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INVESTIGATION OF NUMERICAL SOLUTIONS FOR CHAOTIC LORENZ SYSTEM USING MATHCAD SOFTWARE

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Abstract. In this paper, we investigate the accuracy of numerical solution for chaotic Lorenz system. Mathcad software is used as case-study, built-in algorithms used are Runge-Kutta of fourth order (RK4), Adams backward differential formula (AdamsBDF) and developed series method. The numerical integration that are also checked involves Gauss-quadrature and Simpson's quadrature rules. All the graphical results are showed for all the different numerical methods.

Keywords: Lorenz system; Mathcad® software; Runge-Kutta of fourth order(RK4); series method; AdamsBDF; Simpson rule; Gauss-quadrature rule.

2010 AMS Subject Classification: 74H15, 45G10, 65D32, 65D30, 65G99.

1. INTRODUCTION

Chaos systems are most important theories in applied science. Deterministic system are characterized by sensitive initial conditions,see[1]. Lorenz system is one system that demonstrate chaotic behavior.

Lorenz system behaves like any other family of equations in that it has fixed points, however

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this equation is chaotic for certain parameter values. Semi-analytical methods [2][3][4], are used in science and engineering and to help us understand and approximate many nonlinear equations.

Scientist have used numerical methods in approximating the solution of chaotic systems. [6] used differentiation to the Duffing equation while examine a defined residual function. The idea was applied to different type of mathematical software called Mathcad® while using Duffing equation with periodic excitation as governing equation by [7]. The work of checking the accuracy of numerical solution by means of residual function from mathematical software was initially done by [5].

The idea of this research article is to carry our out a study of investigating numerical solution using two built-in algorithms from Mathcad, namely AdamsBDF and Runge-Kutta of order four (RK-method) using Lorenz system as governing equation. The investigation of numerical solution using series method is also carried out.

2. General Lorenz System

We consider the Lorenz system in general form

$$\begin{aligned} \frac{dx}{dt} &= a(y(t)-x(t)) \\ \frac{dy}{dt} &= bx(t)-cy(t)-dx(t)z(t) \\ \frac{dz}{dt} &= fx(t)y(t)-gz(t) \end{aligned} \tag{1}$$

where a, b, c, d, f, g, are positive scalar parameters. The system has three steady states

$$\begin{pmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{pmatrix} = \begin{pmatrix} 0 & \sqrt{\frac{g(b-c)}{df}} & -\sqrt{\frac{g(b-c)}{df}} \\ 0 & \sqrt{\frac{g(b-c)}{df}} & -\sqrt{\frac{g(b-c)}{df}} \\ 0 & \frac{b-c}{d} & \frac{b-c}{d} \end{pmatrix} \tag{2}$$

Eigenvalues obtained from (1) by its linearization in vicinities of steady-states in (2) are found from characteristic equations with respect to λ:

$$det\begin{pmatrix} -a-\lambda & a & 0 \\ b & -c-\lambda & 0 \\ 0 & 0 & -g-\lambda \end{pmatrix} = 0 \tag{3}$$

for $x_1 = y_1 = z_1 = 0$ and

$$det\begin{pmatrix} -a-\lambda & a & 0 \\ c & -c-\lambda & \pm\sqrt{\frac{dg(b-c)}{f}} \\ \pm\sqrt{\frac{fg(b-c)}{d}} & \pm\sqrt{\frac{fg(b-c)}{d}} & -g-\lambda \end{pmatrix} = 0 \tag{4}$$

For $x_{2,3} = \pm\sqrt{\frac{g(b-c)}{df}}$, $y_{2,3} = \pm\sqrt{\frac{g(b-c)}{df}}$, $z_{2,3} = \frac{b-c}{d}$.

In this paper we consider accuracy of numerical solution of Lorenz system (1) at classical values for parameters $a = 10$, $b = 28$, $c = d = f = 1$, $g = \frac{8}{3}$. In this case all three steady-states (2) are unstable with the corresponding eigenvalues:

where $i^2 = -1$. Hence, all the three steady-states are unstable.

At the iniitial condition

$$\lambda_1^1 \approx 11.828 \qquad \lambda_2^1 \approx -2.667 \qquad \lambda_3^1 \approx -22.828$$
$$\lambda_{1,2}^{2,3} \approx 0.099 \pm 10.195i \qquad \lambda_3^{2,3} \approx -13.858 \tag{5}$$

$\begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ solution of system in (1) obtained by the adapted Runge-Kutta method at tolerance of 10^{-15} at $t \in [0, T = 50]$ is shown in figure 1.

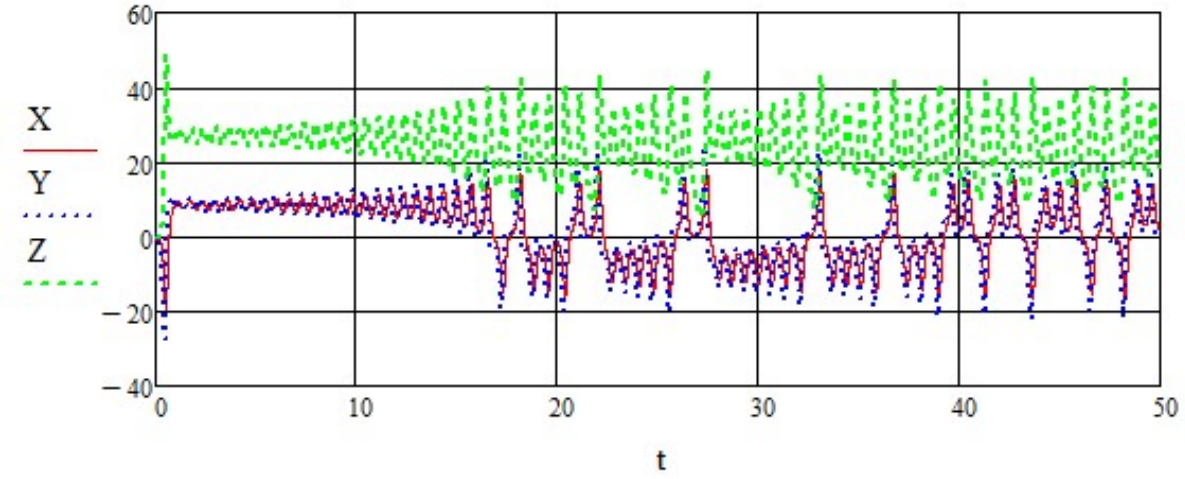


FIGURE 1. Solution of the Lorenz system for $t \in [0, T = 50]$ where $X = x(t)$, $Y = y(t), Z = z(t)$.

For comparison with the same initial value problem (IVP) was solved using the AdamsBDF (Adams backward differential formula) solver at the same tolerance of 10^{-15}. Trajectories of $x(t)$ solutions obtained by the adapted Runge-Kutta and AdamsBDF solvers with tolerance 10^{-15} are showed in figure 2 on the interval $t \in [0, T = 50]$.

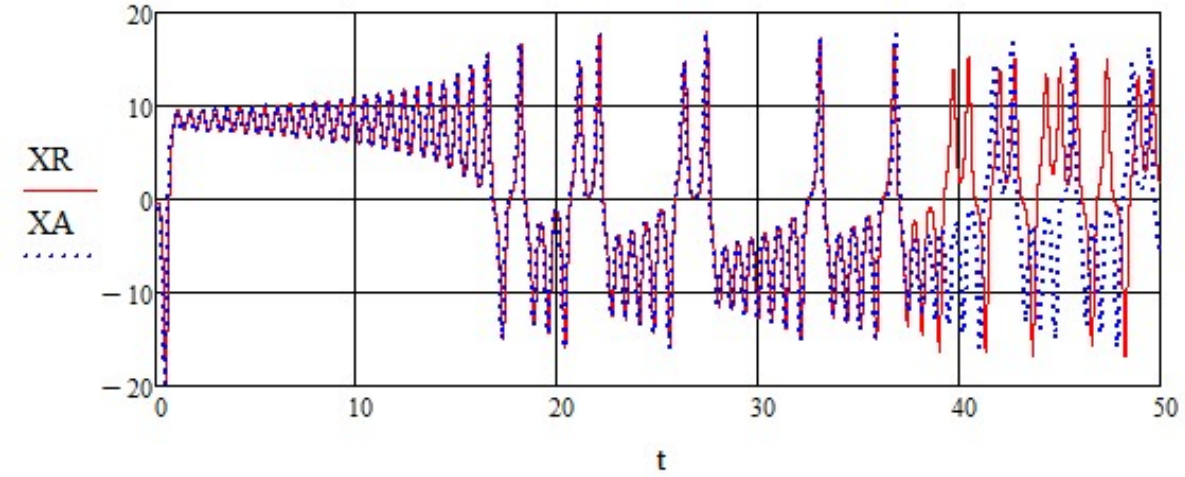


FIGURE 2. Solution of the Lorenz system for $t \in [0, T = 50]$ where $XA = xA(t)$, is the solution of x-variable, obtained bu the AdamsBDF, $XR = xR(t)$ is the solution for x-variable, obtained by the adapted Runge-Kutta method.

Figure 2 we observe, divergence in trajectories at approximately $t = 35$ due to extreme sensitivity of the Lorenz system to initial conditions. This sensitivity is stipulated by above mentioned instability of the steady states. Analogous is the behavior of $y(t)$ and $z(t)$ trajectories. Despite of the trajectory divergence the integral manifold of system (1) Lorenz attractor obtained by both methods is of solution is stable and has well-known "butterfly-shaped "form depicted in below figure 3.

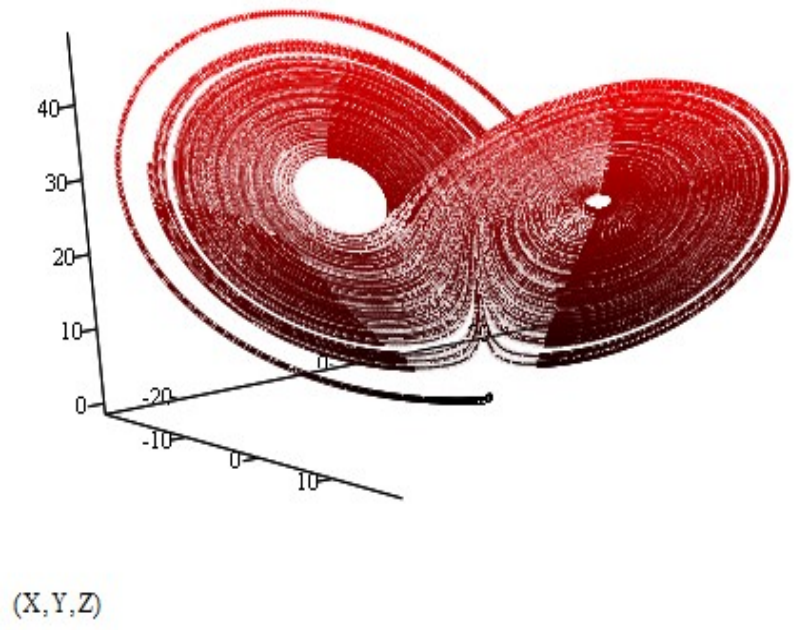


FIGURE 3. Lorenz attractor for $t \in [0, T = 75]$.

Accuracy of the above-mentioned initial value problem (IVP) solution depends on the step of integration h and values of higher derivatives of functions $x(t)$, $y(t)$ and $z(t)$. In the process of solution these derivatives can be found iteratively by means of differentiation of both sides of equations (1), which givens us the following formulas

$$
\begin{aligned}
D^{(m+1)}x &= a[D^m y - D^m x] \\
D^{(m+1)}y &= bD^m x - cD^m y - d\sum_{k=0}^{m} C_m^k D^{(m-k)} x D^k z \qquad (6) \\
D^{(m+1)}z &= f\sum_{k=0}^{m} C_m^k D^{(m-k)} x D^k y - gD^m z.
\end{aligned}
$$

where $D^m u = \frac{d^m u}{dt^m}$, $\quad u = u(t) = (x(t), y(t), z(t))$, $\quad C_m^k = \frac{m!}{k!(m-k)}$, $\quad 0 \le k \le m = 0, 1, 2, , ...$

Graphs of maxima of the fourth, fifth, sixth and seventh derivatives are shown in figure 4 for $t \in [0, 25]$. In the figure max_LDm=max $(|D^m x|, |D^m y|, |D^m z|)$ for $m = 4, 5, 6, 7$.

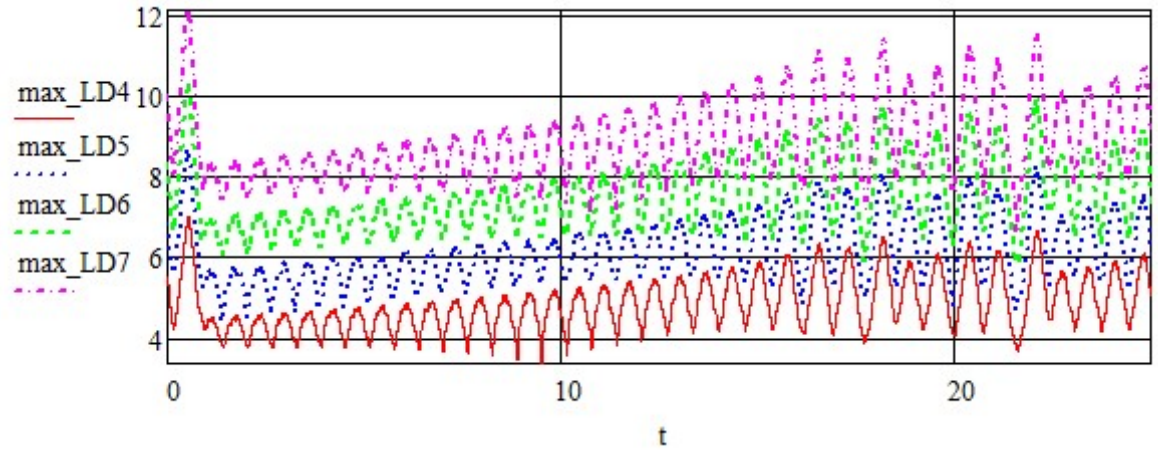


FIGURE 4. Values of maxima of the fourth, fifth, sixth and seventh derivatives pf the Lorenz system for $t \in [0, T = 25]$.

It follows from the this figure that maximum values of the higher derivatives are realized in the vicinity of the first spikes of function $x(t)$, $y(t)$ and $z(t)$ at $t = 1.5$ and have approximate values:

$$\begin{aligned} max\,(max\_LD4) \approx 10^{7} \qquad &, max\,(max\_LD5) \approx 10^{8.5}, \\ max\,(max\_LD6) \approx 10^{10.3} \qquad &, max\,(max\_LD7) \approx 10^{12.2}, \end{aligned} \tag{7}$$

Accuracy of numerical solution of initial value problems depend on highest derivatives of the obtained solution. For the Runge-Kutta method of fourth order $(RK4\_method)$ with fixed step h error of solution can be approximated defined as :

$$|RK\_Error| \approx \frac{h^5}{120} \times max\_LD5. \tag{8}$$

In the case of the Taylor series method of solution with seven terms (which will be used in the future simulations) the highest derivative is the six and the error of solution is estimated as:

$$|Series\_Error| \approx \frac{h^7}{7!} \times max\_LD7. \tag{9}$$

Global errors of the initial value problem solution will be estimated as follows:

$$
\begin{aligned}
|\Delta x(t)| &= |x(t) - x_0 - a\int_0^t [y(\tau) - x(\tau)]\, d\tau| \\
|\Delta y(t)| &= |y(t) - y_0 - \int_0^t [bx(\tau) - cy(\tau) - dx(\tau)z(\tau)]\, d\tau| \\
|\Delta z(t)| &= |z(t) - z_0 - \int_0^t [fx(\tau)y(\tau) - gz(\tau)]\, d\tau|
\end{aligned}
$$

(10)

We used the term “global errors ”because we relate solution, obtained at time t to the initial time instant $t_0 = 0$. In “ideal case”when solution are “exact ”the global errors must be identically equal to zero. Hence, the global errors of $x(t)$, $y(t)$, $z(t)$ solutions can be characterized by expression (10).

As we see from (10) estimation of initial value problem solution accuracy depends not only on accuracy of the obtained $x(t)$, $y(t)$ and $z(t)$ functions, but also on accuracy of numerical integrations, which are performed by approximate quadrature formulas. It is known that error of the Simpson rule is calculated as:

$$
|Simpson\_Error| \approx \frac{h^5}{90} \times max\_LD4. \tag{11}
$$

The most accurate method of numerical integration of smooth functions is the Gauss quadrature with error:

$$
|Gauss\_Error| \approx \frac{2^{(2n+1)} \times \left(n!\right)^4 \times (h)^{(2n+1)}}{(2n+1) \times [(2n)]^3} \times max\_LD2n. \tag{12}
$$

In this case n points are located on interval $[-h, +h]$ with coordinates $h\xi_k$ $(k = 0, 1, 2, ...n)$, where ξ_k are the roots of Legendre polynomials $P_n(\xi) = 0$. In the future we will use the $n = 3$-points on interval $t \in [0, 2h]$ with the following: $\xi_1 = h\left[1 - \sqrt{\frac{3}{5}}\right]$, $\xi_2 = h$, $\xi_3 = h\left[1 + \sqrt{\frac{3}{5}}\right]$. Values of solution are calculated in there three points and multiplied by the weights $w_1 = \frac{5}{9} = w_3$, $w_2 = \frac{8}{9}$ and summed. In this case the error of integral calculations in accordance with (12) is :

$$
|Simpson\_Error|_{n=3} \approx \frac{h^7}{15750} \times max\_LD6
$$

From formulas (8)-(9) and (11) - (12a) it follows that is is worthwhile to select steps, which quadrature the corresponding error as follows:

- for the fixed step RK4-method

$$h\_R \approx \left(\frac{120 \times |RK4 - Error|}{max\_LD5} \right)^{\frac{1}{5}}; \tag{13}$$

- for the Taylor series (with seven terms):

$$h\_T \approx \left(\frac{7! \times |Series - Error|}{max\_LD7} \right)^{\frac{1}{7}}; \tag{14}$$

- for Simpson's quadrature rule:

$$h\_S \approx \left(\frac{90 \times |Simpson - Error|}{max\_LD4} \right)^{\frac{1}{5}}; \tag{15}$$

- for Gauss's quadrature rule (with three points):

$$h\_G \approx \frac{1}{2} \left(\frac{7\,(6!)^3 \times |Gauss - Error|}{(3!)^4\, max\_LD4} \right)^{\frac{1}{7}} \tag{16}$$

For example, it follows from figure 4 that $max\,(max\_LD4) \approx 10^7$, $max\,(max\_LD5) \approx 10^{8.5}$, $max\,(max\_LD6) \approx 10^{10.3}$, $max\,(max\_LD7) \approx 10^{12.2}$. Hence, from equations (13) - (16) the corresponding minimum steps at $|Error| \approx 10^{-17}$ are

$$min\,(h\_R) \approx 2.0 \times 10^{-5}, \qquad min\,(h\_T) \approx 3.3 \times 10^{-4},$$
$$min\,(h\_S) \approx 4.0 \times 10^{-5}, \qquad min\,(h\_G) \approx 5.0 \times 10^{-9}.$$

This means that it is possible that select steps of the Series-method and the subsequent Gauss integration method with approximately order of magnitude larger steps in comparison with the corresponding RK-method with subsequent Simpson's integration. On the basis of figure 4 and formula (14) and (16) we calculated step which quadrature errors of series-method calculations $|Series\_Error| \approx 10^{-16}$ and Gauss-rule calculations $|Gauss\_Error| \approx 10^{-18}$ for the time interval $t \in [0, 25]$. The errors are shown in below figure 5.

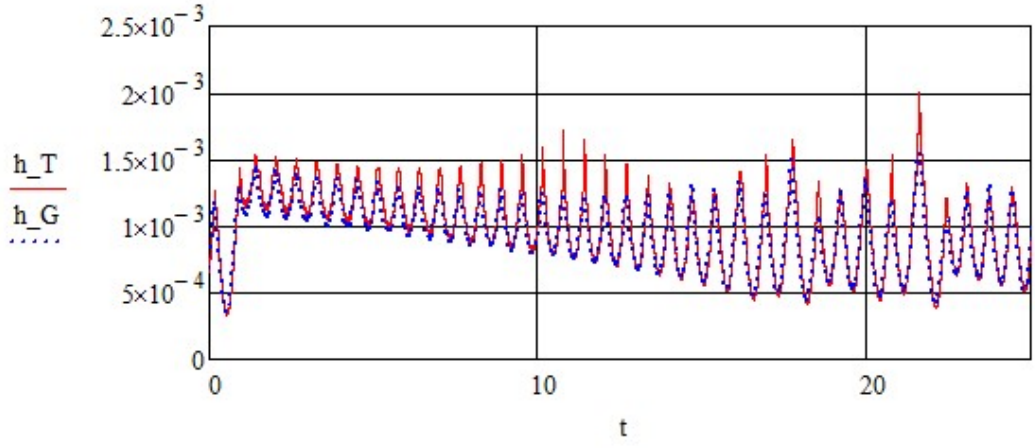


FIGURE 5. Values of steps which guarantee errors of estimation by the Taylor series (h_T) with residual term 10^{-16} Gauss quadrature method (h_G) with error 10^{-18} for $t \in [0, T = 25]$.

This graph shows that for the Taylor series calculations with seven terms which guarantee accuracy $\approx 10^{-16}$ it is necessary to perform calculations with variable step h, which is selected by formula (16) and guarantees accuracy of numerical integration of order $\approx 10^{-18}$ i.e it does not deteriorate accuracy of initial value problem solution. It is necessary to keep in mind that in Math-cad software numerical calculations are performed with sixteen decimal places and hence, the rounding errors are substantial. That is why there are no reasons to perform calculations with accuracy higher that 10^{-17}. As it is follows from figure 5 the 3-points Gauss quadrature rule guarantees accuracy of subsequent integrations. The 3-point Gauss integration of the errors equations of the Lorenz system are as follows:

$$\Delta Sx = x4_0 - x0_0 - ha\Big\{[w_1(x1_1+x3_1)+w_2x2_1] - [w1(x1_0+x3_0)+w_2x2_1]\Big\}$$

$$\Delta Sy = x4_1 - x0_1 - h\Big\{b[w_1(x1_0+x3_0)+w_2x2_0] - c[w1(x1_1+x3_1)+w_2X2_1] - d[w1(x1_0x1_2+x3_0x3_2)+w_2x2_0x2_2]\Big\}$$

$$\Delta Sz = x4_2 - x0_2 - h\Big\{f[w_1(x1_0x1_1+x3_0x3_1)+w_2x2_0x2_1] - g[w1(x1_2+x3_2)+w_2X2_2]\Big\} \tag{17}$$

where $w_1 = w_3 = \frac{5}{9}$, $w_2 = \frac{8}{9}$ are weights of 3-points Gauss quadrature rule, $x1, x2, x3, x4$ are (3×1)-vectors are defined in the next formula, indies "0 ", "1 "and "2 "relate to functions $x = x(t)$, $y = y(t)$ and $z = z(t)$ correspondingly. These expressions characterize local accuracy of particular ODE-Solver with

respect to x, y and z-functions. Algorithm of the Lorenz system IVP solver on time interval $t \in [0, T]$ with adaptive step and local estimator of ODE-solver is as follows:

$$R = t_0 \leftarrow 0$$

$$\tau \leftarrow 0$$

$$s \leftarrow (0 \quad 0 \quad 0)$$

$$R \leftarrow argument\left(t, x^T, s\right) xis(3 \times 1)\text{-vectors io ICS } \tau < T$$

$$x0 \leftarrow x$$

$$\left(Dx^0 \quad Dy^0 \quad Dz^0\right) \leftarrow x$$

$$for\text{m=0,1,2,.....M=6}$$

$$"Evaluation of higher derivatives by (6)"$$

$$"Evaluation of step\text{h} by (16)"$$

$$h1 \leftarrow h\left(1 - \sqrt{\frac{3}{5}}\right)$$

$$x1 \leftarrow ODE_Solver(x, 0, h1, RHSof(1), ToL)$$

$$x2 \leftarrow ODE_Solver(x, 0, h, RHSof(1), ToL)$$

$$x \leftarrow x2$$

$$"Evaluation of higher derivatives by (6)"$$

$$h2 = h\sqrt{\frac{3}{5}}$$

$$x3 \leftarrow ODE_Solver(x, 0, h2, RHSof(1), ToL)$$

$$x4 \leftarrow ODE_Solver(x, 0, h, RHSof(1), ToL)$$

$$x \leftarrow x4$$

$$t_0 \leftarrow t_0 + 2h$$

$$"(\Delta sx \ \Delta sy \ \Delta sz) \leftarrow 3 - points Gauss integration(17)"$$

$$R \leftarrow stack\left(R \ argument\left(t, x^T, S\right)\right)$$

)

In the expression ODE-Solver $(x,0,h,RHSof(1),ToL)$ is a particular solver for initial value problem, where "RHS of (1)"is the right hand side of Lorenz system (1) and x is (3×1)-vector of initial conditions. It solves the system of equations on interval $t\in[0,h]$ with tolerance "ToL ". If the tolerance is not specified the solver uses default value of $Tol=10^{-6}$. The Taylor series ODE-solver is used in the Horner scheme form as follows:

$$\begin{aligned}
X(t=h) &= x_0+h\left(D^{(1)}x_0+\frac{h}{2}\left(D^{(2)}x_0+\frac{h}{3}\left(D^{(3)}x_0+\frac{h}{4}\left(D^{(4)}x_0+\frac{h}{5}\left(D^{(5)}x_0+\frac{h}{6}D^{(6)}x_0\right)\right)\right)\right)\right),\\
Y(t=h) &= y_0+h\left(D^{(1)}y_0+\frac{h}{2}\left(D^{(2)}y_0+\frac{h}{3}\left(D^{(3)}y_0+\frac{h}{4}\left(D^{(4)}y_0+\frac{h}{5}\left(D^{(5)}y_0+\frac{h}{6}D^{(6)}y_0\right)\right)\right)\right)\right),\\
Z(t=h) &= z_0+h\left(D^{(1)}z_0+\frac{h}{2}\left(D^{(2)}z_0+\frac{h}{3}\left(D^{(3)}z_0+\frac{h}{4}\left(D^{(4)}z_0+\frac{h}{5}\left(D^{(5)}z_0+\frac{h}{6}D^{(6)}z_0\right)\right)\right)\right)\right),
\end{aligned}$$

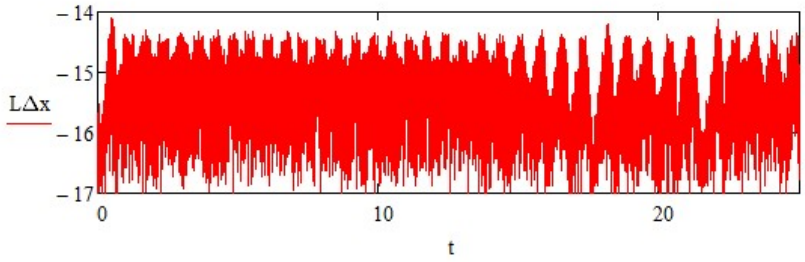
(19)

where $x_0=x(t=0)$, $D^m x_0=\frac{d^m x(t)}{dt^x}|_{t=0}$, $m=1,2,.....,6$. As we mentioned before, real error of calculation is slightly higher than that, which is estimated by (14) due to truncation errors accumulations. That is why it is not recommended to add other terms in the series, which have orders of magnitude less than 10^{-17}. In this case the accuracy of integration must be of order $10^{-17}\div 10^{-18}$. The real local errors of numerical solution of the above-mentioned Lorenz initial value problem are calculated in the decimal logarithmic scale as follows:

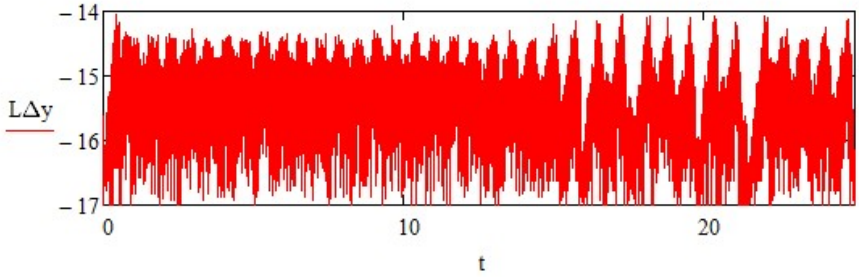
$$\begin{aligned}
L\Delta x(t) &= Log\left|x_{t+2h}-x_t-a\int_0^{t+2h}[y(\tau)-x(\tau)]\,d\tau\right|\\
L\Delta y(t) &= Log\left|y_{t+2h}-y_t-\int_0^{t+2h}[bx(\tau)-cy(\tau)-dx(\tau)z(\tau)]\,d\tau\right|\\
L\Delta z(t) &= Log\left|z_{t+2h}-z_t-\int_0^{t+2h}[fx(\tau)y(\tau)-gz(\tau)]\,d\tau\right|
\end{aligned}$$

(20)

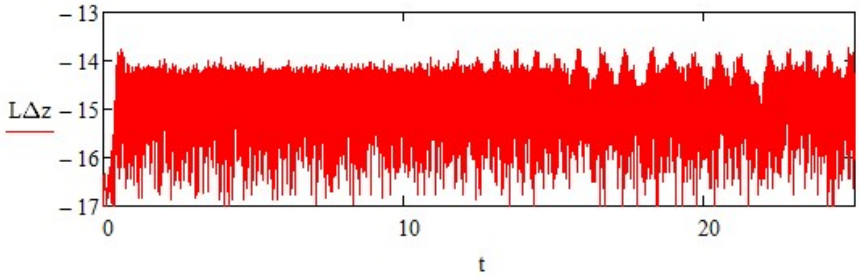
For $t\in[0,T-2h]$ where $h=h(t)$ is the variable step. The local errors in the decimal logarithmic scale of solution of the above mentioned initial valued problem by adapted Runge-Kutta method with tolerance $Tol=10^{-15}$



(a) Local errors of the Lorenz system solution by the adapted RK-method with tolerance 10^{-15} for $x = x(t)$ for $t \in [0,25]$.



(b) Local errors of the Lorenz system solution by the adapted RK-method with tolerance 10^{-15} for $y = y(t)$ for $t \in [0,25]$



(c) Local errors of the Lorenz system solution by the adapted RK-method with tolerance 10^{-15} for $z = z(t)$ for $t \in [0,25]$

FIGURE 6. comparison of adapted RK-method on with 10^{-15}

As we see from figures $(6a)$ to $(6b)$ local errors of calculations of $x(t)$ and $y(t)$ are of order 10^{-15} and 10^{-14} while error of calculation for $z(t)$ figure $(6c)$ is of order 10^{-14}.The global errors of functions $x(t), y(t), z(t)$ calculations in the decimal logarithmic scale can be estimated using formula (10) as follows.

$$
\begin{aligned}
LSx(t) &= Log\left|x(t)-x_0-a\int_0^t [y(\tau)-x(\tau)]\,d\tau\right| \\
LSy(t) &= Log\left|y(t)-y_0-\int_0^t [bx(\tau)-cy(\tau)-dx(\tau)z(\tau)]\,d\tau\right| \qquad (21) \\
LSz(t) &= Log\left|z(t)-z_0-\int_0^t [fx(\tau)y(\tau)-gz(\tau)]\,d\tau\right|
\end{aligned}
$$

These errors are displayed in figure 7 for $t \in [0,25]$

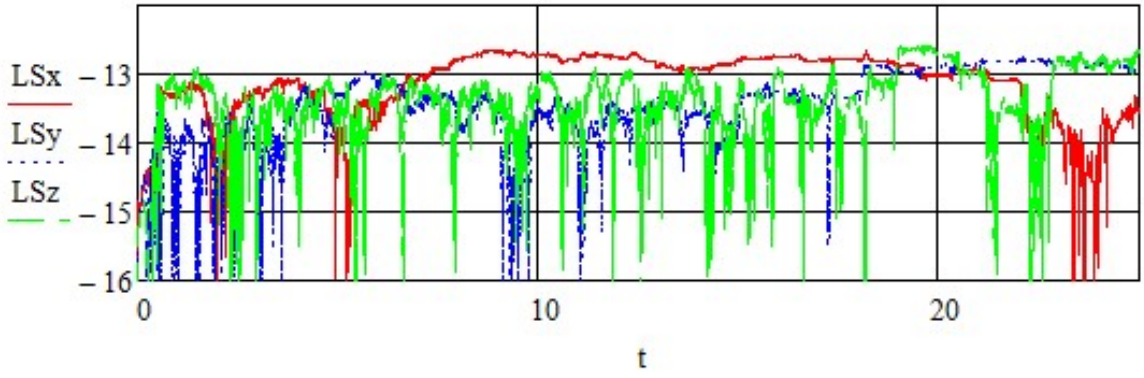
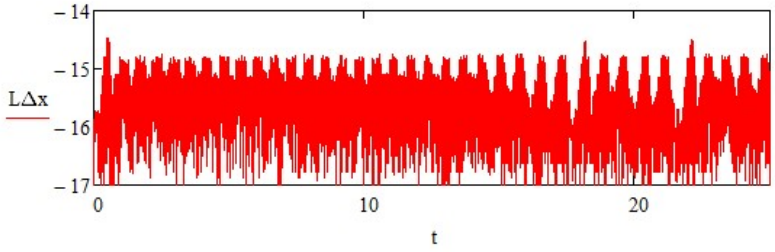
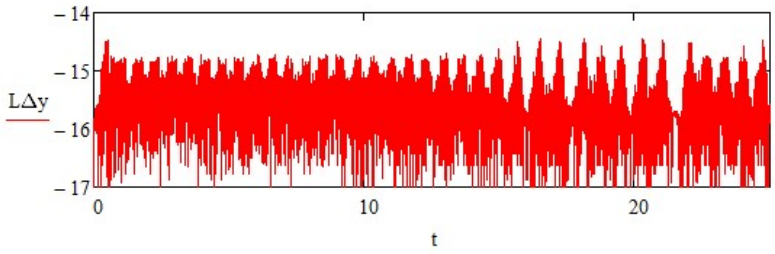


FIGURE 7. Global (accumulated) errors of the Lorenz system solution by the adapted RK-method with tolerance 10^{-15} with $x = x(t), y(t), z = z(t)$ for $t \in [0, T = 25]$.

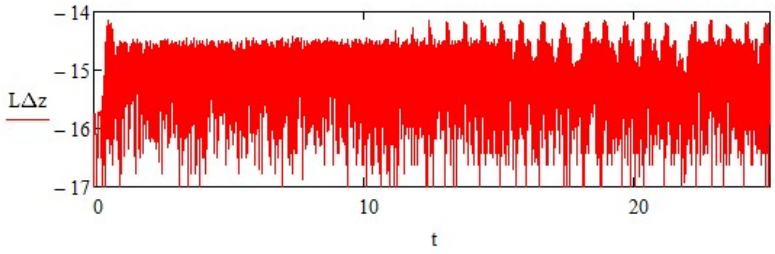
As it follows from this figure 7 the global errors of calculations of $x = x(t), y(t), z = z(t)$ functions are of order $10^{-14} - 10^{-13}$ on the above mentioned time interval. For $T > 25$ the local errors of the solutions have the same order of magnitude as figures $6(a) - 6(c)$, but the global errors are increasing due to random drifts stipulated by the truncation errors. Analogous behavior is demonstrated by the local and global errors of solutions obtained by series method. The results of computational errors obtained by the equations (17) to (19) are shown in figures (8a) to (8c) and figure 9. The local errors of computations have order of magnitude $10^{-15.5} \div 10^{-14}$ for $x(t), y(t)$ functions and $10^{-15} \div 10^{-14}$ for $z(t)$ function and hence slightly better than for the adapted RK-method. Nevertheless, the global errors of solutions obtained by the series-method, are of the same order of magnitude as solutions obtained by the adapted RK-method.



(a) Local errors of the Lorenz system solution by the series method for $x = x(t)$ for $t \in [0,25]$.



(b) Local errors of the Lorenz system solution by the series method for $y = y(t)$ for $t \in [0,25]$



(c) Local errors of the Lorenz system solution by the series method for $z = z(t)$ for $t \in [0,25]$

FIGURE 8. comparison of series method on the interval $t \in [0,25]$

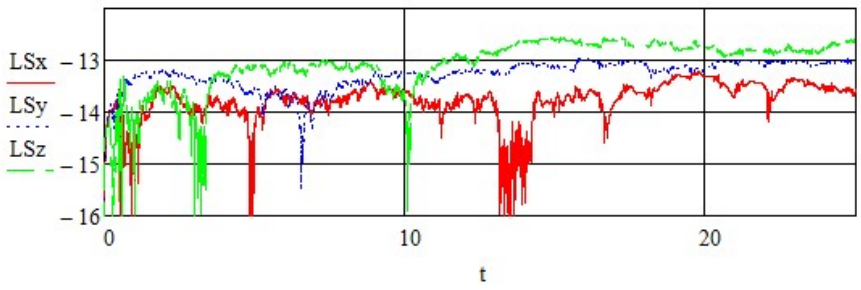
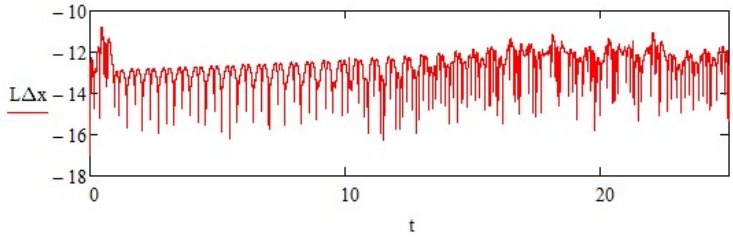
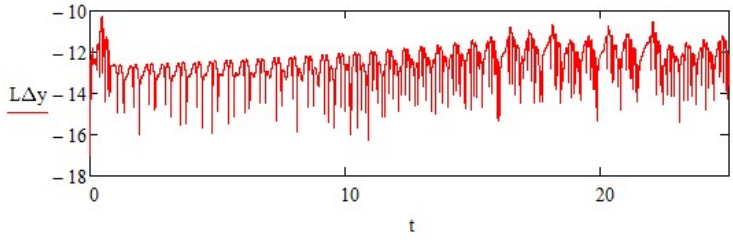


FIGURE 9. Global (accumulated) errors of the Lorenz system solution by the series method with $x = x(t), y(t), z = z(t)$ for $t \in [0, T = 25]$.

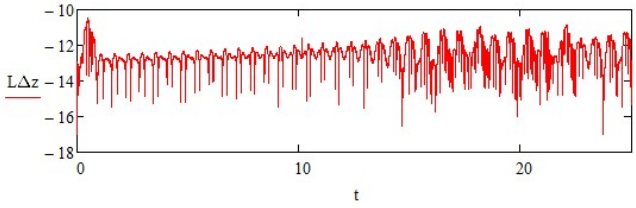
The advantage of the series method of solution in comparison with adapted RK-method in the frames of algorithm (18) is that the series method is two-to-three times faster than the adapted RK-method. Let us now consider the local and global errors of solution obtained by the hybrid AdamsBDF-method. This method is very popular and broadly used because it automatically select regimes for soft and stiff solutions. Moreover this is the default method for ODESolve routine in Math-cad software. The local errors of the initial value problem solution obtained by the AdamsBDF method with tolerance 10^{-15} are shown in figures 10*a*,10*b* and 10*c* in the decimal logarithmic scale. It follows from there graphs that the local errors obtained the AdamsBDF method are of two-three orders of magnitude larger that those obtained by series and adapted Runge-Kutta methods.



(a) Local errors of the Lorenz system solution by the adapted AdamsBDF-method with tolerance 10^{-15} for $x = x(t)$ for $t \in [0,25]$..



(b) Local errors of the Lorenz system solution by the adapted AdamsBDF-method with tolerance 10^{-15} for $y = y(t)$ for $t \in [0,25]$.



(c) Local errors of the Lorenz system solution by the adapted AdamsBDF-method with tolerance 10^{-15} for $z = z(t)$ for $t \in [0,25]$.

FIGURE 10. Comparison of AdamsBDF method on the interval $t \in [0,25]$

The global errors of the AdamsBDF-solution are shown in figure 11. Comparison of this figures 7 and 9 shows that the errors, of global solution obtained by the AdamsBDF-method are, analogously to the local errors, two to three orders of magnitude larger that those obtained by either series or adapted Runge-Kutta method.

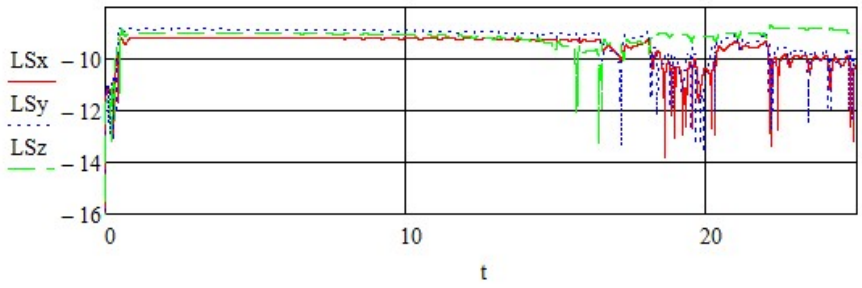


FIGURE 11. Global (accumulated) errors of the Lorenz system solution by the AdamsBDF method with tolerance 10^{-15} for $x = x(t), y(t), z = z(t)$ for $t \in [0, T = 25]$.

In simulations shows in figures 7,9, 11 the steps were the same. Time of evaluation of algorithms (18) with the AdamsBDF-method is three to four times longer that the series-method. If the tolerance of the AdamsBDF-method is not specified the local and global errors of solution are approximately two orders of magnitude larger than for tolerance $Tol = 10^{-15}$. This is illustrated by figure 12, which shows errors of solution obtained by the AdamsBDF-method with default accuracy.

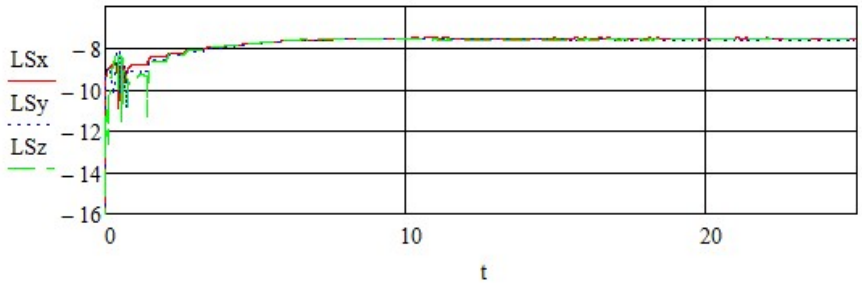


FIGURE 12. Global (accumulated) errors of the Lorenz system solution by the AdamsBDF method with default tolerance for $x = x(t), y(t), z = z(t)$ for $t \in [0, T = 25]$.

3. SUMMARY

In this work, After comparing three numerical procedure from Mathcad using the local and global errors on numerical solution of the Lorenz equation. The critical finding indicates it always advisable to compare two or more numerical procedures when using numerical techniques from built-in algorithms to approximate solutions. Gauss-quadrature methods are the best numerical techniques when approximating integrals numerically. Through investigation of figures 10a, 10b, 10c to 12 shows that initial spike of solution at $t \approx 1.5$ substantially deteriorate global accuracy of solution. This means that the AdamsBDF-method cannot be recommended for solution of chaotic problems, in particular the Lorenz system.

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CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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