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SOME FIXED POINT RESULTS IN $\mathscr{V}-$ FUZZY METRIC SPACE USING RATIONAL CONTRACTION MAPPING

D. POOVARAGAVAN[1,2,†], M. JEYARAMAN[1,*]

[1]PG and Research Department of Mathematics, Raja Doraisingam Government Arts College, Sivagangai, India
Affiliated to Alagappa University, Karaikudi, Tamil Nadu, India

[2]Department of Mathematics, Government Arts College for Women, Sivagangai, Tamil Nadu, India

Abstract. In this paper, we make some fixed point theorems for a new type of generalized contractive mappings including $\mathbf{C}-$ class function, $\gamma_{\mathsf{s}}^{t}-$ admissible type mapping and rational contractive in the casing work of complete $\mathscr{V}-$ Fuzzy Metric Spaces. The outcomes acquired in this work generalize and further develop some fixed point results in this article.

Keywords: $\mathscr{V}-$ fuzzy metric spaces; $\gamma_{\mathsf{s}}^{t}-$ admissible type mapping; triangular $\gamma_{\mathsf{s}}^{t}-$ admissible type mapping; $\mathbf{C}-$ class function; rational contractive mapping.

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1. Introduction

Mustafa and Sims [9] brought the however of the thought of $\mathscr{G}-$ metric spaces as a speculation of metric spaces. Besides, Sedghi et. al [11] presented the idea of $S-$ metric spaces as one of the speculations of the metric spaces. Abbas et. al. [2] broadened the thought of $S-$ metric spaces to $A-$ metric space by stretching out the definition to n - tuple. In 1965, Zadeh [14] at

*Corresponding author
E-mail address: jeya.math@gmail.com
†Part-time Ph.D., Research Scholar
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first presented the idea of fuzzy sets. From that point forward, a few powerful mathematicians thought about the idea of fuzzy sets to present many energizing ideas in the field of science, like fuzzy differential equations, fuzzy logic and fuzzy metric spaces. A fuzzy metric space is notable to be a significant speculation of the metric space. In 1975, kramosil and Michalek [7] utilized the idea of fuzzy sets to present the thought of fuzzy metric spaces. George and Veeramani [3] madified the idea of fuzzy metric spaces in the feeling of Kramosil and Michalek [7]. Sun and Yang [12] begat the possibility of $\mathscr{G}-$ fuzzy metric spaces.

2. PRELIMINARIES

Definition 2.1. [13] Consider Υ be a non-empty set. A triple $(\Upsilon, \mathscr{V}, *)$ is said to be $\mathscr{V}-$ Fuzzy Metric Spaces ($\mathscr{V}-FMS$) where $*$ is a continuous t-norm and $\mathscr{V}$ is a fuzzy set on $\Upsilon^n \times (0,\infty)$ satisfying the following conditions for all $\mathsf{t}, \mathsf{s} > 0$:

($\mathscr{V}$-1) $\mathscr{V}(v, v, \ldots, v, \mu, \mathsf{t}) > 0$ for all $v, \mu \in \Upsilon$ with $v \neq \mu$,

($\mathscr{V}$-2) $\mathscr{V}(v_1, v_1, \ldots, v_1, v_2, \mathsf{t}) \geq \mathscr{V}(v_1, v_2, \ldots, v_n, \mathsf{t})$ for all $v_1, v_2, \ldots, v_n \in \Upsilon$ with $v_2 \neq v_3 \neq \ldots \neq v_n$,

($\mathscr{V}$-3) $\mathscr{V}(v_1, v_2, \ldots, v_n, \mathsf{t}) = 1 \Leftrightarrow v_1 = v_2 = v_3 = \cdots = v_n$,

($\mathscr{V}$-4) $\mathscr{V}(v_1, v_2, \ldots, v_n, \mathsf{t}) = \mathscr{V}(p\{v_1, v_2, \ldots, v_n\}, \mathsf{t})$, where p is a permutation function,

($\mathscr{V}$-5) $\mathscr{V}(v_1, v_2, \ldots, v_n, \mathsf{t}+\mathsf{s}) \geq \mathscr{V}(v_1, v_2, \ldots, v_{n-1}, l, \mathsf{t}) * \mathscr{V}(l, l, \ldots, l, v_n, \mathsf{s})$,

($\mathscr{V}$-6) $\lim\limits_{\mathsf{t}\to\infty} \mathscr{V}(v_1, v_2, \ldots, v_n, \mathsf{t}) = 1$,

($\mathscr{V}$-7) $\mathscr{V}(v_1, v_2, \ldots, v_n, .) : (0,\infty) \to [0,1]$ is continuous.

Example 2.2. Consider $\Upsilon = R$ and (Υ, A) be a A metric space. Define $\mathscr{V} : \Upsilon^n \times (0,\infty) \to [0,1]$ such that

$$\mathscr{V}(v_1, v_2, \ldots, v_n, \mathsf{t}) = \exp\Big(-\frac{A(v_1, v_2, \ldots, v_n)}{\mathsf{t}}\Big)$$

for all $v_1, v_2, \ldots, v_n \in \Upsilon$ and $\mathsf{t} > 0$. Then $(\Upsilon, \mathscr{V}, *)$ is a $\mathscr{V}-$ fuzzy metric space.

Lemma 2.3. [13] *Consider* $(\Upsilon, \mathscr{V}, *)$ *be a* $\mathscr{V}-FMS$. *Then* $\mathscr{V}(v_1, v_2, \ldots, v_n, t)$ *is non-decreasing with respect to t.*

Lemma 2.4. [13] *Consider* $(\Upsilon, \mathscr{V}, *)$ *be a* $\mathscr{V}-FMS$ *such that* $\mathscr{V}(v_1, v_2, \ldots, v_n, \mathsf{kt}) \geq \mathscr{V}(v_1, v_2, \ldots, v_n, \mathsf{t})$ *with* $\mathsf{k} \in (0,1)$. *Then* $v_1 = v_2 = v_3 = \cdots = v_n$.

Definition 2.5. [13] Consider $(\Upsilon, \mathscr{V}, *)$ be a $\mathscr{V}-FMS$. A sequence $\{\nu_r\}$ is said to be convergent to ν if $\lim_{r\to\infty} \mathscr{V}(\nu_r, \nu_r, \ldots, \nu_r,, \nu, \mathsf{t}) = 1$

Definition 2.6. Consider $(\Upsilon, \mathscr{V}, *)$ be a $\mathscr{V}-FMS$. A sequence $\{\nu_r\}$ is said to be a Cauchy sequence if $\mathscr{V}(\nu_r, \nu_r, \ldots, \nu_r, \nu_q, \mathsf{t}) \to 1$ as $r, q, \to \infty$ for all $\mathsf{t} > 0$.

Definition 2.7. Consider $\mathscr{V}-FMS$ $(\Upsilon, \mathscr{V}, *)$ is said to be complete if every Cauchy sequence in Υ is convergent.

Definition 2.8. Consider Υ be a non-empty set, $\gamma : \Upsilon \times \Upsilon \times \overbrace{\cdots}^{n-times} \times \Upsilon \times (0, \infty) \to [0, \infty)$ and $\tau : \Upsilon \to \Upsilon$ is a self mappings. Then τ is said to be $\gamma-$ admissible if for all $\nu_1, \nu_2, \ldots, \nu_n \in \Upsilon, \mathsf{t} > 0$ with $\gamma(\nu_1, \nu_2, \ldots, \nu_n, \mathsf{t}) \geq 1 \Longrightarrow \gamma(\tau\nu_1, \tau\nu_2, \ldots, \tau\nu_n, \mathsf{t}) \geq 1$.

Definition 2.9. Consider Υ be a non-empty set, $\gamma : \Upsilon \times \Upsilon \times \overbrace{\cdots}^{n-times} \times \Upsilon \times (0, \infty) \to [0, \infty)$ and $\tau : \Upsilon \to \Upsilon$ be a self mappings. Then τ is said to be triangular $\gamma-$ admissible if

(1) τ is $\gamma-$ admissible,

(2) $\gamma(\nu_1, \nu_2, \ldots, \nu_{n-1}, \mathsf{l}, \mathsf{t}) \geq 1$ and $\gamma(\mathsf{l}, \mathsf{l}, \ldots, \mathsf{l}, \nu_n, \mathsf{t}) \geq 1$ implies $\gamma(\nu_1, \nu_2, \ldots, \nu_n, \mathsf{t}) \geq 1$.

for all $\nu_1, \nu_2, \ldots, \nu_n, \mathsf{l} \in \Upsilon$.

Definition 2.10. Consider Υ be a non-empty set with $\mathsf{s} \geq 1$ a given real number. $\gamma : \Upsilon \times \Upsilon \times \overbrace{\cdots}^{n-times} \times \Upsilon \times (0, \infty) \to [0, \infty)$ and $\tau : \Upsilon \to \Upsilon$ be mappings. Then τ is called $\gamma-$ admissible type S if for all $\nu_1, \nu_2, \ldots, \nu_n \in \Upsilon, \mathsf{t} > 0$ with $\gamma(\nu_1, \nu_2, \ldots, \nu_n, \mathsf{t}) \geq \mathsf{s} \Longrightarrow \gamma(\tau\nu_1, \tau\nu_2, \ldots, \tau\nu_n, \mathsf{t}) \geq \mathsf{s}$.

Definition 2.11. Consider Υ be a non-empty set with $\mathsf{s} \geq 1$ a given real number. $\gamma : \Upsilon \times \Upsilon \times \overbrace{\cdots}^{n-times} \times \Upsilon \times (0, \infty) \to [0, \infty)$ and $\tau : \Upsilon \to \Upsilon$ be mappings. Then τ is called triangular $\gamma-$ admissible type S if

(1) τ is $\gamma-$ admissible type S,

(2) $\gamma(\nu_1, \nu_2, \ldots, \nu_{n-1}, \mathsf{l}, \mathsf{t}) \geq \mathsf{s}$ and $\gamma(\mathsf{l}, \mathsf{l}, \ldots, \mathsf{l}, \nu_n, \mathsf{t}) \geq \mathsf{s}$ implies $\gamma(\nu_1, \nu_2, \ldots, \nu_n, \mathsf{t}) \geq \mathsf{s}$.

for all $\nu_1, \nu_2, \ldots, \nu_n, \mathsf{l} \in \Upsilon$.

Let Ω denote the class of all functions $\zeta : [0, 1] \to [0, 1]$ such that ζ is non-increasing, continuous and let $\zeta(\mathsf{t}) < \mathsf{t}$ for all $\mathsf{t} \in (0, 1)$.

If $\zeta(0) = 0$ and $\zeta(1) = 1$ additionally hold, then $\zeta(\mathsf{t}) \leq \mathsf{t}, \mathsf{t} \in [0, 1]$ for all functions from Ω.

Definition 2.12. A mapping $\mathscr{F} : [0,1]^2 \to [0,1]$ is called a $\mathsf{C}-$ class function if it is continuous and the following axioms holds:

(1) $\mathscr{F}(\mathsf{s},\mathsf{t}) \leq \mathsf{s}$ for all $\mathsf{s},\mathsf{t} \in [0,1]$,

(2) $\mathscr{F}(\mathsf{s},\mathsf{t}) = \mathsf{s}$ implies either $\mathsf{s} = 1$ or $\mathsf{t} = 1$.

We denote $\mathscr{C}$ the family of $\mathsf{C}-$ class functions.

3. Main Results

In this segment, we present the thought of $\gamma_{\mathsf{s}}^{\iota}-$ rational contraction type mappings and set up the presence and uniqueness aftereffects of the fixed point for this class of mappings. We start by setting up certain outcomes that will be utilized in the confirmation of our primary outcome.

Definition 3.1. Consider Υ be a non-empty set with $\mathsf{s},\iota \geq 1$ a given real number. $\gamma : \Upsilon \times \Upsilon \times \overbrace{\cdots}^{n-times} \times \Upsilon \times (0,\infty) \to [0,\infty)$ and $\tau : \Upsilon \to \Upsilon$ be mappings. Then τ is called $\gamma_{\mathsf{s}}^{\iota}$-admissible type mapping if for all $\nu_1,\nu_2,\ldots,\nu_n \in \Upsilon, \mathsf{t} > 0$ with

$$\gamma(\nu_1,\nu_2,\ldots,\nu_n,\mathsf{t}) \geq \mathsf{s}^{\iota} \Longrightarrow \gamma(\tau\nu_1,\tau\nu_2,\ldots,\tau\nu_n,\mathsf{t}) \geq \mathsf{s}^{\iota}.$$

Definition 3.2. Consider Υ be a non-empty set with $\mathsf{s},\iota \geq 1$ a given real number. $\gamma : \Upsilon \times \Upsilon \times \overbrace{\cdots}^{n-times} \times \Upsilon \times (0,\infty) \to [0,\infty)$ and $\tau : \Upsilon \to \Upsilon$ be self mappings. Then τ is called triangular $\gamma_{\mathsf{s}}^{\iota}-$ admissible type mapping if

(1) τ is $\gamma_{\mathsf{s}}^{\iota}-$ admissible type mapping,

(2) $\gamma(\nu_1,\nu_2,\ldots,\nu_{n-1},\mathsf{l},\mathsf{t}) \geq \mathsf{s}^{\iota}$ and $\gamma(\mathsf{l},\mathsf{l},\ldots,\mathsf{l},\nu_n,\mathsf{t}) \geq \mathsf{s}^{\iota}$ implies $\gamma(\nu_1,\nu_2,\ldots,\nu_n,\mathsf{t}) \geq \mathsf{s}^{\iota}$.

for all $\nu_1,\nu_2,\ldots,\nu_n,\mathsf{l} \in \Upsilon$.

Lemma 3.3. *Consider Υ be a nonempty set and τ be a triangular $\gamma_{\mathsf{s}}^{\iota}-$ admissible mapping. Assume that there exists $\nu_0 \in \Upsilon$, such that $\gamma(\nu_0,\tau\nu_0,\cdots,\tau\nu_0,\mathsf{t}) \geq \mathsf{s}^{\iota}$. Assume the sequence $\{\nu_n\}$ is characterized by $\nu_{n+1} = \tau\nu_n$, then $\gamma(\nu_m,\nu_n,\cdots,\nu_n,\mathsf{t}) \geq \mathsf{s}^{\iota}$ for all $m,n \in N$.*

Proof. Given τ is triangular $\gamma_{\mathsf{s}}^{\iota}-$ admissible mapping and there exists $\nu_0 \in \Upsilon$ such that $\gamma(\nu_0,\tau\nu_0,\cdots,\tau\nu_0,\mathsf{t}) \geq \mathsf{s}^{\iota} \Rightarrow \gamma(\nu_0,\nu_1,\cdots,\nu_1,\mathsf{t}) \geq \mathsf{s}^{\iota}$. Since τ is $\gamma_{\mathsf{s}}^{\iota}-$ admissible mapping,

$$\gamma(\tau\nu_0,\tau\nu_1,\cdots,\tau\nu_1,\mathsf{t}) \geq \mathsf{s}^{\iota} \Rightarrow \gamma(\nu_1,\nu_2,\cdots,\nu_2,\mathsf{t}) \geq \mathsf{s}^{\iota}.$$

Proceeding like this way, we get $\gamma(v_n, v_{n+1}, \cdots, v_{n+1}, \mathrm{t}) \geq \mathrm{s}^{\iota}$ for all $n \in N \cup \{0\}$. Suppose that $m < n$ for all $m, n \in N$, since τ is triangular $\gamma_{\mathrm{s}}^{\iota}-$ admissible mapping,

$$\gamma(v_m, v_{m+1}, \cdots, v_{m+1}, \mathrm{t}) \geq \mathrm{s}^{\iota} \quad \text{and} \quad \gamma(v_{m+1}, v_{m+2}, \cdots, v_{m+2}, \mathrm{t}) \geq \mathrm{s}^{\iota}$$
$$\Longrightarrow \gamma(v_m, v_{m+2}, \cdots, v_{m+2}, \mathrm{t}) \geq \mathrm{s}^{\iota}.$$

Also,

$$\gamma(v_m, v_{m+2}, \cdots, v_{m+2}, \mathrm{t}) \geq \mathrm{s}^{\iota} \quad \text{and} \quad \gamma(v_{m+2}, v_{m+3}, \cdots, v_{m+3}, \mathrm{t}) \geq \mathrm{s}^{\iota}$$
$$\Longrightarrow \gamma(v_m, v_{m+3}, \cdots, v_{m+3}, \mathrm{t}) \geq \mathrm{s}^{\iota}.$$

Proceeding like this way, we get $\gamma(v_m, v_n, \cdots, v_n, \mathrm{t}) \geq \mathrm{s}^{\iota}$. □

Definition 3.4. Consider $(\Upsilon, \mathscr{V}, *)$ be a $\mathscr{V}-FMS$ with $\mathrm{s}, \iota \geq 1$ a given real number. $\gamma : \Upsilon \times \Upsilon \times \overbrace{\cdots}^{n-times} \times \Upsilon \times (0, \infty) \to [0, \infty)$ be a function and τ be a self map on Υ. The mapping τ is called $\gamma_{\mathrm{s}}^{\iota}$ - rational contraction mapping if

$$\gamma(v_1, v_2, \cdots, v_n, \mathrm{t}) \geq \mathrm{s}^{\gamma} \text{ and } n\mathrm{s}^{n-1}\mathscr{V}(v_1, \tau v_1, \cdots, \tau v_1, \mathrm{t}) \geq \mathscr{V}(v_1, v_2, \cdots, v_n, \mathrm{t})$$

$$(1) \quad \Rightarrow \zeta\left(\mathrm{s}^n \mathscr{V}(\tau v_1, \tau v_2, \cdots, \tau v_n, t)\right) \leq \mathscr{F}\Big(\zeta\left(\Xi(v_1, v_2, \cdots, v_n, t)\right), \rho\left(\Xi(v_1, v_2, \cdots, v_n, t)\right)\Big)$$

for all $v_1, v_2, \cdots, v_n \in \Upsilon$, where ζ, ρ are alternating distance functions, $\mathscr{F} \in \mathscr{C}$,

$$\Xi(v_1, v_2, \cdots, v_n, \mathrm{t}) = \min\Bigg\{\mathscr{V}(v_1, v_2, \cdots, v_n, \mathrm{t}), \mathscr{V}(v_1, \tau v_1, \cdots, \tau v_1, \mathrm{t}),$$
$$\mathscr{V}(v_2, \tau v_2, \tau v_3, \cdots, \tau v_n, \mathrm{t}), \frac{\mathscr{V}(v_1, \tau v_1, \cdots, \tau v_1, \mathrm{t}) \cdot \mathscr{V}(v_2, \tau v_2, \tau v_3, \cdots, \tau v_n, \mathrm{t})}{\mathscr{V}(v_1, v_2, \cdots, v_n, \mathrm{t})}\Bigg\}.$$

Theorem 3.5. *Consider $(\Upsilon, \mathscr{V}, *)$ be a complete $\mathscr{V}-FMS$ and $\tau : \Upsilon \to \Upsilon$ be an $\gamma_{\mathrm{s}}^{\iota}$ - rational contraction mapping. Assume the accompanying conditions hold:*

(1) τ is a triangular $\gamma_{\mathrm{s}}^{\iota}$ - admissible type mapping,

(2) there exist $v_0 \in \Upsilon$ such that $\gamma(v_0, \tau v_0, \cdots, \tau v_0, \mathrm{t}) \geq \mathrm{s}^{\iota}$,

(3) τ is continuous.

Then τ has a fixed point.

Proof. Let $\nu_0 \in \Upsilon$ be such that $\gamma(\nu_0, \tau\nu_0, \cdots, \tau\nu_0, \mathrm{t}) \geq \mathrm{s}^{\iota}$. Construct the sequence $\{\nu_n\}$ by $\nu_{n+1} = \tau\nu_n$ for all $n \in N \cup \{0\}$. Suppose that $\nu_{n+1} = \nu_n$, for some $n \in N \cup \{0\}$, get the ideal outcome. Suppose that $\nu_{n+1} \neq \nu_n$, for all $n \in N \cup \{0\}$. Now τ is triangular $\gamma_{\mathrm{s}}^{\iota}$ - admissible type mapping and $\gamma(\nu_0, \nu_1, \cdots, \nu_1, t) = \gamma(\nu_0, \tau\nu_0, \cdots, \tau\nu_0, \mathrm{t}) \geq \mathrm{s}^{\iota}$, also $\gamma(\nu_1, \nu_2, \cdots, \nu_2, \mathrm{t}) = \gamma(\tau\nu_0, \tau\nu_1, \cdots, \tau\nu_1, \mathrm{t}) \geq \mathrm{s}^{\iota}$, proceeding like this way, we obtain that $\gamma(\nu_n, \nu_{n+1}, \cdots, \nu_{n+1}, \mathrm{t}) \geq \mathrm{s}^{\iota}$ for all $n \in N \cup \{0\}$. Since $\gamma(\nu_n, \nu_{n+1}, \cdots, \nu_{n+1}, \mathrm{t}) \geq \mathrm{s}^{\iota}$ and

$$n\,\mathrm{s}^{n-1}\mathscr{V}(\nu_n, \tau\nu_n, \cdots, \tau\nu_n, \mathrm{t}) = n\,\mathrm{s}^{n-1}\mathscr{V}(\nu_n, \nu_{n+1}, \cdots, \nu_{n+1}, \mathrm{t}) > \mathscr{V}(\nu_n, \nu_{n+1}, \cdots, \nu_{n+1}, \mathrm{t})$$

$$\begin{aligned}\zeta(\mathscr{V}(\nu_{n+1}, \nu_{n+2}, \cdots, \nu_{n+2}, \mathrm{t})) &\leq \zeta(\mathrm{s}^n\mathscr{V}(\tau\nu_n, \tau\nu_{n+1}, \cdots, \tau\nu_{n+1}, \mathrm{t})) \\ &\leq \mathscr{F}\Big(\zeta\left(\Xi(\nu_n, \nu_{n+1}, \cdots, \nu_{n+1}, \mathrm{t})\right), \rho\left(\Xi(\nu_n, \nu_{n+1}, \cdots, \nu_{n+1}, \mathrm{t})\right)\Big)\end{aligned} \tag{3.5.1}$$

where

$$\begin{aligned}\Xi(\nu_n, \nu_{n+1}, \cdots, \nu_{n+1}, \mathrm{t}) = \min\Bigg\{&\mathscr{V}(\nu_n, \nu_{n+1}, \cdots, \nu_{n+1}, \mathrm{t}), \mathscr{V}(\nu_n, \tau\nu_n, \cdots, \tau\nu_n, \mathrm{t}), \\ &\mathscr{V}(\nu_{n+1}, \tau\nu_{n+1}, \cdots, \tau\nu_{n+1}, \mathrm{t}), \\ &\frac{\mathscr{V}(\nu_n, \tau\nu_n, \cdots, \tau\nu_n, \mathrm{t})) \cdot \mathscr{V}(\nu_{n+1}, \tau\nu_{n+1}, \cdots, \tau\nu_{n+1}, \mathrm{t})}{\mathscr{V}(\nu_n, \nu_{n+1}, \cdots, \nu_{n+1}, \mathrm{t})}\Bigg\} \\ = \min\Bigg\{&\mathscr{V}(\nu_n, \nu_{n+1}, \cdots, \nu_{n+1}, \mathrm{t}), \mathscr{V}(\nu_n, \nu_{n+1}, \cdots, \nu_{n+1}, \mathrm{t}), \\ &\mathscr{V}(\nu_{n+1}, \nu_{n+2}, \cdots, \nu_{n+2}, \mathrm{t}) \\ &\frac{\mathscr{V}(\nu_n, \nu_{n+1}, \cdots, \nu_{n+1}, \mathrm{t}) \cdot \mathscr{V}(\nu_{n+1}, \nu_{n+2}, \cdots, \nu_{n+2}, t)}{\mathscr{V}(\nu_n, \nu_{n+1}, \cdots, \nu_{n+1}, \mathrm{t})}\Bigg\}\end{aligned}$$

$$\Xi(\nu_n, \nu_{n+1}, \cdots, \nu_{n+1}, \mathrm{t}) = \min\Big\{\mathscr{V}(\nu_n, \nu_{n+1}, \cdots, \nu_{n+1}, \mathrm{t}), \mathscr{V}(\nu_{n+1}, \nu_{n+2}, \cdots, \nu_{n+2}, \mathrm{t})\Big\}$$

Suppose that

$$\begin{aligned}\Xi(\nu_n, \nu_{n+1}, \cdots, \nu_{n+1}, \mathrm{t}) &= \min\Big\{\mathscr{V}(\nu_n, \nu_{n+1}, \cdots, \nu_{n+1}, \mathrm{t}), \mathscr{V}(\nu_{n+1}, \nu_{n+2}, \cdots, \nu_{n+2}, \mathrm{t})\Big\} \\ &= \mathscr{V}(\nu_{n+1}, \nu_{n+2}, \cdots, \nu_{n+2}, \mathrm{t})\end{aligned}$$

then (3.5.1) becomes

$$\begin{aligned}
&\zeta(\mathscr{V}(\nu_{n+1},\nu_{n+2},\cdots,\nu_{n+2},\mathsf{t}))\\
&\le \zeta(\mathsf{s}^n\mathscr{V}(\tau\nu_n,\tau\nu_{n+1},\cdots,\tau\nu_{n+1},\mathsf{t}))\\
&\le \mathscr{F}\Big(\zeta\big(\mathscr{V}(\nu_{n+1},\nu_{n+2},\cdots,\nu_{n+2},\mathsf{t})\big),\rho\big(\mathscr{V}(\nu_{n+1},\nu_{n+2},\cdots,\nu_{n+2},\mathsf{t})\big)\Big)\\
&\le \zeta\big(\mathscr{V}(\nu_{n+1},\nu_{n+2},\cdots,\nu_{n+2},\mathsf{t})\big)
\end{aligned}$$

which implies that

$$\zeta(\mathscr{V}(\nu_{n+1},\nu_{n+2},\cdots,\nu_{n+2},\mathsf{t}))\le \zeta\big(\mathscr{V}(\nu_{n+1},\nu_{n+2},\cdots,\nu_{n+2},\mathsf{t})\big)$$

so that

$$\mathscr{F}\Big(\zeta\big(\mathscr{V}(\nu_{n+1},\nu_{n+2},\cdots,\nu_{n+2},\mathsf{t})\big),\rho\big(\mathscr{V}(\nu_{n+1},\nu_{n+2},\cdots,\nu_{n+2},\mathsf{t})\big)\Big)=\zeta\big(\mathscr{V}(\nu_{n+1},\nu_{n+2},\cdots,\nu_{n+2},\mathsf{t})\big)$$

and by definition of $\mathscr{F}$,

$$\zeta\big(\mathscr{V}(\nu_{n+1},\nu_{n+2},\cdots,\nu_{n+2},\mathsf{t})\big)=1 \ \text{ or } \ \rho\big(\mathscr{V}(\nu_{n+1},\nu_{n+2},\cdots,\nu_{n+2},\mathsf{t})\big)=1.$$

Using the properties of ζ and ρ we get $\mathscr{V}(\nu_{n+1},\nu_{n+2},\cdots,\nu_{n+2},\mathsf{t})=1\Rightarrow \nu_{n+1}=\nu_{n+2}$ which is a contraction. Thus,

$$\begin{aligned}
\Xi(\nu_n,\nu_{n+1},\cdots,\nu_{n+1},\mathsf{t}) &= \min\Big\{\mathscr{V}(\nu_n,\nu_{n+1},\cdots,\nu_{n+1},\mathsf{t}),\mathscr{V}(\nu_{n+1},\nu_{n+2},\cdots,\nu_{n+2},\mathsf{t})\Big\}\\
&= \mathscr{V}(\nu_n,\nu_{n+1},\cdots,\nu_{n+1},\mathsf{t})
\end{aligned}$$

which implies that

$$\mathscr{V}(\nu_n,\nu_{n+1},\cdots,\nu_{n+1},\mathsf{t})\le \mathscr{V}(\nu_{n+1},\nu_{n+2},\cdots,\nu_{n+2},\mathsf{t}) \tag{3.5.2}$$

Thus,

$$\begin{aligned}
\zeta(\mathscr{V}(\nu_{n+1},\nu_{n+2},\cdots,\nu_{n+2},\mathsf{t})) &\le \zeta(\mathsf{s}^n\mathscr{V}(\tau\nu_n,\tau\nu_{n+1},\cdots,\tau\nu_{n+1},\mathsf{t}))\\
&\le \mathscr{F}\Big(\zeta\big(\mathscr{V}(\nu_n,\nu_{n+1},\cdots,\nu_{n+1},\mathsf{t})\big),\rho\big(\mathscr{V}(\nu_n,\nu_{n+1},\cdots,\nu_{n+1},\mathsf{t})\big)\Big)\\
&\le \zeta\big(\mathscr{V}(\nu_n,\nu_{n+1},\cdots,\nu_{n+1},\mathsf{t})\big)
\end{aligned}$$

which implies that

$$\zeta(\mathscr{V}(\nu_{n+1},\nu_{n+2},\cdots,\nu_{n+2},\mathsf{t}))\le \zeta\big(\mathscr{V}(\nu_n,\nu_{n+1},\cdots,\nu_{n+1},\mathsf{t})\big) \tag{3.5.3}$$

using the property of ζ, we obtain

$$\mathscr{V}(\nu_n,\nu_{n+1},\cdots,\nu_{n+1},\mathrm{t})\leq\mathscr{V}(\nu_{n+1},\nu_{n+2},\cdots,\nu_{n+2},\mathrm{t}).$$

Using comparable methodology, we obtain

$$\mathscr{V}(\nu_{n+1},\nu_{n+2},\cdots,\nu_{n+2},\mathrm{t})\leq\mathscr{V}(\nu_{n+2},\nu_{n+3},\cdots,\nu_{n+3},\mathrm{t}).$$

Therefore, $\{\mathscr{V}(\nu_n,\nu_{n+1},\cdots,\nu_{n+1},\mathrm{t})\}$ is a non-decreasing sequence and bounded above. Thus, there exists $0\leq c\leq 1$ such that

$$\lim_{n\to\infty}\mathscr{V}(\nu_n,\nu_{n+1},\cdots,\nu_{n+1},\mathrm{t})=c.$$

Now, suppose that $0\leq c<1$ taking the limit as $n\to\infty$ of (3.5.3), we have that $\zeta(c)=\zeta(c)$ so that $\mathscr{F}(\zeta(c),\rho(c))=\zeta(c)$ and by definition of $\mathscr{F}$, we must have that $\zeta(c)=1$ or $\rho(c)=1$. Using the properties of ζ and ρ, we have that $c=1$. Thus, we have that

$$\lim_{n\to\infty}\mathscr{V}(\nu_n,\nu_{n+1},\cdots,\nu_{n+1},\mathrm{t})=1.$$

Therefore, $\{\nu_n\}$ is Cauchy sequence. Since $(\mathscr{X},\mathscr{V},*)$ is complete $\mathscr{V}-FMS$, there exists $\nu\in\Upsilon$ such that $\lim_{n\to\infty}\nu_n=\nu$. Suppose that τ is continuous, we have that

$$\nu=\lim_{n\to\infty}\nu_n=\lim_{n\to\infty}\nu_{n+1}=\lim_{n\to\infty}\tau\nu_n=\tau\lim_{n\to\infty}\nu_n=\tau\nu.$$

Thus, τ has a fixed point. □

Theorem 3.6. *Suppose that the hypothesis of Theorem 3.5 holds and in addition suppose* $\gamma(\nu,\mu,\cdots,\mu,t)\geq\mathsf{s}^{\iota}$ *for all* $\nu,\mu\in\mathscr{F}(\tau)$, *where* $\mathscr{F}(\tau)$ *is the set of fixed point of* τ. *Then* τ *has a unique fixed point.*

Proof. Let $\nu,\mu\in\mathscr{F}(\tau)$, that is $\tau\nu=\nu$ and $\tau\mu=\mu$ such that $\nu\neq\mu$. Using our hypothesis that $\gamma(\nu,\mu,\cdots,\mu,\mathrm{t})\geq\mathsf{s}^{\iota}$ and $n\,\mathsf{s}^{n-1}\mathscr{V}(\nu,\tau\nu,\cdots,\tau\nu,\mathrm{t})=1\geq\mathscr{V}(\nu,\mu,\cdots,\mu,t)$ we have

(3.6.1)
$$\zeta\left(\mathscr{V}(\nu,\mu,\cdots,\mu,\mathrm{t})\right)\leq\zeta\left(\mathsf{s}^{n}\mathscr{V}(\tau\nu,\tau\mu,\cdots,\tau\mu,\mathrm{t})\right)\leq\mathscr{F}\Big(\zeta\left(\Xi(\nu,\mu,\cdots,\mu,\mathrm{t})\right),\rho\left(\Xi(\nu,\mu,\cdots,\mu,\mathrm{t})\right)\Big)$$

where

$$\Xi(\nu,\mu,\cdots,\mu,\mathrm{t}) = \max\Bigg\{\mathscr{V}(\nu,\mu,\cdots,\mu,\mathrm{t}),\mathscr{V}(\nu,\tau\nu,\cdots,\tau\nu,,\mathrm{t}),$$

$$\mathscr{V}(\mu,\tau\mu,\cdots,\tau\mu,,\mathrm{t}),\frac{\mathscr{V}(\nu,\tau\nu,\cdots,\tau\nu,,\mathrm{t}),\mathscr{V}(\mu,\tau\mu,\cdots,\tau\mu,,\mathrm{t})}{\mathscr{V}(\nu,\mu,\cdots,\mu,\mathrm{t})}\Bigg\}$$

$$= \mathscr{V}(\nu,\mu,\cdots,\mu,\mathrm{t})$$

Using the properties of $\zeta,\rho,(3.6.1)$ becomes

$$\zeta\big(\mathscr{V}(\nu,\mu,\cdots,\mu,\mathrm{t})\big) \leq \zeta\big(\mathscr{V}(\nu,\mu,\cdots,\mu,\mathrm{t})\big)$$

which implies that $\mathscr{F}\Big(\zeta\big(\mathscr{V}(\nu,\mu,\cdots,\mu,\mathrm{t})\big),\rho\big(\mathscr{V}(\nu,\mu,\cdots,\mu,\mathrm{t})\big)\Big) = \zeta\big(\mathscr{V}(\nu,\mu,\cdots,\mu,\mathrm{t})\big)$ and by definition of $\mathscr{F}$, we must that $\zeta\big(\mathscr{V}(\nu,\mu,\cdots,\mu,\mathrm{t})\big) = 1$ or $\rho\big(\mathscr{V}(\nu,\mu,\cdots,\mu,\mathrm{t})\big) = 1$. Using the properties of ζ and ρ we have that $\mathscr{V}(\nu,\mu,\cdots,\mu,\mathrm{t}) = 1$. Thus, $\nu = \mu$. Hence, τ has a unique fixed point. $\square$

Corollary 3.7. *Let* $(\Upsilon,\mathscr{V},*)$ *be a complete* $\mathscr{V}-FMS$ *and* $\tau:\Upsilon\to\Upsilon$ *be mapping satisfying the inequalities* $\gamma(\nu_1,\nu_2,\cdots,\nu_n,\mathrm{t}) \geq \mathsf{s}$ *and* $n\,\mathsf{s}^{n-1}\mathscr{V}(\nu_1,\tau\nu_1,\cdots,\tau\nu_1,\mathrm{t}) \geq \mathscr{V}(\nu_1,\nu_2,\cdots,\nu_n,\mathrm{t})$

$$\Rightarrow \zeta\big(\mathsf{s}^n\mathscr{V}(\tau\nu_1,\tau\nu_2,\cdots,\tau\nu_n,\mathrm{t})\big) \leq \zeta\big(\Xi(\nu_1,\nu_2,\cdots,\nu_n,\mathrm{t})\big) - \rho\big(\Xi(\nu_1,\nu_2,\cdots,\nu_n,\mathrm{t})\big) \tag{3.7.1}$$

for all $\nu_1,\nu_2,\cdots,\nu_n\in\Upsilon$, *where* ζ,ρ *are alternating distance functions,and*

$$\Xi(\nu_1,\nu_2,\cdots,\nu_n,\mathrm{t}) = min\Bigg\{\mathscr{V}(\nu_1,\nu_2,\cdots,\nu_n,\mathrm{t}),\mathscr{V}(\nu_1,\tau\nu_1,\cdots,\tau\nu_1,\mathrm{t}),$$

$$\mathscr{V}(\nu_2,\tau\nu_2,\tau\nu_3,\cdots,\tau\nu_n,\mathrm{t}),\frac{\mathscr{V}(\nu_1,\tau\nu_1,\cdots,\tau\nu_1,\mathrm{t})\cdot\mathscr{V}(\nu_2,\tau\nu_2,\tau\nu_3,\cdots,\tau\nu_n,t)}{\mathscr{V}(\nu_1,\nu_2,\cdots,\nu_n,\mathrm{t})}\Bigg\}.$$

Assume the accompanying conditions hold:

(1) τ *is a triangular* γ *- admissible type mapping,*

(2) there exist $\nu_0\in\Upsilon$ *such that* $\gamma(\nu_0,\tau\nu_0,\cdots,\tau\nu_0,\mathrm{t}) \geq \mathsf{s}$,

(3) τ *is continuous.*

Then τ *has a fixed point.*

Example 3.8. Let $\Upsilon = [0,\infty)$ with $\mathscr{V}(v_1, v_2, \cdots, v_n, \mathsf{t}) = \exp\Big(-\dfrac{\mathscr{A}(v_1, v_2, \cdots, v_n)}{\mathsf{t}}\Big)$. Clearly $(\Upsilon, \mathscr{V}, *)$ is a complete $\mathscr{V}-FMS$. Define the self function $\tau : \Upsilon \to \Upsilon$ by

$$\tau v = \begin{cases} \dfrac{v}{16}, & v \in [0,1] \\ \\ 5v, & v \in (1,\infty), \end{cases}$$

$$\gamma : \Upsilon \times \Upsilon \times \overbrace{\cdots}^{n-times} \times \Upsilon \times (0,\infty) \to [0,\infty)$$

$$\gamma(v_1, v_2, \cdots, v_n, \mathrm{t}) = \begin{cases} n, & if \;\; v_1, v_2, \cdots, v_n \in [0,1] \\ 0, & if \;\; v_1, v_2, \cdots, v_n \in (1,\infty), \end{cases}$$

and $\zeta, \rho : [0,1] \to [0,1]$ by $\zeta(\mathsf{t}) = \frac{\mathsf{t}}{2}$, $\rho(\mathsf{t}) = \mathsf{t}$, $\iota = 1$, $\mathsf{s} = 2$ and $\mathscr{F}(\mathsf{s},\mathsf{t}) = \mathsf{s} - \mathsf{t}$. τ is γ_2^1 rational type mapping and τ satisfy conditions in Corollary 3.7 with a unique fixed point 0.

3. Conclusion

In this paper,verify the existence of unique fixed point for rational contraction mapping in $\mathscr{V}-$ fuzzy metric spaces by using triangular $\gamma_{\mathsf{s}}^{\iota}$ -admissible type mapping.

Conflict of Interests

The author(s) declare that there is no conflict of interests.

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