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ON GENERALIZED (g, h)-DERIVATIONS OF BH-ALGEBRAS

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Abstract: The notion of BCK – algebras was proposed by Imai and Iseki in 1966. In the same year Iseki introduced the notion of BCI-algebras, which is generalization of BCK- algebras. Y.B. Jun, E.H. Roh and H.S. Kim defined the notion of BH-algebras. Motivated by some results on derivations on rings and the generalizations of BCK and BCI-algebras. In 2019, P. Ganesan and N. Kandaraj introduced the notion of the various derivations on BH - algebras. In this paper, we study the notion of generalized (g, h)-derivations on BH – algebras and investigate simple, interesting and elegant results.

Keywords: BH-algebras; BH-sub algebras; generalized (g, h)-derivations on BH-algebras; regular generalized (g, h)-derivations on BH-algebras.

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1. INTRODUCTION

Y. Imai and K. Iseki [8, 9] introduced the on axiom system of propositional calculi and have been extensively investigated by many researchers. K. Iseki and S. Tanaka [10] introduced the theory of BCK-algebras. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. Q. Zhang, Y.B. Jun and E.H. Roh [22] introduced the notion of BH-algebras. Also Y.B. Jun, E.H Roh and H.S. Kim [13] developed BH-Relations in BH-algebras.They investigated several relations between BH-algebras and BCK-algebras. In 1957, E.C. Posner [20] introduced the notion of derivations in Prime rings theory. Also P.H. Lee and T.K Lee [18] developed on derivations of prime rings.The notion of derivations in ring theory is quite old and plays an important role in algebras. N.O Al-Shehri [4] introduced the notion of derivations of B algebras.

Many Research papers have appeared on the derivations of BCI-algebras in different ways. A.M Al-Roqi [3] introduced the notion of on generalized (α, β)-derivations in BCI-algebras. S.M Bawazeer, N.O. Alshehri and R.S. Babusail [6] introduced the notion of generalized derivations of BCC-algebras. Also L. Kamali Ardikani and B. Davvaz [15] developed the properties in generalized derivations of BCI-algebras. Y.B Jun Y and X.L. Xin [14] introduced the notion of derivations of BCI-algebras. M.A Javed and M. Aslam [11] introduced the concept of f-derivations in BCI-algebras. Also J. Zhan and Y.L Liu [21] developed the notion of f-derivations on BCI-algebras. G. Muhiuddin and M. Abdullah Al-roqi [19] introduced on t-derivations of BCI-algebras. Motivated by a lot of Work done on derivations of BH–algebras and on derivations of other related abstract algebraic structures such as d–algebras. The term algebra is used here to denote the algebraic structure defined on a non-empty set with a binary composition satisfying certain laws that resemble the algebra of logic but not the usual algebra.

The notion of the derivations is the same as that in ring theory and the usual algebraic theory. Recently, in the year 2019 Ganesan P and Kandaraj N defined and studied the notion of various derivations such as Derivations, Compositions of derivations, f-derivations, Composition of f-derivations,t-derivations, composition of t-derivations, Generalized derivations, (g, h)-

derivations, and (G, H)-derivations of BH – algebras. Using the idea of regular derivations in BH-algebras and obtained some of its properties. In this paper we introduce the notion of generalized (g, h)-derivations of BH-algebras and investigate simple, interesting and elegant results.

2. PRELIMINARIES

In this section, we summarize some basic concepts which will be used throughout this paper**.**

Let U be a set with a binary operation $*$ and a constant 0. Then (U,$*$, 0) is called a BH-algebra, if it satisfies the following axioms [6].

(1) $u * u = 0$

(2) $u * 0 = u$

(3)If $u * v = 0 \; and \; v * u = 0 \Rightarrow u = v$ for all $u, v \in U$

Define a binary relation $\leq$ on U by taking $u \leq v$ if and only if $u * v = 0$.In this case (U, $\leq$) is a partially ordered set [3].

Let (U, $*$,0) be a BH-algebra and $u \in U$. Define $u * U = \{u * v |\; v \in U\}$.

Then U is said to be edge BH-algebra if for any $u \in U, u * U = \{u, 0\}$

Let S be a nonempty subset of a BH-algebra U. Then S is called Sub algebra of U, if $u * v \in S \; for \; all \; u, v \in S$.

A subset I of a BH-algebra U is called an ideal of U if it satisfies

1.0 $\in$ I

2. $u * v \in I$ and $v \in I$ implies that $u \in I$ for all$u, v \in U$.

In BH-algebra X for all x. y, z $\in$ U, the following Property holds [14].

1. ((u$*$ v) $*$ (u$*$ w)) $*$(w$*$ v) = 0

2. (u$*$ v) $*$ u = 0

3. (u$*$ (u$*$ v)) = v

Every BH-algebra satisfying the condition (u$*$ v) $*$ w = (u$*$ w) $*$ v for all u, v, w$\in$ U is a BCH-algebra.

For a BH-algebra U, We denote $u \wedge v$ for $v * (v * u)$, $\forall x, y \in U$

Let U be a BH-algebra. The set $U_+ = \{u \in U: 0 \leq u\}$ is sub algebra and is called BCK part of U. A BH-algebra U is called proper if$U - U_+ \neq \varphi$.If $U_+ = \{0\}$, then U is called a p- Semisimple BH-algebra.

In any BH-algebra U, for all $u, v \in U$ the following conditions are equivalents [15].

1. U is p-semisimple
2. $u * v = 0$ implies $u = v$
3. $v * (v * u) = u$.

In any p-semisimple BH-algebra U, the following properties are valid, for all $u, v \in U$

1. $(u * w) * (v * w) = u * v$
2. $u * (0 * v) = v * (0 * u)$
3. $u * v = u * w$ implies $v = w$
4. $v * u = w * u$ implies that $v = w$

For a BH-algebra U, the set H (U) = $\{u \in U: 0 * U = U\}$ is called the BH-H part of U.

Note that $H(U) \cap U_+ = \{0\}$.Let U be a BH-algebra and define the binary operation $\wedge$ as $u \wedge v = v * (v * u)$ for all $u, v \in U$ .In particular, we denote $a_u = u \wedge 0 = 0 * (0 * u)$.

An element $a \in U$ is said to be an initial element (p-atom) of U, if $u \leq a$ implies u=a.

We denote by $c_p(U)$ the set of all initial elements (p-atoms) of U, indeed $c_p(U) = \{a \in U | u * a = 0 \text{ implie } u = a, \forall\, u \in U\}$ and we call it the center of U.

Note that $c_p(U) = \{u \in U | a_u = u\}$ which is the p-semisimple part of U and U is a p-semisimple BH-algebra iff $c_p(U) = U$. Let U be a BH-algebra with its center $c_p(U)$ and $a \in c_p(U)$. Then the set $X(a) = \{u \in U |\, a \leq u\}$ is called the branch of U with respect to a.

In BH-algebra U the following results are true.

1. If $u \in X(a)$ and $v \in X(b)$, then $u * v \in X(a * b)$ for all $a, b \in c_p(U)$.
2. If $u \leq v$ then u, v are contained in the same branch of U.
3. If $u, v \in X(a)$ for some $a \in c_p(U)$, then $u * v, v * u \in U_+$.

4. If $a, b \in c_p(U)$ then $a * v = a * b$ for all $v \in X(b)$.

5. $0 * u \in c_p(U)$ for all $u \in U$

6. $a_u \in c_p(U)$, for all $u \in U$. Indeed, $0 * (0 * a_u) = a_u$ for all $u \in U$ which implies that $a_u * y \in c_p(U)$ for all $v \in U$

7. $H(U) \subseteq c_p(U)$.

8. $U * (U * a) = a$ and $a * u \in c_p(U)$ for all $u \in U$ and $a \in c_p(U)$

A self-map g of a BH-algebra U (i.e., a mapping of U into itself) is called an endomorphism of U if $g(u * v) = g(u) * g(v)$ for all $u, v \in U$. Here $g(0) = 0$.

Let g be an endomorphism of a BH-algebra U and let $c_p(U)$ be its center, we have

a. $g(a) \in c_p(U)$ for all $a \in c_p(U)$

b. $g_u * g_v \in c_p(U)$ and $g_{u*v} = g_u * g_v$ for all $u, v \in U$ where $g_u = 0 * (0 * g_u)$

c. $g(a) = 0 * (0 * g(u))$ *for all* $u \in X(a)$.

A BH-algebra U is called commutative if $u \leq v$ implies $u = u \wedge v = v * (v * u)$. It is called branch wise commutative, if $x \wedge y = y \wedge x$ for all $u, v \in X(a)$ and all $a \in c_p(U)$

Note that a BH-algebra U is commutative if and only if it is branch wise commutative.

3. GENERALIZED (g, h)-DERIVATIONS ON BH-ALGEBRAS

In this section we introduce the notion of left-right-generalized (g, h)-derivations and right-left-generalized (g, h)-derivations with associated (g, h)-derivations θ of a BH- algebras and give some example. Also we derive some result related to (l, r) and (r, l)-generalized (g, h)-derivations of a BH-algebras.

Definition 3.1. Let U be a BH-algebra. A map $\varphi: U \to U$ is called a left-right-generalized (g, h) - derivations (briefly, (l, r)-generalized (g, h)-derivations) on U with associated (g, h)-derivation θ, if it satisfies the identity $\varphi(u * v) = (\varphi(u) * g(v)) \wedge (h(u) * \theta(v))$ for all $u, v \in U$.

If φ satisfies the identity $\varphi(u * v) = (g(u) * \varphi(v)) \wedge (\theta(u) * h(v))$ for all $u, v \in U$. Then φ is called a right-left-generalized (g, h)-derivation on U with associated (g, h)-derivation θ. Moreover, If φ is both a (l, r)-generalized (g, h)-derivations and (r, l)-generalized (g, h)-derivation with associated (g, h)-derivation θ, then φ is called a generalized (g, h)-derivation on U.

Example 3.2. Let $X = \{0, a, e\}$ be a BH-algebra with operation * is defined as follows

*	0	a	e
0	0	0	e
a	a	0	e
e	e	e	0

Define a map $\varphi: U \to U$ such that $\varphi(u) = \begin{cases} e & \text{if} \quad u = 0, a \\ 0 & \text{if} \quad u = e \end{cases}$ and

Define a map $\theta: U \to U$ such that $\theta(u) = \begin{cases} e & \text{if} \quad u = 0, e \\ 0 & \text{if} \quad u = a \end{cases}$ and

Define two endomorphism g and h on U as follows

$g(u) = \begin{cases} 0 & \text{if} \quad u = 0, a \\ e & \text{if} \quad u = e \end{cases}$ and $h(u) = \begin{cases} 0 & \text{if} \quad u = 0, e \\ a & \text{if} \quad u = a \end{cases}$

It is easy to checked that φ is a (r, l)-generalized (g, h)-derivation with associated (r,l)-(g,h) derivation θ on U and φ is also a (l, r) – generalized (g, h)-derivation with associated (l, r)- (g, h)-derivation θ on U.

Hence φ is a generalized (g, h)-derivation on U.

Example 3.3. Let $U = \{0, a, e\}$ be a BH-algebra with operation * is given below

*	0	a	e
0	0	0	e
a	a	0	e
e	e	e	0

Define a map $\varphi: U \to U$ such that $\varphi(u) = \begin{cases} e & \text{if} \quad u = 0, a \\ 0 & \text{if} \quad u = e \end{cases}$ and

Define a map $\theta: U \to U$ such that $\theta(u) = \begin{cases} a & \text{if} \quad u = 0, a \\ 0 & \text{if} \quad u = e \end{cases}$ and

Define two endomorphism g and h on U as follows

$g(u) = \begin{cases} 0 & \text{if} \quad u = 0, a \\ e & \text{if} \quad u = e \end{cases}$ and $h(u) = \begin{cases} e & \text{if} \quad u = 0, a \\ 0 & \text{if} \quad u = e \end{cases}$

Now it is easy to checked that φ is a (r, l)-generalized (g, h)-derivation with associated (r, l)-(g, h)-derivation θ on U and φ is also a (l, r)-generalized (g, h)-derivation with associated (l, r)- (g, h)-derivation θ on U. Hence φ is a Generalized (g, h)-derivation on U.

Example 3.4. Let $U = \{0, a, e\}$ be a BH-algebra with operation * is given below

*	0	a	e
0	0	0	e
a	a	0	e
e	e	e	0

Define a map $\varphi: U \to U$ such that $\varphi(u) = \begin{cases} e & \text{if} \quad u = 0, a \\ 0 & \text{if} \quad u = e \end{cases}$ and

Define a map $\theta: U \to U$ such that $\theta(u) = \begin{cases} 0 & \text{if} \quad u = 0, e \\ a & \text{if} \quad u = a \end{cases}$ and

Define two endomorphism g and h on U as follows

$g(u) = \begin{cases} 0 & \text{if} \quad u = 0, a \\ e & \text{if} \quad u = e \end{cases}$ and $h(u) = \begin{cases} e & \text{if} \quad u = 0, a \\ a & \text{if} \quad u = e \end{cases}$

Then It is easy to checked that φ is a (r, l)-generalized (g, h)-derivation with associated (r, l)-(g, h)-derivation θ on U and φ is also a (l, r) – generalized (g, h)-derivation with associated (l, r)- (g, h)-derivation θ on U.

Hence φ is a generalized (g, h)-derivation on U.

Example 3.5. Let $X = \{0, a, e, i, p, q\}$ be a BH-algebra with operation * is defined as follows

*	0	a	e	i	p	q
0	0	0	0	0	0	0
a	a	0	0	0	0	0
e	e	e	0	0	0	0
i	i	i	a	0	0	0
p	p	e	a	a	0	0
q	q	q	i	i	a	0

Define a map $\varphi: U \to U$ such that $\varphi(u) = \begin{cases} 0 & \text{if} \quad u = 0, a, e, i \\ e & \text{if} \quad u = \text{ otherwise} \end{cases}$ and

Define a map $\theta: U \to U$ such that $\theta(u) = \begin{cases} 0 & \text{if} \quad u = 0, a, i, p, q \\ e & \text{if} \quad u = 0\text{therwise} \end{cases}$ and

Define two endomorphism g and h on U as follows

$$\mathrm{g(u) = h(u)} = \begin{cases} 0 & if \quad u = 0 \\ a & if \quad u = a \\ e & if \quad u = e \\ i & if \quad u = i \\ p & if \quad u = p \\ q & if \quad u = q \end{cases}$$

It is easily verified that φ is a (r, l)-generalized (g, h)-derivation with associated (r, l)-(g, h)-derivation θ on U. But φ is not (l, r)-generalized (g, h)-derivation on U.

Since $\varphi(q * a) = \varphi(q) = e$

On the other side, $(\varphi(q) * a) \wedge (q * \theta(a)) = (e * a) \wedge (q * 0) = e \wedge q = i$

Therefore $\varphi(q * a) \neq (\varphi(q) * a) \wedge \big(q * \theta(a)\big)$.

Theorem 3.6. Let U be a BH-algebra with $0 * u = 0$. If $\varphi: U \to U$ is a (l, r)-generalized (g, h)-derivation with associated (g, h)-derivation θ on U, then φ is regular.

Proof. Let $\varphi: U \to U$ is a left-right generalized (g, h)-derivation with associated (g, h)-derivation θ. Now $\varphi(0) = \varphi(0 * u)$

$$=(\varphi(0) * g(u)) \wedge (h(0) * \varphi(u))$$

$$=(\varphi(0) * g(u)) \wedge (0 * \theta(u))$$

$$= (\varphi(0) * g(u)) \wedge 0. = 0 * (0 * (\varphi(0) * g(u))) = 0$$

Hence we get the required result.

Theorem 3.7. Let U be a BH-algebra with $0 * u = 0$. Then every (r, l)-generalized (g, h)-derivation $\varphi: U \to U$ with associated (g, h)-derivation θ is regular.

Proof. Since $\varphi: U \to U$ is a (r, l)-generalized (g, h)-derivation with associated (g, h)-derivation θ. Now $\varphi(0) = \varphi(0 * u)$

$$=\big(g(0) * \varphi(u)\big) \wedge \big(\theta(0) * h(u)\big)$$

$$=0 \wedge \big(\theta(0) * h(u)\big)$$

$$=(\theta(0) * h(u)) * ((\theta(0) * h(u)) * 0)$$

$$=(\theta(0) * h(u)) * (\theta(0) * h(u)) = 0$$

Therefore every generalized (g, h)-derivation is regular.

Remark. From theorem 3.6 and 3.7 we get a generalized (g, h)-derivation φ with associated (g, h)-derivation θ on a BH-algebra U is regular.

Theorem 3.8. Let U be a BH- algebra such that $0 * u = 0$ for all $u \in U$.

a). If φ is a left-right-generalized (g, h)-derivation with associated (g, h)-derivation θ on U, then $\varphi(u) = \varphi(u) \wedge h(u)$ for all $u \in U$.

b). If φ is a (r, l)-generalized (g, h)-derivation with associated (g, h)-derivation θ on U, then $\varphi(u) = g(u) \wedge \theta(u)$ for all $u \in U$.

Proof (a). Let φ be a (r, l)-generalized (g, h)-derivation with associated (g, h)-derivation θ on U, then $\varphi(u) = \varphi(u * 0)$

$$=(\varphi(u) * g(0)) \wedge (h(u) * \theta(0))$$

$$= (\varphi(u) * 0) \wedge (h(u) * 0) = \varphi(u) \wedge h(u).$$

(b). Let φ be a left-right-generalized (g, h)-derivation with associated (g, h)-derivation θ on U, then $\varphi(u) = \varphi(u * 0)$

$$=(g(u) * \varphi(0)) \wedge (\theta(u) * h(0))$$

$$=(g(u) * 0) \wedge (\theta(u) * 0) = g(u) \wedge \theta(u)$$

Remark: From the above theorem, similarly we can prove the following results.

1. If θ is a (l, r)-generalized (g, h)-derivation on U, then $\theta(u) = \theta(u) \wedge h(u)$for all $u \in U$.

2. If φ is a (r, l)-generalized (g, h)-derivation on U, then $\theta(u) = g(u) \wedge \theta(u)$for all $u \in U$.

Theorem 3.9. Let U be a BH-algebra. If φ is a (r, l)-generalized (g, h)-derivation with associated (g, h)- derivation θ of a BH-algebra U, then $\varphi(u) = \theta(u)$.

Proof. Suppose that φ be a (r, l)-generalized (g, h)-derivation.

Then we have $\varphi(u) = \varphi(u * 0)$

$$= \left(g(u) * \varphi(0)\right) \wedge \left(\theta(u) * h(0)\right)$$

$$= (g(u) * 0) \wedge \left(\theta(u) * h(0)\right) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 1$$

Also $\theta(u) = \theta(u * 0)$

$$= (g(u) * \theta(0)) \wedge (\theta(u) * h(0))$$

$$= (g(u) * 0) \wedge (\theta(u) * h(0)) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$$

From 1 and 2, we have $\varphi(u) = \theta(u)$.

Theorem 3.10. Let φ be a self-map and generalized (g, h)-derivation with associated (g, h)-derivation θ on a BH-algebra U. If $\varphi(u) = g(u)$, then

(a) φ is an (g, h)-derivation on U.

(b). $\varphi(u * v) = \varphi(u) * \varphi(v)$.

Proof (a). Let φ be a self-map and generalized (g, h)-derivation with associated (g, h)-derivation θ on a BH-algebra U such that $\varphi(u) = g(u)$,

We have $\varphi(u * v) = g(u * v)$

$$= g(u) * g(v)$$

$$= \varphi(u) * \varphi(v)$$

$$= \big(h(u) * \theta(v)\big) * (\big(h(u) * \theta(v)\big) * (\varphi(u) * g(v)))$$

$$= (\varphi(u) * g(v)) \wedge (h(u) * \theta(v))$$

This implies φ is a (l, r)-(g, h) derivation on U.

Similarly we can prove that φ is a right – left-(g, h)-derivation on U.

(b) Let $u, v \in U$.

Now $\varphi(u * v) = (\varphi(u) * g(v)) \wedge (h(u) * \theta(v))$

$$= \big(g(u) * g(v)\big) \wedge \big(h(u) * \theta(v)\big)$$

$$= \big(h(u) * \theta(v)\big) * \Big(\big(h(u) * \theta(v)\big) * \big(g(u) * g(v)\big)\Big)$$

$$= g(u) * g(v)$$

$$= \varphi(u) * \varphi(v).$$

Lemma 3.11. Let U be BH- algebra with partial order $\leq$ and let φ be a right-left-(g, h)-derivation on U. Then $\varphi(u) \leq g(u)$ for all $u, v \in U$.

Proof. Let φ be a (l, r)-generalized (g, h)-derivation with associated (g, h)-derivation θ on U, then $\varphi(\mathrm{u}) = \varphi(\mathrm{u} * 0)$

$$= \big(\mathrm{g}(\mathrm{u}) * \varphi(0)\big) \wedge \big(\,\theta(\mathrm{u}) * \mathrm{h}(0)\big)$$

$$= (\mathrm{g}(\mathrm{u}) * 0) \wedge (\theta(\mathrm{u}) * 0)$$

$$\varphi(\mathrm{u}) = \mathrm{g}(\mathrm{u}) \wedge \theta(\mathrm{u})$$

$$= \theta(\mathrm{u}) * \big(\,\theta(\mathrm{u}) * \mathrm{g}(\mathrm{u})\big)$$

This gives $\varphi(\mathrm{u}) * \mathrm{g}((\mathrm{u}) = (\,\theta(\mathrm{u}) * \big(\theta(\mathrm{u}) * \mathrm{g}(\mathrm{u})\big)) * \mathrm{g}(\mathrm{u}) = 0$ since $(\,\mathrm{y} * (\mathrm{y} * \mathrm{x})) * \mathrm{x} = 0$

Hence $\varphi(\mathrm{u}) \leq \mathrm{g}(\mathrm{u})$.

Theorem 3.12. Let U be a BH- algebra such that $0 * u = 0$ for all $u \in U$ and φ be a generalized (g, h) derivation on U with associated (g, h)-derivationθ.

Then $\varphi^{n}(\varphi^{n-1}\big(\ldots\big(\varphi^{2}(\varphi'(u)\big)\ldots.\big) \leq g(u)$.

Proof. Let $n = 1$, using lemma 3.11,

we have $\varphi'(\mathrm{u}) \leq \mathrm{g}(\mathrm{u})$

Suppose for any $n \in N$,

$\varphi^{n}(\varphi^{n-1}\big(\ldots\ldots\ldots..\big(\varphi^{2}(\varphi'(u)\big)\ldots\ldots\ldots..\big) \leq g(u)$.

Let $\delta_n = \varphi^{n}(\varphi^{n-1}\Big(\ldots\ldots\ldots..\Big(\big(\varphi'(u)\big)\Big)\ldots\ldots\ldots\Big)$

$i.e\ \delta_n \leq g(u)$.

Now $\varphi^{n+1}(\delta_n) = \varphi^{n+1}(\delta_n * 0)$

$$=(g(\delta_n) * \varphi^{n+1}(0)) \wedge (\varphi^{n+1}(\delta_n) * h(0))$$

$$= g(\delta_n) \wedge \varphi^{n+1}(\delta_n)$$

$$= \varphi^{n+1}(\delta_n) * (\varphi^{n+1}(\delta_n) * g(\delta_n))$$

Therefore $(\varphi^{n+1}(\delta_n) * g(\delta_n)$=$(\varphi^{n+1}(\delta_n) * (\varphi^{n+1}(\delta_n) * g(\delta_n))) * g(\delta_n)$

Thus $\varphi^{n+1}(\delta_n) \leq g(\delta_n)$

By our assumption, we have $\varphi^{n+1}(\delta_n) \leq g(\delta_n) \leq g(u)$. Hence the proof.

Theorem 3.13. Let U be a BH- algebra with partial order $\leq$ and $g(u) = u$ and let φ be a generalized (g, h) -derivation with associated (g, h) derivation θ on U such that $0 * u = 0$ for all

$u \in U$.Then (a) $\varphi(u * v) \leq g(u)\ \varphi(v)$

(b) $\varphi(u * v) \leq \varphi(u)h(v)$

(c) $\varphi\big(g(u) * \varphi(u)\big) = 0$

(d) $\varphi\big(g(u) * \theta(u)\big) = 0$ if $g = h$

(e) $\varphi\big(\theta(u) * g(u)\big) = 0$ if $g = h$

Proof (a). Let φ be a generalized (g, h)-derivation on U.

Then $\varphi(u * v) = \big(g(u) * \varphi(v)\big) \wedge \big(\theta(u) * h(v)\big)$

$$=(\theta(u) * h(v)) * ((\big(\theta(u) * h(v)\big) * (g(u) * \varphi(v)))$$

$$=(g(u) * \varphi(v))$$

Now $\varphi(u * v) * \big(g(u) * \varphi(v)\big) = 0$

Hence $\Phi(u * v) \leq g(u) * \varphi(v)$

(b) Let φ be a (l, r)-generalized (g, h)-derivation on U.

Then $\varphi(u * v) = \big(\varphi(u) * g(v)\big) \wedge \big(h(u) * \theta(v)\big)$

$$=\big(h(u) * \theta(v)\big) * \Big(\big(h(u) * \theta(v)\big) * \big(\varphi(u) * g(v)\big)\Big)$$

$$= \varphi(u) * g(v)$$

Now we have $\varphi(u * v) * \big(\varphi(u) * g(v)\big) = 0$

Therefore $\varphi(u * v) \leq \varphi(u) * g(v)$

(c) Let φ be a left right generalized (g, h)-derivation on U.

Now$\varphi\big(g(u) * \varphi(u)\big) = (\varphi\big(g(u)\big) * g\big(\varphi(u)\big)) \wedge (h\big(g(u)\big) * \theta(\varphi(u)))$

$$= (\varphi(u) * \varphi(u)) \wedge (h(u) * \theta\big(\varphi(u)\big)) \quad \text{since } g(u) = u$$

$$= 0 \wedge (h(u) * \theta\big(\varphi(u)\big))$$

$$=\Big(h(u) * \theta\big(\varphi(u)\big)\Big) * \Big(\Big(h(u) * \theta\big(\varphi(u)\big)\Big) * 0\Big)$$

$$=(h(u) * \theta\big(\varphi(u)\big) * \Big(h(u) * \theta\big(\varphi(u)\big)\Big) = 0$$

(d) Let φ be a (r, l) generalized (g, h)-derivation on U.

Now $\varphi\big(g(u) * \theta(u)\big) = \Big(g\big(g(u)\big) * \varphi\big(\theta(u)\big)\Big) \wedge (\theta\big(g(u)\big) * h(\theta(u)))$

$$= (g(u) * \varphi\big(\theta(u)\big)) \wedge (\theta\big(g(u)\big) * g\big(\theta(u)\big))$$

$$=\Big(u * \varphi\big(\theta(u)\big)\Big) \wedge \big(\theta(u) * \theta(u)\big)$$

$$=0 * \Big(0 * \Big(u * \varphi\big(\theta(u)\big)\Big)\Big) = 0$$

(e) Let φ be a left- right generalized (g, h) derivation on U.

Now $\varphi\big(\theta(u) * g(u)\big) = (\varphi\big(\theta(u)\big) * g\big(g(u)\big)) \wedge (h\big(\theta(u)\big) * \theta\big(g(u)\big))$

$$= \varphi\big(\theta(u) * g(u)\big) \wedge \Big(g\big(\theta(u)\big) * \theta(u)\Big) \text{Since } g = h$$

$$= \varphi(\theta(u) * u) \wedge (\theta(u) * \theta(u))$$

$$= \varphi(\theta(u) * u) \wedge 0$$

$=0 * (0 * \big(\varphi(\theta(u) * u)\big) = 0$ since $0 * u = 0$. Hence the result.

4. CONCLUSION

An algebraic structure that arises from the study of algebraic formulations of propositional logic. Taking different theorems or statements of propositional logic, different algebraic structures could be obtained. The BH-Algebra is one such algebra. The derivation concept is an important and very interesting area of research in the theory of algebraic Structures in Mathematics. The deep theory has been developed for derivations in BCI-algebras [1, 2], BCC-algebras [5], d-algebras [7, 17] and BP-algebras [16]. It plays an important role in algebra, algebraic geometry and linear differential equations.We have considered the concept of generalized (g, h)-derivations on BH-algebras.Finally, we investigated the notion of the regular generalized (g, h)-derivations on BH-algebras.. In future any Researcher can study the notion of generalized derivations in different algebraic Structures which may have a lot of applications in various fields. This work is a foundation for the further study of the Researcher on derivations of algebras.

The future study of derivations on BH-algebras may be the following topics should be covered.

(a) To find the generalized derivations on d-algebras.

(b) To find the t-derivations of Q-algebras, B-algebras and so on so.

(c) To find more results and its applications in derivations on BH-algebras.

(d) To find to investigate how these concepts could be applied to the field of computers for processing information.

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CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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