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J. Math. Comput. Sci. 11 (2021), No. 6, 7832-7843

<https://doi.org/10.28919/jmcs/6676>

ISSN: 1927-5307

HEAT TRANSFER ANALYSIS OF FORCED CONVECTIVE FLOW WITHIN AN ENCLOSURE

VUSALA AMBETHKAR, LAKSHMI RANI BASUMATARY*

Department of Mathematics, University of Delhi, Delhi-110007, India

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Abstract. The paper is to study about the movement of heat analysis with respect to forced convective flow within sideways four-sided enclosure. The mathematical statements of the flow variables are represented using a discrete quantity by virtue of upwind control volume scheme. Non-dimensional parameters are suitably preferred so as to determine \overline{Nu} . This \overline{Nu} for the sake of different fluids are calculated by using a suitable empirical correlation and thereby the transfer of heat between the top wall and forced convective flow of these fluids is examined. It is ascertained that, a substantial rise in \overline{Nu} and hence the transfer of heat within the four-sided enclosure by way of forced convective flow of any fluid substantially increases within the sideways four-sided enclosure. The average Nusselt numbers for fluids such as air and water at different Rayleigh numbers of 1, 10, 100, 1000, 1700 in the range of $Ra \leq 1708$ are calculated and observed to increase linearly.

Keywords: four-sided enclosure; Rayleigh numbers; transfer of heat; control volume; forced convective flow.

2010 AMS Subject Classification: 76R10, 76M12, 80A20.

1. INTRODUCTION

The movement of heat in rectangular enclosures has been analyzed empirically and theoretically owing to an extensive enthusiasm in its numerous engineering applications such as

*Corresponding author

E-mail address: lrbasumatary@gmail.com

Received August 22, 2021

heating and cooling elements in electric-powered and nuclear production companies, building insulation as well as solar energy collection.

Effect of heating and cooling of the vertical walls on natural convective flow inside a rectangle was examined numerically by [1]. Impact of thermal boundary conditions on free convective flow in rectangular enclosures was studied numerically by [3]. Momentum along with exchange of thermal energy as a means to conjugate free convection flow alongside using streamline and heatline was numerically visualized by [4]. Free convective flow in rectangular enclosures by using combined temperature scale was numerically studied by [5].

A mesh having many sided method is used to determine high Reynolds number solutions for 2-D incompressible flow was reported by [6]. In [7] authors have proposed some empirical correlations for Nusselt numbers for different Rayleigh numbers of a natural convection of transfer of heat inside flowing fluids constricted among two sideways layers warmed up originating at underneath. In [8] authors have proposed empirical correlations for average Nusselt number of free convective air and water flow in horizontal rectangular region. The transfer of heat analysis in permeable airfoil in addition to the sample of four-sided enclosure was studied both in analytical and numerical form by Hoseinzadeh et al. [9]. The joint forces of free and forced convective flow of nanofluids within different enclosures was investigated by Lzadi et al. [10]. Pellew and Southwell [11] have investigated convection flow movement in a fluid heated from below. Rasul et al. [13] have examined the result of heat source position on free convective flow over a C-shaped inclusion saturated by a nanofluid in numerical way. Takhar et al. [14] have presented solutions for the problem of merged buoyancy effect of thermal and mass diffusion of moving vertical slender cylinder both analytical and numerically. Ambethkar and Basumatary [16] have investigated the solutions of steady free convection flow inside four-sided region having distinct enclosed temperature and assemblage sources.

An inspiration to the current investigation is that although the transfer of heat within the rectangular enclosure via natural convective flow inside sideways four-sided inclusion has already extensively studied by some investigators, however, the transfer of heat analysis within the four-sided enclosure by way of forced convective flow is still an unattempted task. Therefore, in order to fulfil this task, we are motivated to accomplish it in the present study.

The newness in this work is to examine critically the transfer of heat within the four-sided enclosure due to sole effect of the temperature gradient defined on the upside and downside wall. The average Nusselt numbers (\overline{Nu}) for different fluids are calculated by using suitable empirical correlation and thereby the transfer of heat between the upside wall and forced convective flow of these fluids is examined.

The objective is to discretize the continuity, momentum and heat equations by using the upwind control volume scheme. Movement of heat by virtue of forced convective flow of a fluid (air and water) within the horizontal rectangular enclosure is critically examined.

Problem formulation is presented in section 2. In section 3, we have demonstrated that how the mathematical statements are represented using a discrete quantity by utilizing the upwind control volume scheme [15, 16]. Analysis of results and conclusions from the current study are specified under sections 4 and 5.

2. PROBLEM FORMULATION

2.1. Schematic information. ABCD is a horizontal rectangle of length H and height L in which one fluid substance is heated from the upside wall of enclosure as demonstrated in figure 1. A velocity $\frac{\alpha}{L}$ is given to the bottom wall in positive x -direction while other three walls are stationary with no-slip boundary conditions are defined for velocity on them. T_c and T_h are the temperatures defined on horizontal rectangle.

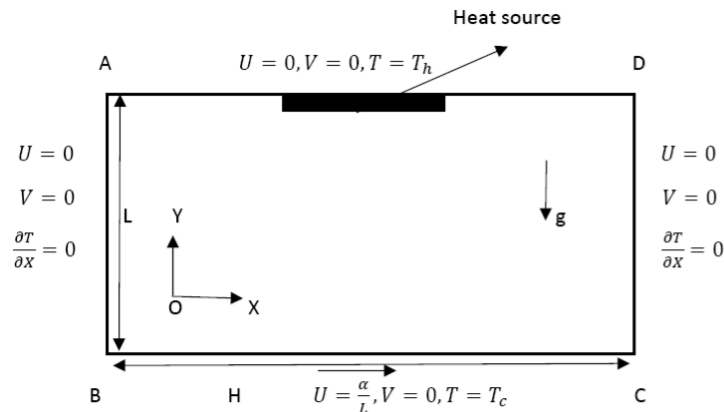


Figure 1. Physical description

2.2. Mathematical statements. The mathematical statements for existing problem in dimensional form are expressed in the following way:

- (1) continuity equation: $\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0,$
- (2) X-momentum equation: $U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{1}{\rho} \frac{\partial p}{\partial X} + \nu \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right),$
- (3) Y-momentum equation: $U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{1}{\rho} \frac{\partial p}{\partial Y} + \nu \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + g\beta_T(T - T_c),$
- (4) energy equation: $U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \alpha \left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} \right).$

where $U, V, X, Y, p, \rho, T, T_c, \alpha, \nu, g$ and β_T are the constituents of velocities through X and Y -axis, independent variables, pressure, density, temperature at the cold wall, thermal diffusivity, kinematic viscosity, standard gravity and coefficient of thermal expansion at constant temperature respectively.

The boundary conditions corresponding to equations (1)–(4) in dimensional form are defined as:

$$(5) \left. \begin{array}{l} \text{on AB: at } X = 0, U = V = 0, \frac{\partial T}{\partial X} = 0, \\ \text{on DC: at } X = H, U = V = 0, \frac{\partial T}{\partial X} = 0, \\ \text{on BC: at } Y = 0, U = \frac{\alpha}{L}, V = 0, T = T_c, \\ \text{on AD: at } Y = L, U = V = 0, T = T_h. \end{array} \right\}$$

where H and L are the length and height of the horizontal rectangle.

The dimensionless variables are set as follows:

$$(x, y) = \frac{(X, Y)}{L}, \quad (u, v) = \frac{(U, V)L}{\alpha}, \quad P = \frac{pL^2}{\rho\alpha^2}, \quad \theta = \frac{T - T_c}{T_h - T_c}, \quad Pr = \frac{\nu}{\alpha},$$

$$Re = \frac{u_0L}{\nu}, \quad Ra = \frac{g\beta_T(T_h - T_c)L^3}{\alpha\nu}.$$

where $x, y, L, u, v, P, \theta, Pr, Re$ and Ra are dimensionless independent variables, height of the rectangle, dimensionless components of velocities along x and y -axis, dimensionless pressure, dimensionless temperature, the Prandtl number, the Reynolds number and the Rayleigh number respectively.

The governing equations (1)–(4) in dimensionless form takes the following form:

$$\begin{aligned}
 (6) \quad & \text{continuity equation:} && \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \\
 (7) \quad & x\text{-momentum equation:} && u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial P}{\partial x} + \frac{1}{Re} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right), \\
 (8) \quad & y\text{-momentum equation:} && u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial P}{\partial y} + \frac{1}{Re} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + RaPr\theta, \\
 (9) \quad & \text{energy equation:} && u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{1}{RePr} \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right).
 \end{aligned}$$

The above boundary conditions in dimensionless form reduces to:

$$(10) \quad \left. \begin{aligned}
 & \text{on AB: at } x = 0, \quad u = v = 0, \quad \frac{\partial \theta}{\partial x} = 0, \\
 & \text{on DC: at } x = 2, \quad u = v = 0, \quad \frac{\partial \theta}{\partial x} = 0, \\
 & \text{on BC: at } y = 0, \quad u = 1, v = 0, \quad \theta = 0, \\
 & \text{on AD: at } y = 1, \quad u = v = 0, \quad \theta = 1.
 \end{aligned} \right\}$$

where AB, DC, BC and AD are the left, right, downside and upside walls of the four-sided enclosure.

3. DISCRETIZATION

3.1. Finite volume discretization. Discretization of continuity, momentum and energy equations is rendered by means of the upwind finite volume scheme [15, p.197-200]. Once momentum and energy equations are discretized, the convective fluxes and diffusive conductances are evaluated at cell faces of the staggered grid figure 2 as given [15, p.196] below.

Continuity equation: The control volume equation at position (I, J) is given by

$$(11) \quad F_e - F_w + F_n - F_s = 0$$

x-momentum: The control volume equation at location (i, J) is

$$(12) \quad a_{i,J} u_{i,J} = \sum a_{nb} u_{nb} + (P_{I-1,J} - P_{I,J}) A_{i,J}$$

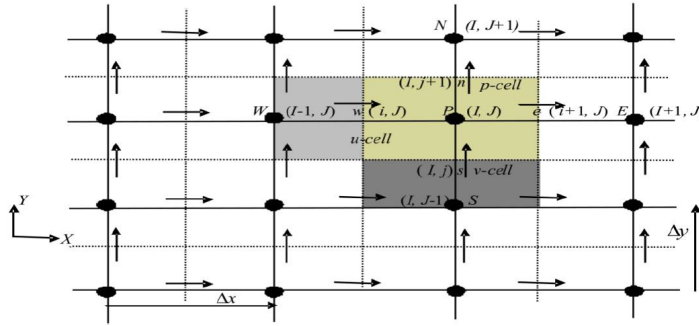


Figure 2. Staggered grid arrangement

where $A_{i,j}$ is area and neighbors of $\sum a_{nb}u_{nb}$ will be $(i+1, j)$, $(i-1, j)$, $(i, j+1)$ and $(i, j-1)$ in congruence with [15, p.197-199]. The coefficients of upwind differencing scheme are

$$(13) \quad a_{i,j} = a_{i-1,j} + a_{i+1,j} + a_{i,j+1} + a_{i,j-1} + \Delta F$$

where

$$\left. \begin{aligned} a_{i+1,j} &= D_{I,J} + \max(-F_{I,J}, 0), & a_{i-1,j} &= D_{I-1,J} + \max(F_{I-1,J}, 0), \\ a_{i,j+1} &= D_{i,j+1} + \max(-F_{i,j+1}, 0), & a_{i,j-1} &= D_{i,j} + \max(F_{i,j}, 0), \\ \Delta F &= (F_e - F_w) + (F_n - F_s) = (F_{I,J} - F_{I-1,J}) + (F_{i,j+1} - F_{i,j}) \end{aligned} \right\}$$

where

$$(14) \quad \left. \begin{aligned} F_e = F_{I,J} &= \frac{F_{i+1,J} + F_{i,J}}{2} = \frac{u_{i+1,J}A_{i+1,J} + u_{i,J}A_{i,J}}{2} \\ F_w = F_{I-1,J} &= \frac{F_{i,J} + F_{i-1,J}}{2} = \frac{u_{i,J}A_{i,J} + u_{i-1,J}A_{i-1,J}}{2} \\ F_n = F_{i,j+1} &= \frac{F_{I,j+1} + F_{I-1,j+1}}{2} = \frac{v_{I,j+1}A_{I,j+1} + v_{I-1,j+1}A_{I-1,j+1}}{2} \\ F_s = F_{i,j} &= \frac{F_{I,j} + F_{I-1,j}}{2} = \frac{v_{I,j}A_{I,j} + v_{I-1,j}A_{I-1,j}}{2} \\ D_e = D_{I,J} &= \frac{A_{I,J}}{Re\Delta x} = \frac{1}{Re\Delta x} \left(\frac{A_{i+1,J} + A_{i,J}}{2} \right) \\ D_w = D_{I-1,J} &= \frac{A_{I-1,J}}{Re\Delta x} = \frac{1}{Re\Delta x} \left(\frac{A_{i,J} + A_{i-1,J}}{2} \right) \\ D_n = D_{i,j+1} &= \frac{A_{i,j+1}}{Re\Delta y} = \frac{1}{Re\Delta y} \left(\frac{A_{I,j+1} + A_{I-1,j+1}}{2} \right) \\ D_s = D_{i,j} &= \frac{A_{i,j}}{Re\Delta y} = \frac{1}{Re\Delta y} \left(\frac{A_{I,j} + A_{I-1,j}}{2} \right) \end{aligned} \right\}$$

y-momentum: The control volume equation at position (I, j) is

$$(15) \quad a_{I,j}v_{I,j} = \sum a_{nb}v_{nb} + (P_{I,J-1} - P_{I,J})A_{I,j} + b_{I,j}$$

where $A_{i,J}$ is area and neighbors of $\sum a_{nb}v_{nb}$ will be $(I+1, j)$, $(I-1, j)$, $(I, j+1)$ and $(I, j-1)$ in congruence with [15, p.199-200].

Now the coefficients of upwind control volume scheme are as follows:

$$(16) \quad a_{I,j} = a_{I+1,j} + a_{I-1,j} + a_{I,j+1} + a_{I,j-1} + \Delta F - S_{I,j}$$

and

$$(17) \quad S_{I,j}v_{I,j} + b_{I,j} = \bar{S}\Delta v$$

where

$$\left. \begin{aligned} a_{I+1,j} &= D_{i+1,j} + \max(-F_{i+1,j}, 0), & a_{I-1,j} &= D_{i,j} + \max(F_{i,j}, 0), \\ a_{I,j+1} &= D_{I,j} + \max(-F_{I,j}, 0), & a_{I,j-1} &= D_{I,J-1} + \max(F_{I,J-1}, 0), \\ \Delta F &= (F_e - F_w) + (F_n - F_s) = (F_{i+1,j} - F_{i,j}) + (F_{I,J} - F_{I,J-1}) \end{aligned} \right\}$$

For v -control volume, the values of F and D , we can determined in a similar way for each of the faces e , w , n and s .

Pressure correction equation as per [15, p.202] reduces to

$$(18) \quad a_{I,J}P'_{I,J} = a_{I+1,J}P'_{I+1,J} + a_{I-1,J}P'_{I-1,J} + a_{I,J+1}P'_{I,J+1} + a_{I,J-1}P'_{I,J-1} + b'_{I,J}$$

where

$$a_{I,J} = a_{I+1,J} + a_{I-1,J} + a_{I,J+1} + a_{I,J-1}$$

and the coefficients are

$$(19) \quad \left. \begin{aligned} a_{I+1,J} &= (dA)_{i+1,J}, & a_{I-1,J} &= (dA)_{i,J}, \\ a_{I,J+1} &= (dA)_{I,j+1}, & a_{I,J-1} &= (dA)_{I,j-1}, \\ d_{i,J} &= \frac{A_{i,J}}{a_{i,J}}, & d_{I,j} &= \frac{A_{I,j}}{a_{I,j}} \\ b'_{I,J} &= (u^*A)_{i,J} - (u^*A)_{i+1,J} + (v^*A)_{I,j} - (v^*A)_{I,j+1} \end{aligned} \right\}$$

The equation (18) represents the discretized continuity equation as an equation of pressure correction P' . The source term b' in this equation is continuity imbalance equation arising from the incorrect velocity field u^* and v^* . By solving equation (19), the pressure correction field P' is obtained at all points.

Energy equation: The control volume equation at (I,J) reduces to

$$(20) \quad a_{I,J}\theta_{I,J} = \sum a_{nb}\theta_{nb}$$

The coefficients of upwind finite volume scheme are

$$(21) \quad a_{I,J} = a_{I+1,J} + a_{I-1,J} + a_{I,J+1} + a_{I,J-1} + \Delta F$$

where

$$\left. \begin{aligned} a_{I+1,J} &= D_{i+1,J} + \max(-F_{i+1,J}, 0), & a_{I-1,J} &= D_{i,J} + \max(F_{i,J}, 0), \\ a_{I,J+1} &= D_{I,j+1} + \max(-F_{I,j+1}, 0), & a_{I,J-1} &= D_{I,j} + \max(F_{I,j}, 0), \\ \Delta F &= (F_e - F_w) + (F_n - F_s) = (F_{i+1,J} - F_{i,J}) + (F_{I,j+1} - F_{I,j}) \end{aligned} \right\}$$

and the values of F and D can also expressed in a similar way as discussed above.

3.2. Solution of flow variables. The numerical solutions are obtained from the discretized equations for appropriate parameters Pr , Re and Ra . We compose an algorithm that enable us to the solve the discretized equations iteratively. Algorithm is summarized as follows:

Algorithm

We have amended original SIMPLE algorithm See [15, p.204] that facilitate to calculate pressure, velocities and temperature in a iterative manner. Main steps involved in this algorithm are summarized as:

1. Guess all flow variables.
2. Solve the equation of momentum for (u^*, v^*) .
3. Solve the equation of pressure-correction for p' .

4. Evaluate P as a sum of P' and P^* . Evaluate both velocities (u, v) from the performed values utilizing the formulas of velocity-correction.

$$(22) \quad \left. \begin{aligned} P_{I,J} &= P_{I,J}^* + P'_{I,J}, \\ u_{i,j} &= u_{i,j}^* + d_{i,j}(P'_{I-1,J} - P'_{I,J}), \\ v_{I,j} &= v_{I,j}^* + d_{I,j}(P'_{I,J-1} - P'_{I,J}). \end{aligned} \right\}$$

5. Interpret temperature discretized equivalence.

6. Using the corrected pressure, velocity and temperature being new guessed pressure, velocity and temperature and go back again in step 2 and continue the whole processor till the solution is obtained.

4. RESULTS AND DISCUSSION

Heat transfer in rectangular enclosures is effected by the tilt angle, aspect ratio and the dimensionless parameters such as the Pr and Ra . A rectangular enclosure is reduced to a horizontal rectangle for a small aspect ratio. In the present case, a horizontal rectangle with the heated top wall and the cooled bottom wall is obtained when the tilt angle is 180° . The objective is to examine critically heat transfer due to sole effect within the rectangle via forced convective flow. In order to accomplish this, since the average Nusselt number (\overline{Nu}) represents the overall convective flow within the rectangular enclosure. The average Nusselt numbers (\overline{Nu}) for different fluids are calculated by using suitable empirical correlation which are tabulated in tables 1 and 2 as given below. It has been noted that for all the boundary values of Ra , the substantial rise in \overline{Nu} and hence the transfer of heat within the four-sided enclosure by way of forced convective flow of a fluid (either air or water) increases substantially. Furthermore, it is pointed out that, the transfer of heat within the enclosure increases gradually.

Table 1. Average Nusselt numbers (\overline{Nu}) for $Pr = 0.7$ at different Ra

Type of fluid	Rayleigh number(Ra)	Average Nusselt numbers (\overline{Nu})
Gas		
air($Pr = 0.7$)	3×10^5	4.49877
	3×10^9	96.923
	4×10^5	4.95153
	5×10^5	5.33388
	6×10^5	5.66809
	7×10^5	5.96695
	7×10^9	128.554

Table 2. Average Nusselt numbers (\overline{Nu}) for $Pr = 7$ at different Ra

Type of fluid	Rayleigh number(Ra)	Average Nusselt numbers (\overline{Nu})
Liquid		
water($Pr = 7$)	3×10^5	5.3345
	3×10^9	114.928
	4×10^5	5.89137
	5×10^5	6.32474
	6×10^5	6.72104
	7×10^5	7.07542
	7×10^9	152.435

Suitable Rayleigh number (Ra) applicable for the case of horizontal rectangle satisfies $Ra \leq 1708$. This \overline{Nu} being different fluids in the vicinity of different Rayleigh numbers of 1, 10, 100, 1000, 1700 in the range of $Ra \leq 1708$ are calculated by using a suitable empirical correlation and are sketched in figures 3 and 4. It has been found from figure 3 that, the average Nusselt numbers for air ($Pr = 0.7$) increases linearly. Similarly, it is observed from figure 4 that, the average Nusselt numbers for water ($Pr = 7$) increases linearly. When $Ra \leq 1708$ (the critical

Rayleigh number [11]), the fluid layer remains stable and hence quiescent. The temperature variation decreases linearly from T_h to T_c . So, for all $Ra \leq 1708$, there is no advection within the rectangular region. When a layer of fluid whose specific volume expands is heated from the upside wall, a producing heat covered field is evolved. Hence, the transfer of heat within the sideways four-sided enclosure exists solely through conduction for liquid (water, $Pr = 7$) and for gas (air, $Pr = 0.7$) through conduction and radiation.

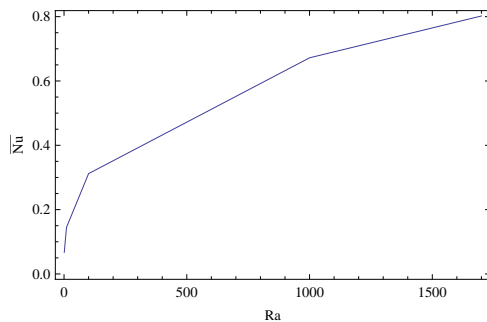


Figure 3. \overline{Nu} for air ($Pr = 0.7$) at $Ra \leq 1708$.

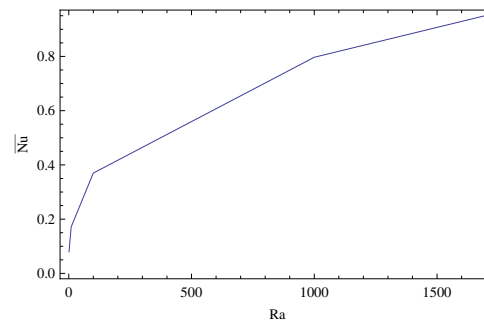


Figure 4. \overline{Nu} for water ($Pr = 7$) at $Ra \leq 1708$.

5. CONCLUSION

From this study, we can conclude that transfer of heat by virtue of forced convective flow of a fluid (air and water) within the sideways four-sided enclosure has been critically examined. It is clear that the mentioned \overline{Nu} increases substantially being non-identical fluids found in distinct Rayleigh numbers. The transfer of heat within the sideways four-sided enclosure by way of forced convective flow increases substantially for any fluid (either air or water).

This \overline{Nu} being different fluids in the vicinity of dissimilar Rayleigh numbers of 1, 10, 100, 1000, 1700 in the range of $Ra \leq 1708$ are increasing linearly. For $Ra \leq 1708$, the fluid layer prevails quiescent, stable and the temperature decreases linearly from the heated top wall to the cooled bottom wall. So, for all those $Ra \leq 1708$, there is no convection within the rectangular region. The heat transfer within the horizontal rectangle exists solely through conduction for liquid (water, $Pr = 7$) and through conduction and radiation for gas (air, $Pr = 0.7$).

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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