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## SOFT SET $\beta$ -CONNECTED MAPPINGS

ALPA SINGH RAJPUT<sup>1,\*</sup>, JYOTI PANDEY BAJPAI<sup>2</sup>, S. S. THAKUR<sup>3</sup>

<sup>1</sup>Department of Science and Humanities, Vignan Foundation for Science, Technology and Research, Guntur,  
522213, India

<sup>2</sup>Department of Applied Mathematics, Jabalpur Engineering College Jabalpur, India

<sup>3</sup>Jabalpur Engineering College Jabalpur, India

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**Abstract.** In this paper, we introduce the concepts of soft  $\beta$ -connectedness between soft sets and soft set  $\beta$ -connected mapping in soft topological spaces. We showed that the concept of  $\beta$ -connectedness between soft sets is stronger than that of semi connectedness between soft sets and pre connectedness between soft sets. Further some of its properties and characterizations of soft  $\beta$ -connectedness between soft sets and soft set  $\beta$ -connected mapping are established.

**Keywords:** soft  $\beta$ -open sets; soft  $\beta$ -closed sets; soft  $\beta$ -connectedness; soft  $\beta$ -connectedness between soft sets; soft set  $\beta$ -connected mapping.

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### 1. INTRODUCTION

The real world is mixed up for our immediate recognizing, many disciplines like medicine, economics, sociology and engineering are extremely dependent on the function of modeling data with uncertainty. The uncertainty is so tricky to specify, classical mathematical models approaches are frequently inadequate to convenient models or derive implicit In the recent past

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\*Corresponding author

E-mail address: [alpasinghrajput09@mail.com](mailto:alpasinghrajput09@mail.com)

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various mathematical theories like fuzzy set theory [34], intuitionistic fuzzy set theory [5], rough set theory [25], vague set theory [12] and probability theory are considered as an operative tools for handling with vagueness and uncertainty in these type of problems. But each of them having its own restrictions of inadequacy of parametrization. In 1999, Molodtsov [23] has been initiated the study of a new theory called soft set theory as a general mathematical tool to deal with uncertainties which is free from the difficulties of the above theories. In ([23, 24]), Molodtsov successfully applied the soft sets concepts in various directions such as , game theory, smoothness of functions, operations research, Riemann integration, Perron integration, probability and theory of measurement. In 2002, Maji, Biswas and Roy [20] gave the practical application of this theory in problem of decision making . In another paper, Maji, Biswas and Roy [19] studied several basic notions of soft set theory. In 2005, Pei and Miao [26] improved the work of Maji, Biswas and Roy ([19, 20]). In the recent years, an increasing number of papers have been written about soft sets theory and its applications in various fields ([6, 7]). In 2011, Shabir and Naz [28] introduced the notion of soft topological spaces as generalization of topological spaces. After the publication of Shabir and Naz [28] paper many authors such as Peyghan et. al. [27], Majumdar and Samanta[22], Georgiou et.al.[13], Chen [8], Hussain[14, 15], Zorlutana et.al. ([35, 36]), Kharal and Ahmad [17], Akdağ , A. Ozkan [1], Arockiarani and Lancy [4], Mahanta and Das [21],Kandil et.al. [16],Krishnaveni and Sekar [18],Yumak et.al. [33] ,Thakur and Rajput [30, 31, 32],Benchalli et.al. [9, 10, 11] have been contributed in the theory of soft topological spaces. In this paper we introduce and study the concepts of soft  $\beta$ -connectedness between soft sets and soft set  $\beta$ -connected mapping in soft topological spaces.

## 2. PRELIMINARIES

Let an initial universe set and a set of parameters is denoted by  $Q$  and  $E$  respectively ,  $P(Q)$  is the power set of  $Q$ .

**Definition 2.1.** [23] A pair  $(X,E)$  is called a soft set over  $Q$ , where  $X$  is a mapping given by  $X: E \rightarrow P(Q)$ .

In other words, a soft set over  $Q$  is a parameterized family of soft sets of the universe  $Q$ . For all  $e \in E$ ,  $X(e)$  may be considered as the set of  $e$ -approximate elements of the soft set  $(X,E)$ .

**Definition 2.2.** [19] Two soft sets  $(X,A)$  and  $(Y,B)$  over  $Q$  where,  $A \subset E$ ,  $B \subset E$ ,  $(X,A)$  is called a soft subset of  $(Y,B)$  and denoted by

$$(X,A) \subseteq (Y,B), \text{ if}$$

$$(a) A \subseteq B \text{ and}$$

$$(b) X(e) \subseteq Y(e) \text{ for all } e \in E.$$

**Definition 2.3.** [19] Two soft sets  $(X,A)$  and  $(Y,B)$  over  $Q$  are called soft equal set denoted by  $(X,A) = (Y,B)$  if  $(X,A) \subseteq (Y,B)$  and  $(Y,B) \subseteq (X,A)$ .

**Definition 2.4.** [22] The complement of a soft set  $(X,E)$  denoted by  $(X,E)^c$ , is defined by  $(X,E)^c = (F^c, E)$ , where  $X^c : E \rightarrow P(Q)$  is a mapping given by  $X^c(e) = Q - X(e)$ , for all  $e \in E$ .

**Definition 2.5.** [19] Let a soft set  $(X,E)$  over  $Q$ .

$$(a) \text{ Null soft set denoted by } \tilde{\phi} \text{ if for all } e \in E, X(e) = \tilde{\phi}.$$

$$(b) \text{ Absolute soft set denoted by } \tilde{Q}, \text{ if for each } e \in E, X(e) = Q.$$

$$\text{Clearly, } \tilde{Q}^c = \tilde{\phi} \text{ and } \tilde{\phi} = \tilde{Q}.$$

**Definition 2.6.** [3] Union of two sets  $(X,A)$  and  $(Y,B)$  over  $Q$  is the soft  $(Z, C)$ , where  $C = A \cup B$ , and for all  $e \in C$ ,

$$Z(e) = \begin{cases} X(e), & \text{if } e \in A - B \\ Y(e), & \text{if } e \in B - A \\ X(e) \cup Y(e), & \text{if } e \in A \cap B \end{cases}$$

**Definition 2.7.** [3] Intersection of two soft sets  $(X,A)$  and  $(Y,B)$  over  $Q$ , is the soft set  $(Z, C)$  where  $C = A \cap B$  and  $Z(e) = X(e) \cap Y(e)$  for each  $e \in E$ .

Let  $M$  and  $N$  be an initial universe sets and  $E$  and  $K$  be the nonnull sets of parameters,  $S(M, E)$ (resp.  $S(N, K)$ ) denotes the family of all soft sets over  $M$ (resp.  $N$ ).

**Definition 2.8.** [28] A subfamily  $\mathfrak{S}$  of  $S(M, E)$  is called a soft topology on  $M$  if:

$$(1) \tilde{\phi}, \tilde{X} \text{ belong to } \mathfrak{S}.$$

$$(2) \text{ The union of any number of soft sets in } \mathfrak{S} \text{ belongs to } \mathfrak{S}.$$

(3) The intersection of any two soft sets in  $\mathfrak{S}$  belongs to  $\mathfrak{S}$ .

The triplet  $(M, \mathfrak{S}, E)$  is called a soft topological space (briefly STS) over  $M$ . The members of  $\mathfrak{S}$  are called soft open sets in  $M$  and their complements are called soft closed sets in  $M$ .

**Definition 2.9.** If STS  $(M, \mathfrak{S}, E)$  and  $(X, E) \in S(M, E)$ .

(a) The closure of  $(X, E)$  is defined as the intersection of all soft closed super sets of  $(X, E)$  and it is denoted by  $Cl(X, E)$ [28].

(b) The interior of  $(X, E)$  is defined as the union of all soft open subsets of  $(X, E)$  and it is denoted by  $Int(X, E)$  [35].

**Definition 2.10.** [28] Let STS  $(M, \mathfrak{S}, E)$  over  $M$  and  $N \subset M$ . Then  $\mathfrak{S}_N = \{(F_N, E) : (X, E) \in \mathfrak{S}\}$  is said to be the soft relative topology over  $N$  and  $(N, \mathfrak{S}_N, E)$  is called a soft subspace of  $(M, \mathfrak{S}, E)$ .

**Definition 2.11.** [35] The soft set  $(X, E) \in S(M, E)$  is called a soft point if there exist  $x \in M$  and  $e \in E$  such that  $X(e) = \{x\}$  and  $X(e') = \phi$  for each  $e' \in E - \{e\}$ , and the soft point  $(X, E)$  is denoted by  $(x_e)_E$ .

**Definition 2.12.** [2, 8, 29] A soft set  $(X, E) \in (M, \mathfrak{S}, E)$  is said to be :

(a) Soft semi-open if  $(X, E) \subseteq Cl(Int(X, E))$ .

(b) Soft pre-open if  $(X, E) \subseteq Int(Cl(X, E))$ .

(c) Soft  $\beta$ -open if  $(X, E) \subseteq Cl(Int(Cl(X, E)))$ .

The complement of soft semi-open set (resp. soft pre-open, soft  $\beta$ -open) set is called soft semi-closed (resp. soft pre-closed, soft  $\beta$ -closed) set.

**Remark 2.13.** [2, 8, 29] (a) Every soft open (resp. soft closed) set is soft pre-open (resp. pre-closed) and soft semi-open (resp. semi-closed) but the converses may not be true.

(b) The concepts of soft semi-open (resp. soft semi-closed) and soft pre-open (resp. soft pre-closed) sets are independent to each other.

(c) Every soft pre-open (resp. pre-closed) and soft semi-open (resp. semi-closed) is soft  $\beta$ -open (resp. soft  $\beta$ -closed) set but the converses may not be true.

**Definition 2.14.** [2] Let  $(F, E)$  be a soft set in a soft topological space  $(M, \mathfrak{S}, E)$ .

(a) The soft  $\beta$ -closure of  $(F, E)$  is defined as the smallest soft  $\beta$ -closed set over which contains  $(F, E)$  and it is denoted by  $\beta Cl(F, E)$ .

(b) The soft  $\beta$ -interior of  $(F, E)$  is defined as the largest soft  $\beta$ -open set over which is contained in  $(F, E)$  and is denoted by  $\beta Int(F, E)$ .

**Definition 2.15.** [9] A soft  $\beta$  separation of soft topological space  $(M, \mathfrak{S}, E)$  is a pair  $(F, E), (G, E)$  of disjoint nonnull soft  $\beta$  open sets whose union is  $\tilde{M}$ .

**Definition 2.16.** [11] A soft topological space  $(M, \mathfrak{S}, E)$  is called a soft  $\beta$  connected, if there does not exist a soft  $\beta$  separation of  $M$ .

**Definition 2.17.** [30, 31, 32] A soft topological space  $(M, \mathfrak{S}, E)$  is called soft connectedness (resp.  $P$ -connected,  $s$ -connectedness) between soft sets  $(F_1, E)$  and  $(F_2, E)$  if and only if there is no soft clopen (resp. preclopen, semiopen) set  $(F, E)$  over  $X$  such that  $(F_1, E) \subset (F, E)$  and  $(F, E) \cap (F_2, E) = \phi$ .

**Remark 2.18.** [30, 31, 32] A soft topological space  $(M, \mathfrak{S}, E)$  is soft connected (resp. preconnected, semiconnected) if and only if it soft connected (resp.  $p$ -connected,  $s$ -connected) between every pair of its nonempty soft sets.

**Definition 2.19.** [17] Let  $S(M, E)$  and  $S(N, K)$  be families of soft sets. Let  $u: M \rightarrow N$  and

$p: E \rightarrow K$  be mappings. Then a mapping  $f_{pu}: S(M, E) \rightarrow S(N, K)$  is defined as:

(a) Let  $(X, A)$  be a soft set in  $S(M, E)$ . The image of  $(X, A)$  under  $f_{pu}$ , written as  $f_{pu}(X, A) = (f_{pu}(X), p(A))$ , is a soft set in  $S(N, K)$  such that

$$f_{pu}(M)(k) = \begin{cases} \bigcup_{e \in p^{-1}(k) \cap A} u(X(e)) & , p^{-1}(k) \cap A \neq \phi \\ \phi & , p^{-1}(k) \cap A = \phi \end{cases}$$

for all  $k \in K$ .

(b) Let soft set  $(Y, B) \in S(N, K)$ . The inverse image of  $(Y, B)$  under  $f_{pu}$ , written as

$$f_{pu}^{-1}(Y, B) = \begin{cases} u^{-1}Y(p(e)) & , p(e) \in B \\ \phi & , otherwise \end{cases}$$

for all  $e \in E$ .

The soft mapping  $f_{pu}$  is called surjective if  $p$  and  $u$  are surjective. The soft mapping  $f_{pu}$  is called injective if  $p$  and  $u$  are injective.

**Definition 2.20.** [35] Let  $(M, \mathfrak{S}, E)$  and  $(N, \mathfrak{U}, K)$  be two STS over  $M$  and  $N$  respectively. A soft mapping  $f_{pu} : (M, \mathfrak{S}, E) \rightarrow (N, \mathfrak{U}, K)$  is called soft continuous if  $f_{pu}^{-1}(Y, K)$  is soft open over  $M$ , for every soft open set  $(Y, K)$  over  $N$ .

**Definition 2.21.** [30, 31] A soft mapping  $f_{pu} : (M, \mathfrak{S}, E) \rightarrow (N, \mathfrak{U}, K)$  is called soft set connected (resp.  $P$ -connected, semi-connected) provided, if soft topological space  $(M, \mathfrak{S}, E)$  is soft connected (resp.  $P$ -connected, semi-connected) between sets  $(X, E)$  and  $(Y, E)$  then soft subspace  $(f_{pu}(M), \mathfrak{U}_{f_{pu}(M)}, K)$  is soft connected (resp.  $P$ -connected, semi-connected) between  $f_{pu}(X, E)$  and  $f_{pu}(Y, E)$ .

**Remark 2.22.** [30, 31] 1. The concepts of soft set  $P$ -connected mappings and soft set-connected mappings are independent.

2. The concepts of soft set  $s$ -connected mappings and soft set-connected mappings are independent.

### 3. $\beta$ -CONNECTEDNESS BETWEEN SOFT SETS

Throughout this paper soft  $\beta$ -clopen set means soft  $\beta$ -closed and soft  $\beta$ -open set and soft connected between the soft sets (soft semi-connectedness between soft sets, soft pre-connectedness between soft sets and soft  $\beta$ -connectedness between soft sets) briefly as cbss (  $s$ -cbss,  $p$ -cbss and  $\beta$ -cbss respectively ).

**Definition 3.1.** A STS  $(M, \mathfrak{S}, E)$  is called soft  $\beta$ -connected between soft sets (briefly  $\beta$ -cbss)  $(X, E)$  and  $(Y, E)$  if and only if there is no soft  $\beta$ -clopen set  $(Z, E)$  over  $M$  such that  $(X, E) \subset (Z, E)$  and  $(Z, E) \cap (Y, E) = \tilde{\phi}$ .

**Theorem 3.2.** Every STS  $(M, \mathfrak{S}, E)$  is soft  $\beta$ -cbss  $(X, E)$  and  $(Y, E)$  is soft cbss  $(X, E)$  and  $(Y, E)$ .

*Proof:* Suppose STS  $(M, \mathfrak{S}, E)$  is not soft cbss  $(X, E)$  and  $(Y, E)$ , there is a soft clopen set  $(Z, E)$  over  $M$  such that  $(X, E) \subset (Z, E)$  and  $(Z, E) \cap (Y, E) = \tilde{\phi}$ . Since each soft clopen set is also soft  $\beta$ -clopen, it follows that  $(M, \mathfrak{S}, E)$  is not soft  $\beta$ -cbss  $(X, E)$  and  $(Y, E)$ . This is a contradiction.

**Remark 3.3.** The converse of Theorem 3.2 may not be true.

**Example 3.4.** Let  $M = \{m_1, m_2, m_3, m_4\}$  be universe set and  $E = \{e_1, e_2\}$  is set of parameter. Let  $(X, E), (X_1, E), (X_2, E), (X_3, E), (F, E)$  and  $(G, E)$  are soft sets over  $M$  defined as follows:  $(X_1, E) = \{(e_1, \{m_1, m_3\}), (e_2, \{m_1, m_3\})\}$ ,  $(X_2, E) = \{(e_1, \{m_2\}), (e_2, \{m_2\})\}$ ,  $(X_3, E) = \{(e_1, \{m_1, m_4\}), (e_2, \{m_1, m_4\})\}$ ,  $(F, E) = \{(e_1, \{m_2\}), (e_2, \phi)\}$ ,  $(G, E) = \{(e_1, \phi), (e_2, \{m_1\})\}$

Let  $\mathfrak{S} = \{\tilde{\phi}, \tilde{M}, (X_1, E), (X_2, E), (X_3, E)\}$  is a soft topology over  $M$ . Then STS  $(M, \mathfrak{S}, E)$  is soft cbss  $(F, E)$  and  $(G, E)$  but not soft  $\beta$ -cbss  $(F, E)$  and  $(G, E)$ .

**Theorem 3.5.** Every STS is soft  $\beta$ -cbss  $(F, E)$  and  $(G, E)$  is soft pre-cbss  $(F, E)$  and  $(G, E)$ .

*Proof.* Suppose STS  $(M, \mathfrak{S}, E)$  is not soft p-cbss  $(F, E)$  and  $(G, E)$ . Then there is a soft pre clopen set  $(X, E)$  over  $M$  such that  $(F, E) \subset (G, E)$  and  $(X, E) \cap (G, E) = \tilde{\phi}$ . Since, each soft pre clopen set is soft  $\beta$ -clopen, it follows that  $(M, \mathfrak{S}, E)$  is not soft  $\beta$ -cbss  $(F, E)$  and  $(G, E)$ . This is a contradiction.  $\square$

**Remark 3.6.** The converse of Theorem 3.5 is not true in general.

**Example 3.7.** Let  $M = \{m_1, m_2, m_3\}$  be universe set and  $E = \{e_1, e_2\}$  is set of parameter. Let  $(X_1, E), (X_2, E), (F, E)$  and  $(G, E)$  are soft sets over  $M$  defined as follows:  $(X_1, E) = \{(e_1, \{m_1\}), (e_2, \{m_1\})\}$ ,  $(X_2, E) = \{(e_1, \{m_1, m_2\}), (e_2, \{m_1, m_2\})\}$ ,  $(F, E) = \{(e_1, \{m_2\}), (e_2, \phi)\}$ ,  $(G, E) = \{(e_1, \phi), (e_2, \{m_1\})\}$ ,

Let  $\mathfrak{S} = \{\tilde{\phi}, \tilde{M}, (X_1, E), (G_2, E)\}$  is a soft topology over  $M$ . Then STS  $(M, \mathfrak{S}, E)$  is soft pre-cbss  $(F, E)$  and  $(G, E)$  but not soft  $\beta$ -cbss  $(F, E)$  and  $(G, E)$ .

**Theorem 3.8.** Every STS is soft  $\beta$ -cbss  $(A, E)$  and  $(B, E)$  is soft semi-cbss  $(A, E)$  and  $(B, E)$ .

*Proof.* Suppose STS  $(M, \mathfrak{S}, E)$  is not soft s-cbss  $(A, E)$  and  $(B, E)$ . Then there is a soft semi clopen set  $(F, E)$  over  $M$  such that  $(A, E) \subset (F, E)$  and  $(F, E) \cap (B, E) = \tilde{\phi}$ . Since, every soft semi clopen set is soft  $\beta$ -clopen, it follows that  $(M, \mathfrak{S}, E)$  is not soft  $\beta$ -cbss  $(A, E)$  and  $(B, E)$ . This is a contradiction.  $\square$

**Remark 3.9.** *The converse of theorem 3.8 is not true in general.*

**Example 3.10.** *Let  $M = \{m_1, m_2, m_3\}$  be universe set and  $E = \{e_1, e_2\}$  is set of parameter. Let  $(X_1, E), (X_2, E), (X_3, E), (A, E)$  and  $(B, E)$  are soft sets over  $M$  defined as follows:  $(X_1, E) = \{(e_1, \{m_1\}), (e_2, \{m_1\})\}$ ,  $(X_2, E) = \{(e_1, \{m_2\}), (e_2, \{m_2\})\}$ ,  $(X_3, E) = \{(e_1, \{m_1, m_2\}), (e_2, \{m_1, m_2\})\}$ ,  $(A, E) = \{(e_1, \{m_3\}), (e_2, \{m_3\})\}$ ,  $(B, E) = \{(e_1, \{m_1, m_2\}), (e_2, \{m_1, m_2\})\}$ ,*

*Let  $\mathfrak{S} = \{\tilde{\phi}, \tilde{M}, (X_1, E), (X_2, E), (X_3, E)\}$  is a soft topology over  $M$ . Then STS  $(M, \mathfrak{S}, E)$  is soft  $s$ -cbss  $(A, E)$  and  $(B, E)$  but not soft  $\beta$ -cbss  $(A, E)$  and  $(B, E)$ .*

**Theorem 3.11.** *A STS  $(M, \mathfrak{S}, E)$  is soft  $\beta$ -cbss  $(X, E)$  and  $(Y, E)$  if and only if there is no soft  $\beta$ -clopen set  $(F, E)$  over  $M$  such that  $(X, E) \subset (F, E) \subset (Y, E)^c$ .*

*Proof:* Follows from Definition 3.1.

**Theorem 3.12.** *If STS  $(M, \mathfrak{S}, E)$  is soft  $\beta$ -cbss  $(X, E)$  and  $(Y, E)$  then  $(X, E) \neq \tilde{\phi} \neq (Y, E)$ .*

*Proof:* If soft set  $(X, E) = \tilde{\phi}$ , then  $\tilde{\phi}$  being soft  $\beta$ -clopen set over  $M$ ,  $(M, \mathfrak{S}, E)$  can not be soft  $\beta$ -cbss  $(X, E)$  and  $(Y, E)$ . This proves the theorem.

**Theorem 3.13.** *If STS  $(M, \mathfrak{S}, E)$  is soft  $\beta$ -cbss  $(X, E)$  and  $(Y, E)$  and  $(X, E) \subset (F, E)$  and  $(Y, E) \subset (G, E)$  then  $(M, \mathfrak{S}, E)$  is soft  $\beta$ -cbss  $(F, E)$  and  $(G, E)$ .*

*Proof:* Suppose STS  $(M, \mathfrak{S}, E)$  is not soft  $\beta$ -cbss  $(F, E)$  and  $(G, E)$  then there is a soft  $\beta$ -clopen set  $(H, E)$  over  $M$  such that  $(F, E) \subset (H, E)$  and  $(H, E) \cap (G, E) = \tilde{\phi}$ . Consequently,  $(M, \mathfrak{S}, E)$  is not soft  $\beta$ -cbss  $(X, E)$  and  $(Y, E)$ .

**Lemma 3.14.** *A soft point  $(x_e)_E \in \beta cl(X, E)$  if and only if  $(X, E) \cap (Y, E) \neq \tilde{\phi}$  for all soft  $\beta$ -open set  $(Y, E)$  containing  $(x_e)_E$  over  $M$ .*

*Proof:* It is obvious.

**Theorem 3.15.** *A STS  $(M, \mathfrak{S}, E)$  is soft  $\beta$ -cbss  $(X, E)$  and  $(Y, E)$  if and only if  $(M, \mathfrak{S}, E)$  is soft  $\beta$ -cbss  $\beta cl(X, E)$  and  $\beta cl(Y, E)$ .*

*Proof:* Necessity: Follows from Theorem 3.13.

*Sufficiency:* If STS  $(M, \mathfrak{S}, E)$  is not soft  $\beta$ -cbss  $(X, E)$  and  $(Y, E)$  then there exists soft  $\beta$ -clopen set  $(A, E)$  over  $M$  such that  $(X, E) \subset (A, E)$  and  $(A, E) \cap (Y, E) = \tilde{\phi}$ . Since  $(A, E)$  is soft  $\beta$ -closed,  $\beta cl(X, E) \subset \beta cl(A, E) = (A, E)$ .



By Lemma 3.14. clearly  $(A, E) \cap \beta cl(Y, E) = \tilde{\phi}$ . Hence,  $(M, \mathfrak{S}, E)$  is not soft  $\beta$ -cbss  $\beta cl(X, E)$  and  $\beta cl(Y, E)$ .

**Theorem 3.16.** If STS  $(M, \mathfrak{S}, E)$  is soft  $\beta$ -cbss  $(X, E)$  and  $(Y, E)$  then  $(M, \mathfrak{S}, E)$  is soft  $\beta$ -cbss  $Cl(X, E)$  and  $Cl(Y, E)$ .

*Proof:* Follows from Theorem 3.13.

**Remark 3.17.** The converse of Theorem 3.16 is not true in general.

**Example 3.18.** Let  $M = \{m_1, m_2, m_3, m_4\}$  be universe set and  $E = \{e_1, e_2\}$  is set of parameter. The soft sets  $(X_1, E), (X_2, E), (X_3, E), (A, E)$  and  $(B, E)$  over  $M$  are defined as follows:  $(X_1, E) = \{(e_1, \{m_1, m_3\}), (e_2, \{m_1, m_3\})\}$ ,  $(X_2, E) = \{(e_1, \{m_2\}), (e_2, \{m_2\})\}$ ,  $(X_3, E) = \{(e_1, \{m_1, m_4\}), (e_2, \{m_1, m_4\})\}$ ,  $(A, E) = \{(e_1, \{m_2\}), (e_2, \phi)\}$ ,  $(B, E) = \{(e_1, \phi), (e_2, \{m_1\})\}$ ,

Let  $\mathfrak{S} = \{\tilde{\phi}, \tilde{M}, (X_1, E), (X_2, E), (X_3, E)\}$  is a soft topology over  $M$ . Then STS  $(M, \mathfrak{S}, E)$  is soft  $\beta$ -cbss  $Cl(A, E)$  and  $Cl(B, E)$  but not soft  $\beta$ -cbss  $(A, E)$  and  $(B, E)$ .

**Theorem 3.19.** A STS  $(M, \mathfrak{S}, E)$  is not soft  $\beta$ -cbss  $(F_{X_0}, E)$  and  $(F_{X_1}, E)$  if and only if there exist soft  $\beta$ -clopen disjoint sets  $(F_0, E)$  and  $(F_1, E)$  such that  $M = (F_0, E) \cup (F_1, E)$  and  $(F_{X_i}, E) \subset (F_i, E)$ ,  $i = 0, 1$ .

*Proof:* This follows from the definition of a soft space soft  $\beta$ -connected between two of its soft subsets.

**Theorem 3.20.** If  $(X_1, E)$  and  $(X_2, E)$  are soft sets over  $M$  and  $(X_1, E) \cap (X_2, E) \neq \tilde{\phi}$ , then STS  $(M, \mathfrak{S}, E)$  is soft  $\beta$ -cbss  $(X_1, E)$  and  $(X_2, E)$ .

*Proof:* If  $(X, E)$  is soft  $\beta$ -clopen set over  $M$  such that  $(X_1, E) \subset (X, E)$ , then  $(X_1, E) \cap (X_2, E) \neq \tilde{\phi} \Rightarrow (X, E) \cap (X_2, E) \neq \tilde{\phi}$ . This proves the theorem.

**Remark 3.21.** The converse of Theorem 3.20 need not be true.

**Example 3.22.** Let  $M = \{m_1, m_2\}$  be universe set and  $E = \{e_1, e_2\}$  is set of parameter. Let  $(X, E), (X_1, E), (X_2, E)$  and  $(G, E)$  are soft sets over  $M$  defined as follows:  $(X, E) = \{(e_1, \{m_1\}), (e_2, \phi)\}$ ,  $(X_1, E) = \{(e_1, \phi), (e_2, \{m_1\})\}$ ,  $(X_2, E) = \{(e_1, \{m_2\}), (e_2, \{m_1\})\}$ ,  $(G, E) = \{(e_1, \phi), (e_2, \{m_2\})\}$ ,

Let  $\mathfrak{S} = \{\tilde{\phi}, \tilde{M}, (X_1, E), (X_2, E)\}$  is a soft topology over  $M$ . Then STS  $(M, \mathfrak{S}, E)$  is soft  $\beta$ -cbss  $(X, E)$  and  $(G, E)$  although  $(X, E) \cap (G, E) = \tilde{\phi}$ .

**Theorem 3.23.** If STS  $(M, \mathfrak{S}, E)$  is neither soft cbss  $(X, E)$  and  $(Y_0, E)$  nor soft  $\beta$ -cbss  $(X, E)$  and  $(Y_1, E)$  then it is not soft  $\beta$ -cbss  $(X, E)$  and  $(Y_0, E) \cup (Y_1, E)$ .

*Proof:* Since STS  $(M, \mathfrak{S}, E)$  is not soft cbss  $(X, E)$  and  $(Y_0, E)$ , there is a soft  $\beta$ -clopen set  $(F_0, E)$  over  $M$  such that  $(X, E) \subset (F_0, E)$  and  $(F_0, E) \cap (Y_0, E) = \tilde{\phi}$ . Also, since  $(M, \mathfrak{S}, E)$  is not soft  $\beta$ -cbss  $(X, E)$  and  $(Y_1, E)$  there exists a soft  $\beta$ -clopen set  $(F_1, E)$  over  $M$  such that  $(X, E) \subset (F_1, E)$  and  $(Y_1, E) \cap (F_1, E) = \tilde{\phi}$ . Put  $(F, E) = (F_0, E) \cap (F_1, E)$ . Since every soft closed set is soft  $\beta$ -closed and intersection of any soft  $\beta$ -closed set is soft  $\beta$ -closed,  $(F, E)$  is soft  $\beta$ -closed. Also,  $(F, E)$  is soft  $\beta$ -open. Therefore  $(F, E)$  is soft  $\beta$ -clopen over  $M$  such that  $(X, E) \subset (F, E)$  and  $(F, E) \cap ((Y_0, E) \cup (Y_1, E)) = \tilde{\phi}$ . Hence,  $(M, \mathfrak{S}, E)$  is not soft  $\beta$ -cbss  $(X, E)$  and  $(Y_0, E) \cup (Y_1, E)$ .

**Remark 3.24.** If STS  $(M, \mathfrak{S}, E)$  is soft  $\beta$ -connected neither between  $(X, E)$  and  $(Y_0, E)$  nor between  $(X, E)$  and  $(Y_1, E)$  then it is not necessarily true that  $(M, \mathfrak{S}, E)$  is not soft  $\beta$ -cbss  $(X, E)$  and  $(Y_0, E) \cup (Y_1, E)$ .

**Example 3.25.** Let  $M = \{m_1, m_2, m_3, m_4\}$ ,  $E = \{e_1, e_2\}$ . Let  $(X, E), (X_1, E), (X_2, E), (X_3, E)$  over  $M$  are defined as follows:  $(X, E) = \{(e_1, \{m_2\}), (e_2, \{m_2, m_4\})\}$ ,  $(X_1, E) = \{(e_1, \{m_2\}), (e_2, \phi)\}$ ,  $(X_2, E) = \{(e_1, \phi), (e_2, \{m_2, m_4\})\}$ ,  $(X_3, E) = \{(e_1, \phi), (e_2, \{m_1\})\}$ ,

Let  $\mathfrak{S} = \{\tilde{\phi}, (X, E), (X_1, E), (X_2, E), \tilde{M}\}$  is topology on  $M$ . Then STS  $(M, \mathfrak{S}, E)$  is soft  $\beta$ -connected neither between the soft sets  $(X_1, E)$  and  $(X_3, E)$  nor  $(X_2, E)$  and  $(X_3, E)$  but it is soft  $\beta$ -cbss  $(X_3, E)$  and  $(X, E)$ .

**Theorem 3.26.** A STS  $(M, \mathfrak{S}, E)$  is soft  $\beta$ -connected if and only if it soft  $\beta$ -connected between every pair of its nonnull soft sets.

*Proof:* Let  $(X, E)$  and  $(Y, E)$  be nonnull soft sets over  $M$ . Suppose STS  $(M, \mathfrak{S}, E)$  is not soft  $\beta$ -cbss  $(X, E)$  and  $(Y, E)$ . Then there is a soft  $\beta$ -clopen set  $(F, E)$  over  $M$  such that  $(X, E) \subset (F, E)$  and  $(Y, E) \cap (F, E) = \tilde{\phi}$ . Since  $(X, E)$  and  $(Y, E)$  are nonnull, it follows that  $(F, E)$  is a nonempty soft proper  $\beta$ -clopen set over  $M$ . Hence,  $(M, \mathfrak{S}, E)$  is not soft  $\beta$ -connected.

Conversely, suppose that  $(M, \mathfrak{S}, E)$  is not soft  $\beta$ -connected. Then, there exists a nonempty soft set  $(F, E)$  over  $M$  which is both soft  $\beta$ -open and  $\beta$ -closed. Consequently,  $(M, \mathfrak{S}, E)$  is not

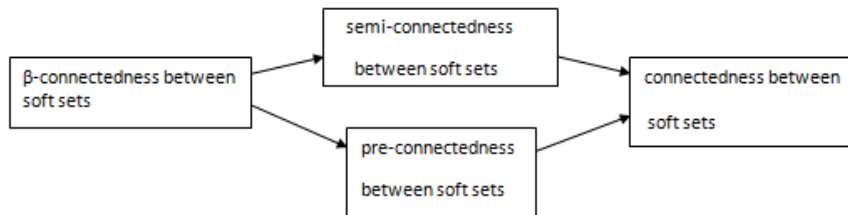
soft  $\beta$ -cbss  $(F,E)$  and  $(F,E)^c$ . Thus  $(M, \mathfrak{S}, E)$  is not soft  $\beta$ -connected between arbitrary pair of its nonempty soft sets.

**Remark 3.27.** If STS  $(M, \mathfrak{S}, E)$  is soft  $\beta$ -connected between a pair of its soft sets, then it is not necessarily soft  $\beta$ -connected between each pair of its soft sets and so is not necessarily soft  $\beta$ -connected.

**Example 3.28.** Let  $M = \{m_1, m_2, m_3, m_4\}$  be universe set and  $E = \{e_1, e_2\}$  is set of parameter. Let  $(X,E), (X_1,E), (X_2,E), (X_3,E), (A,E), (B,E)$  and  $(C,E)$  are soft sets over  $X$  defined as follows:  $(X,E) = \{(e_1, \{m_2, m_3\}), (e_2, \{m_2, m_3\})\}$ ,  $(X_1,E) = \{(e_1, \{m_1, m_3\}), (e_2, \{m_1, m_3\})\}$ ,  $(X_2,E) = \{(e_1, \{m_2\}), (e_2, \{m_2\})\}$ ,  $(X_3,E) = \{(e_1, \{m_1, m_4\}), (e_2, \{m_1, m_4\})\}$ ,  $(A,E) = \{(e_1, \emptyset), (e_2, \{m_1\})\}$ ,  $(B,E) = \{(e_1, \{m_1, m_4\}), (e_2, \{m_1, m_4\})\}$ ,  $(C,E) = \{(e_1, \{m_2\}), (e_2, \emptyset)\}$ ,

Let  $\mathfrak{S} = \{\tilde{\phi}, \tilde{M}, (X_1, E), (X_2, E), (X_3, E)\}$  is a soft topology over  $M$ . Then, STS  $(M, \mathfrak{S}, E)$  is soft  $\beta$ -cbss  $(X,E)$  and  $(B,E)$  but it is not soft  $\beta$ -cbss  $(A,E)$  and  $(C,E)$ . Also the soft topological space  $(M, \mathfrak{S}, E)$  is not soft  $\beta$ -connected.

Thus we reach at the following diagram of implications.



#### 4. SOFT SET $\beta$ -CONNECTED MAPPINGS

**Definition 4.1.** A soft mapping  $f_{pu} : (M, \mathfrak{S}, E) \rightarrow (N, \mathfrak{U}, K)$  is said to be soft set  $\beta$ -connected provided, if soft topological space  $(M, \mathfrak{S}, E)$  is soft  $\beta$ -cbss  $(X, E)$  and  $(Y, E)$  then soft subspace  $(f_{pu}(M), \mathfrak{V}_{f_{pu}(M)}, K)$  is soft  $\beta$ -cbss  $f_{pu}(X, E)$  and  $f_{pu}(Y, E)$  with respect to relative topology.

**Theorem 4.2.** A soft mapping  $f_{pu} : (M, \mathfrak{S}, E) \rightarrow (N, \mathfrak{U}, K)$  is soft set  $\beta$ -connected mapping if and only if  $f_{pu}^{-1}(N, K)$  is a soft  $\beta$ -clopen set over  $M$  for any soft  $\beta$ -clopen set  $(N, K)$  of  $(f_{pu}(M), \mathfrak{V}_{f_{pu}(M)}, K)$ .

*Proof.* Necessity : Let  $f_{pu}$  be soft set  $\beta$ -connected mapping and  $(N,K)$  be soft  $\beta$ -clopen set in  $(f_{pu}(M), \vartheta_{f_{pu}(M)}, K)$ . Suppose  $f_{pu}^{-1}(N,K)$  is not soft  $\beta$ -clopen in  $(M, \mathfrak{S}, E)$ . Then  $(M, \mathfrak{S}, E)$  is soft  $\beta$ -cbss  $f_{pu}^{-1}(N,K)$  and  $(f_{pu}^{-1}(N,K))^c$ . Therefore  $(f_{pu}(M), \vartheta_{f_{pu}(M)}, K)$  is soft  $\beta$ -cbss  $f_{pu}(f_{pu}^{-1}(F,K))$  and  $f_{pu}((f_{pu}^{-1}(F,K))^c)$  because  $f_{pu}$  is soft set  $\beta$ -connected. But,  $f_{pu}(f_{pu}^{-1}(N,K)) = (N,K) \cap (f_{pu}(M), \vartheta_{f_{pu}(M)}, K) = (N,K)$  and  $f_{pu}((f_{pu}^{-1}(N,K))^c) = (N,K)^c$  imply that  $(N,K)$  is not soft  $\beta$ -clopen in  $(f_{pu}(M), \vartheta_{f_{pu}(M)}, K)$ , a contradiction. Hence,  $f_{pu}^{-1}(N,K)$  is soft  $\beta$ -clopen in  $(M, \mathfrak{S}, E)$ .

Sufficiency : Let  $(M, \mathfrak{S}, E)$  be soft  $\beta$ -cbss  $(F,E)$  and  $(G,E)$ . If  $(f_{pu}(M), \vartheta_{f_{pu}(M)}, K)$  is not soft  $\beta$ -cbss  $f_{pu}(F,E)$  and  $f_{pu}(G,E)$  then there exists a soft  $\beta$ -clopen set  $(N,K)$  in  $(f_{pu}(M), \vartheta_{f_{pu}(M)}, K)$  such that  $f_{pu}(F,E) \subset (N,K) \subset (f_{pu}(G,E))^c$ . By hypothesis,  $f_{pu}^{-1}(N,K)$  is soft  $\beta$ -clopen set over  $M$  and  $(F,E) \subset f_{pu}^{-1}(N,K) \subset (G,E)^c$ . Therefore,  $(M, \mathfrak{S}, E)$  is not soft  $\beta$ -cbss  $(F,E)$  and  $(G,E)$ . This is a contradiction. Hence,  $f_{pu}$  is soft set  $\beta$ -connected.  $\square$

**Remark 4.3.** The concepts of soft set  $p$ -connected mappings and soft set  $\beta$ -connected mappings are independent.

**Example 4.4.** Let  $M = \{m_1, m_2, m_3, m_4\}$ ,  $E = \{e_1, e_2\}$  and  $N = \{n_1, n_2, n_3, n_4\}$ ,  $(A,E) = \{(e_1, \{m_2\}), (e_2, \{m_2\})\}$ ,  $(B,E) = \{(e_1, \{m_3\}), (e_2, \{m_3\})\}$ ,  $(C,E) = \{(e_1, \{m_4\}), (e_2, \{m_4\})\}$ ,  $(D,E) = \{(e_1, \{m_2, m_3\}), (e_2, \{m_2, m_3\})\}$ ,  $(I,E) = \{(e_1, \{m_3, m_4\}), (e_2, \{m_3, m_4\})\}$ ,  $(J,E) = \{(e_1, \{m_2, m_4\}), (e_2, \{m_2, m_4\})\}$ ,  $(M,E) = \{(e_1, \{m_2, m_3, m_4\}), (e_2, \{m_2, m_3, m_4\})\}$ ,  $K = \{k_1, k_2\}$ ,  $(F,K) = \{(k_1, \{n_1, n_3\}), (k_2, \{n_1, n_3\})\}$ ,  $(G,K) = \{(k_1, \{n_2\}), (k_2, \{n_2\})\}$ ,  $(H,K) = \{(k_1, \{n_1, n_4\}), (k_2, \{n_1, n_4\})\}$  are soft sets over  $M$  and  $N$ . Let  $\mathfrak{S} = \{\tilde{\phi}, (A,E), (B,E), (C,E), (D,E), (I,E), (J,E), (M,E), \tilde{M}\}$  and  $\mathfrak{U} = \{\tilde{\phi}, (F,K), (G,K), (H,K), \tilde{N}\}$  are topologies on  $M$  and  $N$  respectively. Then soft mapping  $f_{pu} : (M, \mathfrak{S}, E) \rightarrow (N, \mathfrak{U}, K)$  defined by  $u(m_1) = n_3$ ,  $u(m_2) = n_2$ ,  $u(m_3) = n_4$ ,  $u(m_4) = n_1$  and  $p(e_1) = k_1$ ,  $p(e_2) = k_2$  is soft set  $\beta$ -set-connected but it is not soft set  $p$ -connected.

**Example 4.5.** Let  $M = \{m_1, m_2, m_3, m_4\}$ ,  $E = \{e_1, e_2\}$  and  $N = \{n_1, n_2, n_3, n_4\}$ ,  $(F,E) = \{(e_1, \{m_1, m_3\}), (e_2, \{m_1, m_3\})\}$ ,  $(G,E) = \{(e_1, \{m_2\}), (e_2, \{m_2\})\}$ ,  $(H,E) = \{(e_1, \{m_1, m_4\}), (e_2, \{m_1, m_4\})\}$ ,  $(A,K) = \{(k_1, \{n_2\}), (k_2, \{n_2\})\}$ ,  $(B,K) = \{(k_1, \{n_3\}), (k_2, \{n_3\})\}$ ,  $(C,K) = \{(k_1, \{n_4\}), (k_2, \{n_4\})\}$ ,  $(D,K) = \{(k_1, \{n_2, n_3\}), (k_2, \{n_2, n_3\})\}$ ,  $(I,K) = \{(k_1, \{n_3\})\}$

$,n_4 \}$ ),  $(k_2, \{n_3, n_4 \})$ ),  $(J, K) = \{(k_1, \{n_2, n_4 \}), (k_2, \{n_2, n_4 \})\}$ ,  $(M, K) = \{(k_1, \{n_2, n_3, n_4 \}), (k_2, \{n_2, n_3, n_4 \})\}$ ,  $K = \{k_1, k_2\}$  are soft sets over  $M$  and  $N$ . Let  $\mathfrak{S} = \{\tilde{\phi}, (F, E), (G, E), (H, E), \tilde{M}\}$  and  $\mathfrak{U} = \{\tilde{\phi}, (A, K), (B, K), (C, K), (D, K), (I, K), (J, K), (M, K), \tilde{N}\}$  are topologies on  $M$  and  $N$  respectively. Then soft mapping  $f_{pu} : (M, \mathfrak{S}, E) \rightarrow (N, \mathfrak{U}, K)$  defined by  $u(m_1) = n_3$ ,  $u(m_2) = n_2$ ,  $u(m_3) = n_4$ ,  $u(m_4) = n_1$  and  $p(e_1) = k_1$ ,  $p(e_2) = k_2$  is soft set  $p$ -set-connected but it is not soft set  $\beta$ -connected.

**Remark 4.6.** The concepts of soft set  $s$ -connected mappings and soft set  $\beta$ -connected mappings are independent.

**Example 4.7.** Let  $M = \{m_1, m_2, m_3, m_4\}$ ,  $E = \{e_1, e_2\}$ ,  $(A, E) = \{(e_1, \{m_1, m_3\}), (e_2, \{m_1, m_3\})\}$ ,  $(B, E) = \{(e_1, \{m_2\}), (e_2, \{m_2\})\}$ ,  $(C, E) = \{(e_1, \{m_1, m_4\}), (e_2, \{m_1, m_4\})\}$  are soft sets over  $M$  and  $N = \{n_1, n_2, n_3, n_4\}$ ,  $K = \{k_1, k_2\}$ . Let  $\mathfrak{S} = \{\tilde{\phi}, (A, E), (B, E), (C, E), \tilde{M}\}$  and  $\mathfrak{U} = \{\tilde{\phi}, \tilde{N}\}$  are topologies on  $M$  and  $N$  respectively. Then soft mapping  $f_{pu} : (M, \mathfrak{S}, E) \rightarrow (N, \mathfrak{U}, K)$  defined by  $u(m_1) = n_1$ ,  $u(m_2) = n_2$ ,  $u(m_3) = n_3$ ,  $u(m_4) = n_4$  and  $p(e_1) = k_1$ ,  $p(e_2) = k_2$  is soft set  $s$ -connected but it is not soft  $\beta$ -set-connected.

**Example 4.8.** Let  $M = \{m_1, m_2, m_3, m_4\}$ ,  $E = \{e_1, e_2\}$  and  $N = \{n_1, n_2, n_3, n_4\}$ ,  $K = \{k_1, k_2\}$ ,  $(F, K) = \{(k_1, \{n_1, n_3\}), (k_2, \{n_1, n_3\})\}$ ,  $(G, K) = \{(k_1, \{n_2\}), (k_2, \{n_2\})\}$ ,  $(H, K) = \{(k_1, \{n_1, n_4\}), (k_2, \{n_1, n_4\})\}$  are soft sets over  $N$ . Let  $\mathfrak{S} = \{\tilde{\phi}, \tilde{M}\}$  and  $\mathfrak{U} = \{\tilde{\phi}, (F, K), (G, K), (H, K), \tilde{N}\}$  are topologies on  $M$  and  $N$  respectively. Then soft mapping  $f_{pu} : (M, \mathfrak{S}, E) \rightarrow (N, \mathfrak{U}, K)$  defined by  $u(m_1) = n_2$ ,  $u(m_2) = n_1$ ,  $u(m_3) = n_3$ ,  $u(m_4) = n_4$  and  $p(e_1) = k_1$ ,  $p(e_2) = k_2$  is soft set  $\beta$ -set-connected but it is not soft set  $s$ -connected.

**Proposition 4.9.** Every soft mapping  $f_{pu} : (M, \mathfrak{S}, E) \rightarrow (N, \mathfrak{U}, K)$  such that  $(f_{pu}(M), \vartheta_{f_{pu}(M)}, K)$  is a soft  $\beta$ -connected set is a soft set  $\beta$ -connected.

*Proof.* Let  $(f_{pu}(M), \vartheta_{f_{pu}(M)}, K)$  be soft  $\beta$ -connected. Then no nonempty proper soft set of  $(f_{pu}(M), \vartheta_{f_{pu}(M)}, K)$  which is soft  $\beta$ -clopen. Hence,  $f_{pu}$  is soft set  $\beta$ -connected.  $\square$

**Proposition 4.10.** Let  $f_{pu} : (M, \mathfrak{S}, E) \rightarrow (N, \mathfrak{U}, K)$  be a soft set  $\beta$ -connected mapping. If  $(M, \mathfrak{S}, E)$  is soft  $\beta$ -connected set, then  $(f_{pu}(M), \vartheta_{f_{pu}(M)}, K)$  is a soft  $\beta$ -connected set of  $(N, \mathfrak{U}, K)$ .

*Proof.* Suppose  $(f_{pu}(M), \vartheta_{f_{pu}(M)}, K)$  is not soft  $\beta$ -connected in  $(N, \mathcal{U}, K)$ , there is a nonempty proper soft  $\beta$ -clopen set  $(Y, K)$  of  $(f_{pu}(M), \vartheta_{f_{pu}(M)}, K)$ . Since  $f_{pu}$  is soft set  $\beta$ -connected,  $f_{pu}^{-1}(Y, K)$  is a nonempty proper soft  $\beta$ -clopen set over  $M$ . Consequently,  $(M, \mathcal{S}, E)$  is not soft  $\beta$ -connected.  $\square$

**Proposition 4.11.** *Let  $f_{pu} : (M, \mathcal{S}, E) \rightarrow (N, \mathcal{U}, K)$  be a soft set  $\beta$ -connected mapping and  $(X, E)$  is a soft open set over  $M$  such that  $f_{pu}(X, E)$  is soft  $\beta$ -clopen set of  $(f_{pu}(M), \vartheta_{f_{pu}(M)}, K)$ . Then  $f_{pu} / (X, E) : (X, E) \rightarrow (N, \mathcal{U}, K)$  is soft set  $\beta$ -connected mapping.*

*Proof.* Let  $(X, E)$  be soft  $\beta$ -cbss  $(A, E)$  and  $(B, E)$ . Then, by Proposition 4.9,  $(M, \mathcal{S}, E)$  is soft  $\beta$ -cbss  $(A, E)$  and  $(B, E)$ . Since  $f_{pu}$  is soft set  $\beta$ -connected,  $(f_{pu}(M), \vartheta_{f_{pu}(M)}, K)$  is soft  $\beta$ -cbss  $f_{pu}(A, E)$  and  $f_{pu}(B, E)$ . Now, since  $f_{pu}(X, E)$  is soft  $\beta$ -clopen set of  $(f_{pu}(M), \vartheta_{f_{pu}(M)}, K)$ , it follows by Proposition 4.9 that  $f_{pu}(X, E)$  is soft  $\beta$ -cbss  $f_{pu}(A, E)$  and  $f_{pu}(B, E)$ . This proves the proposition.  $\square$

**Theorem 4.12.** *Let  $f_{pu} : (M, \mathcal{S}, E) \rightarrow (N, \mathcal{U}, K)$  be a soft set  $\beta$ -connected surjection. Then for any soft  $\beta$ -clopen set  $(Y, K)$  of  $(N, \mathcal{U}, K)$  is soft  $\beta$ -connected if  $f_{pu}^{-1}(Y, K)$  is soft  $\beta$ -connected in  $(M, \mathcal{S}, E)$ . In particular, if  $(M, \mathcal{S}, E)$  is soft  $\beta$ -connected then  $(N, \mathcal{U}, K)$  is soft  $\beta$ -connected.*

*Proof.* By proposition 4.11  $f_{pu} / f_{pu}^{-1}(F, K) : f_{pu}^{-1}(F, K) \rightarrow (N, \mathcal{U}, K)$  is soft set  $\beta$ -connected. And, since  $f_{pu}^{-1}(F, K)$  is soft  $\beta$ -connected by Proposition 4.10,  $f_{pu} / f_{pu}^{-1}(F, K)(f_{pu}^{-1}(F, K)) = (F, K)$  is soft  $\beta$ -connected.  $\square$

**Proposition 4.13.** *Let  $f_{p_1u_1} : (M, \mathcal{S}, E) \rightarrow (N, \mathcal{U}, K)$  be a surjective soft set  $\beta$ -connected and  $g_{p_2u_2} : (N, \mathcal{U}, K) \rightarrow (Z, \eta, T)$  a soft set  $\beta$ -connected mapping. Then,  $(g_{p_2u_2} \circ f_{p_1u_1}) : (M, \mathcal{S}, E) \rightarrow (Z, \eta, T)$  is soft set  $\beta$ -connected.*

*Proof.* Let  $(A, T)$  be a soft  $\beta$ -clopen set in  $g_{p_2u_2}(N)$ . Then  $g_{p_2u_2}^{-1}(A, T)$  is soft  $\beta$ -clopen over  $N = f_{p_1u_1}(M)$  and so  $f_{p_1u_1}^{-1}(g_{p_2u_2}^{-1}(A, T))$  is soft  $\beta$ -clopen in  $(M, \mathcal{S}, E)$ . Now  $(g_{p_2u_2} \circ f_{p_1u_1})(M) = g_{p_2u_2}(N)$  and  $(g_{p_2u_2} \circ f_{p_1u_1})^{-1}(A, T) = f_{p_1u_1}^{-1}(g_{p_2u_2}^{-1}(A, T))$  is soft  $\beta$ -clopen in  $(M, \mathcal{S}, E)$ . Hence,  $(g_{p_2u_2} \circ f_{p_1u_1})$  is soft  $\beta$ -connected.  $\square$

## 5. CONCLUSION

Connectedness is a major and prime area of topology and it can offer many links between mathematical and other scientific areas and models. The view of connectedness express the idea of tab-togetherness of image objects by given a decision of connectedness to every practicable path between possible pair of image factors. It is a key tool for the plotting of algorithms in image segmentation. Recently, many researcher have modified the soft set theory, which is introduced by Molodtsov [23] and easily put in problems having uncertainties. In this paper, the notion of soft  $\beta$ -cbss in soft topological spaces have been introduced . It is shown that STS is soft  $\beta$ -connected if and only if it is soft  $\beta$ -connected between every pair of its nonempty soft sets. Further new classes of soft mappings called soft set  $\beta$ -connected mappings have been introduced. Hope that the outcome established in this paper will help out researcher to increase and develop the further study on soft topology to carry off a general structure for the progress of information systems.

## CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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