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A NOVEL APPROACH TO FIND THE ENTIRE FEASIBLE SOLUTIONS ON FUZZY LINEAR PROGRAMMING PROBLEM

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Abstract: In this paper, a new method for fuzzy variable linear programming problem is proposed. The optimal solution of the fuzzy variable linear programming problem is derived after finding the feasible solution. A new algorithm is discussed to transfer the infeasible/feasible solution to feasible/optimal solution is also verified.

Keywords: Fuzzy variable Linear Programming Problem, Triangular Fuzzy Number.

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1. Introduction

Fuzzy set theory has been applied to many disciplines such as control theoryand managementsciences, mathematical modeling and industrial applications.The concept of fuzzy linear programming (FLP) on general level was firstproposed by Tanaka et al. [10]in the framework of the fuzzydecision of Bellman and Zadeh [1]. The first formulation of fuzzy linear programming (FLP) was proposed by Zimmermann [17]. A review of the literature concerning fuzzymathematical programming as well as comparison of fuzzy numbers can be seen in Klirand Yuan [6] and also Lai and Hwang [6]. Several authors considered various types ofthe FLP problems and proposed several approaches for solving them [2, 4, 5, 7, 8, 9, 11].In particular, the most convenient methods are based on the concept ofcomparison of fuzzy numbers by use of ranking functions [1,4, 9].

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This Paper has been organized as follows section 2 deals with some basic definitions of the fuzzy concept, section 3 explains the fuzzy variable linear programming problem and the new algorithm, in section 4 three numerical examples with different cases has been solved.

2. Preliminaries

2.1 Fuzzy Set: A fuzzy set $\tilde{A}$ is defined by $\tilde{A}= \{(x, \mu_A(x)): x\in A, \mu_A(x)\in[0,1]\}$. In the pair $(x, \mu_A(x))$, the first element x belong to the classical set A, the second element $\mu_A(x)$, belong to the interval [0, 1], called *Membership function*.

2.2 Fuzzy Number: A fuzzy set $\widetilde{A}$ on R must possess at least the following three properties to qualify as a fuzzy number,

(i) $\widetilde{A}$ must be a normal fuzzy set;

(ii) ${}^{\alpha}\widetilde{A}$ must be closed interval for every $\alpha\epsilon[0,1]$

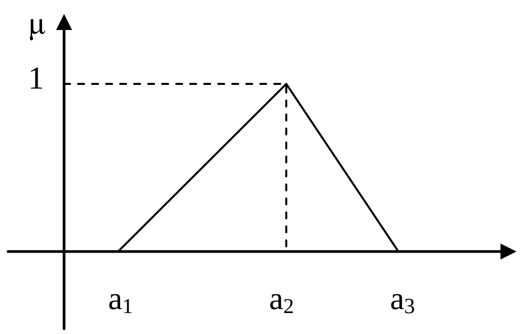
(iii) the support of $\widetilde{A}$, ${}^{o+}\widetilde{A}$, must be bounded.

2.3 Triangular Fuzzy Number:

It is a fuzzy number represented with three points as follows: $\widetilde{A} = (a_1, a_2, a_3)$

This representation is interpreted as membership functions

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0 & for x < a_1 \\ \frac{x-a_1}{a_2-a_1} & for a_1 \le x \le a_2 \\ \frac{a_3-x}{a_3-a_2} & for a_2 \le x \le a_3 \\ 0 & for x > a_3 \end{cases}$$



Triangular fuzzy number $\widetilde{A} = (a_1, a_2, a_3)$

2.4 Operation of Triangular Fuzzy Number Using Function Principle:

The following are the four operations that can be performed on triangular fuzzy numbers:

(i) **Addition**: Let $\widetilde{A} = (a_1, a_2, a_3)$ and $\widetilde{B}= (b_1, b_2, b_3)$ then, $\widetilde{A} + \widetilde{B} = (a_1+b_1, a_2+b_2, a_3+b_3)$.

(ii) **Subtraction**: Let $\widetilde{A} = (a_1, a_2, a_3)$ and $\widetilde{B} = (b_1, b_2, b_3)$ then, $\widetilde{A} - \widetilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$.

(iii) **Multiplication**: Let $\widetilde{A} = (a_1, a_2, a_3)$ and $\widetilde{B} = (b_1, b_2, b_3)$ then,

$\widetilde{A} \bullet \widetilde{B} = (\min(a_1b_1, a_1b_3, a_3b_1, a_3b_3), a_2b_2, \max(a_1b_1, a_1b_3, a_3b_1, a_3b_3))$.

(iv) **Scalar Multiplication:** Let $\widetilde{A} = (a_1, a_2, a_3)$ then $k(\widetilde{A}) = (ka_1, ka_2, ka_3)$ if k is positive and (ka_3, ka_2, ka_1) if k is negative.

(v) **Division**: Let $\widetilde{A} = (a_1, a_2, a_3)$ and $\widetilde{B} = (b_1, b_2, b_3)$ then,

$\widetilde{A} / \widetilde{B} = (\min(a_1/b_1, a_1/b_3, a_3/b_1, a_3/b_3), a_2/b_2, \max(a_1/b_1, a_1/b_3, a_3/b_1, a_3/b_3))$.

3. Problem and Algorithm

3.1 Fuzzy variable Linear Programming Problem:

Consider the following fuzzy variable linear programming problem:

$$\tilde{Z} = c^T \tilde{x}, \qquad \ldots(1)$$

Constraints of the form

$$a_j^T \tilde{x} = \tilde{b}_j, \; j=1,2,\ldots,m \qquad \ldots(2)$$

and the nonnegative conditions of the fuzzy variables $\tilde{x} \geq (0,0,0)$ are also included in the form $-\tilde{x} \leq (0,0,0)$. where $c^T = (c_1,\ldots,c_n)$ is an n-dimensional constant vector, $\tilde{x} = (\tilde{x}_i)$, i=1,2,..n and $\tilde{b}_j$ are non-negative fuzzy variable vectors such that $\tilde{x}_i$ and $\tilde{b}_j \in F(R)$ for all $1 \leq i \leq n$, $1 \leq j \leq m$, is called a fuzzy variable linear programming (FVLP) problem.

Therefore, m>n holds always in this formulation. It is well known that a feasible region of constraints is always a convex set and an optimal solution exists on an extreme point which is consisted of some hyper planes presented by $a_l^T \tilde{x} \leq \tilde{b}_l, l = l_1, l_2, \ldots, l_n$. Then the problem, in other word, is how to find n constraint conditions which constitute the optimal solution in equality. If there is no redundant constraint condition which does not contribute to make a form of a feasible region, there is a hyper plane, $a_k^T \tilde{x} \leq \tilde{b}_k$ on which the optimal solution must be located. If the index $p_j = \frac{c^T a_j}{\|a_j\|}$, (where a_j is coefficient vectors of the equation a_j) is introduced and renumbering p_j's in their order as

$$p_m \leq \ldots \leq p_j \leq \ldots \leq p_2 \leq p_1 \qquad \ldots.(3)$$

Then, it is clear that the first n conditions, p_1, p_2, ..., p_n, in their equality form lead to some solution $\tilde{x}(1)$, which might be optimal, not optimal but feasible, or infeasible solution. Here, the symbol p_j expresses the index value itself or a constrained condition $a_j^T\tilde{x}=\tilde{b}_j$. Substituting $\tilde{x}(1)$ into the constraints (2), these are classified into three types such that

$$a_i^T\tilde{x}(1)=\tilde{b}_i, \quad i=i_1,i_2,\ldots,i_p \quad (2\text{-}1)$$

$$a_j^T\tilde{x}(1)<\tilde{b}_j, \quad j=j_1,j_2,\ldots,j_q \quad (2\text{-}2)$$

$$a_h^T\tilde{x}(1)>\tilde{b}_h, \quad h=h_1,h_2,\ldots,h_r \quad (2\text{-}3)$$

Here, $n \le p$ is always satisfied, since the solution $\tilde{x}(1)$ isobtained by n equality conditions, and$p + q + r = m$. Thenthe problem is how to choose n equations to be solved next.Condition (2-3) is preferred to another, so the first nconditions are selected from r constraints of (2-3). If $n > r$,another $n - r$ equations are selected from (2-1). The indicesare arranged in numeric order as in (3). There are two rules,ascending or descending order, for the selection of nequations.

[Selection Rule 1] The n equations to be solved are, first, selected from the unsatisfying constraints (2-3) in ascending (descending) order. Then, if $n > r$, another $n - r$ equations are selected from (2-1) in descending (ascending) order.

[Selection Rule 2] If the selected n equations have no unique solution, then the last selected equation is changed with the next candidate according to the order of [Selection rule 1].

3.2 Algorithm:

Step1: In Fuzzy Variable Linear Programming Problem, 'n' is the number of fuzzy variables and 'm' is the number of constraints. For this algorithm 'm' must be strictly greater than 'n' (m>n) always.

Step 2: Find all the p_j's(j=1,2…,m) and renumbered as $p_m \le \ldots \le p_j \le \ldots \le p_2 \le p_1$. Where $p_j = \frac{c^T a_j}{\|a_j\|}$.

Step 3: Select the first n constraints and solve the n constraints in their equality form, The selected first set of inequality be denoted by s(k)={1,2,…n}.

Step 4: If n equations have no unique solution, then replace the equations according to the selection rule 2. k=k+1 and set s(k)={Selected Numbers}.

Step 5: Check whether the solution of the equations are feasible or infeasible.

Step 6: If the solution is infeasible, find the new set of n equations based on selection rule 1, k=k+1 and set s(k)= {Selected Numbers}.

Step 7: If s(k)= s(j) for some j ($1 \le j \le k$), then return to step 6 select the new set and goto step 3.

Step 8: If the solution is feasible, then check the Optimality Criterion.

Step 9: If the Optimality test fails, find the adjacent extreme point which improves optimality and return to 8.

3.3 Optimality Criterion:

If $\tilde{x}^*$ is feasible solution such that $a_j^T\tilde{x}^* = \tilde{b}_j$, j=1,2,…,n and $a_j^T\tilde{x}^* < \tilde{b}_j$, j=n+1,n+2,…, m, the optimality of $\tilde{x}^*$ is determined from the solution of $A^Tw=c$, where A is n×n matrix formed by the coefficients of the equations which gives the feasible solution, $w^T=(w_1,w_2,\dots,w_n)$.

(a) If all $w_j>0$, j=1,2,…,n, then $\tilde{x}^*$is unique optimal solution.

(b) If $w_j \geq 0$, j=1,2,…,n and $w_j=0$ at least for some i, $1\leq i\leq n$, then the $\tilde{x}^*$is optimal but not unique solution.

(c) If the solution w includes a negative elements $w_j<0$ the$\tilde{x}^*$ in not an optimal.

Proof: Since $\tilde{x}^*$ is an extreme point as noted above, $a_j^T\tilde{x}^*=\tilde{b}_j$, j=1,2,…,n holds. Now consider the vector $\tilde{x}_N = \tilde{x}^* + \tilde{\beta}\tilde{d}$ which satisfies the equality constraints described above except $a_i^T\tilde{x}^* = \tilde{b}_i$ for a fixed i,$1\leq i\leq n$.

Then, $a_j^T\tilde{x}_N = a_j^T\tilde{x}^* + \tilde{\beta}a_j^T\tilde{d} = \tilde{b}_j$.

In order for $\tilde{x}_N$ to satisfy the constraint $a_i^T\tilde{x} < \tilde{b}_j$, it is necessary that $\tilde{\beta}a_j^T\tilde{d} < 0$.

This means that $\tilde{\beta}>0$ if $a_i^T\tilde{d}< 0$ and $\tilde{\beta}<0$ if $a_i^T\tilde{d}> 0$ are required. On the other hand, since the value of the objective function is $c^T\tilde{x}_N = c^T\tilde{x}^* + \tilde{\beta}c^T\tilde{d}$, it follows that $c^T\tilde{x}_N = c^T\tilde{x}^* = \tilde{\beta}c^T\tilde{d} = w_i\tilde{\beta}a_i^T\tilde{d}$.

Here, the relation $c^T\tilde{d} = (w_1a_1 + w_2a_2 + \cdots + w_na_n)^T\tilde{d} = w_ia_i^T\tilde{d}$ is used.

From $\tilde{\beta}a_j^T\tilde{d} < 0$ and $c^T\tilde{d} = (w_1a_1 + w_2a_2 + \cdots + w_na_n)^T\tilde{d} = w_ia_i^T\tilde{d}$, the optimality criterion is derived as in (a), (b) and (c), since i is any number between 1 and n. The above discussion holds for small $\tilde{\beta}$ in absolute value since the extreme point $\tilde{x}^*$ is an interior point for other constraints except the n constraints from which the extreme point was derived. The value of $\tilde{\beta}$is restricted to:

$\tilde{\beta}=\min\{\tilde{\beta}_j>0: a^T(\tilde{x}^* + \tilde{\beta}\tilde{d})=\tilde{b}_j$, j=n+1,n+2,…,m} if $a_j^T\tilde{d}< 0$, and

$\tilde{\beta}=\max\{\tilde{\beta}_j <0: a^T(\tilde{x}^* + \tilde{\beta}\tilde{d})=\tilde{b}_j$, j=n+1,n+2,…,m} if $a_j^T\tilde{d}> 0$.

Using the result, $\tilde{x}_N = \tilde{x}^* + \tilde{\beta}\tilde{d}$ gives an adjacent extreme point.

4. Numerical Example:

4.1 Example:[In this case the optimal solution is not unique **]**

Maximize $\tilde{Z} = \tilde{x}_1 + \tilde{x}_2 + \tilde{x}_3$

Subject to constraints

$18\tilde{x}_1 + 7\tilde{x}_2 + 6\tilde{x}_3 \le (175,180,185)$,

$-2\tilde{x}_1 + 9\tilde{x}_2 + 10\tilde{x}_3 \le (103,108,113)$,

$-\tilde{x}_1 + 0\tilde{x}_2 + \tilde{x}_3 \le (1,6,11)$,

$-\tilde{x}_1 + 7\tilde{x}_2 + 5\tilde{x}_3 \le (79,84,89)$,

$\tilde{x}_1 \ge (0,0,0)$, $\tilde{x}_2 \ge (0,0,0)$, $\tilde{x}_3 \ge (0,0,0)$.

Solving the problem using the new Algorithm

1) All the p_j's has been found and the constraints are arranged as in (3).

$a_1^T\tilde{x} = 18\tilde{x}_1 + 7\tilde{x}_2 + 6\tilde{x}_3 \le (175,180,185)$,	$p_1=1.532$
$a_2^T\tilde{x} = -\tilde{x}_1 + 7\tilde{x}_2 + 5\tilde{x}_3 \le (79,84,89)$,	$p_2=1.270$
$a_3^T\tilde{x} = -2\tilde{x}_1 + 9\tilde{x}_2 + 10\tilde{x}_3 \le (103,108,113)$,	$p_3=1.249$
$a_4^T\tilde{x} = -\tilde{x}_1 + 0\tilde{x}_2 + \tilde{x}_3 \le (1,6,11)$,	$p_4=0.0$
$a_5^T\tilde{x} = -\tilde{x}_1 + 0\tilde{x}_2 + 0\tilde{x}_3 \le (0,0,0)$,	$p_5=-1.0$
$a_6^T\tilde{x} = 0\tilde{x}_1 - \tilde{x}_2 + 0\tilde{x}_3 \le (0,0,0)$,	$p_6=-1.0$
$a_7^T\tilde{x} = 0\tilde{x}_1 + 0\tilde{x}_2 + \tilde{x}_3 \le (0,0,0)$,	$p_7=-1.0$

2) Solving p_1, p_2 and p_3 in their equality form gives the solution $\tilde{x}^T(1)$= ((2.02,5,7.98), (9,12,15), (−12.27,1,14.27)).

3) The obtained solution is feasible by substituting $\tilde{x}^T(1)$ into all the constraints.

4) Solving $A^Tw=c$ gives the solution of $w_1=0.0625 >0$, $w_2= 0$, and $w_3= 0.0625 >0$.Therefore, the solution $\tilde{s}(1)$ is optimal but not unique.

5) Since $w_2=0$, equation p_2 is going for elimination and a new equation have to enter, to find the new equation a new direction $\tilde{d}^T$=((1,1,1),−(12,12,12),(11,11,11)) is determined. This is perpendicular to the equation p_1 and p_3. The first element of $\tilde{d}$ is set to (1,1,1) in advance.

6) The solution $\tilde{x}(2) = \tilde{x}(1)+\tilde{\beta}_j\tilde{d}$, where $\tilde{\beta}_j$ is obtained from $a_j^T\tilde{x}_j = a_j^T\tilde{x}(1)+\tilde{\beta}a_j^T\tilde{d} = \tilde{b}_j$, $\tilde{\beta}>0$ since $\tilde{d}^T\times$(coeff. vector of equation p_2)=−(30, 30,30) <0 and put $\tilde{\beta}_{j^*}$=min $\{\tilde{\beta}_j>0\}$.Then $\tilde{x}(2) = \tilde{x}(1)+\tilde{\beta}_j\tilde{d}$ gives the nearest extreme point which does not change the value of the objective function.

   In this example j*=4 and $\tilde{x}^T(2)$ =((0.895,6,11.105),(−28.5,0,28.5),(−24.65,12,48.65)) which is the intersection of p_1,p_3 and p_4.

7) Again check the optimality for $\tilde{x}(2)$ by $A^Tw=c$ which gives the solution $w_1=0.0625 >0$, $w_4= 0$, and $w_3= 0.0625 >0$. Since $w_4=0$, it seems as before that new direction has to be searched. But,

it is clear that $\tilde{x}(3) = \tilde{x}(1)$ is resulted (p_4 has been replace by p_2). Finally, the optimal solution is any point on the segment between $\tilde{x}(1)$ and $\tilde{x}(2)$.

4.2 Example: [This is the case with many redundant constraint conditions and circulation **]**

Maximize $\tilde{Z} = \tilde{x}_1 + \tilde{x}_2 + \tilde{x}_3$

Subject to constraints

$12\tilde{x}_1 + \tilde{x}_2 + \tilde{x}_3 \leq (308,312,316)$,

$-8\tilde{x}_1 + 7\tilde{x}_2 + 10\tilde{x}_3 \leq (195,200,205)$,

$-4\tilde{x}_1 + 3\tilde{x}_2 + 6\tilde{x}_3 \leq (895,900,905)$,

$9\tilde{x}_1 + 8\tilde{x}_2 + 2\tilde{x}_3 \leq (115,120,125)$,

$2\tilde{x}_1 + 3\tilde{x}_2 + 5\tilde{x}_3 \leq (65,70,75)$,

$4\tilde{x}_1 + 6\tilde{x}_2 - 3\tilde{x}_3 \leq (45,50,55)$,

$-11\tilde{x}_1 + 14\tilde{x}_2 + 15\tilde{x}_3 \leq (1150,1155,1160)$,

$-\tilde{x}_1 + 3\tilde{x}_2 + 8\tilde{x}_3 \leq (65,70,75)$,

$5\tilde{x}_1 - 2\tilde{x}_2 + 7\tilde{x}_3 \leq (70,75,80)$,

$\tilde{x}_1 \geq (0,0,0)$, $\tilde{x}_2 \geq (0,0,0)$, $\tilde{x}_3 \geq (0,0,0)$.

Solving the problem using the new Algorithm

1) All the p_j's has been found and the constraints are arranged as in (3).

$a_1^T\tilde{x} = 12\tilde{x}_1 + \tilde{x}_2 + \tilde{x}_3 \leq (308,312,316)$, $p_1 = 1.730$

$a_2^T\tilde{x} = -11\tilde{x}_1 + 14\tilde{x}_2 + 15\tilde{x}_3 \leq (1150,1155,1160)$, $p_2 = 1.718$

$a_3^T\tilde{x} = -8\tilde{x}_1 + 7\tilde{x}_2 + 10\tilde{x}_3 \leq (195,200,205)$, $p_3 = 1.712$

$a_4^T\tilde{x} = -4\tilde{x}_1 + 3\tilde{x}_2 + 6\tilde{x}_3 \leq (895,900,905)$, $p_4 = 1.664$

$a_5^T\tilde{x} = 2\tilde{x}_1 + 3\tilde{x}_2 + 5\tilde{x}_3 \leq (65,70,75)$, $p_5 = 1.622$

$a_6^T\tilde{x} = 9\tilde{x}_1 + 8\tilde{x}_2 + 2\tilde{x}_3 \leq (115,120,125)$, $p_6 = 1.556$

$a_7^T\tilde{x} = -\tilde{x}_1 + 3\tilde{x}_2 + 8\tilde{x}_3 \leq (65,70,75)$, $p_7 = 1.162$

$a_8^T\tilde{x} = 5\tilde{x}_1 - 2\tilde{x}_2 + 7\tilde{x}_3 \leq (70,75,80)$, $p_8 = 1.132$

$a_9^T\tilde{x} = 4\tilde{x}_1 + 6\tilde{x}_2 - 3\tilde{x}_3 \leq (45,50,55)$, $p_9 = 0.896$

$a_{10}^T\tilde{x} = -\tilde{x}_1 + 0\tilde{x}_2 + 0\tilde{x}_3 \leq (0,0,0)$, $p_{10} = -1.0$

$a_{11}^T\tilde{x} = 0\tilde{x}_1 - \tilde{x}_2 + 0\tilde{x}_3 \leq (0,0,0)$, $p_{11} = -1.0$

$a_{12}^T\tilde{x} = 0\tilde{x}_1 + 0\tilde{x}_2 - \tilde{x}_3 \leq (0,0,0)$, $p_{12} = -1.0$

2) Solving p_1, p_2 and p_3 in their equality form gives the solution $\tilde{x}^T(1)$=(−(30.02,30.43,30.84), (248.59,252.98,257.36), −(178.07,181.43,184.79)).
3) Substituting the result in all constraints, if follows that i=1,2,3; j=4,5,7,8,11; h=6,9,10,12 using the notation ((2-1),(2-2),(2-3))
4) Now using the selection rule 1 selectp_6,p_9 and p_{10} for next trail and the solution, we obtain is $\tilde{x}^T(2)$= ((0,0,0),(12.08,12.78,13.47), (3.62,8.92,14.18)).
5) Substituting the result in all constraints in this case i=6,9,10; j=1,2,3,4,8,11,12; h=5,7
   Using rule 1 select p_5,p_7 and p_{10} for next trail
6) Solving p_5,p_7 and p_{10} we obtain $\tilde{x}^T(3)$=((0,0,0), (16.12,23.33,30.55), (-3.33,0,3.33)) in this case i=5,7,10; j=1,2,3,4,8; h=6,9.
7) Using Rule 1 the next trials are p_6, $p_9$and p_{10} which is similar to $\tilde{x}^T(2)$ so using rule 2 select p_6,$p_9$and p_7 and solve the equations.
8) $\tilde{x}^T(4)$= ((−0.65,4.48,9.61),(2.74,8.42,14.10),(2.76,6.15,9.55))$\tilde{x}^T(4)$ is feasible solution
9) Solving $A^Tw=c$ gives the solution of $w_6=0.1451 > 0$, $w_9 = -0.0599 < 0$, $w_7=0.066 > 0$. Therefore, the solution $\tilde{x}(4)$ is not optimal.
10) Since $w_9<0$, equation p_9 is going for elimination and a new equation have to enter, to find the new equation a new direction $\tilde{d}^{\,T}$=((56,58,60), − (71.45,74,76.55), (33.79,35,36.21))is determined. This is perpendicular to the equations p_6 and p_7. The first element of $\tilde{d}^T$ is set to (56,58,60) in advance.
11) The solution $\tilde{x}(5) = \tilde{x}(4)+\tilde{\beta}_j d$, where $\tilde{\beta}_j$ is obtained from $a_j^T\tilde{x}_j = a_j^T\tilde{x}(4)+\tilde{\beta}a_j^T\tilde{d} = \tilde{b}_j$, $\tilde{\beta}>0$ since $\tilde{d}^T\times$(coef vector of equation p_9)=−(290.07,317,343.93) <0 and put $\tilde{\beta}_{j^*}=\min\{\tilde{\beta}_j>0\}$. Then $\tilde{x}(5) = \tilde{x}(4)+\tilde{\beta}_j\tilde{d}$ gives the nearest extreme point which does not change the value of the objective function.
   In this example j*=8 and $\tilde{s}^T(5)$ =((−2.15,6.74,15.61),(−4.92,5.53,16.01),(1.85,7.52,10.51)) which is the intersection of p_6,$p_8$and p_7.
12) Again check the optimality for $\tilde{x}(5)$ by $A^Tw=c$ which gives the solution $w_6=0.1039>0$,w_8= 0.0278 >0, and w_9= 0.0746>0. Since all w's are greater than zero so the current solution is Optimal Solution

4.3 Example: [This is the case where selected coefficient vectors are linear dependent and step 4 in algorithm is used]

Maximize $\tilde{Z} = \tilde{x}_1 + \tilde{x}_2 + \tilde{x}_3$

Subject to constraints

$-18\tilde{x}_1 + 12\tilde{x}_2 + 15\tilde{z} \leq (40,50,60)$,

$-20\tilde{x}_1 + 14\tilde{x}_2 + 15\tilde{z} \leq (30,40,50)$,

$5\tilde{x}_1 - 3\tilde{x}_2 - \tilde{z} \leq (10,20,30)$,

$-37\tilde{x}_1 + 25\tilde{x}_2 + 30\tilde{z} \leq (90,100,110)$,

$\tilde{x}_1 \geq (0,0,0)$, $\tilde{x}_2 \geq (0,0,0)$, $\tilde{x}_3 \geq (0,0,0)$.

Solving the problem using the new Algorithm

1) All the p_j's has been found and the constraints are arranged as in (3).

$a_1^T\tilde{x}$=$-18\tilde{x}_1 + 12\tilde{x}_2 + 15\tilde{z} \leq (40,50,60)$, $p_1 = 0.342$

$a_2^T\tilde{x}$= $-37\tilde{x}_1 + 25\tilde{x}_2 + 30\tilde{z} \leq (90,100,110)$, p_2=0.335

$a_3^T\tilde{x}$= $-20\tilde{x}_1 + 14\tilde{x}_2 + 15\tilde{z} \leq (30,40,50)$, $p_3 = 0.314$

$a_4^T\tilde{x}$= $5\tilde{x}_1 - 3\tilde{x}_2 - \tilde{z} \leq (10,20,30)$, $p_4 = 0.169$

$a_5^T\tilde{x}$= $-\tilde{x}_1 + 0\tilde{x}_2 + 0\tilde{z} \leq (0,0,0)$, $p_5 = -1.0$

$a_6^T\tilde{x}$= $0\tilde{x}_1 - \tilde{x}_2 + 0\tilde{z} \leq (0,0,0)$, $p_6 = -1.0$

$a_7^T\tilde{x}$= $0\tilde{x}_1 + 0\tilde{x}_2 - \tilde{z} \leq (0,0,0)$, $p_7 = -1.0$

2) The first three constraintsp_1, p_2 & p_3 in their equality form do not have a unique solution.

3) Now using the rule 2 we select the another set of constraints p_1,p_2&p_4 and the solution in the equality form is

   $\tilde{x}^T$(1)=((-3.33,14.583,62.5),(-73.025,14.583,102.197),(-503.24,9.166,521.57)).

4) Substituting the result in all constraints, if follows that i=1,2,4; j=5,6,7; h=3 using the notation ((2-1),(2-2),(2-3))

5) Now using the selection rule 1 select p_3, p_4 and p_2 for next trail and the solution we obtain is $\tilde{x}^T$(2)= ((-14.44,5.56,25.56),(-31.11,-1.11,28.89),(-188.87,11.13,188.87)).

6) Substituting the result in all constraints in this case i=2,3,4; j=5,7; h=1,6

   Using rule 1 select p_1, p_6 and p_4 for next trail.

7) Solving p_1,p_6 and p_4 we obtain $\tilde{x}^T$(3)=((3.33,6.14,8.95),(0,0,0),(-13.35,10.7,34.75)) which is feasible solution

8) Solving A^Tw=c gives the solution of w_1=0.105>0, w_6=-1.474<0, w_4= 0.579>0. Therefore, the solution $\tilde{x}$(3) is not optimal.

9) Since $w_6<0$, equation p_6 is going for elimination and a new equation have to enter, to find the new equation a new direction $\tilde{d}^T=((10,10,10),\ (17.27,17.27,17.27),\ -(1.82,1.82,1.82))$is determined. This is perpendicular to the equationsp_1 and p_4. The first element of $\tilde{d}^T$ is set to (10,10,10) in advance.

10) The solution $\tilde{x}(4) = \tilde{x}(3)+\tilde{\beta}_j\tilde{d}$, where $\tilde{\beta}_j$ is obtained from $a_j^T\tilde{x}_j = a_j^T\tilde{x}(3)+\tilde{\beta}a_j^T\tilde{d} = \tilde{b}_j$, $\tilde{\beta}>0$ since $\tilde{d}^T\times$(coef vector of equation p_6)$=-(17.27,17.27,17.27)<0$ and put $\tilde{\beta}_{j^*}=\min\{\tilde{\beta}_j>0\}$. Then $\tilde{x}(4) = \tilde{x}(3)+\tilde{\beta}_j\tilde{d}$ gives the nearest extreme point which does not change the value of the objective function. In this example $j^*=3$ and $\tilde{x}^T(4) =((-289.97,7.74,305.34),$ $(-506.53,2.76,511.88),\ (-67.29,10.41,88.13))$ which is the intersection of d_1,$d_4$and d_3

11) Solving $A^Tw=c$ gives the solution of $w_1=-1.58<0, w_4=1.5>0, w_3= 1.75>0$.Therefore, the solution $\tilde{x}(4)$ is not optimal.

12) Since $w_1<0$, equation p_1 is going for elimination and a new equation have to enter, to find the new equation a new direction $\tilde{d}^T=((15,15,15),\ (26.61,26.61,26.61),\ -(4.84,4.84,4.84))$is determined. This is perpendicular to the equationsp_4 and p_3. The first element of $\tilde{d}^T$ is set to (15,15,15) in advance.

13) The solution $\tilde{x}(5) = \tilde{x}(4)+\tilde{\beta}_j d$, where $\tilde{\beta}_j$ is obtained from $a_j^T\tilde{x}_j = a_j^T\tilde{x}(3)+\tilde{\beta}a_j^T\tilde{d} = \tilde{b}_j$, $\tilde{\beta}>0$ since $\tilde{d}^T\times$(coef vector of equation p_1)$=-(23.28,23.28,23.28)<0$ and put $\tilde{\beta}_{j^*}=\min\{\tilde{\beta}_j>0\}$. Then $\tilde{x}(5) = \tilde{x}(4)+\tilde{\beta}_j\tilde{d}$ gives the nearest extreme point which does not change the value of the objective function. In this example $j^*=7$ and $\tilde{x}^T(5) =((-498.47,39.99,578.49),\ (-877.41,59.97,996.45),$ $(-155.43,0,155.43))$ which is the intersection of p_7,$p_3$and p_4.

14) Again check the optimality for $\tilde{x}(5)$ by $A^Tw=c$ which gives the solution $w_7=14.4>0$, $w_4= 3.4>0$, and $w_3= 0.8>0$. Since all w's are greater than zero so the current solution is Optimal Solution.

5. Conclusion:

A novel method is proposed to find the optimal solution of a fuzzy variable linear programming problem. Here in this method we find the feasible solution of the problem and then we find the optimal solution from the feasible solution. This method is very simple and minimum time only required to obtain the optimal solution.

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