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Λ_{rs} - OPEN SETS AND Λ_{rs} - CLOSED SETS IN TOPOLOGICAL SPACES

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Abstract: In [16] Maki has introduced the concept of Λ- sets in topological spaces as the sets that coincide with their kernel. The kernel of a set A is the intersection of all open supersets of A. In this paper we obtain new classes of sets by using regular semi open sets in topological spaces and study their basic properties, and their connections with other kind of topological sets.

Keywords: Λ_{rs}-set; V_{rs}-set; Λ_{rs}-open set; Λ_{rs}-closed set.

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1. INTRODUCTION

In [16] Maki has introduced the concept of Λ- sets in topological spaces as the sets that coincide with their kernel. The kernel of a set A is the intersection of all open supersets of A. Caldas and Dontchev [4] built on Maki's work by introducing Λ_s-sets, V_s-sets, gΛ_s-sets and gV_s-sets using semi open sets and semi closed sets. Ganster et al [10] introduced the notion of pre - Λ- sets and pre - V - sets using pre open sets and pre closed sets. Also M. J. Jeyanthi [13] introduced the concepts of Λ_r- sets and V_r-sets using regular open sets and regular closed sets.

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Georgiou [12] defined and investigated Λ_δ-sets and (Λ,δ) closed sets via δ- open sets and δ- closed sets. Also Caldas [5] introduced Λ_α-sets and (Λ,α) closed sets via α- open sets and α- closed sets. Lellis Thivagar [14] introduced the notions of Λ_a-sets and Λ_a-closed sets via a- open set and a- closed set.

In this paper we introduce the Λ_{rs}- closed set and Λ_{rs}- open set. To define these sets, we are using the set Λ_{rs}- set. These sets are lies between closed sets and semi closed sets and discussed some relationship between other generalized sets.

Throughout this paper, (X, τ) (or simply X) will always represent a topological space on which no separation axioms are assumed, unless otherwise mentioned. When A is a subset of X, cl(A) and Int(A) denote the closure and interior of a set A, respectively. A subset A of a topological space X is said to be semi-regular [8] if it is both semi-open and semi-closed. In [8], it is pointed out that a set is semi-regular if and only if there exists a regular open set U such that U $\subseteq$ A $\subseteq$ cl(U). Cameron [6] called semi regular sets regular semi-open.

2. PRELIMINARIES

Definition 2.1 A subset *A* of a topological space is called:

(1) Semi-open [15] if A $\subseteq$ cl(int(A))

(2) Pre-open [17] if A $\subseteq$ int(cl(A))

(3) b-open [2] if A $\subseteq$ int(cl(A)) $\bigcup$ cl(int(A))

(4) Regular open [20] if A = int(cl(A))

The class of all semi-open (resp. pre -open, b-open and regular open) denoted by SO(X, τ) (resp. PO(X, τ), BO(X, τ) and RO(X, τ))

The complement of these sets called semi-closed (resp. pre-closed, b-closed and regular closed) and the classes of all these sets will be denoted by SC(X, τ) (resp. PC(X, τ), BC(X, τ) and RC(X, τ))

Definition 2.2 A subset *A* of a topological space is called

1) Λ_r-closed [13] if A = T $\bigcap$ C, where T is Λ_r-set and C is closed set.

2) (Λ,b) closed [7] if A = T $\bigcap$ C, where T is Λ_b-set and C is b- closed set.

Definition 2.3 A topological space (X, τ) is said to be locally indiscrete [9] if every open set in it is closed.

Definition 2.4 A subset *A* of a space (X, τ) is called:

(1) Λ -set (resp.$\vee$-set) [16] if it is the intersection (resp, union) of open (resp. closed) sets.

(2) Λ_s -sets (resp. V_s -sets) [4] if it is the intersection (resp. union) of semi-open (resp. semi-closed) sets.

(3) pre-Λ sets (resp. pre-V set) [10] if it is the intersection (resp. union) of pre-open (resp. pre-closed) sets.

Theorem 2.5 [11] every regular semi open set in X is semi open but not conversely.

Theorem 2.6 [11] If A is regular semi open set in X, then X/A is also regular semi open set.

Theorem 2.7 [11] In a space X, the regular closed sets, regular open sets and clopen sets are regular semi open set.

3. Λ_{rs} - SETS AND V_{RS} -SETS

Definition 3.1

Let A be a subset of a topological space (X, τ). We define the sets as follows $\Lambda_{rs}(A)$ and $V_{rs}(A)$ as follows

$$\Lambda_{rs}(A) = \bigcap \{G / G \in RSO(X, \tau) \,\&\, A \subseteq G\}$$

$$V_{rs}(A) = \bigcup \{F / F \in RSC(X, \tau) \,\&\, A \supseteq F\}$$

Example 3.2

Let X ={a, b, c, d}, τ ={∅, X, {a}, {b, c}, {a, b, c}}

RSO(X, τ) = {∅, X, {a},{b, c}, {a, d}, {b, c, d}}

Λ_{rs}(A) = {∅, X, {a},{b, c}, {a, d}, {b, c, d}, {d}}

V_{rs} (A) = {∅, X, {b, c, d}, {a, d}, {b, c}, {a}, {a, b, c}}

Lemma 3.3

For subsets A, B & A_i , i ∈I of a topological space (X, τ), the following properties hold

(i) A $\subseteq \Lambda_{rs}$(A)

(ii) B $\subseteq$ A $\Rightarrow \Lambda_{rs}$(B) $\subseteq \Lambda_{rs}$(A)

(iii) Λ_{rs}(Λ_{rs}(A) = Λ_{rs}(A)

(iv) If A $\in$ RSO (X, τ) then A = Λ_{rs}(A)

(v) $\Lambda_{rs}\left(\bigcup_{i\in I} A_i\right) \supseteq \bigcup_{i\in I} \Lambda_{rs}(A_i)$

(vi) $\Lambda_{rs}\left(\bigcap_{i\in I} A_i\right) \subseteq \bigcap_{i\in I} \Lambda_{rs}(A_i)$

(vii) $\Lambda_{rs}(A^C) = (V_{rs}(A))^C$

Proof

i) Let x $\notin \Lambda_{rs}$(A). Then there exist a regular semi open set G such that A $\subseteq$G and x $\notin$G. Hence x$\notin$A and so A $\subseteq \Lambda_{rs}$(A).

ii) Let x $\notin \Lambda_{rs}$(A). Then there exist a regular semi open set G such that A $\subseteq$G and x $\notin$G. Since, B $\subseteq$ A, B $\subseteq$ G and hence x $\notin \Lambda_{rs}$(B) and so Λ_{rs}(B) $\subseteq \Lambda_{rs}$(A).

iii) From (i) and (ii) we have Λ_{rs}(A)$\subseteq$ Λ_{rs}(Λ_{rs}(A))

For another inclusion, if x$\notin$ Λ_{rs}(A). Then there exist a regular semi open set G such that A $\subseteq$G and x $\notin$G. Hence $\Lambda_{rs}(A) \subseteq G$ and so we have x$\notin \Lambda_{rs}(\Lambda_{rs}(A))$. Therefore Λ_{rs}(A) = Λ_{rs}(Λ_{rs}(A))

iv) By definition & A $\in$ RSO(X,τ), we have $\Lambda_{rs}(A) \subseteq A$.

By (i) we have Λ_{rs}(A) = A.

v) Suppose that there exist a point x $\in$X such that x $\notin \Lambda_{rs}\left(\bigcup_{i\in I} A_i\right)$.

Then there exist a regular semi open set G such that $\bigcup_{i\in I} A_i \subseteq G$ and x $\notin$ G

Thus for each i $\in$ I, we have x $\notin \Lambda_{rs}(A_i)$

Thus x $\notin \bigcup_{i\in I} \Lambda_{rs}(A_i)$.

Therefore $\Lambda_{rs}\left(\bigcup_{i\in I} A_i\right) \supseteq \bigcup_{i\in I} \Lambda_{rs}(A_i)$

vi) Suppose that there exist a point x such that x $\notin \bigcap_{i\in I} \Lambda_{rs}(A_i)$

Then $\forall\, i$ $\in$I, x $\notin \Lambda_{rs}(A_i)$. Hence $\forall\, i$ $\in$I and G $\in$ RSO(X,τ) such that

$A_i \subseteq G$ and x $\notin$G. Thus x $\notin \Lambda_{rs}\left(\bigcap_{i\in I} A_i\right)$.

Therefore $\Lambda_{rs}\left(\bigcap_{i\in I} A_i\right) \subseteq \bigcap_{i\in I} \Lambda_{rs}(A_i)$

vii) Let x $\in$ $\Lambda_{rs}(A^C)$. Then there exist a regular semi open set G such that

$A^C \subseteq G$ and x $\in$G. Hence x $\notin G^C$,for every regular semi closed set G^C and

G^C $\subseteq$A. Therefore x $\notin V_{rs}(A)$ and hence x $\in (V_{rs}(A))^C$.

Similarly $(V_{rs}(A))^C \subseteq \Lambda_{rs}(A^C)$. Therefore $\Lambda_{rs}(A^C) = (V_{rs}(A))^C$

Hence the proof.

Lemma 3.4

For subsets A, B & A_i , i $\in$I of a topological space (X, τ), the following properties hold

(i) $V_{rs}(A) \subseteq A$

(ii) B $\subseteq$ A $\Rightarrow V_{rs}(B) \subseteq V_{rs}(A)$

(iii) $V_{rs}(V_{rs}(A)) = V_{rs}(A)$

(iv) If A $\in$ RSC (X, τ) then A $= V_{rs}(A)$

(v) $V_{rs}\left(\bigcap_{i\in i} A_i\right) \subseteq \bigcap_{i\in I} V_{rs}(A_i)$

(vi) $V_{rs}\left(\bigcup_{i\in I} A_i\right) \supseteq \bigcup_{i\in I} V_{rs}(A_i)$

Remark 3.5

In general we have $\Lambda_{rs}(S \cap Q) \neq \Lambda_{rs}(S) \cap \Lambda_{rs}(Q)$

and $\Lambda_{rs}(S \cup Q) \neq \Lambda_{rs}(S) \cup \Lambda_{rs}(Q)$ as the following example shows.

Example 3.6

Let X = {a, b, c, d}, τ = {∅, X,{c}, {d}, {c,d}, {a,c,d}, {b,c,d}}

RSO(X, τ) = {∅, X,{c}, {d}, {a,c}, {a,d}, {b,c}, {b,d}, {a, b, c}, {a, b, d}}

If S = {a, c}, Q = {c,d}. Here $\Lambda_{rs}(S \cap Q) = \Lambda_{rs}(c) = \{c\}$. But $\Lambda_{rs}(S) = \Lambda_{rs}(a,c) =$ {a, c}

and $\Lambda_{rs}(Q) = \Lambda_{rs}(c,d)$ = X. Also $\Lambda_{rs}(S) \cap \Lambda_{rs}(Q)$ = {a, c}

Example 3.7

In the previous example S = { c}, Q = {d}

$\Lambda_{rs}(S)$ = {c}, $\Lambda_{rs}(Q)$ ={d}

$\Lambda_{rs}(S) \cup \Lambda_{rs}(Q)$ = {c, d} but $\Lambda_{rs}(S \cup Q) = X$

Definition 3.8

In a topological space (X, τ), a subset B is a semi regular Λ- set (resp. semi regular V- set) briefly Λ_{rs} – set (resp. V_{rs} – set) of (X, τ) if B = $\Lambda_{rs}(B)$ (resp. B = $V_{rs}(B)$))

Remark 3.9

By (iv) of lemma 3.3 and 3.4 we have that

a) If B is a Λ_r - set (or) if B ∈ RSO (X, τ), then B is a Λ_{rs} - set.

b) If B is a V_r - set (or) if B ∈ RSC (X, τ), then B is a V_{rs} - set.

Proposition 3.10

For a space (X, τ), the following statements hold

(i) ∅, X are Λ_{rs} - sets and V_{rs} - sets

(ii) Every union of V_{rs} - sets is a V_{rs} - set.

(iii) Every intersection of Λ_{rs} - sets is a Λ_{rs} - set.

Proof

(i) It is obvious

(ii) Let $\{ A_i / i \in I\}$ be a family of V_{rs} - sets in (X, τ).

Then $A_i = V_{rs}(A_i)$ for each i $\in$ I.

Let A = $\bigcup_{i\in I} A_i$

Then $V_{rs}(A) = V_{rs}\left(\bigcup_{i\in I} A_i\right) \supseteq \bigcup_{i\in I} V_{rs}(A_i) = \bigcup_{i\in I} A_i$ = A.

ie $V_{rs}(A) \supseteq A$. By lemma 3.4 (i), we have $V_{rs}(A) \subseteq A$.

Hence A is a V_{rs} - set.

(iii) Similarly we can prove the result using the lemma 3.3 [(i) and (v)]

The following example shows that union of Λ_{rs} - sets need not be Λ_{rs} - set and intersection of V_{rs} - sets need not be a V_{rs} - set.

Example 3.11

Let X = {a, b, c, d, e}, τ = {∅, X, {a}, {c,d}, {a,c,d}}

RSO(X, τ) = {{∅, X, {a}, {a, b}, {a, e}, {c, d}, {a, b, e}, {b, c, d}, {c, d, e}, {b, c, d,e}}

Λ_{rs} (X, τ) = {{∅, X, {a}, {a, b}, {a, e}, {c, d}, {a, b, e}, {b, c, d}, {c, d,e},{b, c, d, e},

{b}, {e}, {b, e}}

Here {a, e} and {c, d} are Λ_{rs} - sets. But {a, e}∪{c ,d} = {a, c, b, e} is not set Λ_{rs} -set.

Example 3.12

Let X and τ be defined as in example 3.6

We have V_{rs} (X, τ) ={∅, X, {c}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {a, b, c},{a, b,d},{c, d}, {a, c, d}, {b, c, d}}

Here {a, c} and {a, d} are V_{rs} -sets. But {a, c } ∩ {a ,d} = {a } is not V_{rs} -set

Remark 3.13

Observe that a subset A is semi regular Λ- set if A^c is semi regular $\vee$-set

Also observe that every semi regular open (semi regular closed) set is a semi regular Λ- set (semi regular $\vee$-set).

Λ_{rs} - closed sets and its properties

Definition 3.14

A subset A of a topological space (X,τ) is called Λ_{rs} - closed if A = T $\cap$ C, where T is a Λ_{rs} - set and C is a closed set.

The collection of all Λ_{rs} - closed sets in a topological space (X, τ) is denoted by Λ_{rs}C (X, τ)

Example 3.15

Let X = {a, b, c}, τ = {∅, X, {a}, { b}, {a, b}},

Λ_{rs}-closed set = {∅, X, {a}, { b}, {a, c}, {b, c}, {c}},

Theorem 3.16

For a subset A of a topological space (X, τ) , the following properties are equivalent.

i) A is – Λ_{rs}-closed

ii) A = T $\cap$ cl(A), where T is a Λ_{rs} - set.

iii) A = Λ_{rs} (A) $\cap$ cl(A)

Proof

(i)$\Rightarrow$**(ii)** Let A = T $\cap$ C, where T is a Λ_{rs} - set rand C is a closed set. Since A $\subseteq$C, we have cl (A) $\subseteq$C and A = T $\cap$ C $\supseteq$T $\cap$cl(A) $\supseteq$A. Therefore A = T $\cap$ cl(A).

ii)$\Rightarrow$**(iii)** Let A = T $\cap$ cl(A), where T is a Λ_{rs} - set. Since A $\subseteq$T, by lemma 3.3 (ii) we have $\Lambda_{rs}(A) \subseteq \Lambda_{rs}(T) = T$ and hence A $\subseteq \Lambda_{rs}(A) \cap cl(A) \subseteq T \cap cl(A) = A.$

Therefore A = Λ_{rs} (A) $\cap$ cl(A)

(iii) $\Rightarrow$ **(i)** Since Λ_{rs} (A) is a Λ_{rs} -set and cl (A) is a closed set and A = Λ_{rs} (A) $\cap$ cl(A). Therefore A is Λ_{rs} - closed.

Lemma 3.17

Every Λ_{rs} - set is a Λ_{rs} - closed set but not conversely.

Proof

Let A be a Λ_{rs} - set. Then A = A $\cap$ X, where A is a Λ_{rs} - set and X is a closed set. Therefore A is a Λ_{rs} - closed set.

Example 3.18

Let X ={a, b, c, d}, τ = {∅, X, {a}}, Λ_{rs} - set = {∅, X},

Λ_{rs} - closed set = {∅, X, {b, c, d}}. Since {b, c, d} is Λ_{rs} - closed set but it is not Λ_{rs} -set.

Corollary 3.19

Every regular semi open is a Λ_{rs} - closed set but not conversely

Example 3.20

In example 3.2 {c} is Λ_{rs} - closed but {c} is not regular semi open.

Lemma 3.21

Every Λ_{r} - closed set is a Λ_{rs} - closed set but not conversely

Proof

Obvious

Example 3.22

In example 3.6 {a, b} is Λ_{rs} - closed but {a, b} is not Λ_{r} - closed.

Definition 3.23

A subset A of a topological space (X, τ) is said to be Λ_{rs} -open if the complement of A is

Λ_{rs} -closed. The collection of all Λ_{rs} -open sets in a topological space (X, τ) is denoted by

Λ_{rs} O(X,τ).

Theorem 3.24

(i) If A_k is Λ_{rs} -closed for each k $\in$ I, then $\bigcap_{k\in I} A_k$ is Λ_{rs} -closed.

(ii) If A_k is Λ_{rs} -open for each k $\in$ I, then $\bigcup_{k\in I} A_k$ is Λ_{rs} -open.

Proof

i) Suppose A_k is Λ_{rs} -closed for each k $\in$ I. Then for each k, there exist a

Λ_{rs} -set T_k and a closed set C_k such that $A_k = T_k \cap C_k$.

We have $\bigcap_{k\in I} A_k = \bigcap_{k\in I}(T_k \cap C_k) = \left(\bigcap_{k\in I} T_k\right) \cap \left(\bigcap_{k\in I} C_K\right)$

By proposition 3.10 $\bigcap_{k\in I} T_k$ is a Λ_{rs} -set and $\bigcap_{k\in I} C_k$ is a closed set.

Therefore $\bigcap_{k\in I} A_k$ is Λ_{rs} -closed.

ii) Let A_k is Λ_{rs} -open for each k $\in$ I.

Then X – A_k is Λ_{rs} -closed and X - $\bigcup_{k\in I} A_k = \bigcap_{k\in I}(X - A_K)$

By (i) X - $\bigcup_{k\in I} A_k$ is Λ_{rs} -closed and hence $\bigcup_{k\in I} A_k$ is Λ_{rs} -open.

Remark 3.25

The union of Λ_{rs} - closed sets need not be Λ_{rs} - closed and the intersection of Λ_{rs} - open sets need not be Λ_{rs} - open as seen in the following example.

Example 3.26

Let X = {a, b, c}, τ = {∅, X, {b}, {c}, {b, c}}, τ^C = {∅, X, {a, c}, {a, b},{a}}

Λ_{rs} - closed = {∅, X, {b}, {c}, {a, c},{a, b}, {a}}

Λ_{rs} - open = {∅, X, {a, c}, {a, b}, {b}, {c}, {b, c}}

Since {b} and {c} are Λ_{rs} - closed sets.

But {b} $\bigcup$ {c}={b, c} is not Λ_{rs} - closed set.

Also {a, c} and {a, b} are Λ_{rs} - open sets. But {a, c} $\bigcap$ {a, b}= {a} is not Λ_{rs} - open set.

Lemma 3.27

Every V_{rs} - set is Λ_{rs} - open.

Proof

Take A $\bigcup \emptyset$, where A is a V_{rs} - set and $\emptyset$ is a open set.

Theorem 3.28

For a subset A of a topological space (X, τ) the following are equivalent

i) A is Λ_{rs} - open.

ii) A = T $\bigcup$ C, where T is a Λ_{rs} - set and C is open

iii) A = T $\bigcup$ Int (A)

iv) A = $V_{rs}(A) \bigcup$ Int (A)

Proof

(i)$\Rightarrow$(ii) Suppose A is Λ_{rs} - open. Then X – A is Λ_{rs} - closed.

Therefore X – A = S $\bigcap$ T, where S is a Λ_{rs} - set and T is a closed set.

Hence A = (X – S) $\bigcup$ (X - T), where (X – S) is a V_{rs} - set and (X - T) is a open set.

(ii)$\Rightarrow$(iii) Let A = T $\bigcup$ C, where T is a V_{rs} - set and C is open.

Since C $\subset$ A and 'C' is open implies C $\subset$ Int (A)

Therefore A = T $\bigcup$ C $\subset$ T $\bigcup$ Int A $\subset$ A

A = T $\bigcup$ Int (A)

(iii)$\Rightarrow$(iv) Let A = T $\bigcup$ Int (A), where T is V_{rs} - set.

Since T $\subset$ A $\Rightarrow$ $V_{rs}(T) \subset V_{rs}(A)$

Since A = T $\bigcup$ Int (A). Here A $\supset V_{rs}(A) \bigcup Int(A) \supset V_{rs}(T) \bigcup Int(A)$ = T $\bigcup$ Int A = A

Therefore A = $V_{rs}(A) \bigcup$ Int (A).

(iv)$\Rightarrow$(i) Let A = $V_{rs}(A) \bigcup$ Int (A). Then X – A = $\left(X - V_{rs}(A)\right) \bigcap (X - IntA)$

= $\Lambda_{rs}(X-A) \bigcap$ cl (X– A).

Since $\Lambda_{rs}(X-A)$ is a Λ_{rs} - set and cl (X– A) is a closed set.

Therefore (X– A) is Λ_{rs} -closed. Therefore A is Λ_{rs} - open.

Theorem 3.29

For a subset A of a topological space (X, τ) the following holds

i) $\Lambda_{rs}(A) \subseteq \Lambda_r(A)$

ii) $\Lambda_b(A) \subseteq \Lambda_{rs}(A)$

iii) $A^{\Lambda_s} \subseteq \Lambda_{rs}(A)$

iv) $\Lambda_\theta^{\Lambda_s}(A) \subseteq \Lambda_{rs}(A)$

Proof

i) Suppose x $\notin \Lambda_r(A)$. Then there exist a regular open set G such that A$\subseteq G$ and x $\notin$G. But we have every regular open set is regular semi open. Then there exist a regular semi open G such that A $\subseteq G$ and x $\notin$G.

Hence x$\notin \Lambda_{rs}(A)$.

ii) Result follows from the fact that every regular semi open is b –open.

iii) Result follows from the fact that every regular semi open is semi open

iv) Result follows from the fact that every regular semi open is semi –θ-open.

Theorem 3.30

i) Every Λ_r - set is Λ_{rs} - set.

ii) Every Λ_{rs} - set is Λ_b - set.

iii) Every Λ_{rs} - set is A^{Λ_s} - set.

iv) Every Λ_{rs} - set is $\Lambda_\theta^{\Lambda_s}$ - set.

Proof

i) Let A be a subset of a topological space (X, τ).

Suppose A is Λ_r - set. Then A = Λ_r (A)

By lemma 3.3 A $\subset \Lambda_{rs}(A)$. By previous theorem $\Lambda_{rs}(A) \subseteq \Lambda_r(A)$ =A

Therefore we have A = Λ_{rs} (A). Therefore A is Λ_{rs} - set.

ii) Suppose A is a Λ_{rs} - set. Then A = Λ_{rs} (A)

By lemma 2.1 [1] A $\subset \Lambda_b(A)$. By previous theorem $\Lambda_b(A) \subseteq \Lambda_{rs}(A)$ =A.

Therefore we have A = Λ_b (A). Therefore A is Λ_b - set.

iii) Suppose A is a Λ_{rs} - set. Then A = Λ_{rs} (A).

By lemma 3.1 [4] A $\subseteq A^{\Lambda_s}$. By previous theorem $A^{\Lambda_s} \subseteq \Lambda_{rs}(A)$ =A.

Therefore we have A = A^{Λ_s}. Therefore A is Λ_s - set.

iv) Suppose A is a Λ_{rs} - set. Then A = Λ_{rs} (A).

By lemma 2.1 [18]A $\subseteq \Lambda_\theta^{\Lambda s}(A)$. By previous theorem $\Lambda_\theta^{\Lambda s}(A) \subseteq \Lambda_{rs}(A)$ = A.

Therefore we have A = $\Lambda_\theta^{\Lambda s}(A)$. Therefore A is a $\Lambda_\theta^{\Lambda s}$ - set.

Definition 3.31

Let A be a subsets of a topological space (X, τ). A point x $\in$ X is called a Λ_{rs} - cluster point of A if for every Λ_{rs} - open set U containing 'x', A $\cap$ U $\neq \phi$. The set of all Λ_{rs} - cluster points is called the Λ_{rs} - closure of A and it is denoted by Λ_{rs} - cl (A).

Lemma 3.32

Let A and B be subsets of a topological space (X, τ). The following properties hold.

i) A $\subset$ Λ_{rs}-cl (A) $\subset$cl(A)

ii) Λ_{rs}-cl (A) = $\bigcap$ {F / A $\subset$F and F is Λ_{rs}-closed}

iii) If A $\subset$ B , then Λ_{rs} - cl (A) $\subset$ Λ_{rs} - cl (B)

iv) A is Λ_{rs} - closed iff A = Λ_{rs} - cl (A)

v) Λ_{rs} - cl (A) is Λ_{rs} - closed.

Proof

i) Let x $\notin \Lambda_{rs}$ -cl (A). Then 'x' is not a Λ_{rs} -cluster point of A. Then there exist a Λ_{rs} - open set U containing 'x' such that A $\bigcap$ U = ϕ.

Therefore x $\notin$ A. The other inclusion follows from the fact that every closed set is Λ_{rs} - closed.

ii) Let x $\notin \Lambda_{rs}$ - cl (A). Then there exist a Λ_{rs} - open set U containing 'x' such that A $\bigcap$ U = ϕ. Take F = U^C. Then F is a Λ_{rs} -closed set, A $\subset$ F and x $\notin$ F.

Hence x $\notin \bigcap$ {F / A $\subset$ F and F is Λ_{rs} - closed}.

Similarly Λ_{rs} - cl(A) $\subset \bigcap$ { F / A $\subset$ F and F is Λ_{rs} - closed}.

iii) Let x $\notin \Lambda_{rs}$ - cl (B). Then there exist a Λ_{rs} - open set U containing 'x' such that B $\bigcap$ U = ϕ. Since A $\subset$ B, A $\bigcap$ U = ϕ and hence 'x' is not a Λ_{rs} -cluster point of A. Therefore x $\notin \Lambda_{rs}$ - cl (A)

iv) Suppose A is $\notin \Lambda_{rs}$ -closed. Let x $\notin$ A, then x $\in$ A^C and A^C is Λ_{rs} - open. Take A^C = U. Then U is a Λ_{rs} - open set containing 'x' and A $\bigcap$ U = ϕ and hence x $\notin \Lambda_{rs}$ - cl (A). By (i) we get A = Λ_{rs} - cl (A).

Conversely suppose that A = Λ_{rs} - cl (A).

Since A = $\bigcap$ { F / A $\subset$ F and F is Λ_{rs} - closed}.

By (ii) A is Λ_{rs} -closed.

v) By (i) and (iii) we have Λ_{rs} - cl (A) $\subset \Lambda_{rs}$ - cl (Λ_{rs} - cl (A)).

Let x $\in$ Λ_{rs} - cl (Λ_{rs} - cl (A)) implies 'x' is a Λ_{rs} - cluster point of Λ_{rs} - cl (A). Therefore for every Λ_{rs} - open set U containing 'x' (Λ_{rs} - cl (A)) $\cap$ U $\neq \phi$.

Let y $\in$ Λ_{rs} - cl (A) $\cap$ U. Then 'y' is a Λ_{rs} - cluster point of A. Therefore for every Λ_{rs} - open set G containing 'y' and A $\cap$ G $\neq$ ϕ. Since U is Λ_{rs} - open and y $\in$ U, A $\cap$ U $\neq$ ϕ and hence x $\in \Lambda_{rs}$ - cl (A).

Hence Λ_{rs} - cl (A) = Λ_{rs} - cl (Λ_{rs} - cl (A)).

By (iv) Λ_{rs} - cl (A) is Λ_{rs} -closed.

Note 3.33

1) ϕ, X are both Λ_{rs} -open and Λ_{rs} -closed.
2) By (ii) and (v) Λ_{rs} -cl(A) is the smallest Λ_{rs} -closed set containing A.

Theorem 3.34

Every closed set is a Λ_{rs} **-** closed set but not conversely.

Proof

Let A be a closed set. Let A = X ∩ A, where X is a Λ_{rs} **- s**et and A is a closed set. Therefore A is a Λ_{rs} -closed set.

Example 3.35

Let X = {a, b, c, d}, τ = {∅, X, {c}, {d}, {c, d}, {a, c, d}, {b, c, d}}

Here A = {a, c} is Λ_{rs} **-** closed but A is not closed**.**

Corollary 3.36

Every θ- closed, δ-closed, regular closed set is a Λ_{rs} **-** closed but not conversely**.**

Proof

Since every θ- closed [19], δ-closed [19], regular closed set is closed and by above theorem

Λ_{rs} - closed.

Example 3.37

X = {1, 2, 3, 4} τ ={φ, X, {1}, {2}, {1, 2}, {1, 2, 3}}

Closed set = { φ, X, {2, 3, 4}, {1, 3, 4}, {3, 4}, {4}}

θ- closed = { φ, X }, δ-closed = { φ, X , {2, 3, 4}, {1, 3, 4}, {3, 4}}

Regular closed = { φ, X , {2, 3, 4}, {1, 3, 4}}

Here A = {2, 3} is Λ_{rs} - closed but A is not θ- closed, δ-closed, regular closed.

Theorem 3.38

i) Every Λ_r -closed is Λ_{rs} -closed

ii) Every Λ_{rs} -closed is (Λ, b) closed

But converse not true.

Proof

i) Let A be a subset of a topological space (X, τ).

Suppose A is Λ_r -closed. Then A = S ∩ C, where 'S' is a Λ_r -set and 'C' is a closed set. By theorem 3.30 (i) 'S' is a Λ_{rs} -set. Therefore A is Λ_{rs} -closed.

ii) Let A be a subset of a topological space (X, τ). Suppose A is Λ_{rs} -closed.

Then A = S ∩ C, where 'S' is a Λ_{rs} -set and 'C' is a closed set.

By theorem 3.30 (ii) Λ_{rs} -set is Λ_b -set and every closed set is b-closed.

Therefore A is Λ_{rs} -closed.

Theorem 3.39

Every Λ_{rs} -closed set is semi closed but not conversely.

Proof

Since every regular semi open set is both semi open and semi closed. Also intersection of semi closed set is semi closed and every closed set is semi closed. Therefore by the definition of Λ_{rs} -closed set, every Λ_{rs} -closed set is semi closed.

Example 3.40

Let X = {a, b, c, d}, τ ={∅, X, {c, d}} since {a} is semi closed but not Λ_{rs} -closed.

Theorem 3.41

If RSO (X, τ) is indiscrete space, then every Λ_{rs} -closed is pre closed and α-closed

Proof

If RSO (X, τ) = {∅, X }. Then only regular semi open sets are ∅, X only. Let A be any subset of X. WKT every closed set is pre closed and α-closed. Therefore by the definition of Λ_{rs} -closed set, every Λ_{rs} -closed is pre closed and α-closed.

Theorem 3.42

If a space X is locally indiscrete, then every locally closed set is Λ_{rs} -closed.

Proof

By the definition of locally indiscrete we have every open set is closed and by theorem 3.34 every closed set is Λ_{rs} -closed.

Theorem 3.43

If a subset A of (X, τ) is regular open, then PInt (A) and Scl(A) is Λ_{rs} -closed

Proof

WKT PInt (A) = A ∩ Int cl (A) and Scl(A) = A ∪ Int cl(A)

since A is regular open, we have A = int cl (A)

Therefore PInt (A) = A ∩ A = A and Scl(A) = A ∪A = A.

WKT every regular open set is regular semi open and every regular open set is Λ_{rs} -set.

Also every Λ_{rs} -set is Λ_{rs} -closed.

Therefore PInt (A) and Scl(A) is Λ_{rs} -closed.

Theorem 3.44

If a subset A of (X, τ) is regular closed, then Pcl (A) and SInt(A) is Λ_{rs} -closed.

Proof

Similar to above

Definition 3.45

A subset A of a space (X, τ) is called regular weakly closed (briefly rw- closed) [3] if cl(A) $\subseteq$U whenever A$\subseteq$U and U is regular semi open in X.

Lemma 3.46

A subset A of a space (X, τ) is RW closed iff cl(A) $\subseteq$ Λ_{rs} (A).

Theorem 3.47

For a subset A of a topological space (X, τ) the following conditions are equivalent.

i) A is closed

ii) A is RW closed and Λ_r -closed

iii) A is RW closed and Λ_{rs} -closed

Proof

i)$\Rightarrow$ii) Every closed set is both RW closed and Λ_r -closed.

ii)$\Rightarrow$iii) by theorem 3.38 (i)

iii)$\Rightarrow$i) A is RW closed so by lemma 3.46 cl (A) $\subseteq$ Λ_{rs} (A).

Also A is Λ_{rs} -closed. By theorem 3.16 we have A = Λ_{rs} (A) $\cap$ cl (A)

Hence A = cl (A). Therefore A is closed.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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