $g^{\#}\alpha$-closed in X.*

Theorem 4.29. *If a map f:$(X,\tau_{iX}) \to (Y,\tau_{iX})$ from a Infra topological space X into a Infra topological space Y is Infra $g^{\#}\alpha$-continuous, then it is Infra gsp-continuous.*

Proof: Let f: $(X,\tau_{iX}) \to (Y,\tau_{iX})$ be Infra $g^{\#}\alpha$-continuous. Let F be any Infra $g^{\#}\alpha$-closed set in Y. Then the inverse image $f^{-1}(F)$ is Infra $g^{\#}\alpha$-closed in X. Since, every Infra $^{\#}g\alpha b$-closed set is Infra gsp-closed, thus $f^{-1}(F)$ is Infra gsp-closed in X. Hence f is Infra gsp-continuous.

Remark 4.30. *The converse of the above theorem need not be true as seen from the following example.*

Example 4.31. *Let X = Y = $\{a, b, c\}$ with the Infra topologies $\tau = \{X, \phi, \{a\}, \{c\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b\}\}$, with the mapping defined by f(a) = a, f(b) = b, f(c) = c. Then for the closed set F = $\{a,c\}$ in Y, $f^{-1}(\{a,c\}) = \{a,c\}$ implies f is not Infra $g^{\#}\alpha$-continuous, since $f^{-1}(\{a,c\})$ is not Infra $g^{\#}\alpha$-closed in X.*

Theorem 4.32. *If a map f: $(X,\tau_{iX}) \rightarrow (Y,\tau_{iX})$ from a Infra topological space X into a Infra topological space Y, then the following statements are equivalent.*

(1) f is Infra $g^{\#}\alpha$-continuous.

(2) The inverse image of each Infra open set in Y is Infra $g^{\#}\alpha$-open in X.

Proof: Assume that f: $(X,\tau_{iX}) \rightarrow (Y,\tau_{iX})$ be Infra $g^{\#}\alpha$-continuous. Let G be Infra open in Y. Then G^c is Infra closed in Y. Since f is Infra $g^{\#}\alpha$-continuous, $f^{-1}(G^c)$ is Infra $g^{\#}\alpha$-closed in X. But $f^{-1}(G^c)$ = X - $f^{-1}(G)$. Thus X - $f^{-1}(G)$ is Infra $g^{\#}\alpha$-closed in X and so $f^{-1}(G)$ is Infra $g^{\#}\alpha$-open in X. Therefore (i) implies (ii).

Conversely assume that the inverse image of each Infra open set in Y is Infra $g^{\#}\alpha$-open in X. Let F be any Infra closed set in Y. The F^c is Infra open in Y. By assumption, $f^{-1}(F^c)$ is Infra $g^{\#}\alpha$-open in X. But $f^{-1}(F^c)$ = X - $f^{-1}(F)$. Thus X - $f^{-1}(F)$ is Infra $g^{\#}\alpha$-open in X and so $f^{-1}(F)$ is Infra $g^{\#}\alpha$-closed in X. Therefore f is Infra $g^{\#}\alpha$- continuous. Hence (ii) implies (i). Thus (i) and (ii) are equivalent.

Theorem 4.33. *If f: $(X,\tau_{iX}) \rightarrow (Y,\tau_{iX})$ and g: $(Y,\tau_{iX}) \rightarrow (Z,\tau_{iX})$ be any two functions, then gof: $(X,\tau_{iX}) \rightarrow (Z,\tau_{iX})$ is Infra $g^{\#}\alpha$-continuous and f is Infra $g^{\#}\alpha$-continuous.*

Proof: Let V be any Infra closed set in Z. Since g is Infra continuous, $g^{-1}(V)$ is Infra closed in Y and since f is Infra $g^{\#}\alpha$-continuous, $f^{-1}(g^{-1}(V))$ is Infra $g^{\#}\alpha$-closed in X. Hence $(gof)^{-1}(V)$ is Infra $g^{\#}\alpha$-closed in X. Thus gof is Infra $g^{\#}\alpha$-continuous.

5. PROPERTIES OF INFRA $g^{\#}\alpha$-IRRESOULUTE MAPS

In this section we set forth the concept of $g^{\#}\alpha$-irresolute function. The relationship between Infra $g^{\#}\alpha$- irresolute function and other defined Infra irresolute functions are explored.

Definition 5.1. *Let f: $(X,\tau_{iX}) \to (Y,\tau_{iX})$ be a Infra topological space X into a Infra topological space Y is called $g^{\#}\alpha$-irresolute, if the inverse image of every Infra $g^{\#}\alpha$- closed set in Y is Infra $g^{\#}\alpha$- closed set in X.*

Theorem 5.2. *A map f: $(X,\tau_{iX}) \to (Y,\tau_{iX})$ is Infra $g^{\#}\alpha$-irresolute if and only if the inverse image of every Infra $g^{\#}\alpha$-open set in Y is Infra $g^{\#}\alpha$-open in X.*

Proof: Assume that f is Infra $g^{\#}\alpha$-irresolute. Let A be any Infra $g^{\#}\alpha$-open set in Y. Then A^c is Infra $g^{\#}\alpha$-closed set in Y. Since f is Infra $g^{\#}\alpha$-irresolute, $f^{-1}(A^c)$ is Infra $g^{\#}\alpha$-closed in X. But $f^{-1}(A^c)$ = X - $f^{-1}(A)$ and so $f^{-1}(A)$ is Infra $g^{\#}\alpha$-open in X. Hence the inverse image of every Infra $g^{\#}\alpha$-open set in Y is Infra $g^{\#}\alpha$-open set in X.

Conversely, assume that the inverse image of every Infra $g^{\#}\alpha$-open set in Y is Infra $g^{\#}\alpha$-open in X. Let A be any Infra $g^{\#}\alpha$-closed set in Y. Then A^c is Infra $g^{\#}\alpha$-open in Y. By assumption, $f^{-1}(A^c)$ is Infra $g^{\#}\alpha$-open in X. But $f^{-1}(A^c)$ = X - $f^{-1}(A)$ and so $f^{-1}(A)$ is Infra $g^{\#}\alpha$-closed in X. Therefore f is Infra $g^{\#}\alpha$-irresolute.

Theorem 5.3. *If a map f: $(X,\tau_{iX}) \to (Y,\tau_{iX})$ is Infra $g^{\#}\alpha$-irresolute, then it is Infra $g^{\#}\alpha$-continuous.*

Proof: Assume that f is Infra $g^{\#}\alpha$-irresolute. Let F be any Infra closed set in Y. Since every Infra closed set is Infra $g^{\#}\alpha$- closed, F is Infra $g^{\#}\alpha$-closed in Y. Since f is Infra $g^{\#}\alpha$-irresolute, $f^{-1}(F)$ is Infra $g^{\#}\alpha$-closed in X. Therefore f is Infra $g^{\#}\alpha$-continuous.

Remark 5.4. *The converse of the above theorem need not be true as seen from the following example.*

Example 5.5. *Let X = Y = $\{a, b, c, d\}$ with the Infra topologies $\tau = \{X, \phi, \{a\}, \{a,b,d\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{a,b\}, \{a,d\}\}$, with the identity mapping. Here f is Infra $g^{\#}\alpha$-continuous. But f is not Infra $g^{\#}\alpha$-irresolute, since for the closed set F = $\{a, b\}$ in Y implies, $f^{-1}(\{a,b\}) = \{a,b\}$ is not Infra $g^{\#}\alpha$-closed in X.*

Theorem 5.6. *Let X, Y and Z be any Infra topological spaces. For any Infra $g^{\#}\alpha$-irresolute map f: $(X,\tau_{iX}) \to (Y,\tau_{iX})$ and any Infra $g^{\#}\alpha$-continuous map g: $(Y,\tau_{iX}) \to (Z,\tau_{iX})$ the composition gof: $(X,\tau_{iX}) \to (Z,\tau_{iX})$ is Infra $g^{\#}\alpha$-continuous.*

Proof: Let F be any Infra closed set in Z. Since g is Infra $g^{\#}\alpha$-continuous, $g^{-1}(F)$ is Infra $g^{\#}\alpha$-closed in Y. Since f is Infra $g^{\#}\alpha$-irresolute, $f^{-1}(g^{-1}(F))$ is Infra $g^{\#}\alpha$-closed in X. But $f^{-1}(g^{-1}(F)) = (gof)^{-1}(F)$. Therefore gof: $(X,\tau_{iX}) \rightarrow (Z,\tau_{iX})$ is Infra $g^{\#}\alpha$-continuous.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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