Available online at http://scik.org
J. Math. Comput. Sci. 11 (2021), No. 6, 7624-7633
https://doi.org/10.28919/jmcs/6634
ISSN: 1927-5307

SOLVING NECROTIZING ENTEROCOLITIS MODEL BY THE TAYLOR SERIES METHOD

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Abstract. The paper represents the convergence analytical solution for the mathematical model of necrotizing enterocolitis using the Taylor series method. This method is simple with solve approximately to the nonlinear equations.

Keywords: Taylor series; analytical method; non-linear equation; numerical simulation.

2010 AMS Subject Classification: 42A10, 34A34, 41A60, 65K05.

1. INTRODUCTION

Most of the mathematical modelling in chemical engineering, environment engineering, biological science, etc., are represented in nonlinear equation form and not keeing exact solution in numerically and analytically. Many researchers' derived the approximate solution for the nonlinear differential equation using asymptotic methods such as Homotopy perturbation method (HPM), Homotopy Analysis method (HAM), He's Variational iteration method(VIM), Adomian decomposition method(ADM), Taylor series method (TSM) and so on [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15]; the derived solution can be useful for the system

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Received August 11, 2021

behaviour. In the asymptotic method, Taylor series method is one of the most powerful methods compared to other method, and many researchers cannot apply to solve nonlinear method because Taylor series method process based on the derivative and useful for calculating the entire function value at any point. In this paper, we can approach Taylor series method to solve the necrotizing enterocolitis model and determine the error calculation. From this process, we conclude that the Taylor series method is one of the fast convergence methods for solving in nonlinear differential equations.

2. Taylor Series Solution to Necrotizing Enterocolitis Model

In this paper, we have solved the following system of nonlinear differential equations in necrotizing enterocolitis model [13].

$$\frac{\partial X}{\partial t} = -\mu X + K_X \frac{X^2}{(1+X^2)(1+Y)} \tag{1}$$

$$\frac{\partial Y}{\partial t} = -\gamma Y + K_Y \frac{X^2}{(1+X^2)(1+Y)} \tag{2}$$

with initial condition

$$t = 0\,;\ \ X(t) = x1\ \ and\ \ Y(t) = y1 \tag{3}$$

To solve the system of the nonlinear equation as follow, Eqns.(1) and (2) can be written as given below form.

$$(1+X^2)(1+Y)\frac{dX}{dt} = -\mu X + k_X X^2 \tag{4}$$

$$(1+X^2)(1+Y)\frac{dY}{dt} = -\gamma X + k_Y X^2 \tag{5}$$

Differentiating Eqns. (4) and (5) with respect to t and putting t =0, we obtain

$$X'(0) = -\mu X_1 + k_X \frac{x_1^2}{(1+X_1^2)(1+Y_1)} \tag{6}$$

$$Y'(0) = -\gamma Y_1 + k_Y \frac{X_1^2}{(1+X_1^2)(1+Y_1)} \tag{7}$$

Continue the same process, we get

$$X"(0) = -\mu X'(0) + k_X \frac{2X_1(1+Y_1)(1+X_1^2) - \left(X_1^2 Y'(0)(1+X_1^2) + 2X_1 X'(0)(1+Y_1)\right)}{(1+X_1^2)^2(1+Y_1)^2}$$

$$Y"(0) = -\gamma Y'(0) + k_Y \frac{2X_1(1+Y_1)(1+X_1^2) - \left(X_1^2 Y'(0)(1+X_1^2) + 2X_1 X'(0)(1+Y_1)\right)}{(1+X_1^2)^2(1+Y_1)^2}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\ldots\ldots\ldots\ldots\ldots\ldots$$

Therefore, The Taylor series method is

$$X(t) = X(0) + \frac{1}{1!}X'(0) + \ldots\ldots \tag{8}$$

$$Y(t) = Y(0) + \frac{1}{1!}Y'(0) + \ldots\ldots \tag{9}$$

We have introduced the error percentage as follows:

$$\%Error = \left|\frac{F(x)_{NM} - F(x_{TSM})}{F(x)_{NM}}\right| \times 100 \tag{10}$$

where $F(x)_{NM}$ and $F(x)_{TSM}$ are the values obtained by numerical method and TSM. Here Eqn.(10) has been applied through functions of Eqns.(8) & (9). F_{NM} has only been defined as the numerical solution (4^{th} order Runge – Kutta method) of the Eqns.(1) and (2) which is obtained from the function pdex1 in MATLAB software.

The relative error between our analytical and numerical results does not exceed 1.5% in the simulation, which is represented in Tables 2.1 and 2.2. Also, we represent the error percentage in Figs. (2.1) and (2.2). Additionally, we have shown that the parameter variation in graph format in Figs.(2.3) and (2.4).

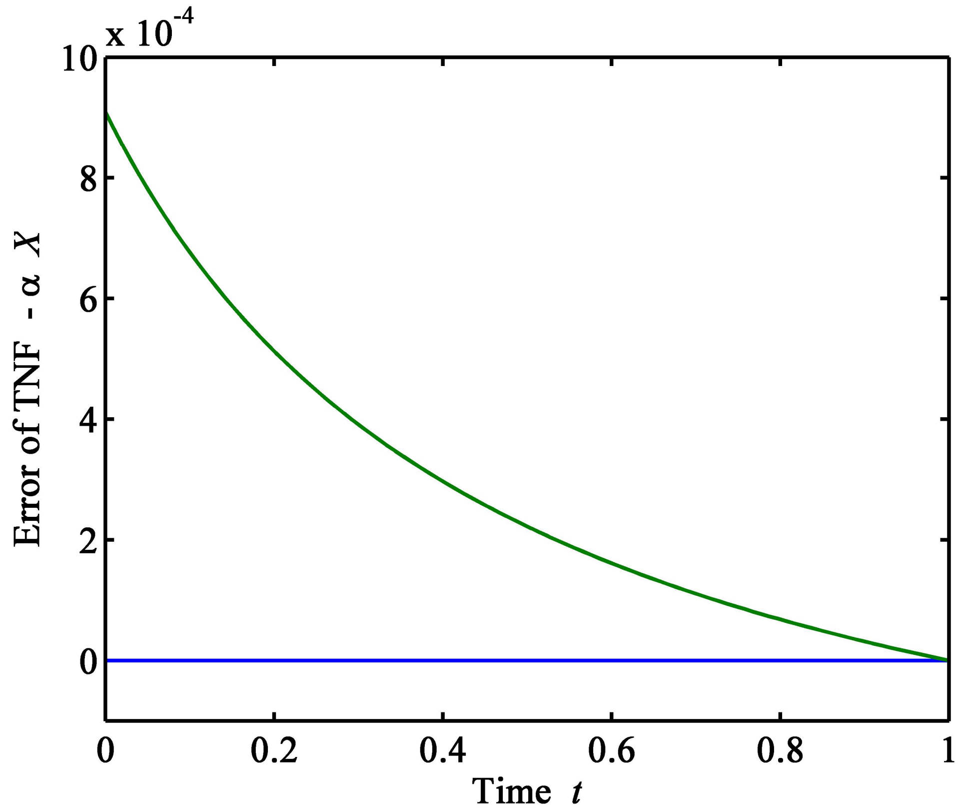

FIGURE 1. Obtained error for TNF - $\alpha\, X$ with time t *when (a)*$\gamma = 0.1$, $\mu = K_X = K_Y = 0.0001$, $x_1 = 10$, $y_1 = 0.1$ *(b)*$\mu = 0.1$, $\gamma = 0.001$, $K_X = K_Y = 10$, $x_1 = y_1 = 1$

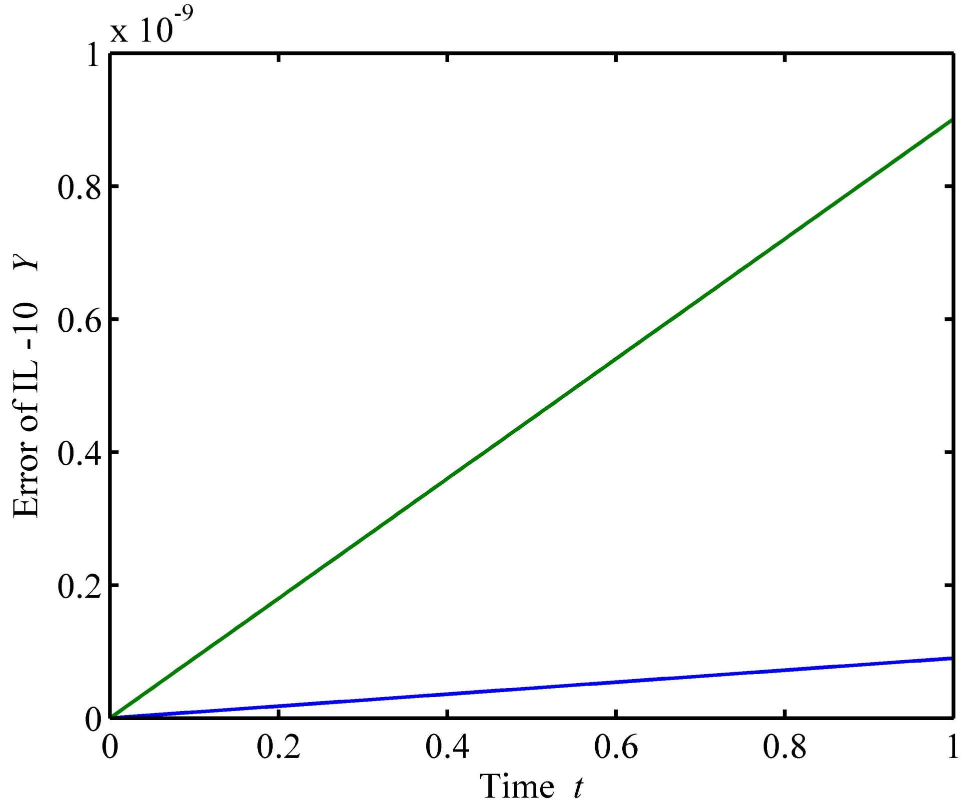

FIGURE 2. Obtained error for IL -10 X with time t *when (a)*$\mu = 0.1,\ \gamma = K_X = 0.001,\ K_Y = 10,\ x_1 = y_1 = 1$ *(b)*$\mu = 0.1,\ \gamma = K_X = K_Y = 0.0001,\ x_1 = 0.1,\ y_1 = 10$.

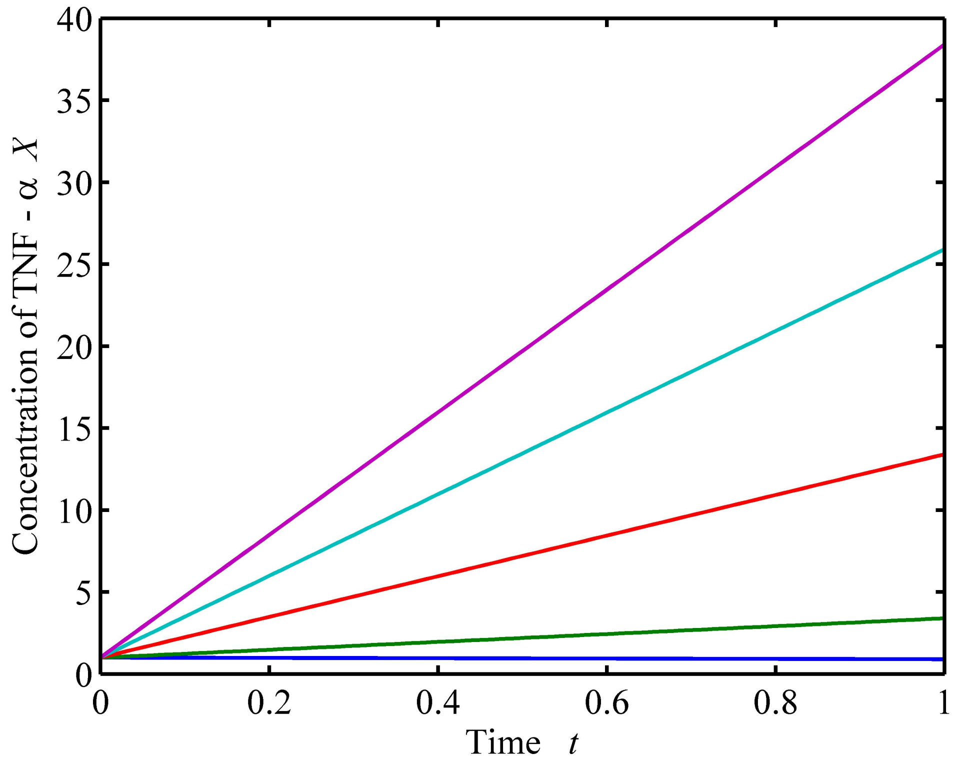

FIGURE 3. Nature for the obtained solution of TNF -$\alpha\, X$ with time t *when* $K_X = 0.001, 10, 50, 100, 150$ *for some fixed parameter* $\mu = \gamma = 0.1$, $K_Y = 0.001$, $x_1 = y_1 = 1$.

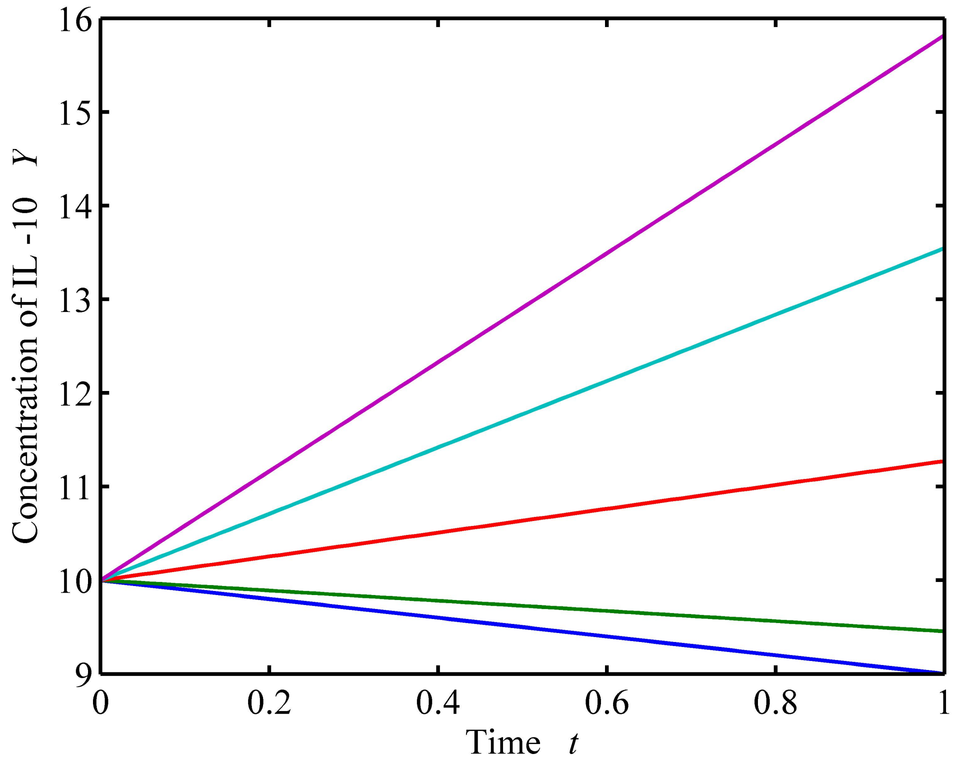

FIGURE 4. Nature for the obtained solution of IL -10 Y with time t *when* $K_y = 0.001, 10, 50, 100, 150$ *for some fixed parameter* $\mu = \gamma = 0.1$, $K_X = 0.001$, $x_1 = 1$, $y_1 = 10$.

μ=0.001, γ=0.1, K_X=0.001, K_Y=0.001, x_1=10, y_1=0.1				μ=0.1, γ=0.001, K_X=10, K_Y=10, x_1=1, y_1=1			
x	Num.	TSM	% of Error	x	Num.	TSM	% of Error
0.0	10.0000	10.0000	0.0000	0.0	1.0000	1.0000	0.0000
0.2	9.9982	9.9982	0.0000	0.2	1.5600	1.5590	0.1560
0.4	9.9964	9.9964	0.0000	0.4	2.0200	2.0100	0.4950
0.6	9.9945	9.9945	0.0000	0.6	2.4800	2.4770	0.1210
0.8	9.9927	9.9927	0.0000	0.8	2.9400	2.9380	0.0680
1.0	9.9909	9.9909	0.0000	1.0	3.4000	3.4000	0.0000
Average Error %			0.0000	Average Error %			0.1400

TABLE 1. Error calculation of TNF - α X with numerical and analytical results.

μ=0.1, γ=0.001, K_X=0.001, K_Y=10, x_1=1, y_1=1				μ=0.1, γ=0.001, K_X=0.001, K_Y=0.001, x_1=0.1, y_1=10			
x	Num.	TSM	% of Error	x	Num.	TSM	% of Error
0.0	1.0000	1.0000	0.0000	0.0	10.0000	10.0000	0.0000
0.2	1.5000	1.4980	0.1334	0.2	9.9980	9.9980	0.0000
0.4	2.0000	1.9960	0.2000	0.4	9.9960	9.9960	0.0000
0.6	2.5000	2.4940	0.2400	0.6	9.9940	9.9940	0.0000
0.8	3.0000	2.9920	0.2667	0.8	9.9920	9.9920	0.0000
1.0	3.5000	3.4900	0.2857	1.0	9.9900	9.9900	0.0000
Average Error %			0.3877	Average Error %			0.0000

TABLE 2. Error calculation of IL - 10 Y with numerical and analytical results.

3. CONCLUSION

In this paper, The Taylor series method has been used for solve the nonlinear differential equation of the necrotizing enterocolitis model. Data from the error figures represent that the obtained solutions with TSM have minor differences with numerical simulations. Furthermore, according to the achieved results, these works are useful to understand the behaviour of the system. In addition, the obtained results are very useful for optimizing the parameters.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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