Available online at http://scik.org
J. Math. Comput. Sci. 11 (2021), No. 6, 8211-8220
https://doi.org/10.28919/jmcs/6795
ISSN: 1927-5307

RELAXATION TECHNIQUE FOR SOLVING FUZZY LINEAR SYSTEMS OF LINEAR FUZZY REAL NUMBERS

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Abstract. The purpose of this paper is to provide an efficient and practical algorithm for solving fuzzy linear systems of linear fuzzy real numbers. Accuracy and efficiency of the method based on relaxation technique are reported in numerical results and compared with the existing fuzzy iterative method. The fuzzy solution sequences of a fuzzy system in the two different methods were visualized and compared in the three-dimensional graphs. Numerical experiments have shown that the new fuzzy method is very efficient and accurate for solving fuzzy linear systems.

Keywords: fuzzy linear system; linear fuzzy real number; iterative method.

2010 AMS Subject Classification: 26E50.

1. INTRODUCTION

Since many real-world systems of equations, such as in mathematics, statistics, social sciences, economics, finance, and engineering, are too complicated to be defined in precise terms, uncertainty is often needed. The concept of fuzzy number and its arithmetic operations were initially introduced by Zadeh [13]. Later, Friedman *et al.* [3] proposed a general model for

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This research was supported by Kumoh National Institute of Technology(2019-104-111).

Received September 17, 2021

solving a fuzzy linear system. Since then, many researches have been done for solving fuzzy linear systems.

Ghanbari and Nuraei [5] applied the homotopy analysis method for solving fuzzy linear systems. Gani and Assarudeen [4] used an algorithmic approach using Fourier Motzkin elimination method. Nasseri and Sohrabi [9] worked on the Householder method for solving fuzzy systems. Mihailović *et al.* [7] presented a method using the block representation of generalized inverses. Malkawi *et al.* [8] porposed implicit Gauss-Cholesky algorithm for solving fuzzy systems. Yin and Wang [12] studied the splitting iterative methods for fuzzy system. Wang and Zheng [11] considered block iterative methods for fuzzy linear systems. Allahviranloo [1] used embedding approach to find non-zero fuzzy number solutions.

One of several different representations of fuzzy number is a linear fuzzy real number [6, 10]. Iterative methods [2] are known to be very efficient for solving large and sparse linear systems. In this paper, a fast iterative algorithm based on relaxation technique is presented for solving fuzzy systems of linear equations with crisp coefficients over linear fuzzy real numbers. We provide numerical and graphical solutions of a fuzzy linear system using the new method.

The paper is organized as follows. In Section 2, we provide some preliminary definitions on the linear fuzzy real numbers. In Section 3, we present a fast iterative algorithm followed by numerical experiments. Lastly, we will make concluding remarks in Section 4.

2. LINEAR FUZZY REAL NUMBERS

As preliminaries, we review some definitions of linear fuzzy real numbers [6, 10] which are used in this paper. We begin to define a linear fuzzy real number with an associated triple of real numbers as follows.

Definition 2.1. [6] (Linear fuzzy real number). Let R be the set of all real numbers. For some real numbers a, b, c, let $\mu : R \to [0,1]$ be a function defined by

$$\mu(x) = \begin{cases} 0, & \text{if } x < a \text{ or } x > c, \\ \frac{x-a}{b-a}, & \text{if } a \leq x < b, \\ 1, & \text{if } x = b, \\ \frac{c-x}{c-b}, & \text{if } b < x \leq c. \end{cases}$$

Then an extended notation $\mu(a,b,c)$ is called a *linear fuzzy real number* with the associated triple of real numbers (a,b,c), where $a \leq b \leq c$, shown in Figure 1. To distinguish it from the set R, the set of all linear fuzzy real numbers is denoted by LFR.

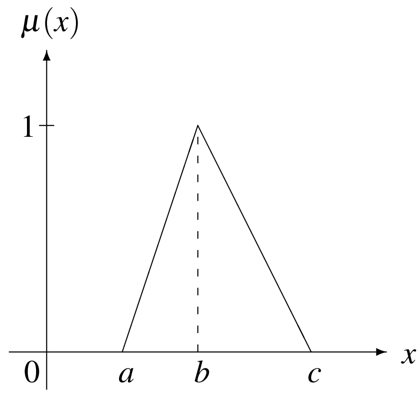


Figure 1. Linear fuzzy real number $\mu(a,b,c)$

Definition 2.2. [6] (Fuzzy arithmetic). Let $\mu_1 = \mu(a_1,b_1,c_1)$ and $\mu_2 = \mu(a_2,b_2,c_2)$ be two linear fuzzy real numbers. Then addition, subtraction, multiplication, and division of μ_1 and μ_2 are linear fuzzy real numbers such that

(1) $\mu_1 + \mu_2 = \mu(a_1+a_2, b_1+b_2, c_1+c_2)$

(2) $\mu_1 - \mu_2 = \mu(a_1-c_2, b_1-b_2, c_1-a_2)$

(3) $\mu_1 \cdot \mu_2 = \mu(\min\{a_1a_2, a_1c_2, a_2c_1, c_1c_2\}, b_1b_2, \max\{a_1a_2, a_1c_2, a_2c_1, c_1c_2\})$

(4) $\frac{\mu_1}{\mu_2} = \mu_1 \cdot \frac{1}{\mu_2}$ where $\frac{1}{\mu_2} = \mu(\min\{\frac{1}{a_2}, \frac{1}{b_2}, \frac{1}{c_2}\}, \text{median}\{\frac{1}{a_2}, \frac{1}{b_2}, \frac{1}{c_2}\}, \max\{\frac{1}{a_2}, \frac{1}{b_2}, \frac{1}{c_2}\})$.

Definition 2.3. [6] (Fuzzy square root). Let $\mu(a,b,c)$ be a linear fuzzy real number, where $a,b,c \geq 0$. Then the square root of $\mu(a,b,c)$ is a linear fuzzy real number which is defined by

$$\sqrt{\mu(a,b,c)} = \mu(\sqrt{a}, \sqrt{b}, \sqrt{c}).$$

Definition 2.4. [6] (Fuzzy sequence). Let $\{\mu^{(k)}\}_{k=0}^{\infty}$ be a sequence in LFR where $\mu^{(k)} = \mu(a^{(k)}, b^{(k)}, c^{(k)})$. The LFR sequence $\{\mu^{(k)}\}$ has the limit $\mu^* = \mu(a^*, b^*, c^*)$ and we write $\lim_{k\to\infty} \mu^{(k)} = \mu^*$, if the sequences $\{a^{(k)}\}$, $\{b^{(k)}\}$, and $\{c^{(k)}\}$ have the limit a^*, b^*, and c^*, respectively. If $\lim_{k\to\infty} \mu^{(k)}$ exists, we say the LFR sequence $\{\mu^{(k)}\}$ is *convergent*. Otherwise, we say the sequence is *divergent*.

Since each real number $t \in R$ is associated with a linear fuzzy real number, denoted by $r(t) = \mu(t,t,t) \in LFR$, we consider $r(t)$ to represent the real number t itself. Thus, we see that $t \cdot \mu(a,b,c) = \mu(t \cdot a, t \cdot b, t \cdot c)$ for $t > 0$.

Definition 2.5. (Fuzzy linear system). Let a_{ij} and b_i be real numbers. Then, for the unknown linear fuzzy real number μ_{x_i},

$$\begin{cases} a_{11}\mu_{x_1} + a_{12}\mu_{x_2} + \cdots + a_{1n}\mu_{x_n} = b_1 \\ a_{21}\mu_{x_1} + a_{22}\mu_{x_2} + \cdots + a_{2n}\mu_{x_n} = b_2 \\ \quad \vdots \qquad\quad \vdots \qquad\qquad\quad \vdots \qquad \vdots \\ a_{n1}\mu_{x_1} + a_{n2}\mu_{x_2} + \cdots + a_{nn}\mu_{x_n} = b_n. \end{cases} \tag{2.1}$$

is called the fuzzy linear system with crisp coefficients over linear fuzzy real numbers.

In matrix notation, we may write the above system as $A\{X\} = \{b\}$, where the coefficient matrix $A = (a_{ij})$, $1 \le i, j \le n$, is a crisp real $n \times n$ matrix, the vector $\{b\} = \{b_i\}$, $1 \le i \le n$, is the crisp real column vector, and the vector $\{X\} = \{\mu_{x_i}\}$, $1 \le i \le n$, is the unknown column vector of linear fuzzy real numbers.

3. Algorithm and Numerical Experiments

In this section, we introduce an efficient iterative algorithm for solving fuzzy linear systems over linear fuzzy real numbers. Relaxation technique to solve the $n \times n$ fuzzy linear system (2.1) over LFR can be done by modification of the crisp successive over relaxation (SOR) method over R whose accuracy and convergence have been proved in [2]. We first start with an initial approximation $\left\{X^{(0)}\right\} = \left\{\mu_{x_i}^{(0)}\right\}$ and then generate a sequence of LFR vectors $\left\{X^{(k)}\right\}_{k=1}^{\infty} = \left\{\mu_{x_i}^{(k)}\right\}_{k=1}^{\infty}$, $1 \le i \le n$, where

$$\mu_{x_i}^{(k)} = \mu_{x_i}^{(k-1)} + \frac{\omega}{a_{ii}}\left[b_i - \sum_{j=1}^{i-1} a_{ij}\mu_{x_j}^{(k)} - \sum_{j=i}^{n} a_{ij}\mu_{x_j}^{(k-1)}\right], \text{ for } i = 1, \cdots, n, \tag{3.1}$$

and a parameter ω. For the sake of simplicity, we assume that the diagonal entries of the coefficient matrix A are all non-zero. Then convergence of the solution sequence of the fuzzy linear system can be seen in the next theorem.

Theorem 3.1. *The sequence* $\left\{X^{(k)}\right\}_{k=1}^{\infty}$ *generated by (3.1) is convergent to the solution* $\{X^*\}$ *of the system (2.1) only if* $0<\omega<2$.

Proof. Suppose that $X^{(k)} = \begin{bmatrix} \mu_{x_1}^{(k)} \\ \mu_{x_2}^{(k)} \\ \vdots \\ \mu_{x_n}^{(k)} \end{bmatrix}$ and $X^* = \begin{bmatrix} \mu_{x_1}^{*} \\ \mu_{x_2}^{*} \\ \vdots \\ \mu_{x_n}^{*} \end{bmatrix}$ be the k-th term of the sequence generated by (3.1) and the solution of the system (2.1), respectively. Let the following linear system of real numbers be the associated crisp linear system corresponding to the fuzzy linear system (2.1) :

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \quad\vdots \qquad\quad \vdots \qquad\qquad\quad\; \vdots \quad\;\; \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n. \end{cases} \tag{3.2}$$

It is well-known [2] that for any initial approximation $\left\{{x_i}^{(0)}\right\}$, the solution sequence $\left\{{x_i}^{(k)}\right\}$ generated by the SOR method of the system (3.2) is convergent to the exact solution $\{{x_i}^*\}$ only if $0<\omega<2$, which is written as $\lim_{k\to\infty} x_i^{(k)} = x_i^*$.

Then for $\mu_{x_i}^{(k)} = \mu(a_i^{(k)}, b_i^{(k)}, c_i^{(k)})$, we get $\lim_{k\to\infty} a_i^{(k)} = x_i^*$, $\lim_{k\to\infty} b_i^{(k)} = x_i^*$, and $\lim_{k\to\infty} c_i^{(k)} = x_i^*$. Therefore,

$$\lim_{k\to\infty} \mu_{x_i}^{(k)} = \lim_{k\to\infty} \mu(a_i^{(k)}, b_i^{(k)}, c_i^{(k)}) = \mu(x_i^*, x_i^*, x_i^*) = \mu_{x_i}^* \text{ for } i = 1, \cdots, n.$$

Thus

$$\lim_{k\to\infty} X^{(k)} = X^*.$$

□

Let D be the diagonal matrix whose diagonal entries are those of A, L be the strictly lower-triangular part of A, and U be the strictly upper-triangular part of A, so that the coefficient matrix of the fuzzy system is written as $A = D + L + U$. As seen in [2], the optimal choice of ω can be selected as

$$\omega = \frac{2}{1 + \sqrt{1 - [\rho(T)]^2}},$$

where $\rho(T)$ is the spectral radius of the matrix $T = -D^{-1}(L+U)$. Now we provide the iterative algorithm based on relaxation technique to solve the fuzzy linear system (2.1) over LFR using (3.1), referred to as the LFR SOR's algorithm.

Algorithm 3.2. (LFR SOR's algorithm)

INPUT: n equations, initial value $\mu_{x_i}^{(0)}$ for all i, parameter ω, integer N

OUTPUT: approximate sol. μ_{x_i} for $i = 1, \cdots, n$.

Step 1: For $k = 1, 2, \cdots, N$ do Step 2.

Step 2: For $i = 1, 2, \cdots, n$ do Step 3.

Step 3: $\mu_{x_i}^{(k)} = \mu_{x_i}^{(k-1)} + \frac{\omega}{a_{ii}} \left[b_i - \sum_{j=1}^{i-1} a_{ij} \mu_{x_j}^{(k)} - \sum_{j=i}^{n} a_{ij} \mu_{x_j}^{(k-1)} \right]$

Step 4: OUTPUT(all $\mu_{x_i}^{(N)}$) and STOP.

Note that since the crisp SOR method at $\omega = 1$ is the same as the Gauss-Seidel(GS) method [2], the LFR SOR method at $\omega = 1$ is referred to as the LFR GS method, which is used as the benchmark method in this paper.

Example 3.3. Consider the following 3×3 fuzzy system of linear equations:

$$\begin{cases} 4\mu_{x_1} + 2\mu_{x_2} - \mu_{x_3} = 8 \\ 2\mu_{x_1} + 4\mu_{x_2} + \mu_{x_3} = 16 \\ -\mu_{x_1} + \mu_{x_2} + 4\mu_{x_3} = 10 \end{cases} \tag{3.3}$$

Let the coefficient matrix A of the system (3.3) be decomposed into $A = D + L + U$ such as

$$\begin{bmatrix} 4 & 2 & -1 \\ 2 & 4 & 1 \\ -1 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ -1 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 2 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

Because

$$T = -D^{-1}(L+U) = -\frac{1}{4} \begin{bmatrix} 0 & 2 & -1 \\ 2 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{2} & \frac{1}{4} \\ -\frac{1}{2} & 0 & -\frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} & 0 \end{bmatrix},$$

we have

$$T-\lambda I=\begin{bmatrix} -\lambda & -\frac{1}{2} & \frac{1}{4} \\ -\frac{1}{2} & -\lambda & -\frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} & -\lambda \end{bmatrix},$$

so

$$\det(T-\lambda I)=-\lambda^3+\frac{3}{8}\lambda+\frac{1}{16}.$$

Thus,

$$\rho(T)=\frac{1}{4}+\frac{\sqrt{3}}{4}\approx 0.6830 \text{ and } \omega=\frac{2}{1+\sqrt{1-[\rho(T)]^2}}\approx 1.1558.$$

In order to solve the fuzzy system (3.3) using the *LFR* SOR's algorithm, we first choose an initial approximation, e.g. $\mu_{x_1}^{(0)}=\mu(0,1,2)$, $\mu_{x_2}^{(0)}=\mu(0,1,2)$, and $\mu_{x_3}^{(0)}=\mu(0,1,2)\in LFR$. Then we generate a solution sequence $\left\{X^{(k)}\right\}_{k=1}^{\infty}$ using the *LFR* SOR's algorithm. As our benchmark, the *LFR* GS method is used to solve the fuzzy system (3.3), too.

In Table 1, we compare the 8 iterations generated by the *LFR* SOR method at ω=1.1558 with the *LFR* GS method. We can see that the solution sequence generated by the *LFR* SOR method converges to the exact solution upto four decimal places within 8 iterations, whereas the *LFR* GS method converges in 13 iterations.

In Figure 2, we provide the three-dimensional graphs to represent the fuzzy solutions, as shown in Table 1, for the system (3.3) generated by the *LFR* GS method and the *LFR* SOR method as visual comparison. We can see that the convergence of the solution sequences from the *LFR* SOR is faster than that from the *LFR* GS. We see that for the iterations to be accurate to four decimal places, the *LFR* GS method requires 13 iterations, as opposed to 8 iterations for the *LFR* SOR.

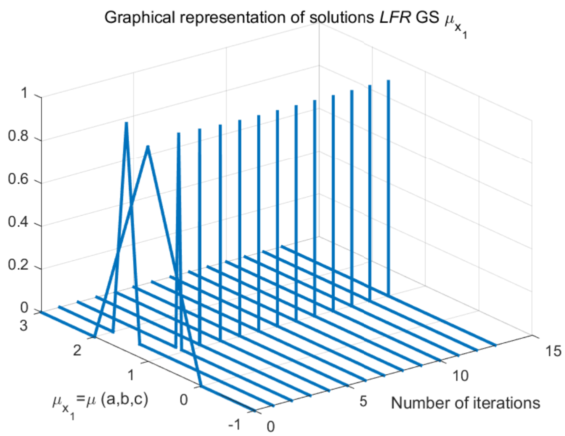
In this research, the coefficient matrix of the fuzzy system is assumed to be a real crisp, whereas an unknown variable vector is set to be linear fuzzy real numbers. In the future, we plan to extend our research to the fuzzy linear systems whose coefficients are fuzzy and work on further mathematical analysis for the algorithm.

4. CONCLUSION

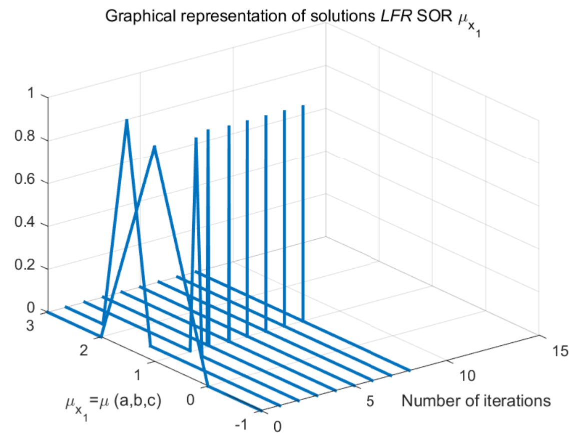
In this paper, we presented a fast iterative algorithm based on relaxation technique for solving fuzzy system of linear equations over linear fuzzy real numbers LFR with a modification of crisp SOR method over real numbers. The numerical experiments show that the LFR SOR method is very efficient and accurate for solving fuzzy linear systems. The fuzzy solution sequences in the two different methods were visualized and compared in the three-dimensional graphs to support the numerical results.

Table 1. Approximate solutions by *LFR* GS $(\omega = 1)$ and *LFR* SOR $(\omega = 1.1558)$

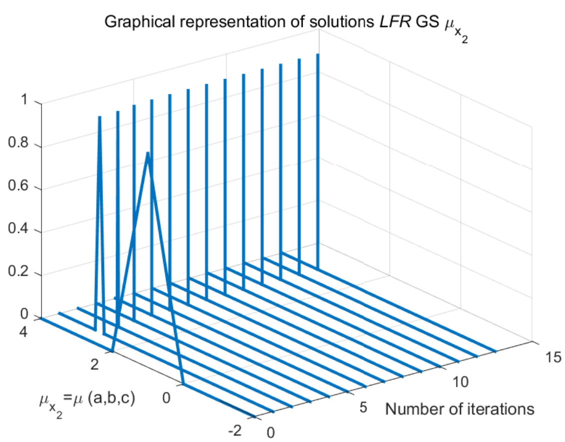
k	Sol. $\mu_{x_i}^{(k)}$ by *LFR* GS	k	Sol. $\mu_{x_i}^{(k)}$ by *LFR* SOR
0	$\mu_{x_1}^{(0)} = \mu(0.0000, 1.0000, 2.0000)$ $\mu_{x_2}^{(0)} = \mu(0.0000, 1.0000, 2.0000)$ $\mu_{x_3}^{(0)} = \mu(0.0000, 1.0000, 2.0000)$	0	$\mu_{x_1}^{(0)} = \mu(0.0000, 1.0000, 2.0000)$ $\mu_{x_2}^{(0)} = \mu(0.0000, 1.0000, 2.0000)$ $\mu_{x_3}^{(0)} = \mu(0.0000, 1.0000, 2.0000)$
1	$\mu_{x_1}^{(1)} = \mu(1.5000, 1.7500, 2.0000)$ $\mu_{x_2}^{(1)} = \mu(2.7500, 2.8750, 3.0000)$ $\mu_{x_3}^{(1)} = \mu(2.1875, 2.2188, 2.2500)$	1	$\mu_{x_1}^{(1)} = \mu(1.4221, 1.8668, 2.3116)$ $\mu_{x_2}^{(1)} = \mu(2.9119, 3.0996, 3.2873)$ $\mu_{x_3}^{(1)} = \mu(2.1474, 2.3775, 2.6076)$
2	$\mu_{x_1}^{(2)} = \mu(1.0625, 1.1172, 1.1719)$ $\mu_{x_2}^{(2)} = \mu(2.8672, 2.8867, 2.9063)$ $\mu_{x_3}^{(2)} = \mu(2.0391, 2.0576, 2.0762)$	2	$\mu_{x_1}^{(2)} = \mu(0.8052, 0.9165, 1.0278)$ $\mu_{x_2}^{(2)} = \mu(2.8923, 2.9237, 2.9551)$ $\mu_{x_3}^{(2)} = \mu(1.8802, 1.9391, 1.9980)$
3	$\mu_{x_1}^{(3)} = \mu(1.0659, 1.0710, 1.0762)$ $\mu_{x_2}^{(3)} = \mu(2.9480, 2.9501, 2.9521)$ $\mu_{x_3}^{(3)} = \mu(2.0295, 2.0302, 2.0310)$	3	$\mu_{x_1}^{(3)} = \mu(1.0211, 1.0395, 1.0580)$ $\mu_{x_2}^{(3)} = \mu(2.9954, 3.0066, 3.0179)$ $\mu_{x_3}^{(3)} = \mu(2.0077, 2.0190, 2.0303)$
4	$\mu_{x_1}^{(4)} = \mu(1.0317, 1.0325, 1.0334)$ $\mu_{x_2}^{(4)} = \mu(2.9759, 2.9762, 2.9764)$ $\mu_{x_3}^{(4)} = \mu(2.0138, 2.0141, 2.0144)$	4	$\mu_{x_1}^{(4)} = \mu(0.9894, 0.9955, 1.0016)$ $\mu_{x_2}^{(4)} = \mu(2.9946, 2.9961, 2.9976)$ $\mu_{x_3}^{(4)} = \mu(1.9938, 1.9969, 2.0000)$
5	$\mu_{x_1}^{(5)} = \mu(1.0154, 1.0154, 1.0155)$ $\mu_{x_2}^{(5)} = \mu(2.9887, 2.9888, 2.9888)$ $\mu_{x_3}^{(5)} = \mu(2.0067, 2.0067, 2.0067)$	5	$\mu_{x_1}^{(5)} = \mu(1.0012, 1.0021, 1.0030)$ $\mu_{x_2}^{(5)} = \mu(2.9997, 3.0003, 3.0009)$ $\mu_{x_3}^{(5)} = \mu(2.0004, 2.0010, 2.0016)$
11	$\mu_{x_1}^{(11)} = \mu(1.0002, 1.0002, 1.0002)$ $\mu_{x_2}^{(11)} = \mu(2.9999, 2.9999, 2.9999)$ $\mu_{x_3}^{(11)} = \mu(2.0001, 2.0001, 2.0001)$	6	$\mu_{x_1}^{(6)} = \mu(0.9995, 0.9998, 1.0001)$ $\mu_{x_2}^{(6)} = \mu(2.9997, 2.9998, 2.9999)$ $\mu_{x_3}^{(6)} = \mu(1.9997, 1.9998, 2.0000)$
12	$\mu_{x_1}^{(12)} = \mu(1.0001, 1.0001, 1.0001)$ $\mu_{x_2}^{(12)} = \mu(2.9999, 2.9999, 2.9999)$ $\mu_{x_3}^{(12)} = \mu(2.0000, 2.0000, 2.0000)$	7	$\mu_{x_1}^{(7)} = \mu(1.0001, 1.0001, 1.0002)$ $\mu_{x_2}^{(7)} = \mu(3.0000, 3.0000, 3.0000)$ $\mu_{x_3}^{(7)} = \mu(2.0000, 2.0001, 2.0001)$
13	$\mu_{x_1}^{(13)} = \mu(1.0000, 1.0000, 1.0000)$ $\mu_{x_2}^{(13)} = \mu(3.0000, 3.0000, 3.0000)$ $\mu_{x_3}^{(13)} = \mu(2.0000, 2.0000, 2.0000)$	8	$\mu_{x_1}^{(8)} = \mu(1.0000, 1.0000, 1.0000)$ $\mu_{x_2}^{(8)} = \mu(3.0000, 3.0000, 3.0000)$ $\mu_{x_3}^{(8)} = \mu(2.0000, 2.0000, 2.0000)$



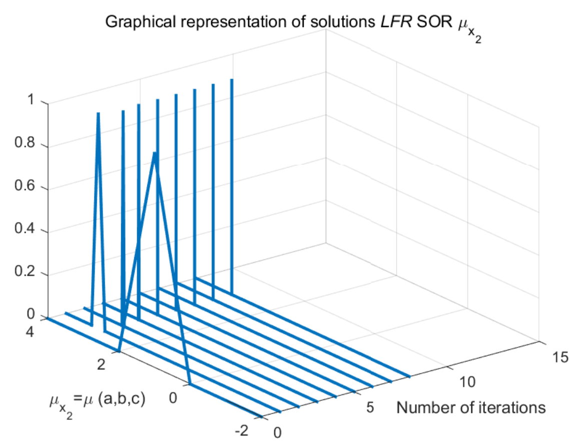
(a–1) Approx. of μ_{x_1} by LFR GS



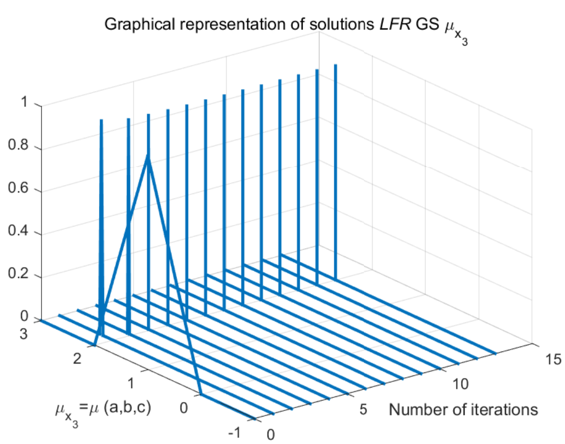
(a–2) Approx. of μ_{x_1} by LFR SOR



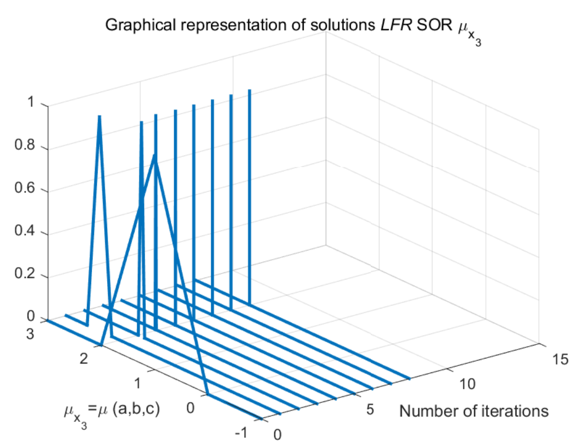
(b–1) Approx. of μ_{x_2} by LFR GS



(b–2) Approx. of μ_{x_2} by LFR SOR



(c–1) Approx. of μ_{x_3} by LFR GS



(c–2) Approx. of μ_{x_3} by LFR SOR

Figure 2. Graphical representation of fuzzy solutions

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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