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STATE OF ART OF DIFFERENT KINDS OF FLUID FLOW INTERACTIONS WITH PIEZO FOR ENERGY HARVESTING CONSIDERING EXPERIMENTAL, SIMULATIONS AND MATHEMATICAL MODELING

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Abstract. In this work, the different kinds of fluid flow interactions with piezo smart materials have been discussed for energy harvesting. The present work has been classified into the following categories: (i) experimental investigations (ii) simulation and (iii) mathematical modelling. In section (i) different experimental set-ups such as harvesting of energy with the help of vortex flow, turbulent flow, cross flow, flow in an open channel and closed channel and flow through nozzles have been examined. In section (ii) simulations studies performed with different tools/software like ANSYS/fluent, COSMOL etc. have been detailed. Lastly, in section (iii) different mathematical equations such as Navier-Stokes equation of motion, Continuity equation, finite element method, numerical methods, transport equations, Bernoulli equation, equation of linear elasticity, fluid structural equations, piezoelectric equations and coupled-wave equations are described for generation of energy with fluid's interaction. The present work has to fulfil two aims: (i) active engineers can choose the best appropriate methodology for their work in the same field (ii) researchers can know about how the area of energy harvesting has been grown in various decades, what are the practical application of this field with real life and the literature gaps of the field.

Keywords: experimental setup; simulation; mathematical modelling; energy harvesting; fluid flow; piezoelectric.

2010 AMS Subject Classification: 93A30, 91A90, 81T80, 74F10, 35Q35, 74M05.

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1. INTRODUCTION

Energy generation is a phenomenon of employing the existing unexploited energy from the atmosphere, converting it suitably and utilize it for upcoming purposes.The obtained energy can be reserved as electrical energy which can be used to operate micro-powered devices in real time or can be stored in the battery for later use. Nowadays, two forms of the battery have been utilized for low-power electronics, specifically primary and secondary. A primary battery is a single-use or ”throwaway” battery that is not reusable in the environment, whereas the secondary battery can be recharged yet again. While these storage devices are harmless and ecologically friendly, their bounded phase cycle, deceptive utility and inadequate life span reduce their utility. So, investigators are discovering a viable solution to generate and utilized energy in real-time as an automatic resource in handy devices, where the power is regularly harvested and utilized. It is vital to offer a different solution to the current requirement of micro-power so that individuals can create their self-supply of power. Some of the micro-scale energy harvesting system discussed by different researchers is an electromagnetic energy generating system, electrostatic energy generating system, thermo-electric energy generating system, solar energy generating system, piezo-electric energy generating system.

Piezoelectric materials are being widely utilized nowadays to power micro devices in real-time because they are lightweight, low cost, flexibility and easily embedded. Piezo-electric materials are efficient in producing electric power when dynamic force is applied to them. The dynamic force can be applied to piezoelectric materials by various means like while walking it can be placed in show sole to run GPS (Global positioning system), charging mobile battery etc.; it can be embedded with automobile tyres to harvest energy; it can be embedded in the roads; aeroplane wings for active vibration control; in boat propellers; in fluid low etc.

Researchers have been showing a great area of interest in fluidic interacted piezo energy harvesting through the arrangement of experimental set-ups, simulations and mathematical modelling. Yadav D et al. designed an open channel for the flowing of the fluid and developed an applied mathematical model [1]. A model of compressible flowing fluid with turbulent characteristics and in this model, the equations for Reynolds Averaged Navier Stokes and piezo-electrics are used by [2]. They also mentioned the mechanical behaviour using a solid mechanical linear basic strain tensor, stress tensor equations.

In the present work, experimental set-ups, simulation studies, and mathematical modelling of fluid flow with piezo material for energy harvesting have been critically reviewed. The energy harvesting using different piezoelectric materials like piezo-polymers/piezo ceramics connected to different circuits like voltage doubler, full-bridge rectifier circuit etc in different systems of fluid flow like vortex flow, turbulent flow, open channel, closed channel have been investigated. The simulations tools like COSMOL, fluent, MATLAB etc and mathematical modelling of fluid interaction with piezo materials have also been discussed to examine the state of the art of recent tools/techniques utilized for energy harvesting with piezoelectric materials.

2. EXPERIMENTAL SET-UPS OF FLUID FLOW WITH PIEZO

The researchers have established various arrangements of set-ups for the generation of electrical energy through smart piezo materials with fluid flow. Fluidic piezo energy harvesting through different types of arrangements such as open channels, closed channel vortex flow, spiral tube type valveless pump etc. have been critically reviewed in this section, as shown in Figure 1.

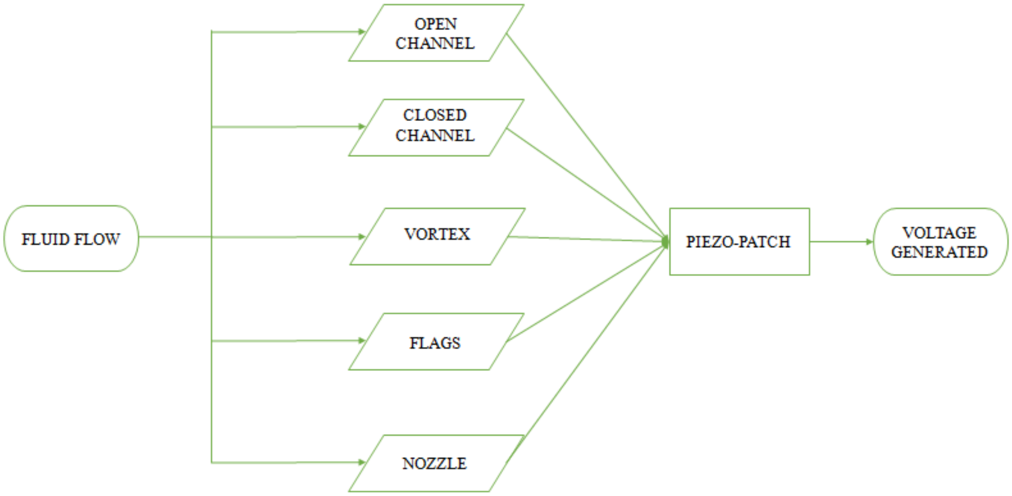

FIGURE 1. Block Diagram of Fluidic Energy Harvesting with Different Experimental Set-ups

2.1 Open Channel Based Interaction

Fluid energy harvesting using the open channel is a rapidly growing area of research, especially for environmentally friendly and renewable resources, as shown in Figure 2. One of the most important model is constructed by placing a PVDF piezo-film in an open channel system having water flow at different rates [3, 4, 5]. The PEHF (piezoelectric energy harvesting with fluid) model is composed of a water tank (66x66x30cm), an open channel (117x23x23cm) with the proper gating system. Two motor pumps to circulate the water, PVDF film hung inside the channel, an electric circuit (four diodes of IN4007 MIC, 1 capacitor of 63v 1 eF), a battery to store the harvested energy. The piezo film connections are properly insulated with the rubber tape to increase their durability and also to make the circuit waterproof. The piezo patch is connected to a rectifier circuit to convert AC (alternating current) into DC (direct current). The layout of the open channel system for PEHF is shown in Figure 2. As the water discharge rate increases, the current and voltages also increase at all stages of experiments. The experimental model produced a $1.03\mu W$ power with a voltage of 0.83V and current $1.24\mu ampere$, when the fluid strikes at a rate of 55.24 litre/min on the patch. The output of the system is measured with the help of a multi-meter with both voltage doubler circuit and full bridge rectifier circuit. This energy harvesting model can be used where the water supply is continuous, ranging from low flow discharge as in household kitchen drains, brook, etc. to high flow water discharge sources such as rivers, lakes, bridges, waterfall, industrial waste disposal, etc.

2.2 Closed Channel Based Interaction

Interaction of fluid-structure through the closed channel has arisen in last few decades since it is easy to cope with other systems, as shown in Figure 3. The investigation has been carried out to generate energy at a micro-scale in different kinds of water flows on piezo-smart materials[6]. In close channel system, applied load is either impact load or continuous fluid force (with turbulent flow). A model for a fluid flowing in a close channel is proposed using different rectifier circuits and piezo patches as shown in Figure 3. The designed model consist a single ceramic piezo patch and produced a maximum voltage with voltage doubler circuit is 5.5V at $33m^3/min$ flow rate, while for the full wave rectifier circuit it is 2.86V [7]. Same model for two patches, the maximum voltage for full wave rectifier circuit and doubler circuits is 2.3V

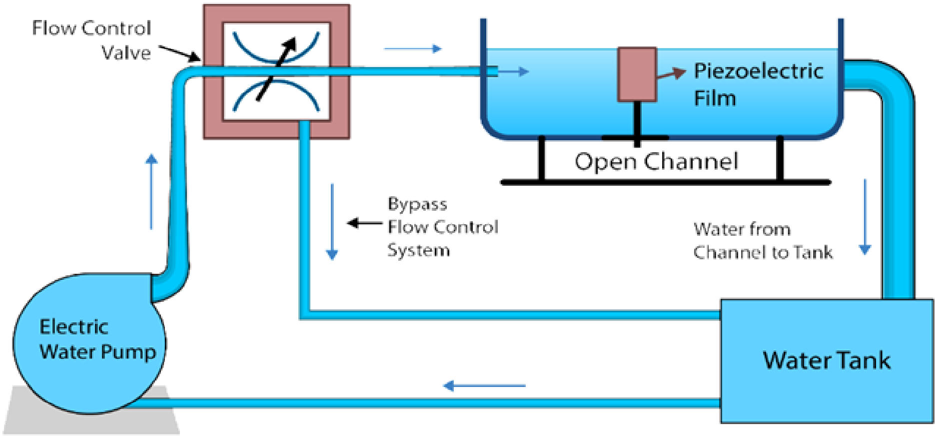

FIGURE 2. Geometrical Layout of Working Model with Open Channel- Based Energy Harvesting System

and 1.76V respectively at a flow rate of $31m^3/min$.Yadav has done further research by developing an experimental set up using water pump arrangement with a fluid flowing measurement device at the inlet side, and the piezo-patch at the outlet [8]. It has been noticed that when the fluid strike with a speed of 20.25 litre/min on a single patch and the most eminent voltage of 3.34 V is produced while double piezo-patch is produced is 2.42 V for an optimal flow speed of 31.26 litre/min.

Further, Akaydin carried out the experimental investigations by placing the PVDF piezoelectric beam in the wind tunnel [9, 10]. They analysed the output in the wake of the cylinder and turbulent boundary layer. The power generated from the circular cylinder is $4\mu W$ when the flow speed is 7.23 m/s and in the boundary layer flow power is $0.06\mu W$ over a different load resistance of $100k\Omega$ and $10M\Omega$ respectively.

2.3 Vortex Based Interaction

Interaction of fluid structure with vortices is a powerful tool for generation of energy at micro level. Goushcha has done a lot of research in this field by subjecting vortices in the form of the wake of bluff bodies to generate electric power with a piezo-electric beam [11]. A power of 5.5

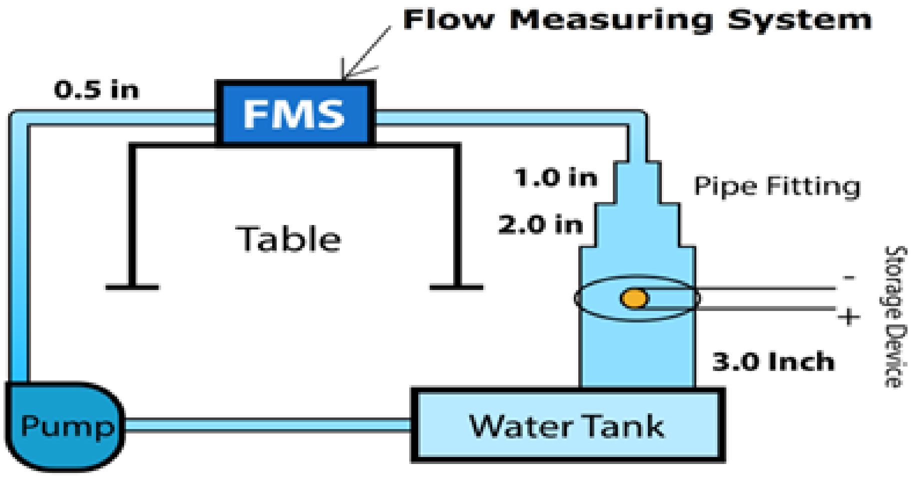

FIGURE 3. Geometrical Layout of Working Model with Closed Channel-Based Energy Harvesting System

W has been generated, and the pressure force's variation with the deflection of the beam has been keenly observed. One of the important techniques of voltage generation is arising from vibrations by the interaction of flowing fluid. Wang developed a model for energy harvesting from flow-induced vibration [12]. They pumped up the tap water in the circular pressure chamber which has PVDF patch arrangement. It is observed with National Instruments data acquisition system that 2.2 V and $0.2\mu W$ be generated when 1.19 kPa is applied under 26 Hz frequency from the proposed set-up.

2.4 Flags Based Interaction

Generation of energy using various placements of piezo-electric flags is a keen area of research. In this aspect scientist Michelin & Doare worked on the energy harvesting effectiveness of the piezo-electric flags placed in axial flow by using numerical technique [13]. Further, the approach of placing piezo-electric flags in axial flow is one of the favourite topics of the generation of energy in research. Li et al. has made further progress by using the phenomena of conversion of ocean wave fluid energy into electricity by placing piezo-electric flags in fluid. A flexible piezo-electric energy generator has been placed in the cross-flow. Hence it caused the strain in structure and output power is produced. The effect of various factors like turning ratio, flow velocity, mass ratio and piezo-electric coupling on the efficiency of harvester has been investigated. According to output, it has been observed that high fluid flow can advance the efficiency and robustness of the energy generating process.

2.5 Nozzle Based Interaction

This is one of the most valuable voltage generation's system using smart materials. Different factors like number of nozzles with different arrangements, circuits, and distance of nozzles from smart materials can influence the voltage produced by the system[14]. Nozzle Based Interaction model has been developed by Rani & Chhabra with number of varying factors like the distance of patch from the nozzle, number of nozzles, angle of the nozzle with horizontal and different circuits. The voltage is generated using dynamic pressure of the fluid flowing through the nozzle and PVDF patch is used for converting this pressure energy into electrical energy [15, 16, 17]. It has been investigated that the highest voltage of 17.54 V has been generated by voltage doubler circuit at 16.92 m/s is the fluid flowing rate at 75 cm far away from the nozzle at an angle of 35°.

2.6 Other Piezoelectric Based Interaction

Some other important mechanism of energy harvesting has been achieved by various researcher. Among these mechanisms one of the classical techniques of generating voltage in which construction of pressure chamber has been done with one side of the chamber is composed of a piezo-electric beam and the smart material piezoceramics are works as a control instrument [18, 19]. Circuit generates voltage with the help of piezoceramic patches and the sensor [20] measured the voltage produced between the piezoceramics materials. A strong proportional relation has been obtained between voltage produced and strain accumulated over the piezoceramics patches. One of the experimental tasks of generating voltage is dynamic [21, 22] loading with patches. Yadav M et al. generated electric energy by using dynamic periodic loading on piezo-smart materials [23]. For this purpose, an experimental model of proposed set-up is constructed by using circular supported boundary conditions for three different loads 33 N, 42 N and 46 N with 20 numbers of strokes. The piezo patch is attached with a voltage doubler circuit and full bridge rectifier circuits. 6.10 V is the highest voltage obtained for 46 N applied load in the circular boundary condition. The energy generation in a fluidic environment by using strips and piezo- smart materials at different frequency ranges is an emerging field of investigation for various researchers in order to generate the energy-yielding capabilities of ionic polymer-metal composite (IPMCS) in the fluid environment [24]. An experiment set up has been modelled and designed with IPMCS strip immersed in water with a resonance frequency in the range of 2-50 Hz under the resistance of 10Ω ,the power obtained through base excitation frequency of polymer is of 1nW. Further piezo-electric micro pumps are acting as a boon for energy harvesting because of their advanced features. Neto [25] developed a self-exciting Poly-methyl-methacrylate valve, less piezo-electric micro pump. It has been observed that the highest value of flow rate is $4.53m^3/min$ with a frequency of 25.2 Hz and correspondingly a voltage of 150 V is generated.

3. SIMULATION OF FLUID FLOW WITH PIEZO

In the previous few decades, various techniques of energy harvesting have come into play [26]. Harvesting of energy from the fluid is the great need of the world because of its eco-friendly behaviour, easy to integrate with other systems and readily available in the environment [27]. So, researchers are swinging towards the pre determination of the interaction between fluid flows with piezo-patches in virtual environment. Using simulation software, we pre-determine the results related to the energy harvesting model using virtual environment. Simulation has been done by various modern software like ANSYS, COMSOL Multiphysics software and FLUENT [28]. The important techniques used in this software for simulation purposes are like finite element method, finite difference method and numerical methods such as the Galerkin method of time and space discretization. Many researchers have used various ways of simulation for different energy harvesting models and for various types of flow analysis.

3.1 Simulation of Piezo-Fluid Interaction with ANSYS/FLUENT

In ANSYS based simulation, a user-defined function code has been created between piezo-circuit and the fluid in ANSYS/FLUENT software. Stress strain analysis, Impact force variation, voltage generation, power production using the constructed model has been pre-analysed using simulation [30, 31]. Simulation has been done with the method of volume meshing in ANSYS software. A comparative analysis between the experiment and simulated models has been done to validate the model.

One of the important ways of generation of power by interaction of unsteady turbulent flow with a piezo-electric generator. In this simulation technique the piezo-electric beam has been located within the turbulent boundary and waked of a circular cylinder, hence the highest output power of $60\mu W$ is recorded by using FLUENT software for flowing speed of 7.12 m/s with pressure of striking fluid, while this power differs from electric power $7\mu W$ obtained from alternating current [29, 30].

Further, the approach of placing piezo-electric flags in axial flow is one of the favourite topics of the generation of energy in research [31, 32]. Li et al. has done the simulation with the method of volume meshing in ANSYS software by placement of piezo-electric energy generator in axial-directional flow resulting strain has been arisen in the structure and output voltage of 4.61 V obtained under deformation of 1.74 mm, while simulating voltage is slightly differ from voltage obtained through the experimental set up of 4.78 V.

Models are refined numerically and analytically to realize the consequence of material criteria on energy producing efficiency of more than one-layer construction [33]. The solution of an equation is done by the technique of variables separation. ANSYS software is used for numerical simulation.

3.2 Simulation of Piezo-Fluid Interaction with COMSOL

Various researchers in today's era heavily investigate different types of flows in different types of channels and fluid-structure interaction mechanism using different simulation tools. Researchers also discussed the ways to deal with the shear over the surface of the structure using COMSOL. Navier-Stokes equations and stress tensor equations are used for simulation while Green's formula and Dirichlet boundary value problem are used for numerical calculations for an incompressible fluid. Sin et al. performs the elastic eel's motion from the wake of deceiving structure in a cross-flow. Collaboration between eels and vortex shedding behind a deceive structure is investigated mathematically [34]. The motto of the research is to upgrade the eel's oscillation by pairing with the varying vortices. COMSOL software is used for simulation in the fluid-structure interaction (FSI) model. In order to generate energy by designing and simulating a valve-less diaphragm based piezo-electric micropump with the interaction of fluid [35]. All the simulation was made using COMSOL Multiphysics software. Piezo-electric device module simulates the deformation of the membrane under different voltage with different arrangements of the membrane. Both Bernoulli's equation and Newton's 2^{nd} law are also used in the simulation. The fluid-structure interaction model was simulated by taking the fluid flow rate as $8.95mm^3/s$ with a frequency of 30 Hz under 30 V voltage.

3.3 Simulation of Piezo-Fluid Interaction with Eulerian and Lagrangian Technique

Sanders et al. did further advancement, by presenting a recent important technique for simulating stiff structure motion in two-phase flow for an incompressible fluid [36]. The investigation aims to numerically define an algorithm for the simulation of stiff structure reacting with incompressible two-phase viscous flowing fluid in the occurrence of liberates surfaces. Numerical simulation has been carried out with the finite element technique and time integration method. The finite difference method is used in the Eulerian and Lagrangian domain of grids structure to calculate the velocity and pressure of the two-phase fluid flowing structure.

To maximize the amount of power, many factors such as the thickness and stiffness of the eel materials, eel length, the bluff body width, the spacing between the body and eel head are considered. Further, the interaction of the fluid with vibrating structures is also the main source for energy harvesting. Afrasiab & Movahhedy carried out the interaction of fluid flowing model with vibrating solid structure in two different micro-pumping devices with various boundary conditions by using the formulation of the Lagrangian-Eulerian approach with VMS finite element simulation technique and thus calculated the efficiency of system [37].

Further advancement in this technique is attained by Ravi & Zilian by proposing a consistent and simultaneous investigation of small-scale energy harvesting systems, which aim toward flow-induced vibrations with trustworthy sensitivity toughness and efficient study of the coupled non-linear system [38]. It includes fluidic energy harvesting at various boundary conditions, structural piezoceramics and electric circuit equations. Simulation is done with the help of the finite element method (FEM) and the Galerkin method of time and space discretization.

3.4 Other Simulation Technique

Interaction between fluid and oscillators is also played a key role in energy generating. Matsiev operated flexural mechanical resonators at a high rate of flowing liquid [39]. Interaction between the oscillator and surrounding fluid is theoretically considered. For numerical simulation, Navier-Stokes equations and continuity equations with suitable boundary restrictions on the surface of the resonator and at infinity are used for ideal fluid.

The interaction of fluid-structure with propagating wave in 3D is also investigated by various researchers [40, 41, 42] to generate a voltage at the micro-level. The finite element method is used for simulation of this particular type of energy generating process. For numerical simulation, Navier-Stokes equations and the continuity equation are used. The concept of the wave has been introduced, so four coupled wave equations for displacement and voltage are also used.

Further advancement has been done by planned a technique for a self-exciting Polymethylmethacrylate (PMMA) valve-less piezo-electric micropump. A software-based on the finite element technique [43] is considered for studying the feasibility of the system. An experimental test has been done for the construction of a less expenditure and extremely efficient micro pump. It has been observed that the rate of fluid flow and frequency during simulation are $4.8m^3/min$ and 26 Hz respectively, while from experiments, these values are $4.53m^3/min$ and 25 Hz, respectively. Both the obtained values show a good agreement among them.

To generate energy using a diffuser nozzle through a piezo-electric micropump with two dissimilar models, the contrast has been made by Jeong [44]. The obtained results are showing that the model is extremely significant for calculating the performance of the diffuser nozzle with piezo-electric micropump. The fluid flowing rate is 2.83 under the deformation of 2.05 µl/s/cm at 50Hz frequency while the flowing rate under the deformation of 3.61 µl/s/cm is 3.76 at 90Hz frequency.

A further investigation has been carried out for the generation of energy from a vertical flow by using a passive heaving foil [45, 46]. An immersed boundary method with the interaction of incompressible fluidic structure is used for doing numerical simulation.

4. MATHEMATICAL MODELING OF FLUID FLOW WITH PIEZO

Generating of energy through piezo-smart materials with the striking of fluid includes various mathematical equations for numerical calculation of electrical voltage [47, 48, 49]. Different types of fluid interaction model different governing equations of mathematics are used as per the requirement of structure.

4.1 PEHF Modelling with Analytical Equations (Navier-Stokes and Continuity Equations)

One of the most familiar techniques of energy generation is the two-phase flow which is mostly used by researchers. Sanders presented a recent technique for simulating stiff structure motion for two-phase flow in an incompressible fluid [36]. The investigation aims to define an algorithm numerically for the simulation of stiff structure, which is reacting with incompressible two-phase viscous flowing fluid in occurrence of liberates stop layers. Equation of conservation of momentum in two-phase incompressible flow is used, and Navier- Stokes equations for incompressible Newtonian viscous fluid is also applicable for numerical calculation. Equation of conservation of momentum in two-phase incompressible flow is given as

$$\rho_f(\frac{Du}{Dt}) = \nabla \cdot \sigma_f + \rho_f \tag{1}$$

where ρ_f is the fluid's density,u is the fluid's velocity and σ_f is the fluid's stress.

In the case of the incompressible Newtonian viscous fluid equation is

$$\sigma_f = -pI + \mu_f(\nabla u + \nabla^T \mu) \tag{2}$$

where p is the hydrostatic pressure of the fluid and μ_f is the viscosity of the fluid.

Navier- Stokes equations for fluid is

$$\rho_f(\frac{\partial u}{\partial t} + u \cdot \nabla u) = -\nabla p + \mu_f(\nabla u) + \rho_f \cdot g \tag{3}$$

where g is the gravity.

Another important technique of energy generation is the interaction of the fluid with turbulent fluid [50, 51]. As far as the fluid is considered, it can be Newtonian or non-Newtonian depending on the characteristic of the taken fluid. Zafiu invented a piezoelectric device that is reacting when flowing fluid striking with its face. The researcher uses the turbulent airflow model fluid, which is compressible by using Reynolds Averaged Navier Stokes equations and stress tensor, strain tensor equations. Mass conservative equation and conservative momentum equation in case of compressible Newtonian fluid are also used.

A conservative mass equation in case of a Newtonian fluid which is compressible as

$$\nabla \cdot (\rho \overrightarrow{u}) = 0 \tag{4}$$

Momentum conservative equation is

$$\rho(\overrightarrow{u} \cdot \nabla)\overrightarrow{u} = \nabla \cdot [-pI + (\eta + \eta_T)(\nabla \overrightarrow{u} + (\nabla \overrightarrow{u})^T) - \frac{2}{3}(\eta + \eta^T)(\overrightarrow{u} \cdot \nabla)I - \frac{2}{3}\rho k I] \tag{5}$$

Transport equation between turbulence kinetic energy k and turbulence dissipation rate ε_1 is given by

$$\rho(\overrightarrow{u} \cdot \nabla)k = \nabla \cdot [(\frac{\eta + \eta_T}{\sigma_k})\nabla k + P_k - \rho \varepsilon_1] \tag{6}$$

$$\rho(\overrightarrow{u} \cdot \nabla)\varepsilon_1 = \nabla \cdot [(\frac{\eta + \eta_T}{\sigma_\varepsilon})\nabla \varepsilon_1 + \frac{c_1 \varepsilon_1}{k P_k} - \frac{c_2 \rho \varepsilon_1^2}{k}] \tag{7}$$

The piezo-electric equation for Cauchy stress tensor and electrical displacement is given by

$$S = d^T \cdot \overrightarrow{E} + \beta \cdot T \tag{8}$$

where S is the strain symmetric tensor, $\overrightarrow{E}$ is the electric velocity field,T is the stress symmetric tensor,β is the elasticity tensor and d^T is the piezoelectric coupling tensor for a strain - charge form.

Further advancement in this field has been done by Akayein by investigating the energy harvesting technique for the unsteady turbulent flow of fluid with a piezo-electric generator [9]. The piezo-electric beam was located within the turbulent boundary and wakes of a circular cylinder and output energy is recorded. The output energy depends upon the position of the generator and the frequency of fluid flow is matched with the natural frequency of piezo-electric harvester. Simulation of the complete system matches with experiment data. Numerical simulation is carried out by continuity equation and Navier–Stokes equations of incompressible fluid for Newtonian fluid as mention below

$$\frac{D\rho}{Dt} + \rho S_{KK} = 0 \tag{9}$$

$$\rho \frac{DV_i}{Dt} = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} \tag{10}$$

where $\rho \frac{D}{Dt} = -\frac{\partial}{\partial t} + V_K \frac{\partial}{\partial x_K}$ is total derivative operator

Where, p is the pressure in the fluid, ρ is the density of the fluid, V_i is the fluid velocity, τ_{ij} is the stress vector and S_{KK} is strain rate tensor.

The generation of energy with the nozzle by using a PZT micropump is a vital area of research for investigation. Further, Jeong numerically simulates the diffuser nozzle, which occupying a piezo-electric micropump having two dissimilar models [44]. A numerical solution of models has been carried out and comparison has been made among them[59, 60, 61]. The results show that the model is extremely significant for calculating the diffuser nozzle-occupying piezo-electric micropump performance. Navier-Stokes equations and continuity equation in the curvilinear coordinates are used. The piezo-electric equations for the piezo-disk are also used.

Equation of Navier-Stokes and continuity in the curvilinear coordinates are

$$\frac{\partial}{\partial t}(\frac{\rho}{J})+\frac{\partial}{\partial \xi_j}(\frac{\rho U_j}{J})=0 \tag{11}$$

$$\frac{\partial}{\partial t}(\frac{\rho u_i}{J})+\frac{\partial}{\partial \xi^j}(\frac{\rho u_i U_j}{J})=-\frac{1}{J}\frac{\partial \xi^j}{\partial x_i}\frac{\partial p}{\partial \xi^j}+\frac{\partial}{\partial \xi^k}[\frac{\mu}{J}\frac{\partial \xi^k}{\partial x_j}(\frac{\partial \xi^l}{\partial x_j}\frac{\partial u_i}{\partial \xi^l}+\frac{\partial \xi^l}{\partial x_i}\frac{\partial u_j}{\partial \xi^l}-\frac{2}{3}\delta_{ij}\frac{u_l}{\xi^m}\frac{\xi^m}{\partial x_l})] \tag{12}$$

where ρ is the fluid density, p is pressure, t is time, x_i is cartesian coordinates, μ is the effective dynamic viscosity, U_j is velocity component in ξ^j direction, J is Coordinate transformation Jacobian and u_i is Cartesian velocity component.

Piezo-electric effect equations for of piezo-disk are

$$\{\sigma\}=[S]\{\gamma\}-[a]\{E\} \tag{13}$$

$$\{D\}=[a]^T\{\gamma\}-[e]\{E\} \tag{14}$$

where $\{\sigma\}$ is stress vector, $[s]$ is stiffness matrix,$\{\gamma\}$ is strain vector, $[a]$ is piezo-electric coupling matrix, $\{D\}$ is electric flux density vector,$[e]$ is dielectric matrix and $\{E\}$ is electric field vector.

With the help of various arrangements of fluid structure different researchers have done tremendous work in this field. Rojas designed and simulated valve-less, diaphragm based piezo-electric micropump [35]. All the simulation was made using COMSOL Multiphysics software. Using the piezo-electric device module simulates the deformation of the membrane under different voltage, using different arrangements of the membrane. Piezo-electric constitutive strain–charge relation and Navier-Stokes equations for incompressible fluid are used. Bernoulli's equation and Newton's 2nd law are also used in simulation.

Newton's 2^{nd} law is

$$\rho\frac{\partial^2 u}{\partial t^2}-\nabla\cdot\sigma=F_v \tag{15}$$

where ρ is the density of the solid, u is solid displacement vector, σ is stress tensor, F_v is body force per unit volume.

Navier-Stokes equations describe incompressible fluid motion as:

$$\rho(\frac{\partial u}{\partial t}+u\cdot\nabla u)=\nabla\cdot[-pI+\eta(\nabla u+\nabla u^T)]+F \tag{16}$$

where ρ is density of the fluid,u is fluid velocity vector,p is pressure, F is body force, I is identity tensor, η is viscosity and F is vector that represents all external forces.

Continuity equation is

$$\nabla\cdot u=0 \tag{17}$$

Bernoulli Eq. is

$$(\frac{u^2}{2}+\frac{P}{\rho}+gz)_1-(\frac{u^2}{2}+\frac{P}{\rho}+gz)_2=-h_P+h_f+h_L \tag{18}$$

where h_P is sum of head gains of the pump, h_f is sum of head losses due to pipe friction and h_L is sum of head losses in accessories. In this sequence of generating energy through PZT valve-less micropump, Johari carried out the periodical flow performance in the valve-less piezo-electric micropump with the Fluid-Structure Interaction (FSI) simulation technique [52]. Their investigation was based on boundary motion, equations of fluid, equations of structural equations, and actuator equations as given below. Navier-Stokes equations:

$$\rho\frac{\partial u_i}{\partial t}+\rho u_{i,j}(u_j-\hat{u}_j)=\sigma_{i,j}+\rho f_i \tag{19}$$

$$u_{j,j}=0 \tag{20}$$

where u_i is the velocity of fluid, ρ is density of fluid, $\sigma_{i,j}$ is stress tensor,f_i is body force per unit mass and $\hat{u}_j$ is mesh velocity. The method of placing piezo-electric flags in a fluidic medium comes in the process from the last few decades. Li investigated the process of how the ocean wave energy or the energy from the fluid has been converted into electricity [32]. Piezo-electric flags and flexible generators have been placed in the cross fluid and thus, maximal strain energy and output power has been obtained. To maximize the amount of power, many factors such as the thickness and stiffness of the eel materials, eel length, the bluff body width, the spacing between the body and eel head are considered. Simulation is done by using ANSYS software. The Navier - Stokes equations for the FSI model and transport equation for fluid is also used here. The coupling solution is done by piezo-electric equation and equation of elasticity. The governing equation for the FSI model is

$$\rho\frac{\partial \overrightarrow{u}_f}{\partial t}+(\overrightarrow{u}_f\cdot\nabla)\overrightarrow{u}_f=\nabla\cdot[-pI+\mu(\nabla\overrightarrow{u}_f+(\nabla\overrightarrow{u})^T)]+\overrightarrow{F} \tag{21}$$

$$\frac{\partial^2\overrightarrow{u_s}}{\partial t^2}-\nabla\sigma=\rho\overrightarrow{F} \tag{22}$$

where $\overrightarrow{u}_f$ is velocity for flowing of fluid,$\overrightarrow{u}_s$ is velocity of the solid mechanics and $\overrightarrow{F}$ is volume force field.

The transport equation for fluid is

$$\frac{\partial}{\partial t}(\rho k)+\frac{\partial}{\partial x_i}(\rho k u_i)=\frac{\partial}{\partial x_j}[(\mu+\frac{u_i}{\sigma_k})\frac{\partial k}{\partial x_j}]+G_k+G_b+S_k-\rho\varepsilon-Y_m \tag{23}$$

where ε is turbulent dissipation factor, G_k is turbulent kinetic energy produced by average velocity gradient, G_b is turbulent kinetic energy produced by buoyancy, Y_m is effect of pulsating expansion on total turbulent kinetic energy incompressible turbulence.

Flow-induced vibrations play a key role in piezo-electric energy generation system in present society. In this field, Ravi proposed a constant and simultaneous concurrent investigation of elegant small-scale energy harvesting techniques aiming for flow-induced vibrations [38]. Their purpose of allowing trustworthy sensitive toughness and efficient study of a coupled non-linear system that includes fluid structural piezoceramics and electric circuits. Simulation is done with the help of the finite element method (FEM). Navier-Stokes equations for incompressible fluid flow are also used.

The incompressible Navier-Stokes equations for fluid flow as

$$\rho[\dot{u}+u\cdot\nabla u]-\nabla\cdot T-f=0 \tag{24}$$

The Cauchy stress tensor T is given by

$$T = 2\mu D(u) - pI \tag{25}$$

where ρ is density of fluid, f is external body forces, u is fluid's velocity, μ is kinematic viscosity, p is hydrostatic pressure, D is strain rate tensor and I is identity tensor.

Flowing of fluid through various kinds of channels is a task that takes revolution in the world of energy harvesting. Particularly the flow arises due to recirculation in a channel is used by many of the researchers for energy generation. Bank carried out the controlling of recirculation flow in a channel using the fluid-structure interaction mechanism [18]. Experimentalists have discussed the ways to deal out the shear over the surface of the structure. For an incompressible fluid, Navier-Stokes equations are used for simulation purposes. The stress tensor equation is also used for numerical calculations.

Incompressible Navier-Stokes equations is

$$\rho_\circ \frac{\partial u_i}{\partial t} + \nabla \cdot (\rho_\circ u_i u) = \nabla \cdot S_i + p_\circ f_i \tag{26}$$

Stress tensor is given as

$$S_{ij} = -\delta_{ij} p + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{27}$$

where u is velocity field, p is pressure, $\rho_\circ$ is mass density, μ is viscosity of the fluid, f is density of external forces.

Matsiev has made further improvements by taking the interaction between the oscillators and surrounding fluid and operated flexural mechanical resonators at high rate throughout liquid [39]. For numerical simulation, Navier-Stokes equations and continuity equations with suitable boundary restrictions on the surface of the resonator and at infinity are used. The concept of Navier-Stokes equations for ideal fluid is also used. Equations of Navier-Stokes and continuity with appropriate boundary conditions on the resonator surface and in infinity are as

$$\rho\left(\frac{\partial \overrightarrow{u}}{\partial t} + (\overrightarrow{u}\nabla)\overrightarrow{u}\right) = -\nabla p + \eta \nabla \overrightarrow{u} + \left(\xi + \frac{\eta}{3}\right)\nabla(\nabla \cdot \overrightarrow{u}) \tag{28}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \overrightarrow{u}) = 0 \tag{29}$$

$$\overrightarrow{u}_s = \overrightarrow{u}_\circ(s), \overrightarrow{u}_\infty = 0 \tag{30}$$

Navier- Stokes equations for ideal fluid is

$$\frac{\partial \overrightarrow{u}}{\partial t} = -\frac{1}{\rho}\nabla p \tag{31}$$

ρ is fluid density, t is time, p is pressure, η is fluid viscosity, $\overrightarrow{u}_\circ(s)$is velocity distribution along resonator surface,$\overrightarrow{u}$ is fluid velocity and S is indexes.

Interaction of fluid-structure with propagating wave is an interesting topic for research in three dimensional [41]. The finite element method is used for this purpose. For numerical simulation, Navier-Stokes equations and continuity equations are used. Four coupled wave equation for displacement and voltage is also used.

Navier-Stokes equations for fluid as:

$$\rho[\frac{\partial v_f}{\partial t} + v_f \cdot \nabla v_f] = -\nabla P + \nabla \cdot T + f_1 \tag{32}$$

The Continuity equation is given as

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho v_f = 0 \tag{33}$$

where ρ is the fluid density, v_f is fluid velocity, f_1 is body forces, P is pressure and T is Deviatoric stress tensor.

Four coupled wave equation for displacement and voltage is given as

$$\sum_{k,l=1}^{3} c_{ijkl}\frac{\partial^2 u_l}{\partial x_j \partial x_k} + \sum_{k=1}^{3} e_{ijk}\frac{\partial^2 \phi}{\partial x_j \partial x_k} = \rho\frac{\partial^2 u_i}{\partial t^2} \tag{34}$$

$$\sum_{j,k,l=1}^{3} e_{jkl}\frac{\partial^2 u_l}{\partial x_j \partial x_k} - \sum_{j,k=1}^{3} \varepsilon_{jk}\frac{\partial^2 \phi}{\partial x_j \partial x_k} = 0 \tag{35}$$

where c is structural elasticity matrix, e is piezo-electric stress matrix and ε is Dielectric matrix at constant electric field.

A further achievement in the field of voltage generation is carried out by Zhu through the interaction of the dynamic fluid with a flapping foil power producer in a linear shear flow environment [53]. Since the concept of linear shear arrived, so 2-D Navier- Stokes equations in vorticity-stream function format is used.
Navier-Stokes equations in vorticity-stream function format is given as

$$\frac{\partial}{\partial t}(J\omega)+\sqrt{J}u\cdot\nabla\omega=\frac{1}{R_e}\nabla^2\omega \tag{36}$$

$$\nabla^2\psi=-J\omega \tag{37}$$

where J is the Jacobian of the transformation, u is the velocity of the fluid,R_e is Reynolds number, ψ is stream function and ω is vorticity.

4.2 Mathematical modelling of PEHF with applied fluid mechanics

Another recent widely used technique of energy generation is the interaction of vortices with a deformable beam [11]. An experimental method is closed to produce numerous controllable vortices for measuring their interaction with an elastic beam. Distribution of pressure is found out for various instants. Numerical simulation is done by the equation of linear elasticity for a mesh moving model.
Equation of linear elasticity for mesh moving model is

$$\nabla\cdot\sigma+f=0 \tag{38}$$

where σ is Cauchy stress tensor and f is external force.
The inclusion of passive foil has arrived with further achievement in this field. The technique of generation of energy from a vortex flow by using a passive foil has been investigated [45]. An immersed boundary technique with the interaction of the fluid network is used for doing numerical simulation. The governing equation for numerical simulation in the case of incompressible flowing fluid is also used as

$$\frac{\partial v}{\partial t}+v\cdot\nabla v=-\nabla P+\frac{1}{R_e}\nabla^2 v+f \tag{39}$$

where v is velocity of the fluid, R_e is Reynold number, P is instantaneous power input to the foil and f is the body forces.

The concept of cross-flow is equally crucial in fluid-structure interaction mechanisms as other methods. Sin performed the elastic eel's motion from the wake of deceiving structure in a cross-flow. In this paper, the collaboration between eels and vortex shedding behind a deceive structure is investigated mathematically. The motto of the paper is to upgrade the eel's oscillation by pairing with the varying vortices. COMSOL software is used for simulation. The equations for fluid-structure interaction (FSI) model are also used.

The governing equation for the FSI model given as

$$\frac{\partial u_f}{\partial t}+(u_f\cdot\nabla)u_f=\nabla\cdot[-pI+\mu(\nabla u_f+(\nabla u_f)^T)]+F \tag{40}$$

$$\nabla\cdot u_f=0 \tag{41}$$

$$\frac{\partial^2 u_s}{\partial t^2}-\nabla\cdot\sigma=F \tag{42}$$

where u_f is velocity of fluid flowing field, u_s is velocity of solid mechanics field, p is pressure, I is identity tensor, F is volume force field, μ is viscosity of fluid and σ is Cauchy stress tensor.

4.3 PEHF Modelling with Other Methods

Further improvement in this area has done by Neto by planning a technique for a self-exciting PMMA valve-less piezo-electric micropump [26]. A software-based on the finite element is considered for studying the feasibility system configuration. The experimental test shown that planned, constructed path harvest a less expenditure and extremely efficient micro pump. Simulation is done by using the finite element method (FEM). Piezo-electric strain charges constitutive relation is also used for simulation are as per equation (13) and (14). Modified control matrix singular value decomposition approach and optimal placement of the piezoelectric pieces enhance the output performance of the system [62, 63, 64].

Zhang investigated a piezo-electric polymer having more than one layer of construction on the elastic substrate in a fluidic environment for mechanical energy harvesting with a bending approach [33]. Models are refined numerically and analytically to realize the effect of material criteria on the energy producing efficiency of more than one-layer construction. The solution of an equation is done by the method of variable's separation. ANSYS software is also playing a vital role in numerical simulation. Basic equation of piezo-electric materials is also used. Further in case of generation of energy at micro level various vital hybrid techniques like Taguchi and heterogeneous 3-D has been used for optimization, simulation, modelling and fabrication in case of low-level vibration applications [54, 55, 56, 57, 58].

TABLE 1. Classification of Interactions of Piezoelectric Materials with Fluid

Type	Criterion	Solving Method	Source	Year
Energy generation from piezo-electric equipment	Analyzed the behavior of a valve-less micropump interaction with fluid-structure	Navier-Stokes Equations and Continuity equations	Johari and Majlis [52]	2008
	Harvesting energy from flow-induced vibration	Finite element method	Wang and Ko [12]	2010
	Simulating stiff structure motion in incompressible two-phase flow	Navier-Stokes Equations and Continuity equations	Sanders et al. [36]	2011
	Analysis of energy harvesting for unsteady turbulent flow of fluid	Navier-Stokes Equations and Continuity equations	Akaydin et al [9]	2010
	Investigated a piezo-electric device which reacts when flowing fluid strike with its face	Navier-Stokes Equations and Continuity equations	Zaffiu, Nistor and paul [50]	2011
	Discussed a high frequency vibrating structure	Finite element method	Afrasiab and Movahhedy [37]	2013
	Analyzed the energy harvesting efficiency of piezo-electric flags	Implicit Method	Michelin and Doare [13]	2013
	Algorithm to detect and quantify defects in piezo-electric plates	Extended Finite element method	Nanthakumar et al. [29]	2013
	Investigated the consequence of piezoelectricity on the interaction of wave in the fluid	Analytical and Numerical Method	Gupta and Vashistha[42]	2014

	Analyzed the interaction of vortices with a deformable beam	Equation of linear elasticity	Goushcha, Andreopoulo [11]	2014
	Studied the energy harvesting from vertical flow by a passive foil	Boundary method	Wei and Zhang [45]	2016
	Discussed the ocean wave energy	ANSYS Software	Li and Wang et al. [32]	2017
	Experimentally investigate the piezoelectric energy harvesting with turbulent flow	Experimentally and analytically	Abrol and Chhabra [4]	2017
Assembling and storing the harvested energy from piezo-electric equipment	Analyzed the frequency design of the nozzles layout	Experimentally and analytically	Feng and Zhou et al.[17]	2005
	Designed and simulated valveless diaphgram based piezo-electric micropump	Comsol multiphysics software	Rojas and Morales[35]	2015
	Discussed a coupled problem consists of an equation of fluid	Navier-Stokes Equations	Zilian and Ravi [38]	2016
	Designed methodology for a self-priming micropump	Finite element method	Neto and Lima et al.[25]	2016
Optimization of energy stored from diffuser nozzle based harvesting through piezo-electric equipment's	Numerically simulate the diffuser nozzle based piezo-electric micropump	Navier-Stokes Equations and Continuity equation	Jeong and Kim [44]	2006
	Optimize the location of piezo-electric actuator in a cantilever beam	Finite element method	Khalatkar et al.[19]	2011
Piezo-electric energy harvesting through flow and vibration	Analytical treatment of ultrasonic waves with piezo-electric	Analytical method	Nayfeh and Chein[40]	1992

	Controlling of the recirculation flow in a channel using fluid	Navier-Stokes Equations and Stress Tensor Equation	Banks and Ito[18]	1994
	Mesh moving technique for fluid-structure interaction	Equation of elasticity	Stein et al. [31]	2003
	Study Analytically and experimentally the energy harvesting capabilities of polymer	Analytical method	Aureli and prince [24]	2010
	Investigate a piezo-electric polymer multilayer structure	ANSYS Software	Zhang et al. [33]	2013
Others piezoelectric energy harvesting fluid interaction papers	Operation of flexural mechanical resonators to high throughout the liquid characterization	Navier-Stokes Equations and Continuity equation	Matsiev [39]	2000
	Fluid-structure interaction of wave propagation in the SAW device	Finite element method	Singh and Subramanian [41]	2009
	Eel's motion from the wake of deceive structure in a cross-flow	COMSOL software	Sin et al. [34]	2012

5. CONCLUSIONS

The present work efficiently classified the different kinds of fluid flow interactions with piezo smart materials for energy harvesting into three categories: (i) experimental investigations (ii) simulation and (iii) mathematical modeling. Experimental investigation results that the piezo-polymers are more flexible and durable as compared to piezoceramics, and also act as better energy harvester. As compared to linear flow, turbulent flow generated by various means like the wake of a circular cylinder, creating vorticity act as an efficient energy harvester. Simulation investigation efficiently elaborates the advantages of Multiphysics simulations/programming methods than computer graphics of any software/tools for various energy harvesting systems. For mathematical modeling of the energy harvesting through piezo material with fluid flow, it is easier to use numerical methods using the Navier-Stokes equation of motion than analytical methods.In case of Bernoulli's equation, considering various losses like friction losses, bend losses, net head on the piezo material can be calculated and using piezoelectricity equation indicates the energy generated by the system.It can be concluded that, harvesting of energy from the sources such as ocean wave, rain water, human muscles and hydraulic pressure of fluid with much density are very hopeful source of energy generation at a micro level in upcoming time with eco-friendly, durable, economical and easily integrated nature.

TABLE 2. Abbreviation Table

Sr No.	Symbol	Meaning	Sr No.	Symbol	Meaning
1	u	Velocity of fluid	21	S_{KK}	Strain rate tensor
2	ρ_F	Density of fluid	22	U_j	Velocity component in ξ^j direction
3	t	Time	23	J	Jacobian of transformation
4	σ_F	Fluid's stress	24	[s]	Stiffness matrix
5	p	Hydrostatic pressure of fluid	25	$\{\gamma\}$	Strain vector
6	μ_f	Viscosity of the fluid	26	[a]	Piezo-electric coupling matrix
7	∇	Gradient	27	$\{D\}$	Electric flux density vector
8	g	Gravity	28	[e]	Dielectric matrix
9	η	Dynamic viscosity	29	$\{E\}$	Electric field vector
10	η_T	Turbulent viscosity	30	F	Body force
11	K	Turbulence kinetic energy	31	I	Identity tensor
12	ε_1	Turbulence Dissipation rate	32	h_p	Sum of head gains of the pump
13	c_1, c_2	Constant	33	h_f	Sum of head losses due to pipe friction

14	P_K	Product term	34	h_L	Sum of head losses in accessories
15	S	Strain symmetric tensor	35	C	Structural elasticity matrix
16	$\overrightarrow{E}$	Electric velocity field	36	ε	Dielectric matrix at constant electric field
17	T	Stress symmetric tensor	37	R_e	Reynolds number
18	β	Elasticity tensor	38	ψ	Stream function
19	d^T	Piezoelectric coupling tensor	39	ω	Vorticity
20	τ_{ij}	Stress vector	40	u_s	Velocity of solid mechanics field

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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