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LINE TRIANGLE INTERSECTION GRAPH

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Abstract. A graph operator is a mapping between two families of graphs. In this paper, a new graph operator called the line triangle intersection graph is introduced. Also, the concept of a quasi regular graph is proposed. Further, various properties of line triangle intersection graph of a graph are investigated including its chromatic number and clique number. It is proved that the line triangle intersection graph of a complete graph is quasi regular. Moreover, partial characterization for a line triangle intersection graph is presented.

Keywords: graph; regular graph; line graph; line-triangle intersection graph; quasi-regular graph.

2010 AMS Subject Classification: 05C76.

1. INTRODUCTION

Even though Whitney [3] used the construction of line graphs, it was Krausz [4] who formulated the concept of a graph operator and that of a line graph. A characterization of line graphs was also given by him. Later, Beineke [5] gave a new characterization of line graph in terms of 9 forbidden subgraphs. Again Šoltés [6] gave forbidden induced subgraphs of a line graph with atleast 9 vertices. Modifying the construction of a line graph, two new graph

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operators were proposed by Gallai [7], which were named as Gallai and anti-Gallai graphs. The notations for the same were proposed by Sun [9]. Many other graph operators are also studied such as intersection graphs. Further, many of the intersection graphs are studied such as path intersection graphs, P_3 intersection graph, C_4 intersection graphs, clique graphs, block graphs etc.

Graph operators are mainly used to study complicated graphs. Most of the graph operators produce graphs which have common properties with the original graph. So, complex graphs can be studied in terms of simpler graphs. In this paper, we introduce a new graph operator called the line triangle intersection graph and investigate various properties of this graph. The structure of the paper is as follows: section 2 revisits some of the preliminary concepts in graph theory, section 3 presents the concept of a line triangle intersection graph and quasi-regular graph and section 4 concludes the paper.

2. PRELIMINARIES

Definition 2.1. [2] Let $G = (V, E)$ be a graph, where V is a nonempty set of elements called vertices and E is a set of elements called edges such that each element $e \in E$ is associated with two vertices in V which are called the end vertices of e .

Definition 2.2. [1] The number of edges incident at v in G is called the *degree of the vertex* v and is denoted as $d(v)$. The minimum and maximum values of the degrees of the vertices of a graph G are denoted by $\delta(G)$ and $\Delta(G)$ respectively.

Definition 2.3. [1] A *path* is a sequence of vertices and edges in a graph such that no vertex is repeated. A path on n vertices is called a path of length n and is denoted as P_n .

Definition 2.4. [1] A *cycle* is a closed path. ie; A path whose initial and terminal vertex coincide is called a cycle. A cycle of length n is denoted as C_n .

Definition 2.5. [1] A cycle of length 3, ie; C_3 is called a *triangle*.

Definition 2.6. [1] The *shortest distance* between any two vertices u and v in G is the length of shortest path connecting u and v in G , denoted by $d(u, v)$

Definition 2.7. [1] A simple graph G is said to be **complete**, if every pair of distinct vertices of G are adjacent in G . A Complete graph on n vertices is denoted as K_n .

Definition 2.8. [1] A **clique** of G is a complete subgraph of G . A clique of G is said to be a **maximal clique** of G if it is not properly contained in another clique of G .

Definition 2.9. [1] A graph G is said to be **k -regular** if every vertex of G has degree k . A graph is said to be regular if it is k -regular for some non-negative integer k .

Definition 2.10. [1] A graph is said to be **bipartite** if its vertex set can be partitioned into two non-empty disjoint subsets X and Y such that each edge of G has one end in X and the other end in Y .

Definition 2.11. [1] A simple bipartite graph is said to be **complete bipartite** if every vertex of X is adjacent to every vertex of Y .

Definition 2.12. [7] The **clique number** $\omega(G)$ is the supremum of all natural number k such that G contains a complete subgraph with k vertices.

Definition 2.13. [1] The **chromatic number** of a graph G is the minimum number of colors needed for a proper vertex coloring of G . Chromatic number of a graph G is denoted as $\chi(G)$.

Definition 2.14. [1] The minimum value k , for which a loopless graph G has a proper k -edge coloring is called the **edge-chromatic number** or chromatic index of G . It is denoted by $\chi'(G)$.

Definition 2.15. [1] A graph G is called a **split graph** if its vertex set can be partitioned into two subsets K and I such that the subgraph $G[K]$ induced by K in G is a clique in G and I is an independent subset of G .

Definition 2.16. [1] Let G be a loopless graph. We construct a graph $L(G)$ in the following way. The vertex set of $L(G)$ is in 1-1 correspondence with the edge set of G and two vertices of $L(G)$ are joined by an edge if and only if the corresponding edges of G are adjacent in G . The graph $L(G)$ is called **line graph**, derived graph or edge graph of G .

Definition 2.17. [7] The *Gallai graph* Γ of a graph G is the graph whose vertex-set is the edge-set of G ; two distinct edges of G are adjacent in $\Gamma(G)$ if they are incident in G , but do not span a triangle in G .

Definition 2.18. [7] The *anti-Gallai graph* of a graph G denoted as $\Delta(G)$ has the edges of G as its vertices; two edges are adjacent in $\Delta(G)$ if they span a triangle in G .

(Throughout this paper $\Delta(G)$ represent maximum degree of a vertex in G)

Theorem 2.19. [7] For every graph G with $\omega(G) \geq 1$,

$$(1) \quad \omega(\gamma(G)) = \begin{cases} \omega(G) - 1, & \text{if } \omega(G) \neq 3 \\ 3, & \text{if } \omega(G) = 3 \end{cases}$$

where $\gamma(G)$ denote anti-Gallai graph of G

Theorem 2.20. [5] The following statements are equivalent for a graph G .

- (1) G is the derived graph of some graph.
- (2) The edges of G can be partitioned into complete subgraphs in such a way that no vertex belongs to more than two of the subgraphs.
- (3) The graph $K_{1,3}$ is not an induced subgraph of G ; and if abc and bcd are distinct odd triangles, then a and d are adjacent.

Theorem 2.21. Brook's Theorem [8]: Every graph G with maximum degree $\Delta(G)$ has a $\Delta(G)$ coloring unless either (i) G contains $K_{\Delta(G)+1}$ or (ii) $\Delta(G) = 2$ and G contains an odd cycle.

3. LINE TRIANGLE INTERSECTION GRAPH

Definition 3.1. Let $G = (V, E)$ be a graph with vertex set V and edge set E . Then the *line-triangle intersection graph* of G denoted by $LT(G)$ is a graph having edges and triangles in G as its vertices and two vertices in $LT(G)$ are adjacent if and only if they corresponds to two adjacent edges in G or an edge and a triangle in G having only one common vertex.

A vertex corresponding to an edge in G is called an *edge vertex* and a vertex corresponding to a triangle in G is called a *triangle vertex*.

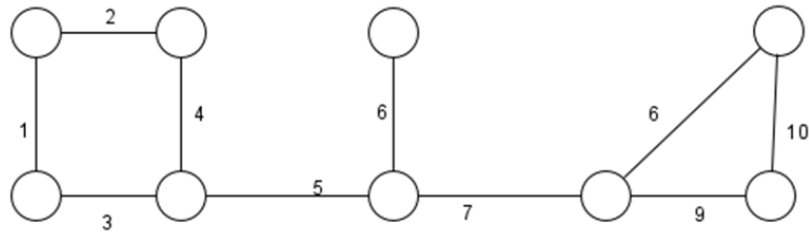


FIGURE 1

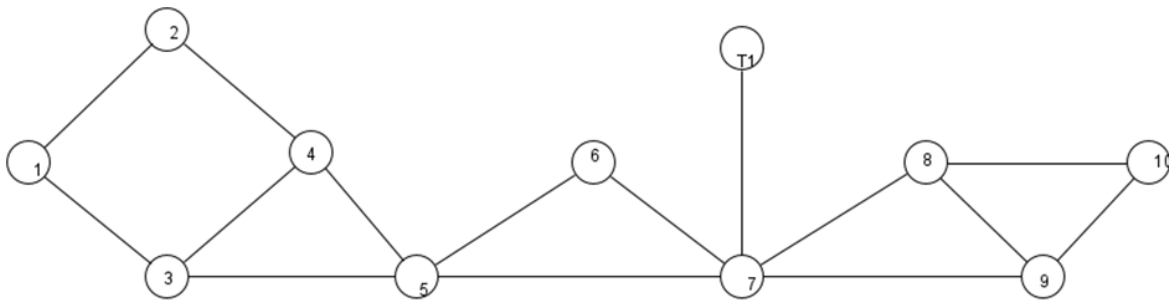


FIGURE 2. $LT(G)$

Example 3.2. The line-triangle intersection graph of the graph G in Figure 1 is the graph in Figure 2. The pendent vertex T_1 in $LT(G)$ is the triangle vertex and all other vertices are edge vertices.

Definition 3.3. A graph having only two distinct non-negative integers in its degree sequence is called a *quasi-regular graph*.

Example 3.4. In Figure 3, the degree of red vertices is 3 and the degree of green vertices is 2. So, it is a quasi-regular graph.

Definition 3.5. The *T-index* of a graph is the number of triangles in it.

Definition 3.6. The number of triangles passing through a vertex of a graph is called the *T-degree* of that vertex and if there is no triangles passes through that vertex T-degree of that vertex is considered to be zero. T-degree of a vertex 'u' is denoted as $T_d(u)$. The number of triangles incident on an edge is called the *T-degree* of that edge. ie; If the number of triangles

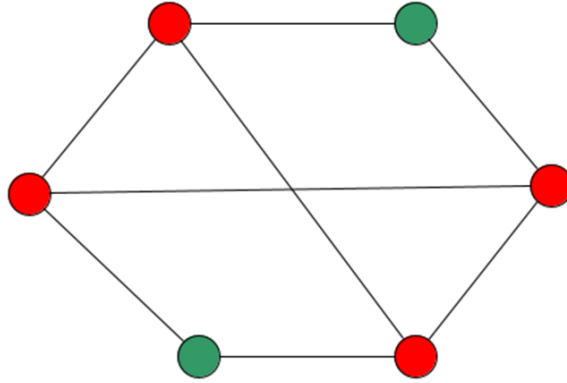


FIGURE 3

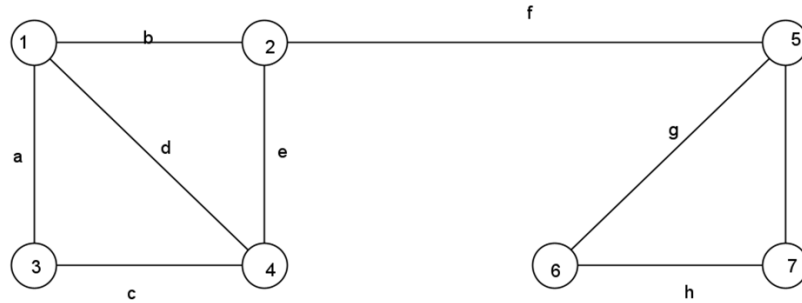


FIGURE 4

incident on an edge is 0, the T-degree of that edge is considered to be zero. The T-degree of an edge 'e' is denoted as $T_d(e)$.

Example 3.7. In figure 4, the T-index of the graph is 3. Also, $T_d(1) = T_d(4) = 2$, $T_d(2) = T_d(3) = T_d(5) = T_d(6) = T_d(7) = 1$, $T_d(a) = T_d(b) = T_d(c) = T_d(e) = 1$, $T_d(d) = T_d(g) = T_d(h) = T_d(i) = 0$ and $T_d(f) = 2$

Theorem 3.8. The T-index of a graph G is $\frac{\sum_{u \in V} T_d(u)}{3}$

Proof. Each triangle in G passes through 3 vertices. So, each triangle is counted once in the T-degree of each of the 3 vertices. Thus, T-index of a graph is given by one-third of the sum of all T-degree of vertices. \square

Remark 1. (1) From the construction itself we can see that the line graph of a graph G is a subgraph of the line triangle intersection graph.

(2) For a triangle free graph G , $LT(G) = L(G)$

Example 3.9. $LT(P_n) = L(P_n)$ and $LT(C_n) = L(C_n)$ for $n \geq 3$

Theorem 3.10. *The maximum number of vertices in the line triangle intersection graph of a graph G with ' n ' vertices is $nC_2 + nC_3$*

Proof. For any graph G , the number of vertices in $LT(G)$ is equal to the the sum of the number of edges in G and the T-index of G . If G is a graph with ' n ' vertices, then the number of edges and the number of triangles (T-index) is maximum when it is a complete graph and for a complete graph, the number of edges is nC_2 and T-index is nC_3 . Therefore, the maximum number of vertices in $LT(G)$ is $nC_2 + nC_3$ \square

Theorem 3.11. *The line triangle intersection graph of a complete graph is a quasi-regular graph.*

Proof. Consider the complete graph K_n on ' n ' vertices. Each vertex of K_n will be of degree ' $n - 1$ '. Consider an edge $e = uv$ in K_n . Then, there are ' $n - 2$ ' more edges having ' u ' as an end vertex and ' $n - 2$ ' more edges having ' v ' as an end vertex. That is, there is a total of $2(n - 2) = 2n - 4$ edges adjacent to the edge ' e '. Also, joining ' u ' to any two of the remaining $(n - 2)$ vertices (other than v), we get a triangle having only one common vertex with ' e '. There are $(n - 2)C_2$ such triangles in K_n having v as a common vertex with ' e '. So, there are $\frac{2(n-2)(n-3)}{2} = (n - 2)(n - 3)$ triangles having only one common vertex with ' e '. Therefore, the degree of any edge vertex in $LT(G)$ is $2(n - 2) + (n - 2)(n - 3) = (n - 2)(n - 1)$.

Now consider a triangle ' uvw ' in K_n . The edges joining u and the remaining $(n - 3)$ vertices (other than ' v ' and ' w ') will have only one common vertex ' u ' with the triangle. There are ' $n - 3$ ' such edges. Similarly there are ' $n - 3$ ' edges having only one common vertex ' v ' with the triangle and ' $n - 3$ ' edges having only one common vertex ' w ' with the triangle. So, each triangle in K_n has exactly $3(n - 3)$ edges having only one common vertex with it. Therefore, the degree of each triangle vertex in $LT(K_n)$ is $3(n - 3)$.

Hence, $LT(K_n)$ is quasi-regular graph with degrees $(n - 1)(n - 2)$ and $3(n - 3)$. \square

Corollary 3.12. The number of edges of $LT(K_n)$ is $\frac{n(n-1)(n-2)^2}{2}$

Proof. From theorem 3.11, each edge vertex has a degree $(n-2)(n-3)$ and there are nC_2 edge vertices in $LT(K_n)$. Similarly, there are nC_3 triangle vertices each of degree $3(n-3)$. So, the degree sum of vertices of $LT(K_n)$ is given by

$$\begin{aligned}\Sigma_e d(e) + \Sigma_T d(T) &= nC_2(n-2)(n-1) + nC_3 3(n-3) \\ &= \frac{n(n-1)}{2}(n-2)(n-1) + \frac{n(n-1)(n-2)}{6} 3(n-3) \\ &= \frac{n(n-1)(n-2)}{2}(n-1+n-3) = n(n-1)(n-2)^2\end{aligned}$$

Therefore, the number of edges in $LT(G)$ is $\Sigma \frac{d(v_i)}{2} = \frac{n(n-1)(n-2)^2}{2}$ \square

Theorem 3.13. $LT(K_{1,n})$ is the only line triangle intersection graph which is also a complete graph.

Proof. By definition itself, $LT(K_{1,n}) = K_n$.

Now if $LT(G) = K_n$ for some G , then G must be triangle free. Also each edge in G should be adjacent to all other edge. This is possible only when $G = K_{1,n}$. \square

Theorem 3.14. $LT(G)$ is isomorphic to G iff $G = \bigcup_{n>3} C_n$ or $G = C_3 \cup K_1$

Proof. Necessary: If G is a triangle free graph, then $LT(G) = L(G)$ and we know that $L(G)$ is isomorphic to G iff $G = \bigcup C_n$. So, let us assume that G is a connected graph having triangles. Now, suppose that $LT(G)$ isomorphic to $L(G)$. This is possible only if $n = m + t$ where m denote number of edges of G and t denotes T-index of G . Now consider the least case when $t = 1$, ie; there is only one triangle. Then, $n = m + 1$ or $m = n - 1$. ie; G is a tree which is a contradiction. When $t > 1$, $n < m - 1$ which means that G is disconnected. Again a contradiction. So, $LT(G)$ is not isomorphic to G except when $G = C_3 \cup K_1$ Sufficient: C_n is a triangle free graph and $LT(C_n) = C_n$. So $LT(\bigcup_{n>3} C_n) = \bigcup_{n>3} C_n$. Also, $LT(C_n \cup K_1) = C_n \cup K_1$ \square

Theorem 3.15. For any graph G , $\chi(LT(G)) \leq \chi'(G) + 1$

Proof. Case 1: If G is a triangle free graph then $LT(G) = L(G)$ and $\chi(L(G)) = \chi'(G)$. Therefore $\chi(LT(G)) = \chi'(G)$.

Case 2: If G is a graph having a triangle, then each edge vertex can be given the same color as the edge corresponding to it in G . Now consider a triangle vertex in $LT(G)$. Corresponding to this vertex there will be a triangle xyz (say) in G . If the color of any of the edges of xyz is different

from the colors of the edges incident with this triangle, that color can be given to the triangle vertex. If all triangle vertices can be colored like this, then $\chi(LT(G)) = \chi'(G)$. Otherwise if there is a triangle in G whose edges have the same color as its incident edges, then the triangle vertex corresponding to this triangle can be given a new color. If there are more than one triangle like this, all such triangle vertices can be given the same color as no two triangle vertices are adjacent. In this case, $\chi(LT(G)) = \chi'(G) + 1$. So, in general $\chi(LT(G)) \leq \chi'(G) + 1$ \square

Theorem 3.16. *If two graphs G_1 and G_2 are isomorphic, then their line-triangle intersection graphs are also isomorphic.*

Proof. If G_1 and G_2 are isomorphic, there is a one-one correspondence between the vertex set of G_1 and G_2 and a one one-one correspondence between the edge set of G_1 and G_2 which preserves the incidence and adjacency relation. So, there will be a one-one correspondence between the triangles of G_1 and G_2 . Hence, in the line-triangle intersection graph of G_1 and G_2 also there will be a one-one correspondence between the vertex sets and the edge sets which preserves adjacency and incidence relation. Hence, $LT(G_1)$ is isomorphic to $LT(G_2)$ \square

Theorem 3.17. *For any graph G with $\omega(G) \geq 1$,*

$$(2) \quad \omega(LT(G)) \geq \begin{cases} \omega(G) - 1, & \text{if } \omega(G) \neq 3, 4 \\ \omega(G), & \text{if } \omega(G) = 3, 4 \end{cases}$$

Proof. Let $\omega(G) \neq 3, 4$. If K_n is a complete subgraph of G where $n \neq 3, 4$, then ' $n - 1$ ' edges of K_n have a common endpoint in G . These ' $n - 1$ ' edges form a complete graph in $LT(G)$. Therefore, $\omega(LT(G)) \geq \omega(G) - 1$ if $\omega(G) \neq 3, 4$.

If $\omega(G) = 3, 4$. clearly $\omega(LT(G)) \geq \omega(G)$ \square

Theorem 3.18. $\omega(LT(G)) = \Delta(G)$ where $\Delta(G)$ is maximum degree of the graph G and $\omega(G)$ is clique number of G .

Proof. Let $\Delta(G) = m$. Let ' u ' be a vertex of degree ' m '. Then the ' m ' edge vertices in $LT(G)$ corresponding to these ' m ' edges in G make a complete graph in $LT(G)$.

So $\omega(LT(G)) \geq m \dots \dots (1)$.

Now let us prove that $\omega(LT(G)) \leq m$.

Let us assume the contrary. ie; Let $\omega(LT(G)) > m$. Then there exist a clique of order atleast $m + 1$. Here there are two cases.

Case 1: All the vertices of that clique are edge vertices of $LT(G)$. Then the corresponding $m + 1$ edges in G have a common vertex say u' . Then the degree of this vertex u' will be $m + 1$, which is a contradiction. So, in this case $\omega(LT(G)) \leq m$.

Case 2: One vertex in the clique is a triangle vertex say T' . (No two triangle vertex belongs to same clique as they will not be adjacent in $LT(G)$.) Then, the edges in G corresponding to the remaining m' edges of the clique will be incident to a vertex u' of the triangle T' . Then, the degree of $u' = m + 2$ (m' edges incident to T' together with the 2 edges of the triangle). Again, it is a contradiction to the fact that $\Delta(G) = m$. Thus, in any case $\omega(LT(G)) \leq m$(2)

Combining (1) and (2) we can say that $\omega(LT(G)) = \Delta(G)$ □

Theorem 3.19. For any graph G , $\Delta(G) \leq \chi(LT(G))$

Proof. By Brook's theorem, $\chi(G) \leq (\Delta(G) + 1)$. Also, we know that $\omega(G) \leq \chi(G)$.

So, for any graph G , $\omega(G) \leq \chi(G) \leq \Delta(G) + 1$. $\therefore \omega(LT(G)) \leq \chi(LT(G)) \leq \Delta(LT(G)) + 1$.

From theorem 2.19 $\Delta(G) = \omega(LT(G))$. $\therefore \Delta(G) \leq \chi(LT(G))$ □

Theorem 3.20. For any graph G ,

$$(3) \quad \chi(LT(G)) \leq \begin{cases} 2\Delta(G), & \text{if } \Delta(G) \leq 5 \\ 3\Delta(G) - 5, & \text{if } \Delta(G) > 5 \end{cases}$$

Proof. Let P be the set of edge vertices and T be the set of triangle vertices of $LT(G)$.

Here, there are two cases.

Case 1: G is a triangle free graph.

Then, the vertex set of $LT(G)$ consists of P only. Hence, each vertex p in $LT(G)$ corresponds to an edge uv in G .

So, the degree of p in $LT(G)$, $d(p) = d(u) + d(v) - 1$

$\therefore \max_{p \in P} d(p) = \max_{u \in (V(G))} d(u) + \max_{v \in (V(G))} d(v) - 1$

$$\Rightarrow \Delta(LT(G)) \leq \Delta(G) + \Delta(G) - 1$$

$$\leq 2\Delta(G) - 1$$

Case 2 : There is a triangle vertex t in $LT(G)$ such that $T = uvw$ in G .

Then, the degree of t in $LT(G)$, $d(t) = d(u) + d(v) + d(w) - 6$

$$\begin{aligned} \therefore \max_{t \in T} d(t) &\leq \max_{u \in (V(G))} d(u) + \max_{v \in (V(G))} d(v) + \max_{w \in (V(G))} d(w) - 6. \\ &\leq \Delta(G) + \Delta(G) + \Delta(G) - 6. \\ &\leq 3\Delta(G) - 6. \end{aligned}$$

Therefore, $\Delta(LT(G)) \leq \max\{\max_{p \in P} d(p), \max_{t \in T} d(t)\}$

$$\begin{aligned} &\leq \max\{2\Delta(G) - 1, 3\Delta(G) - 6\} \\ &\leq \begin{cases} 2\Delta(G) - 1, & \text{if } \Delta(G) \leq 5 \\ 3\Delta(G) - 6, & \text{if } \Delta(G) > 5 \end{cases} \end{aligned}$$

According to Brook's theorem, $\chi(LT(G)) \leq \Delta(LT(G)) + 1$, and hence

$$(4) \quad \chi(LT(G)) \leq \begin{cases} 2\Delta(G), & \text{if } \Delta(G) \leq 5 \\ 3\Delta(G) - 5, & \text{if } \Delta(G) > 5 \end{cases}$$

□

Theorem 3.21. *Let G be a graph with n vertices. Then, $LT(G)$ can be partitioned into edge disjoint complete graphs in such a way that no vertex is common to more than $(n-2)^2$ of such subgraphs.*

Proof. Case 1: Let t be a triangle vertex. Then, vertices in $LT(G)$, corresponding to the edges incident at each vertex of the triangle corresponding to t in G , induces a complete subgraph along with t in $LT(G)$. Since t has exactly 3 vertices, t is common to atmost 3 complete subgraphs in $LT(G)$, for $n \geq 6$

Case 2(a): Let e be an edge in G which is not incident with any triangle in G . Then, the edges incident at each end vertex of ' e ' induces a complete subgraph in $LT(G)$. So, ' e ' will be common to maximum of two complete subgraphs of $LT(G)$.

Case 2(b): Let ' e ' be an edge in G which is incident with one or more triangles in G . Then, each vertex corresponding to a triangle incident with e , alongwith the vertices corresponding to e and the other edges incident on the common vertex of e and the triangle, induce a complete subgraph in $LT(G)$. Now, maximum number of triangles incident on each end vertex of ' e ' will be ${}^{(n-1)}C_2$. Of these, ${}^{(n-1)}C_2 + {}^{(n-1)}C_2$ triangles, ${}^{(n-2)}C_1$ triangles will be common. Therefore, the maximum number of triangles incident on an edge ' e ' in a graph will be ${}^{(n-1)}C_2 + {}^{(n-1)}C_2 -$

$$\begin{aligned}
& \binom{n-2}{1} C_1. \\
& = 2 \binom{n-1}{2} C_2 - \binom{n-2}{1} C_1 \\
& = \frac{2(n-1)!}{2!(n-3)!} - \frac{(n-2)!}{1!(n-3)!} \\
& (n-2)(n-1) - (n-2) = (n-2)(n-2) = (n-2)^2.
\end{aligned}$$

Hence, the maximum number of edge disjoint complete subgraphs common to an edge vertex is $(n-2)^2, \forall n \geq 3$. \square

3.1 Partial Characterization of LT Graph.

Theorem 3.22. *Let G be the LT graph of a graph H having atleast two triangles. Then G is a graph whose vertex set can be partitioned into two sets V_1 and V_2 where V_1 induces a connected graph and V_2 is an independent set. Again, the edges of the connected graph induced by V_1 can be partitioned into complete subgraphs in such a way that not more than two of them have a common vertex.*

Proof. Without any loss of generality, let us assume that G is connected. Let G be the LT graph of a connected graph H having atleast two triangles, then there are two types of vertices in G - edge vertices and triangle vertices. Now, consider a partition of vertices in G into two sets V_1 and V_2 such that V_1 contains only edge vertices and V_2 contains only triangle vertices. As two triangle vertices are not adjacent, V_2 is an independent set. Now consider V_1 . As V_1 is the set of edge vertices, the graph induced by the V_1 is the line graph of H . ie, $L(H)$. So by characterization of line graphs given by Lowell.W.Beinkee [5], the edges of G can be partitioned into complete subgraphs in such a way that not more than two such complete subgraphs have a common vertex. \square

4. CONCLUSION

Here, we introduced a new graph operator called line-triangle intersection graph and studied various properties of it. Also, we introduced the concept of a quasi-regular graph. Graph operators are used in various fields. They are very useful in the study of complex graphs. Similarly, we can use LT graph in various fields to explore complex structures. Also line-triangle intersection graphs can be helpful in traffic controlling. Quasi-regular graphs give a

partition of graphs. We can further study about various properties of quasi-regular graph and line-triangle graph.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

REFERENCES

- [1] R. Balakrishnan, K. Ranganathan, A Text Book of Graph Theory, Springer, 1991.
- [2] J. Clark, D.A. Holton, A First Look at Graph Theory, World Scientific, 1991.
- [3] H. Whitney, Congruent graphs and the connectivity of graphs, Hassler Whitney Collected Papers, 1992, 61–79.
- [4] J. Krausz, Démonstration nouvelle d’une théoreme de Whitney sur les réseaux, Mat. Fiz. Lapok, 1 (1943), 75–85.
- [5] L.W. Beineke, Characterizations of derived graphs, J. Comb. Theory, 2 (1970), 129–135.
- [6] L. Šoltés, Forbidden induced subgraphs for line graphs, Discrete Math. 132 (1-3) (1994), 391–394.
- [7] V.B. Le, Gallai graphs and anti-Gallai graphs, Discrete Math. 159 (1996), 179–189.
- [8] D.W. Cranston, L. Rabern, Brooks’ theorem and beyond, J. Graph Theory, 3 (2015), 199–225.
- [9] L. Sun, Two classes of perfect graphs, J. Comb. Theory, Ser. B, 2 (1991), 273–292.