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COMMON FIXED POINT THEOREMS IN COMPLEX VALUED S_b METRIC SPACES

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Abstract. The aim of this paper is to present some common fixed point results for four mappings satisfying generalized contractive condition in a complex valued S_b-metric space using weak compatibility. Our result generalizes, extends and improve existing results of Priyobarta, Rohen and Mlaiki [9] and several researchers (see 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15 and references their mentioned in) in context of b- metric, S_b metric space, complex valued metric space, complex valued b- metric space, complex valued S-metric space to complex valued S_b-metric spaces.

Keyword. common fixed point; compatible mapping; weakly compatible mapping; complex valued S_b metric space.

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1. INTRODUCTION

Azam et al. [1] developed complex valued metric spaces. As an extension of metric spaces, Bakhtin [3] proposed the notion of b-metric space. Rao et al. [10] integrated these ideas and developed complex valued b-metric spaces as extensions of these spaces, proving fixed point theorems in a complex valued b-metric space. Kang et al. [5] proposed the concept of complex valued G-metric spaces and demonstrated the theory of contraction in this space. Mlaiki [6] developed the complex valued S-metric space and demonstrated the presence and uniqueness of a shared fixed point in

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this space for two self-mappings. Although complex valued metric spaces are a subset of cone metric spaces, our contraction, which has a product and quotient of S_b -metric spaces, cannot be extended to a cone metric space. Recently, Souayah and Mlaiki [15] proposed the idea of S_b-metric spaces as a generalisation of metric space, which was followed by Sedghi et al [12] 's introduction of s- metric space. More, recently Priyobarta, Rohen and Mlaiki [9] introduced the concept of complex valued S_b-metric space and prove some interesting and new results. Our result generalizes, extends and improve existing results of Priyobarta, Rohen and Mlaiki [9].

2. Basic Definitions and Preliminaries

Definition.2.1[6]. Let X be a nonempty set and let b≥1 be a given number. A function $S: X^3 \rightarrow [0,\infty)$ is said to be S_b-metric if and only if for all $x,y,z,t \in X$ the following conditions are satisfied:

(i) $S(x,y,z) = 0$ if and only if $x = y = z$;

(ii) $S(x,x,y) = S(y,y,x)$ for all $x,y \in X$;

(iii) $S(x,y,z) \leq b[S(x,x,t) + S(y,y,t) + S(z,z,t)]$.

The pair (X,S) is called a S_b-metric space.

Definition 2.2[6]. Let (X,S) be an S_b-metric space and $\{x_n\}$ be a sequence in X. Then

(i) a sequence $\{x_n\}$ is called convergent if and only if there exists $z \in X$ such that $S(x_n,x_n,z) \rightarrow 0$ as $n \rightarrow \infty$. In this case we write $\lim_{n} x_n = z$,

(ii) a sequence $\{x_n\}$ is called a Cauchy sequence if and only if $S(x_n,x_n,x_m) \rightarrow 0$ as $n,m \rightarrow \infty$.

(iii) (X,S) is said to be a complete S_b-metric space if every Cauchy sequence $\{x_n\}$ converges to a point $x \in X$ such that $\lim_{n,m\rightarrow\infty} S(x_n,x_n,x_m) = \lim_{n,m\rightarrow\infty} S(x_n,x_n,x) = S(x,x,x)$.

The following definition is recently introduced by Azam et al. [1].

Definition 2.3[1]. Let $\mathbb{C}$ be the set of complex numbers and $z_1, z_2 \in \mathbb{C}$. Define a partial order $\precsim$ on $\mathbb{C}$ as follows:

$z_1 \precsim z_2$ if and only if Re(z_1) ≤ Re(z_2), Im(z_1) ≤ Im(z_2).

Consequently, one can infer that $z_1 \precsim z_2$ if one of the following conditions is satisfied:

(i) Re(z_1) = Re(z_2), Im(z_1) < Im(z_2),

(ii) Re(z_1) < Re(z_2), Im(z_1) = Im(z_2),

(iii) Re(z_1) < Re(z_2), Im(z_1) < Im(z_2),

(iv) Re(z_1) = Re(z_2), Im(z_1) = Im(z_2),

In particular, we write $z_1 \lnsim z_2$ if $z_1 \neq z_2$ and one of (i), (ii), and (iii) is satisfied and we write $z_1 \prec z_2$ if only (iii) is satisfied. Notice that $0 \precsim z_1 \lnsim z_2 \Rightarrow |z_1| < |z_2|$, and $z_1 \precsim z_2, z_2 \prec z_3 \Rightarrow z_1 \prec z_3$.

Definition 2.4.[9] Let X be a nonempty set and let b≥1 be a given real number. Suppose that a mapping $S: X^3 \to \mathbb{C}$ satisfies:

(CS_b1) $0 \prec S(x, y, z)$ for all $x, y, z \in X$ with $x \neq y \neq z \neq x$.

(CS_b2) $S(x, y, z) = 0$ if and only if $x = y = z$;

(CS_b3) $S(x, x, y) = S(y, y, x)$ for all $x, y \in X$;

(CS_b4) $S(x, y, z) \precsim b[S(x, x, a) + S(y, y, a) + S(z, z, a)]$ for all $x, y, z, a \in X$.

Then S is called a complex valued S_b -metric and the pair (X, S) is called a complex valued S_b-metric space.

Definition 2.5. [9] Let (X, S) be a complex valued S_b-metric space and let $\{x_n\}$ be a sequence in X.

(i) A sequence $\{x_n\}$ is a complex valued S_b –convergent to x if for every $a \in \mathbb{C}$ with $0 < a$ there exists $k \in \mathbb{N}$ such that $S(x_n, x_n, x) \prec a$ or $S(x, x, x_n) \prec a$ for all $n \geq k$ and denoted by $\lim_{n \to \infty} x_n = x$.

(ii) A sequence $\{x_n\}$ is called a complex valued S_b –Cauchy if for every $a \in \mathbb{C}$ with $0 < a$ there exists $k \in \mathbb{N}$ such that $S(x_n, x_n, x_m) \prec a$ for each $m, n \geq k$.

(iii) If every complex valued S_b –Cauchy sequence is complex valued S_b –convergent in (X, S), then (X, S) is said to be complex valued S_b –complete

Definition 2.6[4]. Let f and g be two self-maps defined on a set X, then f and g are said to be weakly compatible if they commute at coincidence points.

3. MAIN RESULTS

Theorem 3.1 Let A, B, f and g be four self mappings of a complete complex valued S_b-metric space (X, S) with the coefficient $b \geq 1$ and satisfying

(3.1) $A \subset g$, $\quad B \subset f$

(3.2) $S(Ax, Ax, By) \precsim \alpha_1 \left\{\frac{S(fx,fx,gy)S(By,By,fx)}{2S(fx,fx,gy)+ S(By,By,gy)}\right\} + \alpha_2\{S(Ax, fx, fx) + S(By, By, gy)\}$

$$+ \alpha_3\{S(By, By, fx) + S(Ax, Ax, gy)\}+ \alpha_4 S(fx, fx, gy).$$

If $2S(fx, fx, gy) + S(By, By, gy) \neq 0$, for all $x, y \in X$, $\alpha_i \geq 0$ $(i = 1, 2, 3, 4)$ with at least one α_i is nonzero and $b\alpha_1 + 2\alpha_2 + 3b\alpha_3 + \alpha_4 < 1$. If one of A, B, f or g is a complete subspace of X, then

(i) (A, f) and (B, g) have a unique coincidence point in X,

(ii) if (A, f) and (B, g) are weakly compatible, then A, B, f and g have a unique common fixed point in X.

Proof. Let x_0 be an arbitrary point in X. We define a sequence $\{y_{2n}\}$ in X such that

$$y_{2n} = Ax_{2n} = gx_{2n+1}$$

$$y_{2n+1} = Bx_{2n+1} = fx_{2n+2} \qquad ; n = 0,1,2, \ldots .$$

Then

$$S(y_{2n}, y_{2n}, y_{2n+1}) = S(Ax_{2n}, Ax_{2n}, Bx_{2n+1})$$

$$S(y_{2n}, y_{2n}, y_{2n+1}) \precsim \alpha_1 \left\{\frac{S(fx_{2n},fx_{2n},Tx_{2n+1})\, S(Bx_{2n+1},Bx_{2n+1},fx_{2n})}{2S(fx_{2n},fx_{2n},gx_{2n+1})+ S(Bx_{2n+1},Bx_{2n+1},gx_{2n+1})}\right\}$$

$$+ \alpha_2\{S(Ax_{2n}, Ax_{2n}, fx_{2n}) + S(Bx_{2n+1}, Bx_{2n+1}, gx_{2n+1})\}$$

$$+ \alpha_3\{S\big(Bx_{2n+1}, Bx_{2n+1}, fx_{2n}\big) + S(Ax_{2n}, Ax_{2n}, gx_{2n+1})\}$$

$$+ \alpha_4\, S(fx_{2n}, fx_{2n}, gx_{2n+1})$$

$$S(y_{2n}, y_{2n}, y_{2n+1}) \precsim \alpha_1 \left\{\frac{S(y_{2n-1},y_{2n-1},y_{2n})\, S(y_{2n+1},y_{2n+1},y_{2n-1})}{2S(y_{2n-1},y_{2n-1}, y_{2n})+ S(y_{2n+1},y_{2n+1},y_{2n})}\right\}$$

$$+ \alpha_2\, \{S(y_{2n}, y_{2n}, y_{2n-1}) + S(y_{2n+1}, y_{2n+1}, y_{2n})\}$$

$$+ \alpha_3\{S(y_{2n+1}, y_{2n+1}, y_{2n-1}) + S(y_{2n}, y_{2n}, y_{2n})\}$$

$$+ \alpha_4 S(y_{2n-1}, y_{2n-1}, y_{2n})$$

By condition, (CS_b3) and (CS_b4) in the definition of the complex S_b metric spaces, we get

$$S(y_{2n+1},y_{2n+1},y_{2n-1}) = S(y_{2n-1},y_{2n-1},y_{2n})$$
$$\precsim b[S(y_{2n-1},y_{2n-1},y_{2n}) + S(y_{2n-1},y_{2n-1},y_{2n}) + S(y_{2n+1},y_{2n+1},y_{2n})]$$
$$\precsim b[2S(y_{2n-1},y_{2n-1},y_{2n}) + S(y_{2n+1},y_{2n+1},y_{2n})]$$

Therefore,

$$S(y_{2n},y_{2n},y_{2n+1}) \precsim b\alpha_1 S(y_{2n-1},y_{2n-1},y_{2n}) + \alpha_2\{S(y_{2n},y_{2n},y_{2n-1}) + S(y_{2n+1},y_{2n+1},y_{2n})\}$$
$$+ \alpha_3 S(y_{2n+1},y_{2n+1},y_{2n-1}) + \alpha_4 S(y_{2n-1},y_{2n-1},y_{2n})$$
$$\precsim b\alpha_1 S(y_{2n-1},y_{2n-1},y_{2n}) + \alpha_2\{ S(y_{2n-1},y_{2n-1},y_{2n}) + S(y_{2n},y_{2n},y_{2n+1})\}$$
$$+ \alpha_3 b\{ 2S(y_{2n+1},y_{2n+1},y_{2n}) + S(y_{2n-1},y_{2n-1},y_{2n})\}$$
$$+ \alpha_4 S(y_{2n-1},y_{2n-1},y_{2n})$$
$$S(y_{2n},y_{2n},y_{2n+1}) \precsim (b\alpha_1 + \alpha_2 + b\alpha_3 + \alpha_4) S(y_{2n-1},y_{2n-1},y_{2n})$$
$$+ (\alpha_2 + 2b\alpha_3)\, S(y_{2n},y_{2n},y_{2n+1})$$
$$S(y_{2n},y_{2n},y_{2n+1}) \precsim \frac{b\alpha_1+\alpha_2+b\alpha_3+\alpha_4}{1-\alpha_2-2b\alpha_3}\, S(y_{2n-1},y_{2n-1},y_{2n}).$$

Let $k = \frac{b\alpha_1+\alpha_2+b\alpha_3+\alpha_4}{1-\alpha_2-2b\alpha_3}$ and since $b\alpha_1 + 2\alpha_2 + 3b\alpha_3 + \alpha_4 < 1$ and $b \geq 1$ then we get $k < 1$.

Thus

$$S(y_{2n},y_{2n},y_{2n+1}) \precsim k\, S(y_{2n-1},y_{2n-1},y_{2n}).$$

Similarly

$$S(y_{2n+1},y_{2n+1},y_{2n+2}) \precsim k\, S(y_{2n},y_{2n},y_{2n+1}).$$

If $k < 1$, then

$$|S(y_{2n+1},y_{2n+1},y_{2n+2})| \leq k\, |S(y_{2n},y_{2n},y_{2n+1})| \leq k^2\, |S(y_{2n-1},y_{2n-1},y_{2n})|$$
$$\leq \cdots \leq k^{2n+1}\, |S(y_0,y_0,y_1)|.$$

Thus for any $m > n$, $m,n \in N$ we get

$$|S(y_{2n},y_{2n},y_{2m})| \leq 2b\, |S(y_{2n},y_{2n},y_{2n+1})| + b\, |S(y_{2n+1},y_{2n+1},y_{2m})|$$
$$|S(y_{2n},y_{2n},y_{2m})| \leq 2b|S(y_{2n},y_{2n},y_{2n+1})| + 2b^2|S(y_{2n+1},y_{2n+1},y_{2n+2})|$$
$$+ b^2|S(y_{2n+2},y_{2n+2},y_{2m})|$$
$$\leq 2b|S(y_{2n},y_{2n},y_{2n+1})| + 2b^2|S(y_{2n+1},y_{2n+1},y_{2n+2})|$$
$$+ 2b^3|S(y_{2n+2},y_{2n+2},y_{2n+3})| + \cdots$$

$$+2b^{m-n}|S(y_{2m-1}, y_{2m-1}, y_{2m})|$$

$$|S(y_{2n}, y_{2n}, y_{2m})| \leq 2b\, k^{2n}|S(y_0, y_0, y_1)| + 2b^2k^{2n+1}|S(y_0, y_0, y_1)|$$
$$+ 2b^3k^{2n+2}|S(y_0, y_0, y_1)| + \cdots + 2b^{m-n}k^{2m-1}|S(y_0, y_0, y_1)|$$
$$\leq 2(bk^{2n} + b^2k^{2n+1} + \cdots + b^{m-n}k^{2m-1})|S(y_0, y_0, y_1)|$$
$$= \frac{2bk^{2n}}{1-bk}\ |S(y_0, y_0, y_1)|.$$

Since $bk, k < 1$, we have

$$|S(y_{2n}, y_{2n}, y_{2m})| \leq \frac{2bk^{2n}}{1-bk}\ |S(y_0, y_0, y_1)| \to 0 \text{ as }, n \to \infty.$$

Hence $\{y_{2n}\}$ is a S_b-Cauchy sequence in X. Since X is complete, there exist a point $u \in X$ such that

$$y_{2n} = Ax_{2n} = gx_{2n+1} \to u \text{ as } n \to \infty.$$

Suppose that f is complete subspace of X, there exists a point $v \in X$ such that $fv = u$.

Consequently, we find $Av = u$.

Consider

$$S(Av, Av, y_{2n+1}) = S(Av, Av, Bx_{2n+1})$$
$$\precsim \alpha_1 \left\{\frac{S(fv,fv,gx_{2n+1})\ S(Bx_{2n+1},Bx_{2n+1},fv)}{2S(fv,fv,gx_{2n+1}) + S(Bx_{2n+1},Bx_{2n+1},gx_{2n+1})}\right\}$$
$$+ \alpha_2 \{S(Av, Av, fv) + S(Bx_{2n+1}, Bx_{2n+1}, gx_{2n+1})\}$$
$$+ \alpha_3\{S(Bx_{2n+1}, Bx_{2n+1}, fv) + S(Av, Av, gx_{2n+1})\}$$
$$+ \alpha_4 S(fv, fv, gx_{2n+1}).$$

Letting $n \to \infty$, we get

$$S(Av, Av, u) \precsim \alpha_2 S(Av, Av, u) + \alpha_3 S(Av, Av, u)$$
$$(1 - \alpha_2 - \alpha_3)S(Av, Av, u) \precsim 0$$
$$S(Av, Av, u) \precsim 0.$$

This implies that $S(Av, Av, u) = 0$, that is $Av = u$.

Thus, we have $fv = Av = u$, so that u is a coincidence point of A and f.

Now since $u = Av$ and $A \subset g$, there exists a point $w \in X$ such that $u = gw$.

We claim that $Bw = u$.

Consider

$S(u,u,Bw) = S(Av,Av,Bw)$

$$S(u,u,Bw) \precsim \alpha_1 \left\{\frac{S(fv,fv,gw)S(Bw,Bw,fv)}{2S(fv,fv,gw)+ S(Bw,Bw,gw)}\right\} + \alpha_2\{S(Av,Av,fv) + S(Bw,Bw,gw)\}$$
$$+ \alpha_3 \{S(Bw,Bw,fv) + S(Av,Av,gw)\} + \alpha_4 S(fv,gw)$$

$\Rightarrow S(u,u,Bw) \precsim \alpha_2 S(Bw,Bw,u) + \alpha_3 S(Bw,Bw,u),$

$\Rightarrow (1-\alpha_2-\alpha_3)S(u,u,Bw) \precsim 0$ which gives $S(u,u,Bw) = 0$, that is $Bw = u$.

Hence, we have $u = gw = Bw$, so that u is a coincidence point of B and g.

Now, suppose that u' is another coincidence point of A and f, that is $u' = fv' = Av'$ for some $v' \in X$.

Consider

$S(u',u',u) = S(Au',Au',Bw)$

$$S(u',u',u) \precsim \alpha_1 \left\{\frac{S(fv',fv',gw)S(BW,Bw,fv')}{2S(fv',fv',gw)+ S(Bw,Bw,gw)}\right\} + \alpha_2\{S(Av',Av',fv') + S(Bw,Bw,Tw)\}$$
$$+ \alpha_3\{S(Bw,Bw,fv') + S(Av',Av',gw)\} + \alpha_4 S(fv',fv',gw)$$
$$\precsim \alpha_1 \left\{\frac{S(u',u',u)S(u,u,u')}{2S(u',u',u)+ S(u,u,u)}\right\} + \alpha_2\{S(u',u',u') + S(u,u,u)\}$$
$$+ \alpha_3\{S(u,u,u') + S(u',u',u)\} + \alpha_4 S(u',u',u)$$

$S(u',u',u) \precsim (\frac{1}{2}\alpha_1 + 2\alpha_3 + \alpha_4)\, S(u',u',u).$

This implies that $|S(u',u',u)| < (\frac{1}{2}\alpha_1 + 2\alpha_3 + \alpha_4)\ |S(u',u',u)|$, which leads us to a contradiction. Hence $|\,S(u',u',u)| = 0$, that is $u' = u$.

Now, suppose that $u"$ is another coincidence point of B and g, that is $u" = Bw' = gw'$ for some $w' \in X$, we have

$S(u,u,u") = S(Av,Av,Bw')$

$S(u,u,u") \precsim (\frac{1}{2}\alpha_1 + 2\alpha_3 + \alpha_4)\, S(u,u,u").$

This implies that $|S(u,u,u")| < (\alpha_1 + 2\alpha_3 + \alpha_4)|S(u,u,u")|$, which leads us to a contradiction. So that $|S(u,u,u")| = 0$, that is $u = u"$.

Thus, u is the unique coincidence point of (A,f) and (B,g).

Now since pairs (A, f) and (B, g) are weakly compatible on X and $u = fv = Av = gw = Bw$, and $Afv = fAv$, $Bgw = gBw$.

Implies that

$$Au = Afv = fAv = fu$$

$$Bu = Bgw = gBw = gu.$$

Now consider

$$S(Au, Au, Bu) \precsim \alpha_1 \left\{\frac{S(fu, fu, gu)\, S(Bu, Bu, fu)}{2S(fu, fu, gu) + S(Bu, Bu, gu)}\right\} + \alpha_2\{S(Au, Au, fu) + S(Bu, Bu, gu)\}$$

$$+ \alpha_3\{S(Bu, Bu, fu) + S(Au, Au, gu)\} + \alpha_4 S(fu, fu, gu)$$

$$\precsim \frac{1}{2}\alpha_1 S(Bu, Bu, Au) + 2\alpha_3\{S(Bu, Bu, Au) + \alpha_4 S(Au, Au, Bu)$$

$S(Au, Au, Bu) \precsim (\frac{1}{2}\alpha_1 + 2\alpha_3 + \alpha_4) S(Au, Au, Bu)$

Implies that $Au = Bu$, that is

$Au = fu = Bu = gu.$

Again consider

$$S(Av, Av, Bu) \precsim \alpha_1 \left\{\frac{S(fv, fv, gu)\, S(Bu, Bu, fv)}{2d(fv, fv, gu) + d(Bu, Bu, gu)}\right\} + \alpha_2\{S(Av, Av, fv) + S(Bu, Bu, gu)\}$$

$$+ \alpha_3\{S(Bu, Bu, fv) + S(Av, Av, gu)\} + \alpha_4 S(fv, fv, gu)$$

$$S(Av, Av, Bu) \precsim \alpha_1 \left\{\frac{S(Av, Av, Bu)\, S(Bu, Bu, Av)}{2S(Av, Av, Bu) + S(Bu, Bu, Bu)}\right\} + \alpha_2\{S(Av, Av, Av) + S(Bu, Bu, Bu)\}$$

$$+ \alpha_3\{S(Bu, Bu, Av) + S(Av, Av, Bu)\} + \alpha_4 S(Av, Av, gu)$$

$S(Av, Av, Bu) \precsim (\frac{1}{2}\alpha_1 + 2\alpha_3 + \alpha_4)\, S(Av, Av, Bu),$

we get $Av = Bu$, that is $u = Bu$, which implies that

$u = Au = Bu = fu = gu.$

Hence u is the unique common fixed point of A, B, f and g.

Remark 3.1. All the conditions of Theorem 3.1 remain true if we replace the contraction condition (3.2) by one of following conditions by equating α_1, α_2, α_3 and α_4 to zero suitably in Theorem 3.1.

Corollary 3.1 Let A, B, f and g be four self mappings of a complete complex valued S_b-metric space (X, S) with the coefficient $b \geq 1$ and satisfying condition (3.1) and satisfying one of the following contraction conditions:

(i) $S(Ax, Ax, By) \precsim \alpha_1 \left\{\frac{S(fx,fx,gy)S(By,By,fx)}{2S(fx,fx,gy)+ S(By,By,gy)}\right\} + \alpha_2\{S(Ax, fx, fx) + S(By, By, gy)\}$

$$+ \alpha_3\{S(By, By, fx) + S(Ax, Ax, gy)\}$$

If $2S(fx, fx, gy) + S(By, By, gy) \neq 0$, for all $x, y \in X$, $\alpha_i \geq 0$ $(i = 1, 2, 3,)$ with at least one α_i is nonzero and $b\alpha_1 + 2\alpha_2 + 3b\alpha_3 < 1$.

(ii) $S(Ax, Ax, By) \precsim \alpha_1 \left\{\frac{S(fx,fx,gy)S(By,By,fx)}{2S(fx,fx,gy)+ S(By,By,gy)}\right\} + \alpha_2\{S(Ax, fx, fx) + S(By, By, gy)\}$

$$+ \alpha_4 S(fx, fx, gy).$$

If $2S(fx, fx, gy) + S(By, By, gy) \neq 0$, for all $x, y \in X$, $\alpha_i \geq 0$ $(i = 1, 2, 4)$ with at least one α_i is nonzero and $b\alpha_1 + 2\alpha_2 + \alpha_4 < 1$.

(iii) $S(Ax, Ax, By) \precsim \alpha_1 \left\{\frac{S(fx,fx,gy)S(By,By,fx)}{2S(fx,fx,gy)+ S(By,By,gy)}\right\} + \alpha_3\{S(By, By, fx) +$

$$S(Ax, Ax, gy)\} + \alpha_4 S(fx, fx, gy).$$

If $2S(fx, fx, gy) + S(By, By, gy) \neq 0$, for all $x, y \in X$, $\alpha_i \geq 0$ $(i = 1, 3, 4)$ with at least one α_i is nonzero and $b\alpha_1 + 3b\alpha_3 + \alpha_4 < 1$.

(iv) $S(Ax, Ax, By) \precsim \alpha_1 \left\{\frac{S(fx,fx,gy)S(By,By,fx)}{2S(fx,fx,gy)+ S(By,By,gy)}\right\} + \alpha_2\{S(Ax, fx, fx) + S(By, By, gy)\}$

If $2S(fx, fx, gy) + S(By, By, gy) \neq 0$, for all $x, y \in X$, $\alpha_i \geq 0$ $(i = 1, 2,)$ with at least one α_i is nonzero and $b\alpha_1 + 2\alpha_2 < 1$.

(v) $S(Ax, Ax, By) \precsim \alpha_1 \left\{\frac{S(fx,fx,gy)S(By,By,fx)}{2S(fx,fx,gy)+ S(By,By,gy)}\right\} + \alpha_3\{S(By, By, fx) + S(Ax, Ax, gy)\}$

If $2S(fx, fx, gy) + S(By, By, gy) \neq 0$, for all $x, y \in X$, $\alpha_i \geq 0$ $(i = 1, 2, 3, 4)$ with at least one α_i is nonzero and $b\alpha_1 + 3b\alpha_3 < 1$.

(vi) $S(Ax, Ax, By) \precsim \alpha_1 \left\{\frac{S(fx,fx,gy)S(By,By,fx)}{2S(fx,fx,gy)+ S(By,By,gy)}\right\}$

If $2S(fx, fx, gy) + S(By, By, gy) \neq 0$, for all $x, y \in X$, $\alpha_i \geq 0$ $(i = 1)$ with at least one α_i is nonzero and $b\alpha_1 < 1$.

Remark 3.2. By putting $A = f, B = g$ and $A = B = I$ (identity mapping) in Theorem 3.1, we can obtain the following result:

Theorem 3.2 Let f and g be two self mappings of a complete complex valued S_b-metric space (X, S) with the coefficient $b \geq 1$ and satisfying

(i) $f \subset g$,

(ii) $S(fx, fx, gy) \precsim \alpha_1 \left\{\frac{S(x,x,gy)S(y,y,fx)}{2S(x,x,gy)+ S(y,y,gy)}\right\} + \alpha_2\{S(x, fx, fx) + S(y, y, gy)\}$

$$+ \alpha_3\{S(y, y, fx) + S(x, x, gy)\}+ \alpha_4 S(x, x, y).$$

If $2S(x, x, gy) + S(y, y, gy) \neq 0$, for all $x, y \in X$, $\alpha_i \geq 0$ $(i = 1, 2, 3, 4)$ with at least one α_i is nonzero and $b\alpha_1 + 2\alpha_2 + 3b\alpha_3 + \alpha_4 < 1$. If one of f or g is a complete subspace of X, then the pair (f, g) have a unique coincidence point in X. Also, if pair (f, g) is weakly compatible, then f and g have a unique common fixed point in X.

Remark 3.3. All the conditions of Theorem 3.2 remain true if we replace the contraction condition in Theorem 3.2 by one of following conditions by equating α_1, α_2, α_3 and α_4 to zero suitably in Theorem 3.2.

Corollary 3.2 Let f and g be two self mappings of a complete complex valued S_b-metric space (X, S) with the coefficient $b \geq 1$ and satisfying condition $f \subset g$ and satisfying one of the following contraction conditions:

(i) $S(fx, fx, gy) \precsim \alpha_1 \left\{\frac{S(x,x,gy)S(y,y,fx)}{2S(x,x,gy)+ S(y,y,gy)}\right\} + \alpha_2\{S(x, fx, fx) + S(y, y, gy)\}$

$$+ \alpha_3\{S(y, y, fx) + S(x, x, gy)\}$$

If $2S(x, x, gy) + S(y, y, gy) \neq 0$, for all $x, y \in X$, $\alpha_i \geq 0$ $(i = 1, 2, 3)$ with at least one α_i is nonzero and $b\alpha_1 + 2\alpha_2 + 3b\alpha_3 < 1$.

(ii) $S(fx, fx, gy) \precsim \alpha_1 \left\{\frac{S(x,x,gy)S(y,y,fx)}{2S(x,x,gy)+ S(y,y,gy)}\right\} + \alpha_2\{S(x, fx, fx) + S(y, y, gy)\}$

$$+ \alpha_4 S(x, x, y).$$

If $2S(x, x, gy) + S(y, y, gy) \neq 0$, for all $x, y \in X$, $\alpha_i \geq 0$ $(i = 1, 2, 4)$ with at least one α_i is nonzero and $b\alpha_1 + 2\alpha_2 + \alpha_4 < 1$.

(iii) $S(fx, fx, gy) \precsim \alpha_1 \left\{\frac{S(x,x,gy)S(y,y,fx)}{2S(x,x,gy)+ S(y,y,gy)}\right\} +$

$$\alpha_3\{S(y,y,fx)+S(x,x,gy)\}+\alpha_4 S(x,x,y).$$

If $2S(x,x,gy)+S(y,y,gy)\neq 0$, for all $x,y\in X$, $\alpha_i\geq 0$ $(i=1,3,4)$ with at least one α_i is nonzero and $b\alpha_1+2\alpha_2+3b\alpha_3+\alpha_4<1$

(iv) $S(fx,fx,gy)\precsim \alpha_1\left\{\frac{S(x,x,gy)S(y,y,fx)}{2S(x,x,gy)+S(y,y,gy)}\right\}+\alpha_2\{S(x,fx,fx)+S(y,y,gy)\}$

If $2S(x,x,gy)+S(y,y,gy)\neq 0$, for all $x,y\in X$, $\alpha_i\geq 0$ $(i=1,2)$ with at least one α_i is nonzero and $b\alpha_1+2\alpha_2<1$.

(v) $S(fx,fx,gy)\precsim \alpha_1\left\{\frac{S(x,x,gy)S(y,y,fx)}{2S(x,x,gy)+S(y,y,gy)}\right\}+\alpha_3\{S(y,y,fx)+S(x,x,gy)\}$

If $2S(x,x,gy)+S(y,y,gy)\neq 0$, for all $x,y\in X$, $\alpha_i\geq 0$ $(i=1,3)$ with at least one α_i is nonzero and $b\alpha_1+3b\alpha_3<1$.

(vi) $S(fx,fx,gy)\precsim \alpha_1\left\{\frac{S(x,x,gy)S(y,y,fx)}{2S(x,x,gy)+S(y,y,gy)}\right\}$

If $2S(x,x,gy)+S(y,y,gy)\neq 0$, for all $x,y\in X$, $\alpha_i\geq 0$ $(i=1)$ with at least one α_i is nonzero and $b\alpha_1<1$.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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