Available online at http://scik.org
J. Math. Comput. Sci. 2022, 12:6
https://doi.org/10.28919/jmcs/6727
ISSN: 1927-5307

EFFECT OF ADVANCE PAYMENT ON AN INVENTORY MODEL FOR A DETERIORATING ITEM CONSIDERING EXPIRATION TIME OF PRODUCT, RESTRICTED SHELF SPACE AND AGE DEPENDENT SELLING PRICE WITH DISCOUNT FACILITY

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Abstract: The life-time of a deteriorating product is a major issue in inventory management. In practise the quality as well as quantity of deteriorating product deteriorates over time and the quality (freshness) of the product influences the customer's demand. With the consideration of product life time, we develop an inventory model with price and freshness dependent demand under advance payment system with discount facility. The selling price is freshness sensitive. Goal of our model are twofold. The first one is customer's demand is dependent upon the newness/freshness state of the product as well as selling price of the product. In the last one stock level at the end of cycle is relaxed. The solution process of projected optimization model is illustrated theoretically. Couple of numerical examples and sensitivity analysis are provided to demonstrate the feature of the profit function. Concavity of the average profit function is shown by plotting graphs. This study shows that all parameters in the proposed maximization model significantly influence the optimal solution.

Keywords: advance payment; freshness sensitive selling price; restricted shelf space; freshness dependent demand; price sensitive demand.

2010 Subject Classification: 90B05.

1. INTRODUCTION

To maintain proper balance between production supply and customer's demand, we observe that inventory or stock management is essential in any kind of business sector or industries. In the classical economic order quantity (EOQ) model, it was often assumed that purchaser

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Received September 3, 2021

pays the cost of items or products at the time of release of the product from the seller. Goyal [1] was the first person who introduced the concept of delay-in-payments in his EOQ model to encounter inventory backlog problem due to the retailer's financial constraints for the time being. There is no interest charge if the cost of items is paid with in recognized delay period. On the other hand, if the payment is not paid in full by termination of the permissible delay period, interest is charged on the unpaid amount. There have been extensive researches concerning the EOQ with permissible delay in payments. Aggarwal & Jaggi [2] extended Goyal's [1] model by considering perishable products. Chang et al. [3] developed an economic order quantity model under the assumption that delay period is linked with ordering quantity. De and Goswami [4] provided the probabilistic EOQ model for declining items under tolerable delay in payments.

Offering delay period in payment sometimes leads to several disadvantages, one of being that the supplier does not receive money instantly and this may lead to a supply crisis. In order to address this issue a different tactic practised in the marketplace is advanced payment. The reverse of delay-in-payment is advance payment strategy. Sometimes it becomes much difficult for business person or seller to carry on himself/herself in the competition when the stock level of the particular product is volatized in the market due to higher demand of the product and its insufficient supply. To catch the attention of the consumers in addition to make the potential consumers in to the regular buyers, the business person use different types of price cut scheme, for example, price concession, seasonal price cut, discount due to prepayment or advance payment, etc. At the present time, the price cut due to the advance payment provision becomes a new inclination in the market dynamics. When a retailer places an order for the product, the supplier demands money in advance. Next, supplier provides some price discounts for advance payment. Some suppliers also tolerate the retailer to disburse a part of the purchased cost by instalment. Thus, from the last few decades, researchers or academicians are truly engrossed to study the EOQ model with price cut under advance payment provision. Gupta et al. [5] firstly proposed the advance payment scheme in their EOQ model. Maiti et al. [6] also introduced the concept of advance payment in an inventory model by considering selling price dependent demand and stochastic lead time. Thangam[7] discussed advance payment policy to find out optimal lot-sizing strategy for the perishable goods. Taleizadeh [8] well thought-out a number of prepayments for declining items with shortages. Teng et al. [9] developed an EOQ model with expiration time dependent deterioration rate and advance payment strategy. Diabat et al. [10] introduced partial downstream delayed payment, partial up-stream advance payment in their model for deteriorating items with partial back ordering. Recently Mashud et al. [11] established joint pricing inventory model of deteriorating products with expiration time dependent deterioration rate under the effect of advance payment with discount facility. Rahman et al. [12] also developed a hybrid price and stock dependent EOQ model for deteriorating items with advance payment related to price cut facility under preservation technology. Duary et al. [13] invented a price-discount inventory model for deteriorating product with partially backlogged shortage under the joint effect of advance and delay in payment. In this projected work we incorporate advance payment policy with price discount to promote sales and decrease supply crunch simultaneously.

Deterioration is a common occurrence of the most of the goods, so researchers can not overlook it to make strong inventory model. It is well known that most of the products of grocery shop, vegetable and fruit shop, dairy farm, medicine shop, alcohol shop etc., will spoil or damage or expire over time. The life time of the deteriorating item is limited and these have an expiration date. Specifically, the rate of deterioration accelerates over time and the item will completely spoils at the expiration date and has no utility for consumers. Therefore time dependent deterioration rate should be well thought-out to develop an inventory policy. Researchers developed several extensive models for declining items with an expiration date or utmost life time. Sarkar and Sarkar [14] established a model for deteriorating item with stock dependent demand and time dependent deterioration rate. Teng et al. [9] studied another inventory model of deteriorating products in which the deterioration rate is a function of time and life time of the product. Wu et al [15] considered an inventory model of deteriorating items in which deterioration rate function approaches the full item value near the expiration date. Tiwari et al. [16] invented a supply chain EOQ model with an expiration date to find out the optimum cost and ordering cycle. Recently, Iqbal and Sarkar [17] studied deteriorating items with life-time-reliant demand rate and incorporated the consequence of preservation technology. Mashud et al. [11] also established an inventory model of deteriorating products with expiration time dependent deterioration rate. In the projected work, we formulate and look into an inventory model where the deterioration rate is a function of time and life time of the product.

Consumers are generally aware to quality changes of fresh produce and foods. In view of supermarket consumers, they will like better to purchase fresh products instead of old ones. When price is the same, they will have a first choice to the newer ones. In our present effort, we think about a demand which decreases with the age of the product. To best of our information, there are just a small number of papers in the unpreserved inventory invented story that take into consideration the declining effectiveness of perishable products all through their life time. Fujiwara and Perera [18] was the pioneer who well thought-out declining effectiveness of perishable products related with lifetime. Though, they make use of a constant demand rate. Bai and Kendall[19] established an inventory model where demand rate is dependent on the displayed stock and the freshness of the product. Amorim et al. [20] considered multi-item production based inventory model for deteriorating products where demand is age dependent. Chen et al. [21] established EOQ model with positive stock level at the end of cycle with stock level dependent and linearly declining demand function with age of the product. Dobson et al. [22] formulated an inventory model using an only age reliant demand function that decreases linearly with age of the product until stock of item vanishes.

Price is one of the key factors in a consumer's purchasing decision. Buyers like to purchase from a shop which has minimum selling price. It is evident that the less selling price makes the demand high. If the seller raises the selling price of the manufactured goods, the clients would shift other shopping places to fulfil their demand. As a consequence, demand for perishable goods is dependent upon the joint effect of selling price and product newness. There are several investigations have been made on the consequence of price variations. Kotler [23] included business policies into inventory decisions and debated the connection among economic order quantity and pricing decision. Wee [24] discussed a replenished plan for perishable products where demand is price sensitive and deterioration follows Weibull distribution. Papers related to this field are Shah et al.[25], Ranganayki et al. [26], etc.

Another most important issue of demand is changeable selling price. Alturki and Alfares[27] established a storehouse assortment model where selling price is dependent on time. In general, demand of a product declines with raise in selling-price and vice versa. Also, life span of such goods affects the selling-price. To endorse to sell products of little life seller uses low selling price. Although not too many research papers have been published in this area. Iqbal and Sarkar [28] established a supply chain model where selling price is dependent upon the life time of the product. Recently, De [29] developed an EOQ model where selling price has a reverse association with the newness of the manufactured goods to increase the demand of the product left over in store.

This article presents an inventory model of deteriorating products having maximum lifetime with following considerations: i) the deterioration rate increases over time and the goods are entirely deteriorated at the date of expiration. ii) demand of the product is dependent on selling price and freshness of the product, after exceeding life time there is no demand i.e., cycle time must be shorter than product's life time iii) freshness sensitive selling price of the product iii) the retailer prepays a portion of his purchase cost as an advance through equal multiple instalments to the supplier before receiving the product iv) the inventory level at the end of each cycle may be positive or zero v) the shelf space for holding the product of the retailer is limited. The purpose of this paper is to find out the utmost earnings of this model. The next part of the paper is designed to arrange as cited. The assumptions and notations of the model are presented in section 2. Following the section 3, we have established a mathematical maximization problem of this model. In section 4, theoretical outcome for maximization of the total profit function is discussed. In section 5, we present numerical solution process and algorithm for the projected model. In section 6, some numerical examples and graphical representation are carried out. The sensitivity analysis is recorded in section 7. In the last, we have finished with conclusion and suggest some future research scope in section 8.

Author(s)	Demand	Selling price	Payment mode	Deterioration rate	Discount	Life-time
Goyal[1]	Const	Const	Delay	No	No	No
Aggarwal &Jaggi[2]	Const	Const	Delay	Const	No	No
De and Goswami[4]	Probabilistic	Const	Delay	Const	No	No
Maiti et al.[6]	Price sensitive	Const	Advance	No	No	No
Thangam[7]	Price sensitive	Const	Advance & Delay both partially	No	No	No
Taleizadeh[8]	Const	Const	Advance	Const	No	No
Teng et al.[9]	Const	Const	Advance	Time and expiration time dependent	No	Yes
Rahman et al.[12]	Price and stock dependent	Const	Advance	Const	No	Yes
Duary et al.[13]	Advertisement , time and stock dependent	Const	Advance & Delay both partially	Const	Yes	No
Sarkar and Sarkar [14]	Stock dependent	Const	No	Time dependent	No	No
Tiwari et al.[16]	Price sensitive	Const	Delay	Time dependent	No	Yes
Chen et al. [21]	Freshness and stock sensitive	Const	No	Time and expiration time dependent	No	Yes
Dobson et al. [22]	Freshness and stock sensitive	Const	No	Const	No	Yes
Alturki and Alfares[27]	Price sensitive	Time dependent	No	Const	No	Yes
Iqbal and Sarkar [28]	Price sensitive	Life-time dependent	Advance	Time dependent	No	Yes
Mashud et al.[11]	Price sensitive	Const	Advance	Time and expiration time dependent	Yes	Yes
De [29]	Freshness and price sensitive	Life-time /freshness dependent	Delay	Const	No	Yes
This paper	Freshness and price sensitive	Life-time /freshness dependent	Advance	Time and expiration time dependent	Yes	Yes

Table 1: Comparison between a number of earlier research work and this proposed work

2. ASSUMPTIONS AND NOTATIONS

To build up the present model, the succeeding assumptions and notations are used all through this paper.

2.1 Assumptions:

i) Infinite replenishment rate and zero lead time are considered.

ii) There is no shortage allowed

iii) No chance for substitute or renovate in this model for the single perishable item is considered.

iv) Product's selling price p is depended upon age of the product i.e., $p = p_0\left(1 - \frac{t}{L}\right)$, where p_0&L are initial selling price and life of the product respectively.

v) Selling price and freshness sensitive customer's demand $D = (\alpha - \beta p)\left(1 - \frac{t}{L}\right)$, where $\alpha, \beta > 0$. The product is fresh and there is no age effect on the demand at the beginning of the cycle.. Then the product loses its newness with time, so the demand for the product decreases. After the expiration date of the product customer's demand becomes nil.

vi) The deterioration rate $\theta(t)$ of the product is time reliant. We have supposed that $\theta(t) = \frac{1}{1+L-t}$, $0 \le t \le T \le L$ following Teng et al.[9] and Mashud et al.[11]

vii) At the end of each cycle the inventory level is allowed to be zero or positive and the residual stock (if any) is disposed of.

viii) Due to the especially seasonal and deteriorating substance, the supplier seeks advance payment in instalment of an exact percentage of the product's purchasing cost before the time of delivery from the retailer. The left over balance is paid at the time of release or delivery of the purchasing product.

ix) To attract advance payment policy, the supplier also offers a discount or concession on the cost of the purchasing products. To receive the discount the retailer accepts the condition and pays $\gamma \times total\ purchasing\ cost$ on n equal instalments during M years before delivery.

2.2 Notations:

i) K: replenishment cost per order

ii) α: an invariable factor in the demand function$(\alpha > 0)$

iii) β: price reliant demand rate factor$(\beta > 0)$

iv) p: the selling price per unit of product

v) p_0: the initial selling price per unit of product

vi) L: life /expiration time of the product

vii) W: capability of shelf space for the retailer

viii) C_h: holding cost for per unit product per unit time

ix) C_p: purchasing cost for every unit of product

x) Q: amount of the product ordered by the retailer in each cycle

xi) s: salvage cost of the for each unit disposed product

xii) M: time range for the retailer to pay in advance, where $M > 0$

xiii) r: interest rate imposed on the total purchased cost per unit time

xiv) n: number of identical instalments

xv) x: concession rate for a single instalment of the prepayment

xvi) I_r: banking interest rate on the loan

xvii) γ: a fraction that the retailer has to prepay from the purchasing cost before the reception of the product, $0 \leq \gamma \leq 1$

xviii) $I(t)$: inventory level at time t

xix) $\theta(t)$: deterioration rate at time t

xx) AT: average turnover/profit for each unit time

Decision Variables

i) T: length of each cycle, it have to be not more than product life i.e., $T \leq L$

ii) q: inventory level at the end of cycle $(q \geq 0)$.

3. MATHEMATICAL MODEL FORMULATION

In this paper, two models with advance payments have been discussed. In the first case, we have supposed that the prepayment is completed in a number of equal instalments.

In the 2nd case, the retailer will pay in advance the entire cost at the specified time to enjoy a cash reduction on that cost by borrowing some funds for his purchasing cost from some financial organization like bank.

3.1 Case 1(Advance payment is done in instalments)

In the EOQ model developed in case 1, γ fraction of the entire purchased cost of the product is been prepaid by n equal instalments within M years before the delivery and the rest amount of the purchased cost must be paid at the time of delivery. After paying all purchased cost, instantaneously Q units of product is been delivered by the supplier to the retailer. Now the inventory level $I(t)$ starting with Q units decreases gradually due to both demand and deterioration and reaches to $q(\geq 0)$.

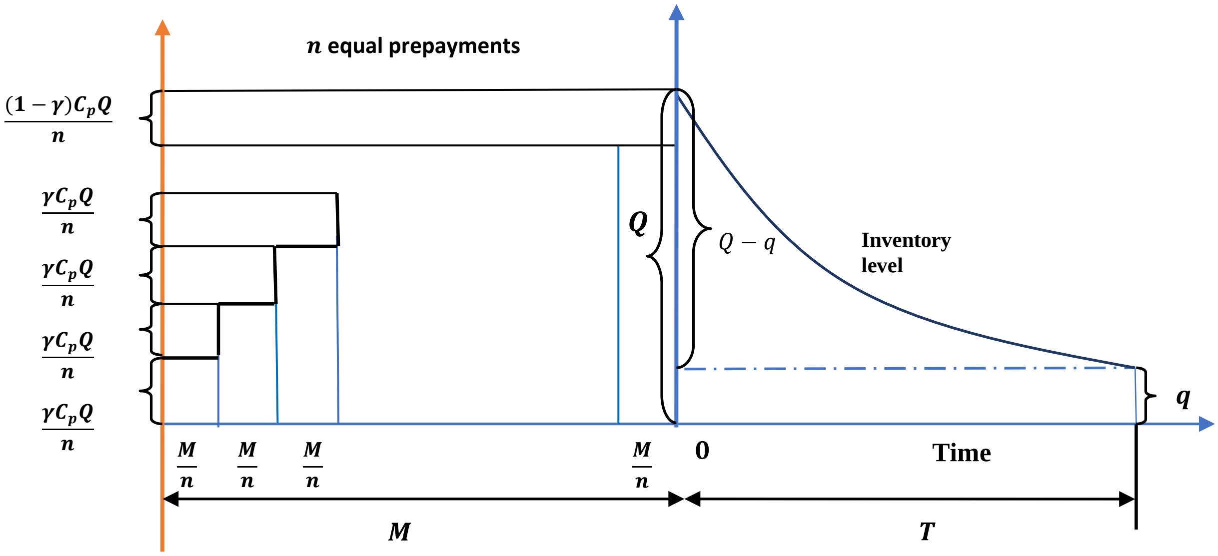

Fig 1. Graphical representation of the inventory system when prepayment is done in instalments

The governing differential equation is as follows:

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -D, \qquad 0 \leq t \leq T \leq L \tag{1}$$

Where, $D = (\alpha - \beta p)\left(1 - \frac{t}{L}\right), p = p_0\left(1 - \frac{t}{L}\right)$ and $\theta(t) = \frac{1}{1+L-t}, 0 \leq t \leq L$

With boundary conditions $I(T) = q \geq 0$ and $I(0) = Q \leq W$.

The obtained solution of equation (1) is

$$I(t) = A\{(1+L-t)^3 - (1+L-t))(1+L-T)^2\} + B\{(1+L-t)^2 - (1+L-t)(1+L-T)\} + C\{(1+L-t)\log(1+L-t) - (1+L-t)\log(1+L-T)\} + q\frac{1+L-t}{1+L-T} \tag{2}$$

Where $A = -\frac{\beta p_0}{L^2}$, $B = \frac{\alpha}{L} + \frac{2\beta p_0}{L^2}, C = -\left(\frac{\alpha}{L} + \frac{\beta p_0}{L^2}\right)$

Using the initial condition $I(0) = Q$, from equation (2) we get

$$Q = A\{(1+L)^3 - (1+L)(1+L-T)^2\} + B\{(1+L)^2 - (1+L)(1+L-T)\} + C(1+L)\log\frac{(1+L)}{(1+L-T)} + q\frac{1+L}{1+L-T} \tag{3}$$

Based on the above expressions and assumptions, the profit function in each cycle consists following terms:

Total sales revenue (SR) per cycle $= \int_0^T p\,(\alpha - \beta p)\left(1 - \frac{t}{L}\right) dt$

$$=\frac{L\alpha p_0}{3}\left\{1-\left(1-\frac{T}{L}\right)^3\right\}-\frac{L\beta {p_0}^2}{4}\left\{1-\left(1-\frac{T}{L}\right)^4\right\} \quad (4)$$

Replenishment cost (RC) per cycle$=K$ (5)

Total inventory cost (IHC)per cycle$=C_h\int_0^T I(t)dt$

$$= AC_h\left[\frac{(1+L)^4}{4}+\frac{(1+L-T)^4}{4}-(1+L)^2\frac{(1+L-T)^2}{2}\right]+BC_h\left[\frac{(1+L)^3}{3}+\frac{(1+L-T)^3}{6}-(1+L)^2\frac{(1+L-T)}{2}\right]+CC_h\left[\frac{(1+L)^2}{2}\log\frac{(1+L)}{(1+L-T)}-\frac{(1+L)^2}{4}+\frac{(1+L-T)^2}{4}\right]+C_hq\frac{T}{1+L-T} \quad (6)$$

Total purchase cost (PC) per cycle $=C_pQ=C_pA\{(1+L)^3-(1+L))(1+L-T)^2\}+C_pB\{(1+L)^2-(1+L)(1+L-T)\}+C_pC(1+L)\log\frac{(1+L)}{(1+L-T)}+C_pq\frac{1+L}{1+L-T}$ (7)

Total salvage value for the disposed product(SV) per cycle$=sq$ (8)

The retailer pays in advance γ fraction of the entire purchased cost$(=C_pQ)$ of the product in n uniform instalment before M years of release of the product on request of the supplier.

Total interest paid (IP_1) on the purchased cost per cycle due to early or advance pay $=\frac{\gamma C_pQ}{n}\times r\times\frac{M}{n}\times[1+2+3+\cdots..+n]=\frac{(n+1)\gamma C_pQMr}{2n}$ (9)

To receive the maximum x $(0\leq x\leq 100)$ percent discount on purchase cost, the retailer must pay γ fraction of the entire purchased cost in n uniform instalment before delivery. As x is the discount rate for a single instalment of the prepayment, so the discount rate $=\frac{x}{n}$.

The total discount (DC) for the advance payment $=\frac{x}{n}C_pQ$ (10)

So average profit or turnover $(AT_1(q,T))$ per unit time $=\frac{1}{T}[SR+DC+SV-RC-IHC-PC-IP_1]$

$$=\frac{1}{T}\left[\frac{L\alpha p_0}{3}\left\{1-\left(1-\frac{T}{L}\right)^3\right\}-\frac{L\beta {p_0}^2}{4}\left\{1-\left(1-\frac{T}{L}\right)^4\right\}+sq-K-AC_h\left[\frac{(1+L)^4}{4}+\frac{(1+L-T)^4}{4}-(1+L)^2\frac{(1+L-T)^2}{2}\right]-BC_h\left[\frac{(1+L)^3}{3}+\frac{(1+L-T)^3}{6}-(1+L)^2\frac{(1+L-T)}{2}\right]-CC_h\left[\frac{(1+L)^2}{2}\log\frac{(1+L)}{(1+L-T)}-\frac{(1+L)^2}{4}+\frac{(1+L-T)^2}{4}\right]-C_hq\frac{T}{1+L-T}+C_p\left(\frac{x}{n}-1-\frac{(n+1)\gamma Mr}{2n}\right)\left(A\{(1+L)^3-(1+L))(1+L-T)^2\}+B\{(1+L)^2-(1+L)(1+L-T)\}+C(1+L)\log\frac{(1+L)}{(1+L-T)}+q\frac{1+L}{1+L-T}\right)\right] \quad (11)$$

3.2 Case 2(Advance payment is done at a time)

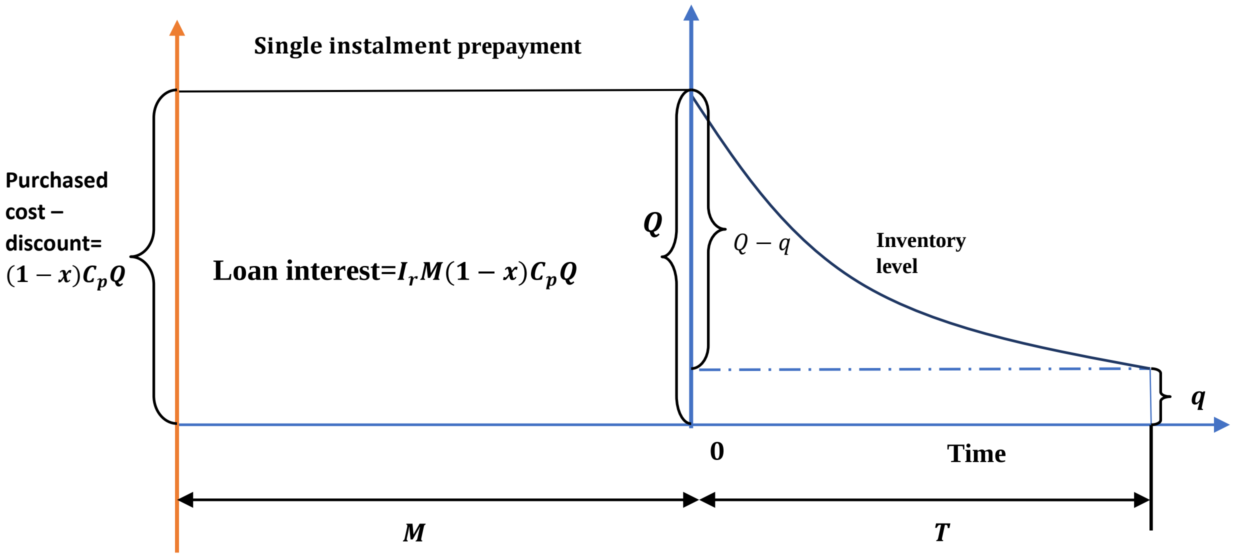

Fig 2. Graphical representation of the inventory system when prepayment is done at a time

From Fig 2, it is clear that supplier offers x percent discount rate to the retailer for complete advance payment before delivery. When the retailer does not enough money in hand during the time M year, he may take loan from a bank or any financial sector at an interest of I_r%.

So here total discount (DC) for the advance payment=$x\ C_pQ$ (12)

Total interest paid (IP_2) to the bank on the purchased cost per cycle due to advance payment=$I_rM(1-x)C_pQ$ (13)

So average profit or turnover ($AT_2(q,T)$) per unit time $=\frac{1}{T}[SR+DC+SV-RC-IHC-PC-IP_2]$

$$= \frac{1}{T}\Bigg[\frac{L\alpha p_0}{3}\left\{1-\left(1-\frac{T}{L}\right)^3\right\}-\frac{L\beta {p_0}^2}{4}\left\{1-\left(1-\frac{T}{L}\right)^4\right\}+\ sq-K-AC_h\left[\frac{(1+L)^4}{4}+\frac{(1+L-T)^4}{4}-(1+L)^2\frac{(1+L-T)^2}{2}\right]-BC_h\left[\frac{(1+L)^3}{3}+\frac{(1+L-T)^3}{6}-(1+L)^2\frac{(1+L-T)}{2}\right]-CC_h\left[\frac{(1+L)^2}{2}\log\frac{(1+L)}{(1+L-T)}-\frac{(1+L)^2}{4}+\frac{(1+L-T)^2}{4}\right]-C_hq\frac{T}{1+L-T}+C_p(x-1)(1+MI_r)\left(A\{(1+L)^3-(1+L))(1+L-T)^2\}+B\{(1+L)^2-(1+L)(1+L-T)\}+\ C(1+L)\log\frac{(1+L)}{(1+L-T)}+q\frac{1+L}{1+L-T}\right)\Bigg] \tag{14}$$

Now our objective is to obtain optimal cycle time T^* and inventory remaining q^* at end of cycle in order to maximize the average profit per unit time.

4. THEORETICAL RESULT FOR OPTIMALITY

Case1. (Advance payment is done in instalments)

Taking 1st and 2nd derivatives of $AT_1(q,T)$in (11) with respect to q, we find

$$\frac{\partial\left(AT_1(q,T)\right)}{\partial q}=\frac{s}{T}+C_p\left(\frac{x}{n}-1-\frac{(n+1)\gamma Mr}{2n}\right)\frac{1+L}{T(1+L-T)}-C_h\frac{T}{T(1+L-T)} \tag{15}$$

and $\frac{\partial^2\left(AT_1(q,T)\right)}{\partial q^2}=0$ (16)

Here we see that $AT_1(q,T)$ is linear function of q. So $AT_1(q,T)$ is either rising or declining function in q. Therefore, two cases may arise.

Subcase 1. When $\frac{\partial\left(AT_1(q,T)\right)}{\partial q}>0$, $AT_1(q,T)$ is strictly increasing function in q. Thus $AT_1(q,T)$ attains its maximum when q reaches its maximum.

Now as $Q\leq W$, so by the help of equation (3), maximum value of q is $W\left(\frac{1+L-T}{1+L}\right)-A\{(1+L)^2(1+L-T)-(1+L-T)^3\}-B\{(1+L)(1+L-T)-(1+L-T)^2\}$ - $C(1+L-T)\log\frac{(1+L)}{(1+L-T)}$ (17)

Putting the expression $W\left(\frac{1+L-T}{1+L}\right)-A\{(1+L)^2(1+L-T)-(1+L-T)^3\}-B\{(1+L)(1+L-T)-(1+L-T)^2\}$- $C(1+L-T)\log\frac{(1+L)}{(1+L-T)}$ in place of $q$in equation (11), we obtain the following expression of the profit function as a function of Tonly:

$$AT_{11}(T)=\frac{1}{T}\left[\frac{L\alpha p_0}{3}\left\{1-\left(1-\frac{T}{L}\right)^3\right\}-\frac{L\beta {p_0}^2}{4}\left\{1-\left(1-\frac{T}{L}\right)^4\right\}+s\left(W\left(\frac{1+L-T}{1+L}\right)-A\{(1+L)^2(1+L-T)-(1+L-T)^3\}-B\{(1+L)(1+L-T)-(1+L-T)^2\}-C(1+L-T)\log\frac{(1+L)}{(1+L-T)}\right)-K-AC_h\left[\frac{(1+L)^4}{4}+\frac{(1+L-T)^4}{4}-(1+L)^2\frac{(1+L-T)^2}{2}\right]-BC_h\left[\frac{(1+L)^3}{3}+\frac{(1+L-T)^3}{6}-(1+L)^2\frac{(1+L-T)}{2}\right]-CC_h\left[\frac{(1+L)^2}{2}\log\frac{(1+L)}{(1+L-T)}-\frac{(1+L)^2}{4}+\frac{(1+L-T)^2}{4}\right]-C_h\left(W\left(\frac{1+L-T}{1+L}\right)-A\{(1+L)^2(1+L-T)-(1+L-T)^3\}-B\{(1+L)(1+L-T)-(1+L-T)^2\}-C(1+L-T)\log\frac{(1+L)}{(1+L-T)}\right)\frac{T}{1+L-T}+C_p\left(\frac{x}{n}-1-\frac{(n+1)\gamma Mr}{2n}\right)\left(A\{(1+L)^3-(1+L))(1+L-T)^2\}+B\{(1+L)^2-(1+L)(1+L-T)\}+C(1+L)\log\frac{(1+L)}{(1+L-T)}+\left(W\left(\frac{1+L-T}{1+L}\right)-A\{(1+L)^2(1+L-T)-(1+L-T)^3\}-B\{(1+L)(1+L-T)-(1+L-T)^2\}-C(1+L-T)\log\frac{(1+L)}{(1+L-T)}\right)\frac{1+L}{1+L-T}\right)\right] \tag{18}$$

Again Hessian matrix for $AT_1(q,T)$ is $\begin{bmatrix}\frac{\partial^2 AT_1}{\partial q^2} & \frac{\partial^2 AT_1}{\partial q\partial T}\\ \frac{\partial^2 AT_1}{\partial T\partial q} & \frac{\partial^2 AT_1}{\partial T^2}\end{bmatrix}=\begin{bmatrix}0 & \frac{\partial^2 AT_1}{\partial q\partial T}\\ \frac{\partial^2 AT_1}{\partial T\partial q} & \frac{\partial^2 AT_1}{\partial T^2}\end{bmatrix}=-\left(\frac{\partial^2 AT_1}{\partial q\partial T}\right)^2<0$

So, there exists unique T^* at which the profit function $AT_{11}(T)$ attains a maximum.

Subcase 2. When $\frac{\partial(AT_1(q,T))}{\partial q} \leq 0$, $AT_1(q,T)$ is non- increasing function in q. Thus $AT_1(q,T)$ attains its maximum when q reaches its minimum i.e., when $q = 0$.

Putting $q = 0$ in equation (11), we obtain the following expression of the profit function as a function of Tonly:

$$AT_{12}(T) = \frac{1}{T}\left[\frac{L\alpha p_0}{3}\left\{1-\left(1-\frac{T}{L}\right)^3\right\} - \frac{L\beta {p_0}^2}{4}\left\{1-\left(1-\frac{T}{L}\right)^4\right\} - K - AC_h\left[\frac{(1+L)^4}{4} + \frac{(1+L-T)^4}{4} - (1+L)^2\frac{(1+L-T)^2}{2}\right] - BC_h\left[\frac{(1+L)^3}{3} + \frac{(1+L-T)^3}{6} - (1+L)^2\frac{(1+L-T)}{2}\right] - CC_h\left[\frac{(1+L)^2}{2}\log\frac{(1+L)}{(1+L-T)} - \frac{(1+L)^2}{4} + \frac{(1+L-T)^2}{4}\right] + C_p\left(\frac{x}{n} - 1 - \frac{(n+1)\gamma Mr}{2n}\right)\left(A\{(1+L)^3 - (1+L))(1+L-T)^2\} + B\{(1+L)^2 - (1+L)(1+L-T)\} + C(1+L)\log\frac{(1+L)}{(1+L-T)}\right)\right] \tag{19}$$

As Hessian matrix for $AT_1(q,T)$ is less than zero, so there exists unique T^* at which the profit function $AT_{12}(T)$ attains a maximum.

Case2. (**Advance payment is done at a time)**

Similarly, taking 1st and 2nd derivatives of $AT_2(q,T)$in (14) with respect to q, we find

$$\frac{\partial(AT_2(q,T))}{\partial q} = \frac{s}{T} + C_p(x-1)(1+MI_r)\frac{1+L}{T(1+L-T)} - C_h\frac{T}{T(1+L-T)} \tag{20}$$

and $\frac{\partial^2(AT_2(q,T))}{\partial q^2} = 0$ (21)

Here also we see that $AT_2(q,T)$ is linear function of q. So $AT_2(q,T)$ is either increasing or decreasing function in q.

Subcase 1. $\frac{\partial(AT_2(q,T))}{\partial q} > 0$.

Similarly, putting the expression $W\left(\frac{1+L-T}{1+L}\right) - A\{(1+L)^2(1+L-T) - (1+L-T)^3\} - B\{(1+L)(1+L-T) - (1+L-T)^2\}$- $C(1+L-T)\log\frac{(1+L)}{(1+L-T)}$ in place of q in equation (14), we obtain the following expression of the profit function as a function of Tonly:

$$AT_{21}(T) = \frac{1}{T}\left[\frac{L\alpha p_0}{3}\left\{1-\left(1-\frac{T}{L}\right)^3\right\} - \frac{L\beta {p_0}^2}{4}\left\{1-\left(1-\frac{T}{L}\right)^4\right\} + s\left(W\left(\frac{1+L-T}{1+L}\right) - A\{(1+L)^2(1+L-T) - (1+L-T)^3\} - B\{(1+L)(1+L-T) - (1+L-T)^2\} - C(1+L-T)\log\frac{(1+L)}{(1+L-T)}\right) - K - AC_h\left[\frac{(1+L)^4}{4} + \frac{(1+L-T)^4}{4} - (1+L)^2\frac{(1+L-T)^2}{2}\right] - BC_h\left[\frac{(1+L)^3}{3} + \frac{(1+L-T)^3}{6} - (1+L)^2\frac{(1+L-T)}{2}\right] - CC_h\left[\frac{(1+L)^2}{2}\log\frac{(1+L)}{(1+L-T)} - \frac{(1+L)^2}{4} + \frac{(1+L-T)^2}{4}\right] - C_h\left(W\left(\frac{1+L-T}{1+L}\right) - A\{(1+L)^2(1+L-T) - (1+L-T)^3\} - B\{(1+L)(1+L-T) - \right.$$

$(1+L-T)^2\}-\ C(1+L-T)\log\frac{(1+L)}{(1+L-T)}\Big)\frac{T}{1+L-T}+C_p(x-1)(1+MI_r)\Big(A\{(1+L)^3-(1+L))(1+L-T)^2\}+B\{(1+L)^2-(1+L)(1+L-T)\}+\ C(1+L)\log\frac{(1+L)}{(1+L-T)}+\Big(W\left(\frac{1+L-T}{1+L}\right)-A\{(1+L)^2(1+L-T)-(1+L-T)^3\}-B\{(1+L)(1+L-T)-(1+L-T)^2\}-\ C(1+L-T)\log\frac{(1+L)}{(1+L-T)}\Big)\frac{1+L}{1+L-T}\Big]$ (22)

Again, as Hessian matrix for $AT_2(q,T)$ isless than zero, so there exists unique T^* at which the profit function $AT_{21}(T)$ attains a maximum.

Subcase 2. $\frac{\partial(AT_2(q,T))}{\partial q}\le 0$

For this case, putting $q=0$ in equation (14), we obtain the following expression of the profit function as a function of Tonly:

$$AT_{22}(T)=\frac{1}{T}\left[\frac{L\alpha p_0}{3}\left\{1-\left(1-\frac{T}{L}\right)^3\right\}-\frac{L\beta p_0{}^2}{4}\left\{1-\left(1-\frac{T}{L}\right)^4\right\}-K-AC_h\left[\frac{(1+L)^4}{4}+\frac{(1+L-T)^4}{4}-(1+L)^2\frac{(1+L-T)^2}{2}\right]-BC_h\left[\frac{(1+L)^3}{3}+\frac{(1+L-T)^3}{6}-(1+L)^2\frac{(1+L-T)}{2}\right]-CC_h\left[\frac{(1+L)^2}{2}\log\frac{(1+L)}{(1+L-T)}-\frac{(1+L)^2}{4}+\frac{(1+L-T)^2}{4}\right]+C_p(x-1)(1+MI_r)\Big(A\{(1+L)^3-(1+L))(1+L-T)^2\}+B\{(1+L)^2-(1+L)(1+L-T)\}+\ C(1+L)\log\frac{(1+L)}{(1+L-T)}\right] \quad (23)$$

Here also there exists unique T^* at which the profit function $AT_{22}(T)$ attains a maximum.

5. ALGORITHM

Now we sketch the algorithm to achieve optimal solution for both cases of the projected model.

<u>For case 1</u>

Step1. Find solution of the equation $\frac{d(AT_{11}(T))}{dT}=0$ and denote it as T_{11}^* and find $AT_{11}(T_{11}^*)$

Step 2 Find solution of the equation $\frac{d(AT_{12}(T))}{dT}=0$ and denote it as T_{12}^* and find $AT_{12}(T_{12}^*)$

Step 3. Set $AT_1(T^*)=\text{Max}\ \{AT_{11}(T_{11}^*),AT_{12}(T_{12}^*)\}$

Step 4. Calculate the corresponding value of q^*(either 0 or $W\left(\frac{1+L-T}{1+L}\right)-A\{(1+L)^2(1+L-T)-(1+L-T)^3\}-B\{(1+L)(1+L-T)-(1+L-T)^2\}$ - $C(1+L-T)\log\frac{(1+L)}{(1+L-T)}$) and Q^*(from equation (3))

<u>For case 2</u>

Step1. Find solution of the equation $\frac{d(AT_{21}(T))}{dT}=0$ and denote it as T_{21}^* and find $AT_{21}(T_{21}^*)$

Step 2 Find solution of the equation $\frac{d(AT_{22}(T))}{dT} = 0$ and denote it as T_{22}^* and find $AT_{22}(T_{22}^*)$

Step 3. Set $AT_2(T^*) = \text{Max}\ \{AT_{21}(T_{21}^*), AT_{22}(T_{22}^*)\}$

Step 4. Calculate the corresponding value of q^*(either 0 or $W\left(\frac{1+L-T}{1+L}\right) - A\{(1+L)^2(1+L-T) - (1+L-T)^3\} - B\{(1+L)(1+L-T) - (1+L-T)^2\}$ - $C(1+L-T)\log\frac{(1+L)}{(1+L-T)}$) and Q^*(from equation (3))

6. NUMERICAL ILLUSTRATIONS

To demonstrate different cases of our developed model, four numerical examples are cited by means proper values of parameters.

Example 1. Given the inventory system for **case 1** with the subsequent parameters: $L = 2$ yrs., $W = 500$, $\alpha = 250$, $\beta = 0.04$, $K = 1250\$$,$C_h = 0.5$ \$, $C_p = 5\$$, $p_0 = 25.75\$$, $s = 6.4\$$, $r = 0.01\$$,$n = 8$, $M = 0.5$ yrs, $x = 0.25$, γ =0.2

Step1. Solution of $\frac{d(AT_{11}(T))}{dT} = 0$ is 0.522481. Hence T_{11}^*=0.522481 and $AT_{11}(T_{11}^*)$=114.47

Step 2. Solution of $\frac{d(AT_{12}(T))}{dT} = 0$ is 0.674826. Hence T_{12}^*=0.674826 and $AT_{12}(T_{12}^*)$=67.6123

Hence optimum cycle time $T^* = 0.522481$ year and optimum profit is 114.47\$. Correspondingly q^*=192.889, Q^*=500

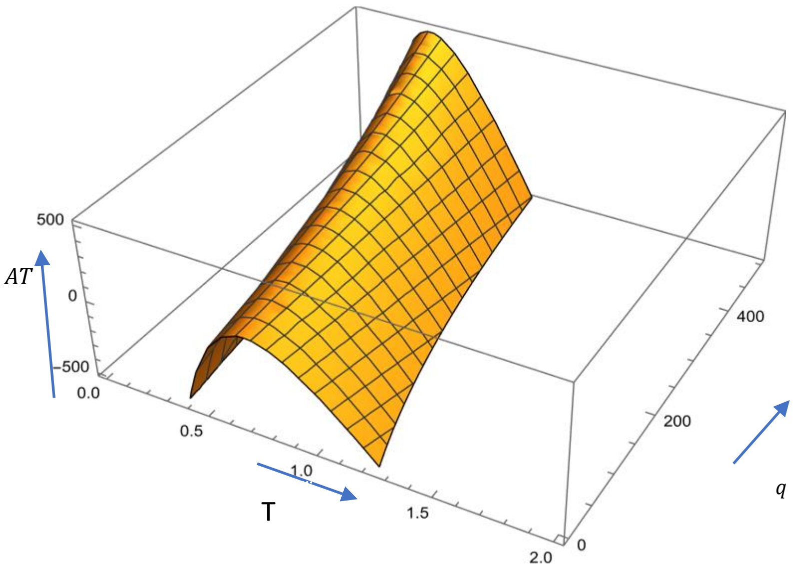

Fig 3. Profit per unit time vs T and q of example 1

Example 2. For Case1, in this example, all parameters are same as example 1 except $K = 1200\$$,$s = 6.0\$$, $M = 0.4$ yrs.

Step1. Solution of $\frac{d(AT_{11}(T))}{dT}=0$ is 0.60549. Hence T_{11}^{*}=0.60549 and $AT_{11}(T_{11}^{*})$=81.5731

Step 2. Solution of $\frac{d(AT_{12}(T))}{dT}=0$ is 0.659352. Hence T_{12}^{*}=0.659352 and $AT_{12}(T_{12}^{*})$=142.855

Hence optimum cycle time $T^{*}=0.659352$ year and optimum profit is 142.855$. Correspondingly q^{*}=0, Q^{*}=338.816

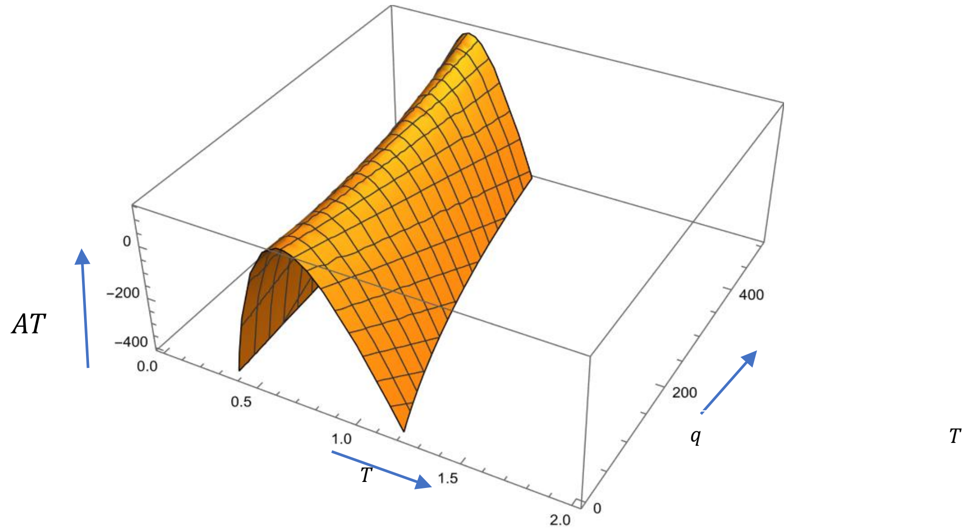

Fig 4. Profit per unit time vs *T* and *q* of example 2

Example 3. For Case 2, here all parameters are considered same as example 1 except $K=$ 1400$, $I_r=0.3$

Step1. Solution of $\frac{d(AT_{21}(T))}{dT}=0$ is 0.441908. Hence T_{21}^{*}=0.441908 and $AT_{21}(T_{21}^{*})$=357.482

Step 2. Solution of $\frac{d(AT_{22}(T))}{dT}=0$ is 0.722605. Hence T_{22}^{*}=0.722605 and $AT_{22}(T_{22}^{*})$=127.945

Hence optimum cycle time $T^{*}=0.441908$ year and optimum profit is 357.482$. Correspondingly q^{*}=235.02, Q^{*}=500

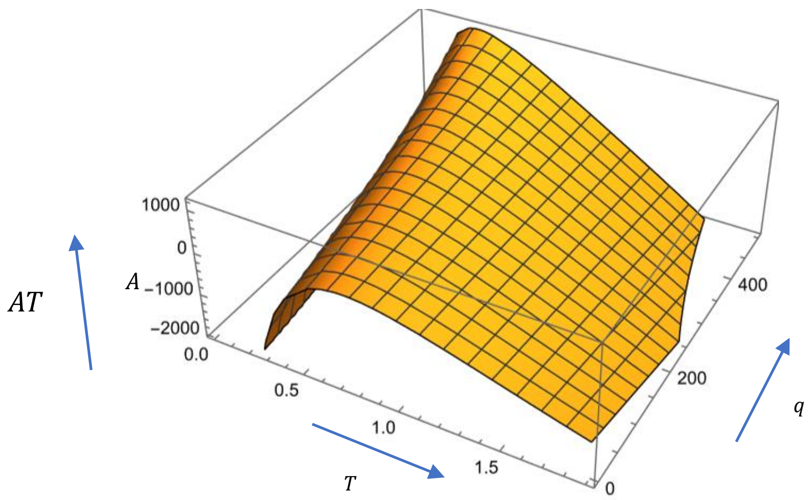

Fig 5. Profit per unit time vs *T* and *q* of example 3

Example 4. For Case 2, here all parameters are considered same as example 1 except $K = 1300\$$, $s = 5.4\$$, $I_r = 0.3$

Step1. Solution of $\frac{d(AT_{21}(T))}{dT} = 0$ is 0.662258. Hence T^*_{21}=0.662258 and $AT_{21}(T^*_{21})$=214.208

Step 2. Solution of $\frac{d(AT_{22}(T))}{dT} = 0$ is 0.692444. Hence T^*_{22} =0.692444 and $AT_{22}(T^*_{22})$=269.293.509

Hence optimum cycle time T^* =0.692444 year and optimum profit is 269.293$.

Correspondingly q^*=0, Q^*=356.509

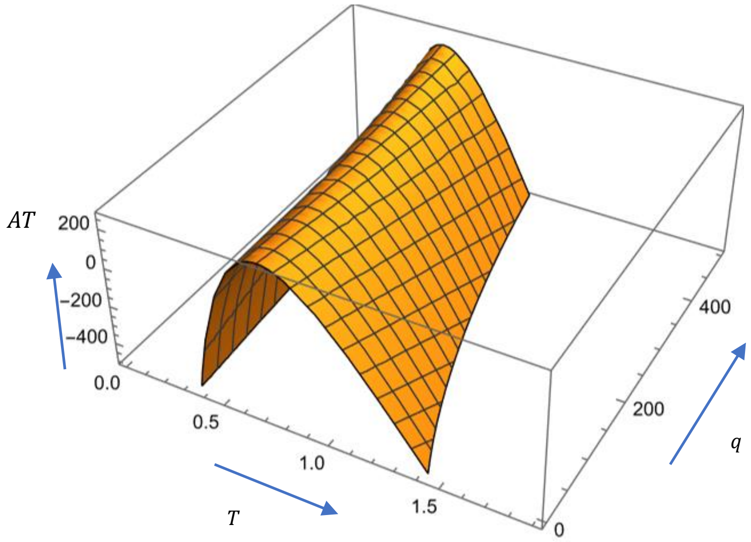

Fig 6. Profit per unit time vs T and q of example 4

7. Sensitivity Analysis

To inspect the sensitivity of the model, we study the impact of changes in different inventory parameters against optimal solutions (T, q), optimal order quantities and average profit for the example1(for case 1) and example 4 (for case 2) by varying the value of one parameter at a time and fixing other left over parameters, the analysis has been completed. The results of sensitivity analysis are shown in Table 1.

paramete	Case 1(Advance payment is done in instalments) (Example 1)						Case 2(Advance payment is done at a time) (Example 4)					
	Original value	New value	T^*	q^*	Q^*	AP	Original value	New value	T^*	q^*	Q^*	AP
K	1250	1210	0.495614	206.723	500	193.059	1300	1260	0.680171	0	349.93	327.576
		1230	0.509121	199.742	500	153.246		1280	0.686324	0	353.23	298.304
		1270	0.535711	186.156	500	76.668		1320	0.698535	0	359.77	240.536
		1290	0.548825	179.533	500	39.785		1340	0.704595	0	363.02	212.028
α	250	240	0.535559	195.255	500	27.904	250	240	0.708923	0	350.67	184.087

		245	0.528902	194.081	500	71.0741		245	0.700542	0	353.60	226.466
		255	0.516284	191.683	500	158.086		255	0.684614	0	359.39	312.552
		260	0.510298	190.462	500	201.914		260	0.677036	0	362.25	356.232
β	0.04	0.01	0.521913	192.402	500	118.258	0.04	0.01	0.691504	0	357.18	273.469
		0.02	0.522102	192.565	500	116.995		0.02	0.691817	0	356.96	272.076
		0.08	0.52324	193.539	500	109.423		0.08	0.693701	0	355.61	263.734
		0.16	0.524763	194.844	500	99.338		0.16	0.696227	0	353.82	252.649
C_h	0.5	0.3	0.524734	191.739	500	155.367	0.5	0.3	0.699396	0	360.23	302.753
		0.4	0.523604	192.316	500	134.915		0.4	0.695891	0	358.35	285.982
		0.6	0.521365	193.460	500	94.032		0.6	0.689054	0	354.69	252.683
		0.7	0.520256	194.027	500	73.601		0.7	0.685718	0	352.90	236.152
C_p	5	4.6	0.38465	266.129	500	543.516	5	4.6	0.564137	141.291	500	495.953
		4.8	0.456218	227.397	500	312.723		4.8	0.693211	0	356.92	358.107
		5.2	0.674014	0	346.65	-32.094		5.2	0.691682	0	356.10	180.482
		5.4	0.673206	0	346.21	-131.795		5.4	0.690922	0	355.70	91.675
p_0	25.75	25.25	0.532066	187.985	500	19.763	25.75	25.25	0.699525	0	360.33	183.150
		25.50	0.527200	190.471	500	67.156		25.50	0.695959	0	358.40	226.181
		26.00	0.517895	195.245	500	161.900		26.00	0.688980	0	354.64	312.485
		26.25	0.513438	197.542	500	209.496		26.25	0.685565	0	352.79	355.757
s	6.4	6.2	0.674826	0	347.08	67.612	5.4	5.0	0.692444	0	356.51	269.293
		6.3	0.552415	177.729	500	79.9893		5.2	0.692444	0	356.51	269.293
		6.5	0.491222	209.005	500	154.119		5.6	0.692444	0	356.51	269.293
		6.6	0.458402	226.239	500	199.957		5.8	0.558812	143.883	500	312.112
W	500	400	0.622789	63.901	400	72.6929	500	400	0.692444	0	356.51	269.293
		450	0.573255	126.886	450	87.6317		450	0.692444	0	356.51	269.293
		550	0.469720	262.431	550	156.119		550	0.692444	0	356.51	269.293
		600	0.413921	336.307	600	217.029		600	0.692444	0	356.51	269.293
L	2	1.8	0.504407	186.771	500	-233.09	2	1.8	0.661872	0	360.78	-53.628
		1.9	0.513493	190.044	500	-51.694		1.9	0.677346	0	358.43	114.877
		2.1	0.531359	195.373	500	267.336		2.1	0.707189	0	354.94	411.426
		2.2	0.540117	197.548	500	408.516		2.2	0.721600	0	353.68	542.780
x	0.25	0.15	0.543101	182.417	500	55.812	0.25	0.15	0.689915	0	355.97	-26.727
		0.20	0.532828	187.619	500	84.858		0.20	0.691175	0	355.82	121.277
		0.30	0.512055	198.232	500	144.678		0.30	0.580829	133.222	500	445.731
		0.35	0.501542	203.653	500	175.509		0.35	0.495226	175.486	500	713.238
M	0.5	0.3	0.522107	193.080	500	115.547	0.5	0.3	0.598787	124.633	500	392.745
		0.4	0.522294	192.985	500	115.009		0.4	0.692944	0	356.77	327.215
		0.6	0.522668	192.794	500	113.932		0.6	0.691947	0	356.24	211.372
		0.7	0.522855	192.698	500	113.394		0.7	0.691450	0	355.97	153.454
γ	0.2	0.10	0.522014	193.128	500	115.817						
		0.15	0.522247	193.009	500	115.143						
		0.25	0.522715	192.770	500	113.798						
		0.30	0.522948	192.651	500	113.125						
n	8	6	0.505092	201.819	500	165.058						
		7	0.515057	196.690	500	135.941						
		9	0.528227	189.958	500	97.980						
		10	0.532807	187.629	500	84.916						
r	0.01	0.0025	0.521780	193.247	500	116.490						
		0.005	0.522014	193.128	500	115.817						
		0.02	0.523416	192.412	500	111.781						
		0.04	0.525282	191.459	500	106.417						
I_r							0.3	0.1	0.555356	145.57	500	522.752
								0.2	0.693278	0	356.95	365.831
								0.4	0.691616	0	356.06	172.760
								0.5	0.690791	0	355.62	76.231

Table 2. Sensitivity Analysis

From Table 2, the following analysis can be made.

(i) It is observed from the Table 2 that, superior value in prepayment time (M) gives lesser profit of the inventory system especially for case 2. This is happening because due to bigger prepayment the retailer needs to wait for a product for longer time and have to pay more banking interest in taking that time.

(ii) Another noticeable observation is that, large number of instalments (n) reduces the total profit of the system as the retailer will be given a smaller discount rate.

(iii) If we notice the sensitivity of one of the most important parameter L, life time of the product, we will see that, higher profit will be obtained for a product having bigger life time as the product deteriorates slowly.

(iv) From Table 2, it is readily observed that, the increment of shelf space (W) increases the total profit as the retailer may purchase higher amount of quantity.

(v) There is positive impact on the profit (AT) with respect to the value of the parameter $\alpha,\ p_0, s, x$ that is AT increases when the values of $\alpha,\ p_0, s, x$ increase, while for the parameters $K, \beta, C_h, C_p, \gamma, r, I_r$ there is negative impact on AT.

(vi) The optimal cycle time T^* is dependent on parameters $K, \beta, L, C_p, M, n, r$ in a positive way but it depends on parameters $p_{0,} \alpha, C_{h,} s, x, \gamma, I_r, W$ in negative way.

(vii) The impact on the optimal ordering quantity Q^* and end inventory level q^* at end in both cases with respect to the value of all parameters is not easy to say. Sometimes it may have positive impact, sometimes negative. Sometimes optimal solution will occur by taking zero end inventory model sometimes positive end inventory models. But it can be definitely concluded that relaxation of end inventory level produces improved result.

8. CONCLUSIONS

In this article, we discussed an EOQ model for deteriorating item with expiration date, time dependent deterioration rate and freshness and price sensitive demand. We have assumed that the selling price of the product is freshness sensitive. We have also assumed that, the retailer pays the purchase cost of the product in advance either in equal instalments or in a single payment. In the first case, the retailer will receive a price discount when he prepays the some portion of purchase cost before the delivery time, while in the second case he prepays the same at a time before delivery of the product to get price discount. The present article extends the earlier existing work by incorporating these two concepts such as freshness and price sensitive demand and freshness dependent selling price. The conventional assumption of zero ending inventory is relaxed in our model to any positive amount of inventory. Shelf space for holding inventory is restricted and salvage revenue of dumping items are been incorporated. After developing the model mathematically, solution procedure has been developed to determine optimal cycle length and stock of disposal items. With the help of MATHEMATICA 12 software four different numerical examples are presented for demonstration purpose. The concave nature of the profit function is justified by drawing graphs in three dimensions. To check the changes in the decision variables for changes in different parameters, a sensitivity analysis is also carried out.

There are some limitations in the present study. One limitation is that shortages are not allowed, which would not be true in fact. In addition, we also assume that the lead time is zero. In reality, the lead time is not zero. So more research is need to be carried out in the future.

CONFLICT OF INTERESTS

The author declares that there is no conflict of interests.

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